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by

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Abstract

This paper examines structural changes of the empirical matching function in Japanese labor market. Employing the two-state Markov regime switching model, we found that the matching function changed frequently between increasing and constant returns regime, and expected durations were very short for both regimes.

These results suggest that the matching relations among unemployed labor forces and unfilled vacancies would be more fragile than commonly thought, implying the importance of addressing the process through which search frictions in the labor market is endogenously formulated.

JEL Classification Number: C53, E24, J41, J60.

1 Introduction

For the last decade the theory of equilibrium unemployment has grown to constitute prominent paradigms in both macroeconomics and labor economics. The matching function, which is its fundamental building block therein, neatly abstracts from the model underlying informational, spatial, and institutional difficulties of finding desirable jobs or workers, i.e., search frictions in the labor market. In virtue of this useful device, we have gained in-depth understanding of the mechanism governing the dynamics of unemployment flows and accompanying considerations such as relations between growth and unemployment, the real business cycle model embodying search frictions, and policy analysis based on them[1].

Continuing success of the equilibrium unemployment theory and its branches depends, clearly, much on its empirical validity. Since the influential study of Blanchard and Diamond (1989), literature on estimating empirical matching functions has exploded[2]. Foci of them are mainly put on the degree of returns to scale the function exhibits, because the theory suggests that there would exist multiple, Pareto-rankable equilibria in the case of increasing returns. Applying any possible parameterization strategies to wide variety of data sets, the bulk of previous studies present quantitative evidences supporting assumptions on the properties of the matching function which the equilibrium unemployment theory requires.

[1] See Pissarides (2000) and its reference.

[2] See Petrongolo and Pissarides (2001) for the survey of recent theoretical and empirical development of the matching function as well as its quite long history.

However, little attention has been paid to the stability of estimated parameters of the matching function[3]. Many authors in this paradigm, due to its convenient property mentioned above, preferably analogize the role of the aggregate matching function to that of the aggregate production function in analyzing macroeconomy. Indeed, if we start with doubts on the reliability of building blocks themselves, any arguments based on them would collapse. Although we are likely to forget this point, however, there lies an obvious difference between these functions; unlike the production function, the matching function does not capture any physical or engineering relations between inputs and output. Therefore, it is natural to expect that the relation among inputs (unemployment and unfilled vacancies) and output (new hiring), characterized by parameters of the function, is more flexibly changing over time. This notion motivates us to doubt that the matching function could be as stable as believed by proponents of the equilibrium unemployment theory.

From this skeptical standing point, this paper investigates if there are frequent structural changes in the empirical matching function of Japanese labor market. For this purpose we use two-state Markov regime switching (hereafter MRS) model which is developed by Hamilton (1989) and extensively applied to empirical analysis of finance and macroeconomics.

The advantage of using MRS is that we need not impose *ad hoc* breaking point(s) on the model to be estimated. Instead, it estimates the possibility that the

[3] An exception is Gross (1997)'s study on Germany. He estimates the function splitting whole sample into two sub-samples, the first and the second half, and finds significant difference of returns to scales between them.

economy is lying in a certain state on each date via an algorithm fully exploiting properties of the Markov process as well as assumptions on the distribution of the explained variable, the so-called Hamilton filter. In other words, in estimating MRS model, the timing of regime switches, or equivalently of structural changes, is of interest. In particular, with unknown breaking points, MRS is quite effective when frequent structural changes are expected within a given sample period.

Employing this newly developed technique, we will show that Japanese aggregate matching function has changed frequently between increasing and constant returns regime over the time. That is, it is found that the expected duration of a regime during which the economy stays is very short. These result also suggests the importance of addressing the mechanism through which search frictions occur in the labor market.

The remaining part of this paper is constructed as follows. Section 2 introduces the matching function and explains our MRS specification, comparing our specification with previous studies. Section 3 presents the estimation result of the MRS as well as that of conventional OLS (actually GLS). Section 4 concludes our analysis.

2 The estimation model of the matching function

In the equilibrium unemployment theory and related literature, the matching function is given generally by

$$H = m(U, V),$$

where H denotes the number of newly hired persons, U unemployed who are seeking their jobs, and V unfilled job vacancies, respectively. It is assumed that the function is increasing with respect to both arguments, both of which are essential for achievement of matching, i.e.,

$$\frac{\partial m(U,V)}{\partial U} > 0,\ \frac{\partial m(U,V)}{\partial V} > 0,\ m(0,V) = m(U,0) = 0.$$

These assumptions imply that the economy does "produce" new hiring in each period by combining a set of inputs, the stocks of unemployed and vacancies. In addition, the homogeneiety of degree one is commonly imposed on the function.

For $t = 1, ..., T$, existing studies usually adopt a log-linear Cobb-Douglas form with linear time trend,

$$\log H_t = \beta_0 + \beta_1 \log U_{t-1} + \beta_2 \log V_{t-1} + \beta_3\, trend + e_t, \tag{1}$$

so that the function is characterized by elasticities of hiring with respect to unemployment and vacancies, and is estimated by OLS. Here $e_t \sim i.i.d.(0, \sigma^2)$ is an error term. Note that in the equilibrium unemployment model, the outflow of unemployment is determined by new hiring[4]. This fact suggests the possibility of simultaneous equation bias when we estimate the matching function with contemporaneous explanatory variables by OLS. However lagging explanatory variables will allow us to treat them as predetermined variables, and thus we can presumably resolve this problem[5].

[4]See, for example, Pissarides (2000) for the baseline structure of the model.

[5]Another strategy to avoid this problem is the instrumental variables method or 2SLS, though lagging explanatory variables is commonly used in the literature of estimating matching functions. See Blanchard and Diamond (1989).

Alternative specification of the matching function we propose here is given by

$$\log H_t = \beta_{0,t} + \beta_{1,t} \log U_{t-1} + \beta_{2,t} \log V_{t-1} + \beta_{3,t}\, trend + e_t, \tag{2}$$

where $e_t \sim i.i.n.(0, \sigma^2)$ is an error term[6]. The only, but crucial deference between expression (1) and (2) is that all the coefficients are dependent on time in the latter, while they are not so in the former. Further, in our analysis we indicate the "regime" at which the economy stays by a dummy variable S_t, which is zero when the economy stays at regime 0 and unity when at regime 1. It is assumed that coefficients shift corresponding to the change of regimes, namely,

$$\beta_{j,t} = (1 - S_t)\beta_j^0 + S_t\beta_j^1 \qquad (j = 0, 1, 2, 3). \tag{3}$$

If we know entirely when S_t takes zero (or one) in advance, equation (2) is obviously nothing but a conventional dummy variable model, and it is estimated easily by OLS. However, here, the timing of regime switch is not revealing. Therefore, as mentioned above, it is required to estimate the probability with which S_t equals to 0 or 1 for all t. Hereafter we express this probability as $Pr[S_t = i]$, $i = 0, 1$. Note that unlike S_t itself, which takes the exact value of either 0 or 1, $Pr[S_t = i]$ ranges between 0 and 1.

Provided that S_t obeys the first order Markov process with constant transition probabilities given by

$$Pr\{S_t = 0|S_{t-1} = 0\} = p_{00},\ Pr\{S_t = 0|S_{t-1} = 1\} = 1 - p_{00},$$

$$Pr\{S_t = 1|S_{t-1} = 1\} = p_{11},\ Pr\{S_t = 1|S_{t-1} = 0\} = 1 - p_{11}, \tag{4}$$

[6]The normality of the error term is assumed by the MRS.

$Pr[S_t = i]$ is estimated by the Hamilton filter. See Hamilton (1989) as well as Kim and Nelson (1999) for details of this filtering algorithm.

Now, equations (2), (3), and (4) constitute our MRS expression of the matching function. The number of parameters to be estimated is eleven; two transition probabilities, 2×4 coefficients, and a variance[7]. Let us summarize them as

$$\theta = \{p_{00}, p_{11}, \beta_0^0, \beta_1^0, \beta_2^0, \beta_3^0, \beta_0^1, \beta_1^1, \beta_2^1, \beta_3^1, \sigma^2\}.$$

One advantage of the MRS over the conventional dummy variables method is that, exploiting the estimated probability weights $Pr[S_t = i]$, we obtain expected values of parameters defined by

$$E[\beta_{j,t}] = Pr[S_t = 0] \times \beta_j^0 + Pr[S_t = 1] \times \beta_j^1, \tag{5}$$

for $j = 0, ..., 3$. Hence, the MRS can be viewed as a version of time-varying parameter model since $E[\beta_{j,t}]$ is varying over the time according to the variation of the probability weight, $Pr[S_t = i]$. We will take advantage of $E[\beta_{j,t}]$ later.

3 Estimation results of the matching function

3.1 Empirical matching function without regime switching

Let us start with the data description. The data we used is drawn form "Employment Referral Statistics" in *Year Book of Labour Statistics,* issued by The Ministry of Labour, Japan. Our variables correspond to the series of this data source

[7]We assume the variance does not exhibits regime switches because its variation is not of interest here.

as follows, where names officially refereed to therein (translated to English) are in parenthesis,

H; Filled vacancies (Placement) ,

U; Stock of job seekers (Active applications) ,

V; Stock of job vacancies (Active openings) ,

New school leavers are excluded while part time workers are included for all variables. They are all monthly data and seasonally-adjusted by the moving average method. Sample period is from $1964 : 07$ to $2000 : 10$, which was the maximum length available when our research began.

For variable U we choose the population of effective job seekers, not of unemployed, because of the following two reasons. First, the equilibrium unemployment theory requires that U should mean those who are not employed *and* participating in the labor market. Second, in our setting, all the variables are consistent in that thay are collected by the same establishment, i.e., employment referral offices[8].

Before proceeding to MRS, we present the estimation result of the matching function in the conventional form, i.e., equation (1). Table 1 shows GLS estimates *via* the two step Prais-Winsten procedure because we find a serial correlation in

[8]However we must notice that the number of effective job seeker (called active applicant) might be more sensitive to business cycle shocks than aggregate unemployment, since those who take advantage of employment referral offices tend to increase at recessional periods. So, there emerges solicitude that it may not represent the “aggregate” unemployment best. Nevertheless, we use this series as U_t because there are no statistics on unfilled vacancies which conceptually exactly match with the aggregate unemployment.

Table 1: Matching function without regime switching[1]

	$constant$	$\log U$	$\log V$	$trend$	R^2
Estimate	-1.095	0.625	0.411	-0.692	0.992
(s. e.)	(0.406)	(0.046)	(0.026)	(0.040)	-

[1] Note: Standard errors are in parentheses.

the OLS residuals. (The Durbin-Watson statistics is 0.007 and estimated AR(1) coefficient of the error term is 0.94.) Numbers in parentheses are standard errors.

Estimated elasticities of unemployment and unfilled vacancies show correct signs and are statistically significant. The point estimate of the returns to scale of the matching function given by $\beta_1 + \beta_2$ is 1.04. The t-value for testing the null hypothesis that $H_0 : \beta_1 + \beta_2 = 1$ is 0.113, so we cannot reject the constant returns to scale. Hence, it is found that the matching function in Japanese labor market satisfies conditions required by the equilibrium unemployment theory as a whole sample period.

In the next subsection, however, we will show that the matching function exhibits increasing and decreasing returns to scale in different subperiods within a sample period when we estimate it by the MRS.

3.2 Empirical matching function with regime switching

Our MRS model of the matching function is estimated by the maximum likelihood method. We employ the expectation-maximization (EM) algorithm tailored to MRS, following Hamilton (1990). However we replace his smoothing algo-

Table 2: Matching function with regime switching[1].

Variables		*Regime 0*	*Regime 1*
p_{00}	0.384^{*}	-	-
	(0.128)		
p_{11}	0.385^{*}	-	-
	(0.127)		
$constant$	-	-3.400^{**}	-0.323
		(0.679)	(0.936)
$\log U$	-	0.970^{**}	0.397^{**}
		(0.079)	(0.102)
$\log V$	-	0.452^{**}	0.516^{**}
		(0.043)	(0.065)
$trend$	-	-0.976^{**}	-0.896^{**}
		(0.072)	(0.095)
$\sigma^2 \times 10^3$	0.205^{**}	-	-
	(0.045)		
$Log\text{-}likelihood$	436.405		

[1] Note: $*$ and $**$ denote that the estimates are significant at 5% and 1% levels, respectively. Standard errors are in parentheses.

rithm at each expectation step with Kim (1994)'s one from the viewpoint of computational efficiency[9].

Table 2 presents the result. Numbers in parentheses denote asymptotic standard errors of parameters[10]. Except the coefficient of the constant term at regime 1, all the estimates are statistically significant. The coefficient of unemployment

[9] See also Kim and Nelson (1999), Chapter 4.

[10] It is often claimed that, unlike numerical optimization methods such as Newton-Raphson type algorithms, EM algorithm does not yield information matrix, and inevitably corresponding asymptotic variance-covariance matrix is not gained as a byproduct of maximization process. However, in effect, we are able to obtain the information matrix by the conventional way, namely, by computing the inner product of gradient vectors of the likelihood function evaluated at estimates. See Ruud (1991) for this point.

for regime 0 is clearly larger than that for regime 1, though there is no apparent difference between vacancies' coefficients in both regimes.

Large difference in the elasticities of unemployment, consequently, leads to non-negligible gap of returns to scale between two regimes. For regime 1, the returns to scale is 0.91 (and its s.e. is 0.155) and the t-value for testing the null hypothesis of the constant returns is 0.133. So the null hypothesis is not rejected at 1% in regime 1. On the other hand, it is 1.42 (s.e. is 0.113) for regime 0 and the t-value for testing the null hypothesis of the constant returns is 3.734. So the null hypothesis is rejected at 1% significant level in regime 0. Hence our result suggests that there can be periods of constant and of increasing returns, i.e., periods of single and of possibly multiple natural rate(s) regimes, in the history of Japanese labor market.

The expected durations of regime 0 and 1, defined by $D_0 = (1 - p_{00})^{-1}$ and $D_1 = (1 - p_{11})^{-1}$, are 1.62 and 1.63 months, respectively. These estimates mean that when the economy starts from a particular regime, it would remain at the same regime only for less than two months on average. This is a statistical evidence which supports our initial guess on the instability of the matching function.

In order to visualize the regime switches in our MRS model, we plot $Pr[S_t = 0]$, the probability that the economy stays at regime 0 in Figure 1. This is the probability that the economy exhibits the increasing returns at time t. Based on this probability weight we obtain the expected returns to scale given by

$$E[RTS_t] = E[\beta_{1,t}] + E[\beta_{2,t}],$$

where RTS denotes returns to scale, and $E[\beta_{j,t}]$, $j = 1, 2$, is defined by the equation (5). It is also plotted in Figure 2.

From both figures we find the following marked features of the regime switching. First, during the first decade of the sample period, the matching function shifts frequently between two regimes, and then stays at constant returns regime during the next decade. Although the function constantly exhibits increasing returns for the remaining period, there exists a “trough” between 1988 and 1993. Notice that major breaking points in the function, first around 1974 and then 1993, are years of big economic events; the former corresponds to the first oil crisis, while the latter corresponds to the end of the so-called “ bubble economy” era.

4 Concluding remarks

In this paper we have showed that the aggregate matching function in Japanese labor market is subject to non-negligible structural changes empirically, and it changes frequently between increasing and constant returns regime. Moreover, the expected durations of both regimes are very short. These findings are in favor of our initial guess on the instability of the matching relation in the labor market.

In the context of equilibrium unemployment theory, the aggregate matching function is often said to be analogous to the aggregate production function in that unemployment and unfilled vacancies can be viewed as “inputs” and new hiring can be viewed as “output” in the matching function, and that the properties of the matching function are stated by the same notions of the production function. However, we must be more careful about a clear difference between them; the matching relation is neither a physical nor an engineering one. Our result supports this view empirically. For almost all the sample regions and periods in previous studies, aggregate production functions exhibit constant returns to scales empirically (for example, see Douglas, 1976). However, our result shows that it is not the case in the matching function.

Our result also suggests the importance of exploring the endogenous formulation of the process where search frictions occur in the labor market (for the study of this formulation, see, for example, Lagos, 2000). Further theoretical investigation of the “black box” would contribute to explain the mechanism of changing matching relations between unemployment and unfilled vacancies, though it is not fully addressed in this paper.

Of course there remains the possibility that observed instability of the matching function is the peculiarity of the Japanese labor market. Therefore it will be worthwhile investigating the instability of matching functions of other countries by the method of this paper.

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