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A Re-consideration on Technical Inefficiency in Stochastic Frontier Production Models

by

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Abstract:

We first derive that conditional expectation for technical inefficiency in stochastic frontier production models is an increasing function of the individual firm's inefficiency effect which is defined as the mean of the normal distribution that is truncated at zero. The finding provides a theoretical basis for using the Battese and Coelli (1995) type specification. Second, we derive a confidence interval of prediction for technical inefficiency, which is shown to be useful for evaluating the reliability about prediction through numerical illustrations and an empirical study on the Japanese pharmaceutical industry.

1. Introduction

Since the stochastic frontier production function was independently proposed by Aigner, Lovell and Schmidt (1977) and Meeusen and van den Broeck (1977), there has been considerable research to extend and apply the model. The stochastic frontier model presumes the existence of technical inefficiencies of production for individual firms. Kumbhakar, Ghosh and McGuckin (1991) and Reifschneider and Stevenson (1991) proposed stochastic frontier models in which the inefficiency effects are expressed as an explicit function of firm specific variables. Battese and Coelli (1995) extended their models for panel data. Recent empirical studies including Battese, Heshmatiand and Hjalmarsson (2000) and Kim (2001) extensively adopted this type of specification for technical efficiency.

Battese and Coelli (1988) define the technical efficiency of a given firm at a given time period as the ratio of its mean production (conditional on its levels of factor inputs and firm effects) to the corresponding mean production if the firm utilizes its levels of inputs most efficiently. The technical inefficiency (i.e., technical efficiency subtracted from one) is a random variable which takes the values between zero and one, and usually predicted by its conditional expectation on the observations. Battese and Coelli (1995) and their followers implicitly postulate that the inefficiency effects ($\mu_{it}$by the notation in Section 2) is positively related to the predictor for technical inefficiency. Even though their postulate looks intuitively reasonable, it is not trivial to see how the inefficiency effects affect the predictor because the shift of the former affects the latter in a highly nonlinear fashion.

On the other hand, the predicted values for technical inefficiency have been reported in many empirical studies. However, none of the previous researches, seems to have discussed the reliability about prediction.

The purpose of the paper is twofold. First, we analytically investigate the relationship between the inefficiency effects and the predictor for technical inefficiency in the model of Battese and Coelli (1995). We find that the predictor is an increasing function of the inefficiency effects. This finding confirms Battese and Coelli (1995)'s implicit postulate, and provides a theoretical basis for using their specification for technical inefficiency. Second, we propose a method for constructing confidence interval for prediction in order to evaluate the reliability about the predicted values. For any given significance levels, the lower and upper confidence limits are an increasing function of the inefficiency effect. Numerical illustrations indicate that prediction can be very inaccurate for some cases depending on the choice of parameter sets for the model. We will give a simple example of stochastic frontier model using panel data from the Japanese pharmaceutical industry in order to examine the empirical significance of our confidence interval argument.

The paper is organized as follows. Section 2 describes the model. Section 3 discusses the relationship between inefficiency effects and the conditional expectation for technical inefficiency, proposes a confidence interval for prediction, and finally gives numerical illustrations. Section 4 examines a simple empirical example from the Japanese pharmaceutical industry. Section 5 states concluding remarks.

2. The Stochastic Frontier Production Model

We consider a stochastic frontier production function:

$$Y_{it} = X_{it}\beta + \varepsilon_{it} , \qquad (1)$$
$$\varepsilon_{it} = V_{it} - U_{it} , \; i = 1,..,N ; \; t = 1,..,T,$$

where Y_{it} denotes the output (in logarithms) for the *i-th* firm at the *t-th* time period; X_{it} is a 1×k vector of k input variables (in logarithms); β is a k×1 vector of k unknown parameters; N is the number of firms and T is the number of time periods. Random variables V_{it} are assumed to have iid $N(0,\sigma_V^2)$ distribution and non-negative random variables U_{it} are assumed to have iid $N(\mu_{it},\sigma_U^2)$ distribution truncated at zero,

$$\mu_{it} = Z_{it}\delta , \qquad (2)$$

where Z_{it} is a $1 \times p$ vector of variables which influence the inefficiency of a firm and δ is a p×1 vector of unknown parameters. The V_{it} are assumed to be independent of U_{it} . The U_{it} accounts for technical inefficiency in production for the *i-th* firm at the *t-th* time. The μ_{it} is the mean of the normal distribution that is truncated at zero, and indicates the inefficiency effects for the *i-th* firm at the *t-th* time.

This model specification, which was developed by Battese and Coelli (1995), allows the mean to be a function of the characteristics for the individual firms at time t. It encompasses a number of other specifications as special cases. Variables such as firm size and the ratio of external capital could be related to μ_{it}. Battese and Coelli (1995) implicitly postulate that μ_{it} is positively related to the expectation of technical inefficiency for production (i.e., the expectation of equation (3)) conditional on ε_{it}. However, it is not trivial to see how μ_{it} affects the expected value because the shift of μ_{it} affects it in a highly nonlinear fashion. We examine this problem in the next section.

3. Technical Inefficiency

3. 1 Technical Inefficiency and Its Distribution

Battese and Coelli (1988, p.389) define the rate of technical efficiency of production for the *i-th* firm at the *t-th* time as a ratio of its mean production to the corresponding mean with $U_{it} = 0$:

$$TE_{it} \equiv \frac{E(Y_{it}^* \mid U_{it}, X_{it})}{E(Y_{it}^* \mid U_{it} = 0, X_{it})}, \tag{3}$$

where Y_{it}^* (= $\exp(X_{it}\ \beta + V_{it} - U_{it})$) denotes the value of production in original units. Necessarily, it holds taht $0 \leq TE_{it} \leq 1$. Alternatively we can discuss the problem in terms of the technical inefficiency defined by $TIE_{it} = 1 - TE_{it}$. Then, the rate of technical inefficiency becomes

$$TIE_{it} = 1 - \exp(-U_{it}), \quad (4)$$

which is a random variable taking the values between zero and one. We simply call TIE_{it} the technical inefficiency of production as well as U_{it} . There should be no confusion.

The main concern of the paper is to predict the values of TIE_{it} on the basis of information for Y_{it} and X_{it}. We are only able to estimate ε_{it} (= V_{it} - U_{it}). The distribution of TIE_{it} conditional on ε_{it} is derived in the next theorem. All proofs of theorems and corollaries are given in Appendix.

Theorem 1: The distribution and density functions of TIE_{it} conditional on ε_{it} are respectively given by

$$F_{TI}(w \mid \varepsilon_{it}; \mu_{it}^*, \sigma^*) = Pr\{ TIE_{it} \leq w \mid \varepsilon_{it}\}$$

$$= 1 - \frac{\Phi((\mu_{it}^* + \log(1-w))/\sigma^*)}{\Phi(\mu_{it}^*/\sigma^*)} \text{ for w } (0 < w < 1), \quad (5)$$

and

$$f_{TI}(w \mid \varepsilon_{it}; \mu_{it}^*, \sigma^*) = \frac{\phi((\mu_{it}^* + \log(1-w))/\sigma^*)}{\sigma^* \Phi(\mu_{it}^*/\sigma^*)(1-w)} \quad \text{for w } (0 < w < 1), \quad (6)$$

where $\mu_{it}^* \equiv (\sigma_V^2 \mu_{it} - \sigma_U^2 \varepsilon_{it})/(\sigma_V^2 + \sigma_U^2)$, $\sigma^* \equiv \sqrt{\sigma_U^2 \sigma_V^2 / (\sigma_V^2 + \sigma_U^2)}$, $\Phi(\cdot)$ and $\phi(\cdot)$ denote the standard normal distribution and density functions.

The distribution in Theorem 1 is the sole source for inference on technical inefficiency. For the completeness of discussion, we derive the unconditional distribution of TIE_{it}.

Corollary 1: The unconditional distribution and density functions of TIE_{it} are respectively given by

$$F_{TI}(w\ ;\mu_{it},\sigma_U) = Pr\{\ TIE_{it} \leq w\}$$

$$= 1 - \frac{\Phi((\mu_{it} + \log(1-w))/\sigma_U)}{\Phi(\mu_{it}/\sigma_U)} \text{ for w } (0 < w < 1), \qquad (7)$$

and

$$f_{TI}(w\ ;\mu_{it},\sigma_U) = \frac{\phi((\mu_{it} + \log(1-w))/\sigma_U)}{\sigma_U \Phi(\mu_{it}/\sigma_U)(1-w)} \qquad \text{for w } (0 < w < 1). \qquad (8)$$

We note that the unconditional distribution of TIE_{it} is identical to the conditional distribution with μ_{it}^* and σ^* replaced by μ_{it} and σ_U.

3.2 Prediction for Technical Inefficiency

A natural predictor for technical inefficiency of production for the *i-th* firm at the *t-th* time conditional on the value of ε_{it} is given by its conditional expectation:

$$E(TIE_{it} \mid \varepsilon_{it}) \equiv ETIE(\mu_{it} \mid \varepsilon_{it})$$

$$= 1 - \{\exp(-\mu_{it}^* + \sigma^{*2}/2)\}\{\Phi((\mu_{it}^*/\sigma^*) - \sigma^*)/\Phi(\mu_{it}^*/\sigma^*)\}. \qquad (9)$$

Equation (9) was derived by Battese and Coelli (1993, p.20). We note that μ_{it}^* and σ^{*2} are given by (6) and the former is a function of μ_{it}. The values of (9) depend on the observations of Y_{it} and X_{it} through ε_{it}.

The following theorem clarifies the relation of the predictor for technical inefficiency given by (9) to μ_{it}.

Theorem 2: For all μ_{it}, the following relationship holds:

$$\partial ETIE(\mu_{it} \mid \varepsilon_{it}) / \partial \mu_{it} > 0. \qquad (10)$$

Theorem 2 implies that the predictor of technical inefficiency for the i-th firm at the t-th time is an increasing function of μ_{it}. The evaluation of μ_{it} is justified in order to analyze the technical inefficiency. Theorem 1 clarifies the role of μ_{it} as an inefficiency measure for the individual firms in the Battese and Coelli (1995) specification, and provides a theoretical basis for using their model. Although Battese and Coelli (1993) derived the equation (9), they did not develop the relationship between μ_{it} and the predictor.

If T = 1 and $\mu_{it} = \mu$ for all i, the model of (1) and (2) reduces to the Stevenson (1980) specification. Although this is a more general error specification than a half-normal distribution originally proposed by Aigner, Lovell and Schmidt (1977), Stevenson (1980) did not explore its implication for technical inefficiency. Theorem 2 implies that μ is positively related to the amount of technical inefficiency of production. If we specify $\mu_{it} = \mu_i$ for all t, then μ_i stands for a location of an inefficiency random variable $U_{it} \sim |N(\mu_i, \sigma_U^2)|$, which in turn indicates the individual firm effects.

Corollary 2: If we condition the error terms at $\varepsilon_{it} = \varepsilon_{jt}$ $(\equiv \varepsilon_t)$, then the individual firm effects satisfy the relation:

$$\mu_i > \mu_j \text{ if and only if } ETIE(\mu_i \mid \varepsilon_t) > ETIE(\mu_j \mid \varepsilon_t). \tag{11}$$

Corollary 2 implies that the production technology for the *i-th* firm is less efficient than that of the *j-th* firm if and only if $\mu_i > \mu_j$ in the sense that the conditional expectation of inefficiency for the i-th firm is greater than that for the j-th firm.

So far we have concerned with the prediction conditional on the value of ε_{it}. The unconditional version of Theorem 2 may be of theoretical interest. By an analogous manner to (9), we define unconditional quantities $ETIE(\mu_{it}) \equiv E(TIE_{it}) = 1 - E(\exp(-U_{it}))$. Then, we can obtain

$$ETIE(\mu_{it}) = 1 - \frac{\Phi((\mu_{it}/\sigma_U) - \sigma_U)}{\Phi(\mu_{it}/\sigma_U)} \exp\left\{ -\mu_{it} + \sigma_U^2/2 \right\}. \tag{12}$$

The right hand sides of (12) is solely determined by the distribution of U_{it}, and do not incorporate the observed information on Y_{it} and X_{it}.

Theorem 3: For all μ_{it} , the following relationship holds:

$$\partial ETIE(\mu_{it}) / \partial \mu_{it} > 0 \quad . \tag{13}$$

Theorem 3 has parallel interpretation to that of Theorem 2.

Corollary 3: If we specify $\mu_{it} = \mu_i$ for all t, then the individual firm effects satisfy the following relation:

$$\mu_i > \mu_j \text{ if and only if } ETIE(\mu_i) > ETIE(\mu_j) . \tag{14}$$

Theorem 2 and Corollary 2 may not be meaningful from the view point of empirical study because they do not reflect the information on the observations of Y_{it} and X_{it}. However, they articulate the nature of technical inefficiency for the Battese and Coelli (1995) specification in the stochastic frontier models.

3.3 A Confidence Interval for Technical Inefficiency

In this section, we construct a confidence interval of prediction for technical inefficiency. First, we define the quantile point w for a given level α ($0 < \alpha < 1$) as $\alpha = \Pr\{ TIE_{it} \le w \mid \varepsilon_{it}\}$ Then, we obtain from (5)

$$w \equiv c(\mu_{it} \mid \varepsilon_{it};\alpha) = 1 - \exp\{-\mu_{it}* + \sigma * \Phi^{-1}((1-\alpha)\Phi(\mu_{it} * / \sigma*))\} . \tag{15}$$

The value of (15) depends on the observations of Y_{it} and X_{it} through ε_{it}. Theorem 4 clarifies the relationship between μ_{it} and the quantile point stated in (15).

Theorem 4: For all μ_{it}, the following relationship holds:

$$\partial c(\mu_{it} \mid \varepsilon_{it};\alpha) / \partial \mu_{it} > 0 \qquad for \quad any \; 0 < \alpha < 1 . \tag{16}$$

Theorem 4 implies that the conditional quantile point for the i-th firm at the t-th time is an increasing function of μ_{it} for any fixed value of α. The distribution function shifts to the right along with μ_{it}.

Corollary 4: If we specify $\mu_{it} = \mu_i$ for all t and condition the error terms at $\varepsilon_{it} = \varepsilon_{jt}$ ($\equiv \varepsilon_t$), then the individual firm effects satisfy the relation:

$$\mu_i > \mu_j \text{ if and only if } c(\mu_i \mid \varepsilon_t; \alpha) > c(\mu_j \mid \varepsilon_t; \alpha) \text{ for any } 0 < \alpha < 1. \quad (17)$$

The inefficiency measure of (15) has the same properties as the predictor ($ETIE(\mu_i \mid \varepsilon_{it})$ defined by (9)), concerning the relationship to μ_{it}.

Second, we use the quantile points for constructing a confidence interval of prediction. We define a confidence interval with a confidence level of 1 - 2α by $[c_L,\ c_U]$. The lower and upper confidence limits (c_L and c_U) are respectively calculated from $\alpha = \Pr\{\ TIE_{it} \le c_L \mid \varepsilon_{it}\}$ and $\alpha = \Pr\{\ TIE_{it} \ge c_U \mid \varepsilon_{it}\}$. Then, we obtain from (15)

$$c_L = c(\mu_{it} \mid \varepsilon_{it}; \alpha), \quad c_U = c(\mu_{it} \mid \varepsilon_{it}; 1-\alpha). \quad (18)$$

The values c_L and c_U depend on the observations through ε_{it} and can be estimated by replacing the unknown parameters with their estimates. By Theorem 4, both c_L and c_U are increasing in μ_{it}.

The unconditional quantile point w for a given level α (0< α < 1) is defined by $\alpha = \Pr\{\ TIE_{it} \le w\}$ Then, we obtain

$$w \equiv c(\mu_{it}; \alpha) = 1 - \exp\{-\mu_{it} + \sigma_U\ \Phi^{-1}((1-\alpha)\Phi(\mu_{it}/\sigma_U))\}\ . \quad (19)$$

The right hand side of (19) does not incorporate the observed information on Y_{it} and X_{it}.

Theorem 5: For all μ_{it}, the following relationship holds:

$$\partial c(\mu_{it};\alpha)/\partial\mu_{it} > 0 \qquad \textit{for any } 0 < \alpha < 1. \tag{20}$$

The unconditional confidence interval can be defined in a similar manner to (18).

3.4 Numerical Illustrations

This section numerically illustrates the prediction for technical inefficiency and its confidence limits. The conditional distribution is completely characterized by μ_{it}^* and σ^*. Hence, the predictor and the confidence limits with a specified confidence level (1- 2α) are completely determined by them.

Figures 1(a) and (b) draw the density functions of TIE_{it} conditional on ε_{it} for some sets of parameter values of μ_{it}^* and σ^*. The densities are quite sensitive to those parameters. If μ_{it}^* is fixed constant, the distribution becomes more flat as σ^* increases. If σ^* is fixed, the distribution shifts towards one as μ_{it}^* increases.

Figures 2(a)-(d) illustrate the prediction, the lower and upper confidence limits with confidence level of 0.90 (α= 0.05) against μ_{it}^* for the cases of $\sigma^* = 1.0$, 0.5, 0.2 and 0.1 respectively. As have been proved in Theorems 1 and 2, we see that the predictor and the lower and upper confidence limits are increasing in μ_{it}^*. Depending on the values of σ^* and μ_{it}^*, the prediction for technical inefficiency can exhibit quite diverse values. As σ^* goes up large, the technical inefficiency shifts upward, and become possibly very large. Table 1 shows the predicted values and 90% lower and upper confidence limits for some specific sets of parameters. For example, suppose that $\mu_{it}^* = 0.0$. Then, if $\sigma^* = 1.0$, the predicted technical inefficiency is as large as 0.477, the lower limit is 0.061 and the upper limit is 0.868. In this case the prediction is very inaccurate. On the other hand, if $\sigma^* = 0.1$, the prediction is reasonably accurate.

We do not know the value of σ^* in advance. In the empirical study about agricultural production by Battese and Coelli (1995), we can calculate the value as $\sigma^* = 0.18$. Their study is close to the situation illustrated in Figure 2(c). The technical inefficiency in Battese and Coelli (1995) is reasonably small, and the confidence limits may not be far from the predicted value if they are compared with the cases (a) and (b) in Figure 2.

We can say that the confidence limits provide useful information on the reliability about prediction.

4. An Empirical Example

We investigate an empirical example of stochastic frontier model using panel data from the Japanese pharmaceutical industry. We do not intend to analyze the production function for this industry in detail. The purpose of this section is only to illustrate the empirical significance of our previous arguments. We choose four firms (Sankyo Co., Takeda Chemical Industries, Shionogi & Co., Tanabe Seiyaku Co.) from the pharmaceutical industry during the data periods from the fiscal years 1979 to 1998. We employ a simple Cobb-Douglas type production function. We assume that μ_{it} represents only the individual firm's effect and is independent from time periods (i.e., $\mu_{it} = \mu_i$). The model presumes that all the four firms have the same production technology through observation periods, but the technical inefficiency (TIE_{it} in the equation (4)) differs through individual firms and time periods.

The single output of the firm is defined by *"operating profits"*, the capital input (K) is *"total fixed assets minus land values"* .These variables are measured in terms of a million yen. The labor input (L) is *"number of officers and personals"* . The data are complied from "NIKKEI ZAIMU DATA" published by The Nihon Keizai Shimbun.

The calculation is carried out by the computer program, FRONTIER 4.1, written by Coelli (1996). The maximum-likelihood estimates and their standard errors (in parentheses) are reported according to the format of Battese and Coelli (1995):

Stochastic Frontier:

$$\text{Log}(Y_{it}) = -\ 3.839 \ + \ 1.003\ \text{Log}(L_{it}) \ + \ 0.508\ \text{Log}(K_{it}) \qquad (21)$$

$$(0.85) \qquad (0.18) \qquad (0.092)$$

Inefficiency Mean:

$$\mu_i = \ -1.117 \ + 1.718D_{2i} + 1.841D_{3i} + 1.821D_{4i} \qquad (22)$$

$$(0.092) \quad (0.062) \quad (0.037) \quad (0.012)$$

Variance Parameters: $\hat{\sigma}^2 = 0.0825, \quad \hat{\gamma} = 0.224$ (23)

$$(0.0076) \qquad (0.044)$$

Log(likelihood) = -16.81

In equation (23), σ and γ are defined as $\sigma^2 = \sigma_V^2 + \sigma_U^2$ and $\gamma = \sigma_U^2 / (\sigma_V^2 + \sigma_U^2)$. We note that the dummy variables D_{ki} (= 1 if k = i; = 0 otherwise; k, i = 2, 3 and 4)) indicates the individual firm effect on μ_i. The estimated coefficients of the stochastic frontier have signs and magnitudes anticipated from economic theory. The sum of the coefficient of labor (1.003) and that of capital (0.508) is greater than one, which indicates increasing returns to scale for production.

The values of μ_i are of particular concern. The all coefficients on the dummy variables are significant and take values around 1.8. The μ_1 of the firm 1 is less than other three firms. The firm 1 is technically more efficient than others in terms of the mean μ_i. We can calculate $\sigma* = \{\gamma(1-\gamma)\sigma^2\}^{1/2} = \{0.224 \times (1 - 0.224) \times 0.0825\}^{1/2} = 0.120$. This value roughly corresponds to the case of Figure 2(d). We evaluate the predictor of (9) for each firm at the average of $\mu_{it}*$ over time,

$$\bar{\mu}_i * = \frac{1}{T}\sum_{t=1}^{T} \mu^*_{it} = (1-\gamma)\mu_i - \gamma\bar{\varepsilon}_i \quad , \quad \bar{\varepsilon}_i = \frac{1}{T}\sum_{t=1}^{T}\varepsilon_{it} \ . \qquad (24)$$

Table 2 shows the predicted values and the lower and upper confidence limits calculated from the equations (9) and (18). The inefficiency for the firm 1 is 0.015. We can see from Figure 2(d) that the confidence interval for this predicted value is very narrow even if the confidence limits are not directly calculated in Table 2. The firms 2, 3 and 4 are technically less efficient. The ETIE of the firm 2 is 0.447, and its 90% confidence interval consists of [0.331, 0.549]. The predicted value is less accurate compared with the firm 1. The empirical example in this section indicates that the confidence interval exhibits a measure on reliability for prediction.

5. Conclusion

Battese and Coelli (1995) specification is flexible for expressing the technical inefficiency for an individual firm at a time period in stochastic frontier models. Their model has been applied for many empirical studies. However, their model implicitly lays an important postulate that the inefficiency effect (μ_{it}) is positively related to the expectation of the technical inefficiency for production. The paper analytically confirms their postulate. This finding presents a theoretical basis for using their specification.

Many previous empirical studies for stochastic frontier models reported the predicted value for technical inefficiency, but never discussed reliability about prediction. The paper derives a confidence interval of prediction for technical inefficiency. The confidence interval is shown to provide a useful measure for evaluating the reliability about prediction through numerical illustrations and an empirical study on the Japanese pharmaceutical industry.

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Appendix

We prove theorems and corollaries stated in the text without proofs.

Proof of Theorem 1: Battese and Coelli (1993, p.20) derive the density of U_{it} conditional on ε_{it} as

$$f_U(U_{it} \mid \varepsilon_{it}) = \frac{\phi((U_{it} - \mu_{it}^*)/\sigma^*)}{\sigma^* \Phi(\mu_{it}^* / \sigma^*)}, U_{it} \geq 0. \tag{A.1}$$

Then, we have

$$Pr\{ TIE_{it} \leq w \mid \varepsilon_{it} \}$$

$$= \int_{\infty}^{a} f_U(u \mid \varepsilon) du, \qquad a = -\log(1 - w)$$

$$= \int_{\infty}^{a^*} \frac{\phi(t)}{\Phi(\mu_{it}^* / \sigma^*)} dt, \tag{A.2}$$

where $a^* = (a - \mu_{it}^*)/\sigma^*$. This leads to (5). Differentiating (5) with respect to w, we have (6). ■

Proof of Corollary 1: The unconditional distribution of U is identical to (A.1) with μ_{it}^* and σ^* replaced by μ_{it} and σ_U. Hence, (7) and (8) are derived by a similar manner to Theorem 1. ■

We show a preliminary result in the next lemma which is useful for proving Theorems 2 and 4.

Lemma 1: Let us define two functions as $g(x) = \phi(x) / \Phi(x)$ and $h(x) = x + g(x)$, where $\phi(\cdot)$ and $\Phi(\cdot)$ respectively represent the density function and the distribution function of a standard normal random variable. Then, the following inequalities hold:

$$\frac{dg(x)}{dx} < 0 \quad \text{and} \quad \frac{dh(x)}{dx} > 0, \quad \text{for all x.} \tag{A.3}$$

Proof : From Battese and Coelli (1992, pp.163), we have

$$E(U) = \sigma_U \, h(\mu / \sigma_U) > 0, \tag{A.4}$$

$$Var(U) = \sigma_U^2\{1 - g(\mu/\sigma_U)h(\mu/\sigma_U)\} > 0, \tag{A.5}$$

where U is a nonnegative random variable from $|N(\mu,\sigma_U^2)|$, which denotes a truncation (at zero) of normal distribution with mean μ and variance σ_U^2. The inequality in (A.4) follows from U ≥ 0. Differentiating g(x) and h(x) with respect to x, we have

$$\frac{dg(x)}{dx} = -g(x)h(x) \text{ and } \frac{dh(x)}{dx} = 1 - g(x)h(x) . \tag{A.6}$$

The first and second inequalities in (A.3) respectively follow from (A.4), (A.5) and (A.6). ■

Proof of Theorem 2: After differentiation of (9) with respect to μ_{it} and some calculations, we have

$$\partial ETIE(\mu_{it} \mid \varepsilon_{it}) / \partial \mu_{it}$$

$$= \frac{1}{\sigma_U^2} \exp(\sigma^{*2}/2) \frac{\Phi((\mu_{it}^*/\sigma^*) - \sigma^*)}{\Phi(\mu_{it}^*/\sigma^*)} \left\{ h(\mu_{it}^*/\sigma^*) - h((\mu_{it}^* - \sigma^{*2})/\sigma^*) \right\} > 0. \tag{A.7}$$

The inequality holds because h(•) is increasing by Lemma 1. ■

Proof of Theorem 3: In a similar manner to the proof of Theorem 2, we can obtain (13) after differentiating (12) with respect to μ_{it}. ■

Proof of Theorem 4: Differentiating (5) with respect to μ_{it}, we obtain

$$\partial F_{TI}(w \mid \varepsilon_{it}; \mu_{it}, \sigma^*) / \partial \mu_{it}$$

$$= \frac{\sigma^*}{\sigma_U^{\ 2}} \frac{\Phi((\mu_{it}^* + \log(1-w))/\sigma^*)}{\Phi(\mu_{it}^*/\sigma^*)} \left\{ g((\mu_{it}^*)/\sigma^*) - g((\mu_{it}^* + \log(1-w))/\sigma^*) \right\} < 0. \tag{A.8}$$

The inequality holds since g(•) is decreasing by Lemma 1. Then, we have

$$\frac{dw}{d\mu_{it}} = -\frac{\partial F_{TI}}{\partial \mu_{it}} \Bigg/ \frac{\partial F_{TI}}{\partial w} > 0 . \tag{A.9}$$

The inequality holds from (A.8) and $\partial F_{TI} / \partial w > 0$ because $F_{TI}(\cdot)$ is the distribution function. ■

Proof of Theorem 5: In a similar manner to the proof of Theorem 4, we can obtain (20) after differentiating (7) with respect to μ_{it}. ■

Figure1. Density Function of Inefficiency

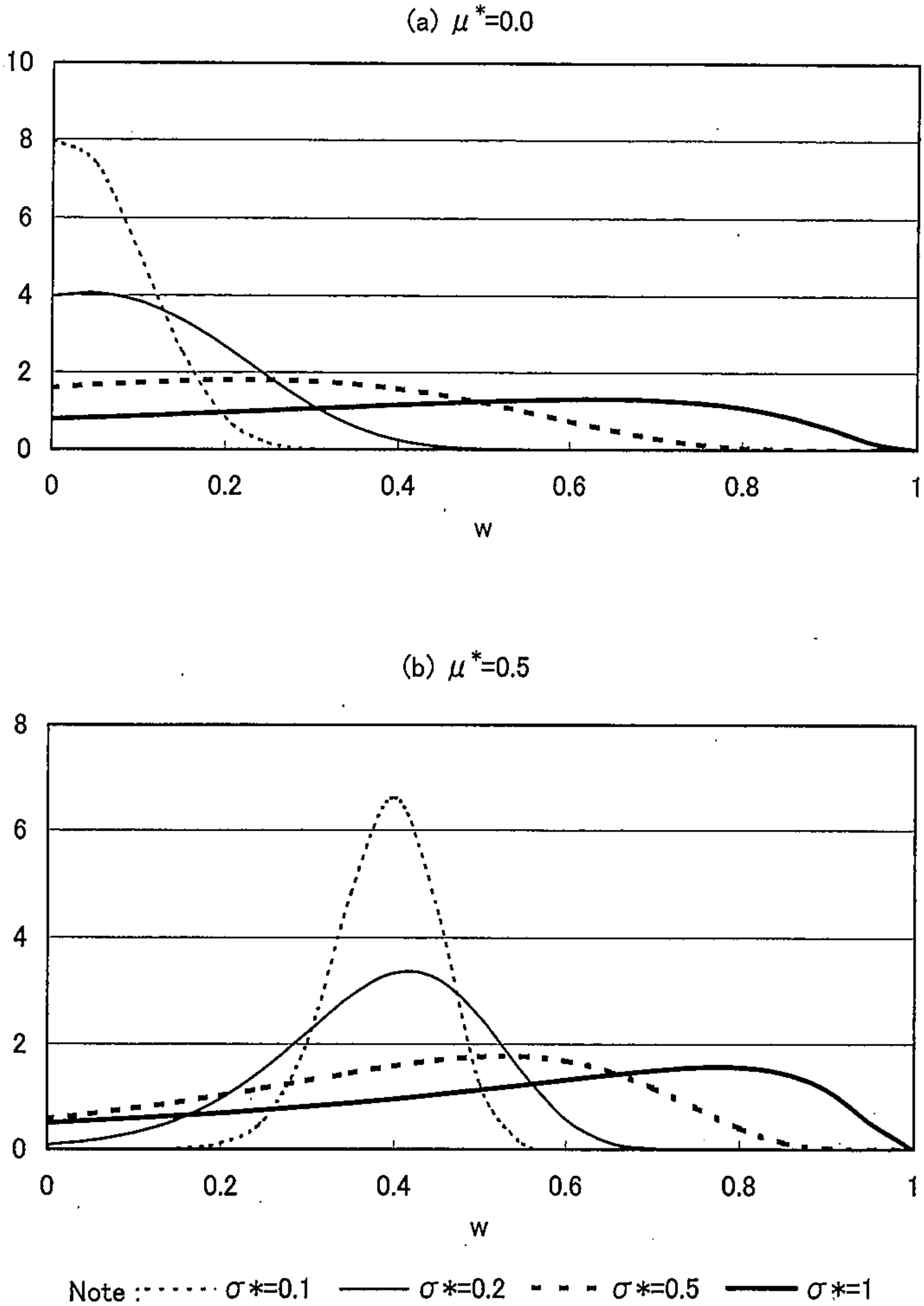
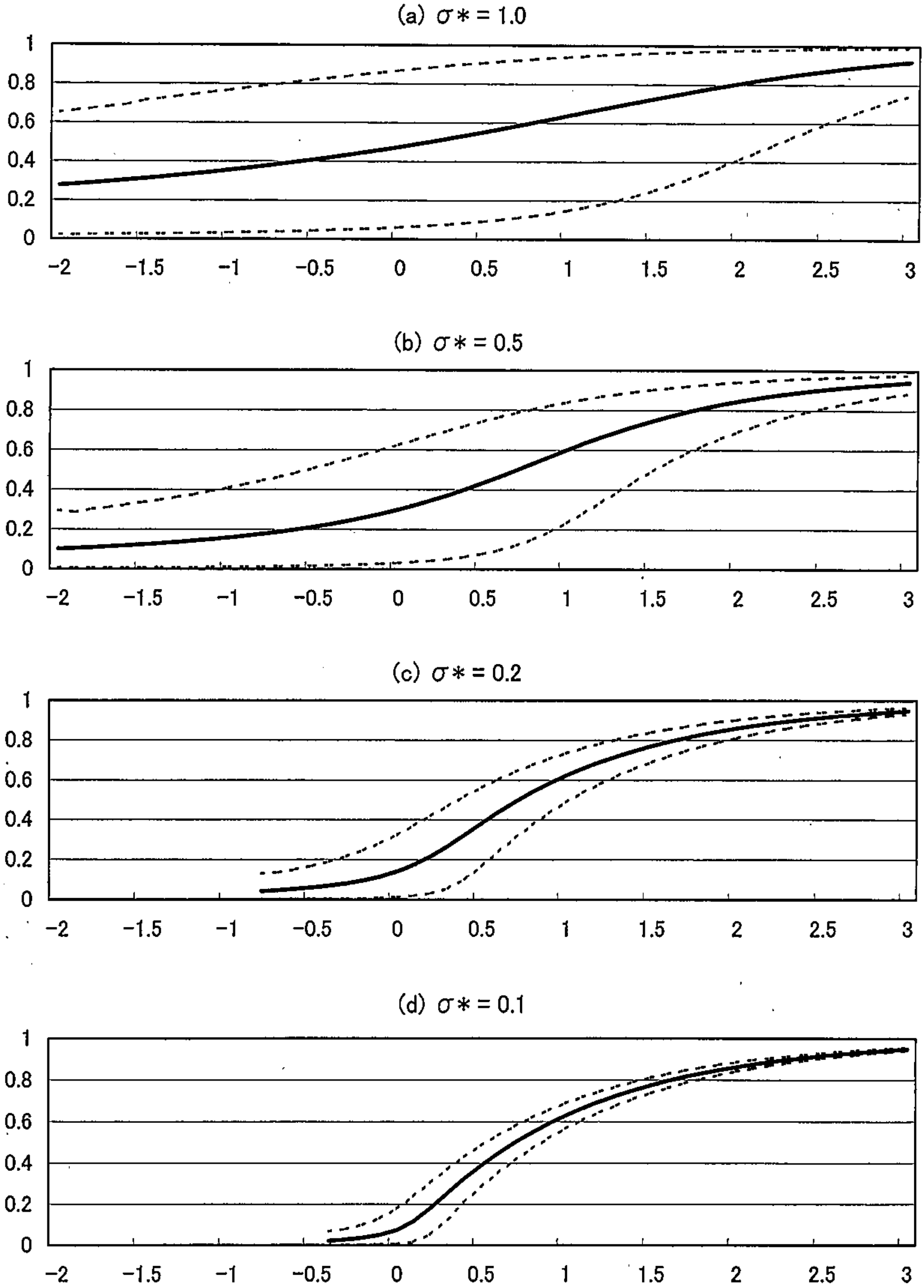


Figure2. Prediction of Technical Inefficiency and Confidence Limit against $\mu*$



Note : —— prediction ----- lower and upper limits

Table1. Prediction of Technical Inefficiency and Confidence Limit

σ^*	$\mu^* = 0.0$			$\mu^* = 0.5$		
	C_L	ETIE	C_U	C_L	ETIE	C_U
1.0	0.061	0.477	0.868	0.092	0.554	0.909
0.5	0.031	0.301	0.625	0.077	0.435	0.744
0.2	0.012	0.142	0.324	0.166	0.384	0.564
0.1	0.006	0.075	0.178	0.285	0.390	0.485

Note : The enties are calcurated by EXCEL.

Table2. 90% Confidence Interval for Prediction

firm	μ^*	C_L	ETIE	C_U
1	−0.863	– [a)]	0.015	– [a)]
2	0.600	0.331	0.447	0.549
3	0.721	0.407	0.510	0.601
4	0.699	0.395	0.499	0.592

Note : The enties are calcurated by EXCEL.

a) EXCEL does not calcurate the quantile point in equation (15) for this value of μ^*