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A Subjective Equilibrium Model of the Agricultural Household with Demographic Behavior--A Methodological Note--

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1. Introduction

The objective of this methodological note is to introduce an approach linking the economic behavior of the household, such as the family's production and consumption activities, with its demographic behavior. The framework for the analysis is provided by the agricultural household in LDCs for reasons that refer to data availability (the Mindanao Survey) but especially because it is there that the three decisions -- demographic, production and consumption -- take place under the same roof.

The analysis of fertility behavior has been enriched in the years since Becker [1965] and Lancaster [1966, 1971] developed a new theory of household economics. In this new approach to consumption theory, the consumption activity of the household is regarded as a production process where time and purchased goods are combined to produce "commodities" which yield utility. The number and the "quality" of children [Willis, 1973], or the "child services" which are simply a product of the number and quality of children [De Tray, 1973], are regarded as nondurable goods which produce "utility" or "satisfaction" to the parents. This utility or satisfaction is considered as the "value of children".

In agricultural households in LDCs children often make a direct contribution to production, besides their contribution of providing satisfaction to their parents. The participation of children in agricultural production activities, starting as early as the tender age of six, is often impressive. Children participate in the labor market for certain farming activities.[1] In such cases, the direct contribution of children to the production process may be an important economic factor determining the fertility behavior of the household. Even more important is that the economic value of children can be measured directly in such cases, as opposed to the indirect contribution of children to the parents' utility, which can only be conjectured.

By incorporating children into the production and consumption behavior of the household, this methodological paper offers an operational handle for measuring more fully the benefits and costs of children. At this stage the demographic behavior of the household is still exogenous, although the calculus of the contributions of children may be an important determinant of fertility. The next step, that will not be pursued in this paper, will be to make the fertility decision endogenous in the demographic-economic model of the household.

2. The Subjective Equilibrium of the Agricultural Household

Agricultural households are at the same time producers of agricultural products and consumers of purchased final goods as well as of home-produced agricultural products. In this sense they can be regarded as "firm-household complexes". For the present study, the theory of subjective equilibrium of the agricultural household contributes a set of behavioral hypotheses for the analysis of the agricultural household which possesses such production and consumption characteristics.

The theory of the subjective equilibrium of the agricultural household was first introduced in Japan by Tanaka [1951] based on Chayanov's[1923, 1925] theory of the peasant economy. It was further developed by Nakajima [1957, 1958] to explain the economic behavior of the agricultural household as a firm-household complex. In this theory of subjective equilibrium, there are two important postulates. First, just as profit maximization is postulated in the theory of the firm and utility maximization in the theory of the consumer choice, utility maximization is likewise postulated in the theory of subjective equilibrium of the agricultural household. Second, economic rationality is postulated. Economic rationality in this context implies that the agricultural household as a firm-household complex performs economic activities so as to meet its objective, i.e., utility maximization, within the scope of the subjective possibilities for economic choice.

This early version of the theory of the subjective equilibrium was further developed in the 1960's to provide an explanation of labor supply in LDCs by Mellor [1966], Tanaka [1967], Sen [1966], Nakajima [1969], and Maruyama [1975] among others. In this form it was brought to bear on the zero-marginal-productivity-of-labor hypothesis of Nurkse [1953], Lewis [1954], and Ranis and Fei [1964].

A further refinement was introduced in the theory of the subjective equilibrium by Sasaki and Maruyama [1966] and Jorgenson and Lau [1969]. They provided for the separability between the production and consumption sides of the behavior of the agricultural household and explicitly introduced the equilibrium of the firm on the production side. This implies that the production behavior of the agricultural household (as a firm) can be analyzed independently of its consumption behavior, as long as there exist competitive markets for agricultural labor. The consumption behavior, however, may not be analyzed independently of the production behavior, since the income constraint depends on the outcome of the agricultural production in the form of profits.

Empirical applications of the theory of the subjective equilibrium of the agricultural household based on the above refinement have been accumulating in recent years. For the analysis of production behavior of the agricultural household, the profit and factor demand functions approach has been applied to Taiwan [Lau and Yotopoulos, 1971, 1972, 1973], Japan [Kuroda and Yotopoulos, 1978], India [Sidhu, 1974], Malaysia [Tamin, 1979], Thailand [Lerttamrab, 1975], Turkey [Somel, 1979], and Guatemala [Sevilla, 1980]. For the analysis of the consumption side, the linear logarithmic expenditure system were estimated for Taiwan [Lau, Lin, and Yotopoulos, 1978] and Japan [Kuroda and Yotopoulos, 1980].

The present model of the subjective equilibrium of the farm household is based on an application of the profit and factor demand functions as well as the linear logarithmic expenditure system. The point of departure is, however, the explicit introduction of child labor in the production function of the agricultural household. As will be seen later, through the estimation of the profit and factor demand functions, the demand function of child labor can be explicitly estimated along with the demand functions for other variable inputs. Furthermore, from the indirect estimation of the production function, the magnitude of the contribution of child labor to the agricultural production can be estimated.

On the consumption side, the number of children is introduced as an exogenous variable. In this formulation, although we cannot estimate the demand function for children, we can still analyze at least indirectly the effect of an increase (or decrease) in the number of children on the agricultural production, the demand for home-produced agricultural products and its complement, the supply of marketed surplus.

3. A Formal Model of Subjective Equilibrium of the Agricultural Household

3.1 The Household Utility Function

Our hypothesis regarding the behavior of the agricultural household is that it maximizes its utility subject to the production function, the time constraint, and the budget constraint. The household chooses the levels of consumption of goods and leisure, the levels of adult and child labor supplied to agricultural production, and the quantities of variable inputs including hired labor, given the prices of agricultural output, purchased consumer final goods and variable inputs, and the quantities of fixed inputs.

We divide the family members of the agricultural household into three categories: (1) adult members older than 16 years of age who are engaged in agricultural production activities; (2) child members between 6 and 15 years of age who may also engage in agricultural production activities; and (3) dependents who do not engage in any production activities. In this last category are included family members who are old and retired and also children under 5 years of age. Furthermore, given the empirical evidence of the Mindanao Survey, we assume that competitive markets exist for both adult and child labor. However, it is assumed that adult and child labor are not substitutes but family, hired, and exchange labor are perfectly substitutable within each category.

The conventional theory of the subjective equilibrium of the agricultural household has often specified the utility function as a function only of leisure and income. This income is usually interpreted as a Hicksian [1946] composite of all consumption goods. By subdividing the Hicksian composite into two categories of consumer goods, home-produced agricultural products and purchased consumer final goods, the explanatory power of the subjective equilibrium model is greatly improved [Torii, 1971]. That is, instead of obtaining the demand functions only for leisure and income, we may derive through the first-order conditions of utility maximization two demand functions for the two different groups of goods in addition to the demand functions for leisure (or conversely the supply functions of labor).

Finally, to distinguish between households with different characteristics, we assume that the household utility function depends parametrically on the household composition. Thus, the numbers of family members in the three categories mentioned above are introduced as shift parameters into the household utility function.

The utility function of the agricultural household can now be written as

(3-1) $U = U(Z_1, Z_2, Z_3, A, C; a_1, a_2, a_3)$

where

Z_1 : leisure time of adult laborers,

Z_2 : leisure time of child laborers,

Z_3 : leisure time of dependents,

A : amount of consumption of home-produced agricultural products,

C : amount of consumption of purchased consumer final goods,

a_1 : the number of adult laborers,

a_2 : the number of child laborers aged between 6 and 15, and

a_3 : the number of dependents including children younger than 5 years old and old and retired family members.

The utility function given in (3-1) is assumed to possess the following properties. (i) It is a continuous, twice-differentiable, monotonically increasing function of the three kinds of leisure, home-produced agricultural goods, and purchased consumer final goods, with the numbers of the family members of the three different groups being exogenous variables. (ii) It is quasi-concave (in Z_1, Z_2, Z_3, A, and C) so that there exists an inner solution of equilibrium. (iii) There exists non-increasing marginal utility for the three kinds of leisure (Z_1, Z_2, and Z_3) and the two bundles of goods (A and C). (iv) No satiation level exists.

3.2 The Agricultural Production Technology

We assume the following agricultural production function for the agricultural household.[2]

(3-2) $Q = F(X_1, X_2, X_i; K_j)$, $i = 3, \ldots, m$, $j = 1, 2, \ldots, n$,

where

- Q : the total amount of agricultural output,
- X_1 : the adult labor time,
- X_2 : the child labor time,
- X_i : the i-th variable input other than labor such as fertilizer and agri-chemicals, $i = 3, \ldots, m$,
- K_j : the j-th fixed input such as capital and land, $j = 1, 2, \ldots, n$.

The assumptions made for the production function are as follows. (i) It is strictly non-decreasing, continuous, and twice-differentiable function of two kinds of labor, other variable inputs, and fixed inputs.[3] (ii) It possesses diminishing marginal products with respect to all inputs. (iii) It is strictly quasi-concave so that the unique output supply and factor demand functions may be derived.

As mentioned earlier, it is assumed that competitive labor markets exist separately for adult and child labor as well as for the other variable inputs and agricultural output. In addition, the agricultural household in the present model is considered to consume part of the agricultural output and to sell the rest of it in a market. Furthermore, the agricultural household both purchases and sells variable factors of production including adult and child farm labor. We assume that both output marketed and consumed, and inputs purchased and sold are homogeneous and perfectly substitutable. That is, no difference is assumed in quality of output marketed and sold and inputs purchased and sold. With this assumption there exists for each household only one price for output and one price for each input.

3.3 The Time Constraint

The time constraint of the agricultural household is given as follows. First, for the adult laborers in the household,

(i) $$a_1\bar{Z}_1 - Z_1 = X_1^f,$$

for the child laborers,

(ii) $$a_2\bar{Z}_2 - Z_2 = X_2^f,$$

and for the dependents,

(iii) $$a_3\bar{Z}_3 - Z_3 = 0,$$

where

$\bar{Z}_i$: the quantity of leisure per family member in each category,

X_1^f : the total family labor time of the adult laborers, and

X_2^f : the total family labor time of the child laborers.

The total adult and child labor time on the farm is given respectively as,

(3-3) $$X_1 = (a_1\bar{Z}_1 - Z_1) + X_1^h + X_1^e = X_1^f + X_1^h + X_1^e,$$

(3-4) $$X_2 = (a_2\bar{Z}_2 - Z_2) + X_2^h + X_2^e = X_2^f + X_2^h + X_2^e,$$

where

X_1^h: the net of hired-in adult labor time,

X_1^e: the net of adult exchange labor time,

X_2^h: the net of hired-in child labor time, and

X_2^e: the net of child exchange labor time.

Note here that we are assuming homogeneity and perfect substitutability between family, hired, and exchange labor in each category of adult and child labor, so that we have only one price of farm labor in each category.

3.4 The Total Income and Expenditure Constraint

Finally, the total income and expenditure constraint may be given as follows.

$$(3\text{-}5) \qquad (P_A Q - q_1' X_1 - q_2' X_2 - \sum_{i=3}^{m} q_i' X_i) + q_1'(a_1 \bar{Z}_1 - Z_1)$$
$$+ q_2'(a_2 \bar{Z}_2 - Z_2) + I_A = P_A A + P_C C,$$

where

P_A : the agricultural output price,

q_1' : the agricultural wage rate for adult labor,

q_2' : the agricultural wage rate for child labor,

q_i' : the price of the i-th variable input other than labor, $i = 3, 4, \ldots, m$,

I_A : non-labor asset income, and

P_C : the price of a Hicksian composite of purchased consumer final goods.

The first term on the LHS of (3-5) represents the farm non-labor income of the agricultural household which accrues to fixed inputs such as entrepreneurship, capital equipment, and land after subtracting the costs of variable inputs from the total value of output. We define this as the farm profit of the agricultural household. The second and third terms on the LHS are the total wage income accruing respectively to the adult and child labor in the household. In this case, we are explicitly imputing the price of unpaid family and exchange labor for both categories by the respective market wage rates of hired labor by assuming that there exist competitive markets for both adult and child labor.

Since $(a_1\bar{Z}_1 - Z_1) = X_1^f$ and $(a_2\bar{Z}_2 - Z_2) = X_2^f$, the first three terms of (3-5) can be rewritten as,

$$P_AQ - q_1' \ (X_1^f + X_1^h + X_1^e) - q_2' \ (X_2^f + X_2^h + X_2^e) - \sum_{i=3}^{m} q_i'X_i$$

$$+ q_1'X_1^f + q_2'X_2^f$$

$$= P_AQ - q_1' \ (X_1^h + X_2^e) - q_2' \ (X_2^h + X_2^e) - \sum_{i=3}^{m} q_i'X_i.$$

That is, although the labor costs related to the family labor are, in our model, explicitly treated as part of variable costs in the firm's agricultural production, they are in fact implicitly treated as an important source of family income. This implies that although family labor is employed in the self-management of the firm as unpaid labor, it is assumed to yield wage income to the household as evaluated by the market wage rates.

Following Becker [1965], we may rewrite the income constraint given in (3-5) as,

$$\text{(3-6)} \qquad (P_AQ - q_1'X_1 - q_2'X_2 - \sum_{i=3}^{m} q_i'X_i) + q_1'a_1\bar{Z}_1 + q_2'a_2\bar{Z}_2 + I_A$$

$$= q_1'Z_1 + q_2'Z_2 + P_AA + P_CC.$$

The LHS denotes the "full income" and the RHS the "full expenditure" in Becker's terminology.

3.5 Maximization of the Household Utility Function

The agricultural household is assumed to maximize its utility subject to the constraints. By following Becker [1965], we can solve this problem by maximizing the household utility function given in (3-1) subject to the income constraint given in (3-5) which includes the production function constraint given in (3-2) and the time constraints given in (3-3) and (3-4).

A Lagrange multiplier method is used for maximization. The Lagrangean equation is given as,

(3-7) $$U^* = U(Z_1, Z_2, Z_3, A, C; a_1, a_2, a_3) + \lambda\{(P_A Q - q_1' X_1 - q_2' X_2 - \sum_{i=3}^{m} q_i' X_i) + q_1'(a_1 \bar{Z}_1 - Z_1) + q_2'(a_2 \bar{Z}_2 - Z_2) + I_A - P_A A - P_C C\}$$

The first-order conditions are:

(3-8) $$\frac{\partial U^*}{\partial Z_1} = U_{Z_1} - \lambda q_1' = 0,$$

(3-9) $$U_{Z_2} - \lambda q_2' = 0,$$

(3-10) $$U_{Z_3} - 0 = 0,$$

(3-11) $$U_A - \lambda P_A = 0,$$

(3-12) $$U_C - \lambda P_C - 0,$$

(3-13) $$P_A F_{X_1} - q_1' = 0$$

(3-14) $$P_A F_{X_2} - q_2' = 0$$

(3-15) $$P_A F_{X_i} - q_i' = 0,\ i = 3, 4, \ldots, m,$$

(3-16) $$(P_A Q - q_1'X_1 - q_2'X_2 - \sum_{i=3}^{m} q_i'X_i) + q_1'(a_1\bar{Z}_1 - Z_1) + q_2'(a_2\bar{Z}_2 - Z_2) + I_A - P_A A - P_C C = 0.$$

(3-17) $$Q = F(X_1, X_2, X_i; K_j),\ i = 3, 4, \ldots, m,\ \ j = 1, 2, \ldots, n.$$

The equations given in (3-8) through (3-17) constitute a system of (9 + m - 2) simultaneous equations in the endogenous variables, Z_1, Z_2, Z_3, A, C, Q, X_1, X_i, and λ (i = 3, 4, ..., m). The exogenous variables in the system are q_1', q_2', P_A, P_C, K_j, I_A, a_1, a_2, and a_3(j = 1, 2, ..., n). Since the number of the endogenous variables is equal to the number of equations, (7 + m), the system of simultaneous equations can be solved for the equilibrium values of the endogenous variables.

However, equations given in (3-13), (3-14), (3-15), and (3-17) may be solved jointly for the equilibrium values of Q, X_1, X_2, and X_i (i = 3, 4, ..., m), given q_1'/P_A, q_2'/P_A, q_i'/P_A(i = 3, 4, ..., m), and K_j (j = 1, 2, ..., n), without reference to the other equations in the complete simultaneous equations system. Given competitive markets for agricultural output and labor as well as the other variable inputs, Sasaki and Maruyama [1966] and Jorgenson and Lau [1969] have shown that the production behavior of the agricultural household is completely independent of the consumption choice. That is, given the prices of the agricultural output and the variable inputs, the agricultural household as a firm first chooses the level of production which maximizes its profit. Then, it assigns the maximized profit as the agricultural non-labor income to the consumption side of the household.

Using this agricultural non-labor income together with income that may accrue from the other sources, the agricultural household is now assumed to behave so as to maximize the household utility. In this manner, the agricultural household model in the present study is separable for the production behavior and the consumption behavior in that the production choice is completely independent of the consumption choice but the latter in turn depends on the activities of agricultural production in the form of profit. That is, the system is "block-recursive" [Jorgenson and Lau, 1969].

The block-recursive nature of the model can be further clarified. Equations (3-13), (3-14), and (3-15) are the equilibrium conditions of the two types of labor and the other variable inputs for a firm. By solving these equations simultaneously, we may obtain the equilibrium quantities of the labor inputs, X_1^* and X_2^*, and the other variable inputs, X_i^* $(i = 3, 4, \ldots, m)$, given P_A, q_1', q_2', and q_i' $(i = 3, 4, \ldots, m)$. Up to this point, the behavior of the agricultural household is completely equivalent to the profit-maximizing behavior of a firm; i.e.,

$$\Pi' = P_A Q - q_1' X_1 - q_2' X_2 - \sum_{i=3}^{m} q_i' X_i$$

$$= P_A F(X_1, X_2, X_i; K_j) - q_1' X_1 - q_2' X_2 - \sum_{i=3}^{m} q_i' X_i,$$

$$i = 3, 4, \ldots, m, \quad j = 1, 2, \ldots, n,$$

then the first-order conditions of profit maximization are,

$$\frac{\partial \Pi'}{\partial X_1} = P_A F_{X_1} - q_1' = 0,$$

$$\frac{\partial \Pi'}{\partial X_1} = P_A F_{X_2} - q_2' = 0, \text{ and}$$

$$\frac{\partial \Pi'}{\partial X_i} = P_A F_{X_i} - q_i' = 0, \ i = 3, 4, \ldots, m,$$

which are identical to equations (3-13), (3-14), and (3-15) in the complete simultaneous equations system. Given these equilibrium quantities of the variable inputs, the agricultural household as a firm can determine the equilibrium quantity of output, Q^*, and hence the maximized profit, Π^*.

In sum, the behavior of the agricultural household in our model may be considered in two stages: at the first stage, the agricultural household as a firm maximizes its profit and determines the equilibrium quantities of X_1^*, X_2^*, and X_i^* ($i = 3, 4, \ldots, m$), and Q^* and hence Π^*. At the second stage, the agricultural household behaves as a utility maximizer with the agricultural non-labor income (i.e., the maximized profit, Π^*) and the income from other sources being the budget constraint. That is, subject to the budget constraint (3-16) in its rewritten form,

$$(3\text{-}18) \qquad \Pi^* + q_1'(a_1\bar{Z}_1 - Z_1) + q_2'(a_2\bar{Z}_2 - Z_2) + I_A - P_A A - P_C C = 0$$

the agricultural household behaves so as to maximize the utility in (3-1) and determines the equilibrium quantities Z_1^*, Z_2^*, Z_3^*, A^*, and C^*. Given these equalibrium quantities, the maximum level of utility of the agricultural household can be attained.

3.6 The Production Side

By making use of the duality theorem, we may obtain the profit (or output supply) and variable factor demand functions for the system constructed by equations (3-13), (3-14), (3-15), and (3-17). The reduced-form functions for the subjective equilibrium of the agricultural household as a firm may be derived as follows:

(i) The profit function ,

(3-19) $\Pi^* = \Pi^*(P_A, q_1', q_2', q_i', K_j)$, $i = 3, \ldots, m$, $j = 1, \ldots, n$,

(ii) The labor demand functions,

(3-20) $X_1^* = X_1^*(P_A, q_1', q_2', q_i', K_j)$, $i = 3, \ldots, m$, $j = 1, \ldots, n$,

(3-21) $X_2^* = X_2^*(P_A, q_1', q_2', q_i', K_j)$, $i = 3, \ldots, m$, $j = 1, \ldots, n$,

(iii) The demand functions for the variable inputs other than labor,

(3-22) $X_i^* = X_i^*(P_A, q_1', q_2', q_i', K_j)$, $i = 3, \ldots, m$, $j = 1, \ldots, n$.

Simultaneous estimation of these reduced-form equations will give us the reduced-form elasticities of the profit and the demand for labor and the other variable inputs. Note here that we explicitly estimate the demand function for child labor given in (3-21). This provides an estimate of the benefits from children which are among the determinants of the agricultural household's fertility decisions.

3.7 The Consumption Side

As shown earlier, the agricultural household's problem is to maximize its utility in (3-1) subject to the budget constraint in (3-18). The household then determines the equilibrium quantities of the demand for leisure (or conversely the supplies of labor), for home-produced agricultural products, and for purchased consumer final goods.

The reduced-form demand equations for leisure (Z_1, Z_2, and Z_3) and commodities (A and C) may be obtained from the simultaneous equations system composed of equations (3-8), (3-9), (3-10), (3-11), (3-12), and (3-18). Note here that since the consumption choice depends on the outcome of the production behavior in the form of the maximized profit, Π^*, equation (3-18) instead of (3-16) should be used in the simultaneous equations system for the consumption side.

The reduced-form demand functions may be given as,

(3-23) $$Z_1 = Z_1(q_1', q_2', P_A, P_C, \Pi^*, I_A; a_1, a_2, a_3),$$

(3-24) $$Z_2 = Z_2(q_1', q_2', P_A, P_C, \Pi^*, I_A; a_1, a_2, a_3),$$

(3-25) $$Z_3 = Z_3(q_1', q_2', P_A, P_C, \Pi^*, I_A; a_1, a_2, a_3),$$

(3-26) $$A = A(q_1', q_2', P_A, P_C, \Pi^*, I_A; a_1, a_2, a_3),$$

(3-27) $$C = C(q_1', q_2', P_A, P_C, \Pi^*, I_A; a_1, a_2, a_3).$$

Observe that equations (3-23) through (3-27) constitute a simultaneous equations system with each equation having the same exogenous variables. Moreover, the size of each group of the family members enters in these equations. In this way, we may analyze the effects of an increase (or decrease) in the number of children on the demand for leisure and goods. In particular, the effect of a decrease in the number of children on the supply of adult labor and on the marketed surplus (which are immediately obtained from the estimates of the demand functions for leisure Z_1 and for home-produced agricultural output A) are of great interest. By examining the estimates of elasticities in these equations, we may gain a better understanding of the agricultural household's behavior with demographic decisions not only in the consumption but also, at least indirectly, in the production.

With the specifications of appropriate functional forms for the production and utility functions, we may estimate the reduced-form equations on both the production and consumption sides of the agricultural household.

For this purpose, we will specify the production function given in (3-2) as a Cobb-Douglas type from which the Cobb-Douglas profit and factor demand functions can immediately be derived by applying the duality theorem. This system of the profit and factor demand functions will then be estimated for a sample of agricultural households.

On the consumption side, the indirect utility function of the utility function given in (3-1) will first be derived by making use of the duality theorem. Then, by assuming that the indirect utility function is a homogeneous transcendental logarithmic function with a second-order approximation, the linear logarithmic expenditure system will be derived and statistically estimated.

4. A Case with Child-Specific Z-Production Activities

The model of the previous section, while distinguishing between adult and child labor, assumed the production of one (composite) agricultural commodity in which both kinds of labor were utilized in a relationship of complementarity. Evidence from the Mindanao Survey suggests that certain production activities, such as caring for animals or tending the family garden plot, are carried out almost exclusively by children. We can thus introduce a second agricultural production function into the model that is child-labor specific.

Based on the nature of children's participation in production and on the existence or absence of markets for child labor, we can distinguish three cases for such a model of the agricultural household.

4.1 Case I

It is assumed that child labor is utilized for certain activities of the adult-specific production such as weeding and harvesting. Child labor is also exclusively utilized for the child-specific production mentioned above. A competitive market is assumed to exist for adult labor but not for child labor. Moreover, it is assumed that adult labor and child labor in the adult-specific production are not substitutes. However, within the category of adult labor, family, hired, and exchange labor are assumed to be perfect substitutes, so that only one wage rate, q_1', exists.

The agricultural household is assumed to consume part of both production outputs, the adult-specific and the child-specific, and to sell in markets the rest. We assume further that the markets exist separately for the adult-specific production output and the child-specific production output, so that there exist two different output prices, P_A^1 and P_A^2, respectively.

The complete system of economic behavior of the agricultural household in this case may be written as follows.

(4-1) $U = U(Z_1, Z_2, Z_3, A^1, A^2, C; a_1, a_2, a_3)$,

(4-2) $Q^1 = F^1(X_1, X_2^1, X_i; K_j)$, $i = 3, 4, \ldots, m$, $j = 1, 2, \ldots, n$,

(4-3) $Q^2 = F^2(X_2^2)$,

(4-4) $$Z_1 = a_1\bar{Z}_1 - X_1^f,$$

(4-5) $$Z_2 = a_2\bar{Z}_2 - X_2^f,$$

(4-6) $$Z_3 = a_3\bar{Z}_3,$$

(4-7) $$(P_A^1Q^1 - q_1'X_1 - \sum_{i=3}^{m} q_i'X_i) + P_A^2Q^2 + q_1'(a_1\bar{Z}_1 - Z_1) + I_A$$

$$= P_A^1A^1 + P_A^2A^2 + P_CC,$$

where

(4-8) $$X_1 = X_1^f + X_1^h + X_1^e,$$

(4-9) $$X_2 = X_2^1 + X_2^2 = X_2^f.$$

The variables not defined earlier are as follows:

A^1 : amount of consumption of output of adult-specific agricultural production,

A^2 : amount of consumption of output of child-specific agricultural production,

Q^1 : the total amount of output of adult-specific agricultural production,

Q^2 : the total amount of output of child-specific agricultural production,

X_2^1 : the child labor time for adult-specific agricultural production,

X_2^2 : the child labor time for child-specific agricultural production,

X_2^f : the total family child labor time,

P_A^1 : the output price of adult-specific agricultural production, and

P_A^2 : the output price of child-specific agricultural production.

The utility function given in (4-1) is different from the one given in (3-1) in section 3 in that three types of consumption goods are distinguished. The nonagricultural commodity, C, remaining as previously, one which is consumed but not produced. The two agricultural commodities, on the other hand, A^1 and A^2, are both produced and consumed by the household. Furthermore, both commodities can be traded at existing perfect markets. The two commodities differ by their respective production functions, Q^1 and Q^2 in that the latter employs only child labor while Q^1 as given in (4-2) is identical with the production function given in (3-2) in section 3. We make the assumption, therefore, that adult and child labor are not substitutes within the Q^1 production. However, we assume that within the category of adult labor, perfect substitution possibilities exist for family, hired, and exchange labor. In this manner, we have only one wage rate, q_1', for adult labor. On the other hand, since we assume no competitive market for child labor in the present case, the child labor employed for the Q^1 production may be assumed to be only family child labor and thus we have no market wage rate for child labor. We also assume that there exist separate markets for the two kinds of agricultural outputs, Q^1 and Q^2, and therefore there exist two different market prices, P_A^1 and P_A^2.

The maximization problem of the agricultural household in the present case may be solved by assuming that the household maximizes its utility given in (4-1) subject to the child time constraint and the budget constraint given respectively in (4-5) and (4-7).

The Lagrangean equation for the maximization is given as,

$$(4\text{-}10)\qquad U^* = U + \delta[Z_2 - a_2\bar{Z}_2 + X_2^1 + X_2^2] + \lambda[(P_A^1Q^1 - q_1'X_1 - \sum_{i=3}^{m} q_i'X_i)$$

$$+ P_A^2Q^2 + q_1'(a_1\bar{Z}_1 - Z_1) + I_A - P_A^1A^1 - P_A^2A^2 - P_CC].$$

The first-order conditions are:

$$(4\text{-}11)\qquad U_{Z_1} - \lambda q_1' = 0\ ,$$

$$(4\text{-}12)\qquad U_{Z_2} + \delta = 0\ ,$$

(4-13) $U_{Z_3} - 0 = 0,$

(4-14) $U_{A^1} - \lambda P_A^1 = 0$,

(4-15) $U_{A^2} - \lambda P_A^2 = 0$

(4-16) $U_C - \lambda P_C = 0$,

(4-17) $P_A^1 F_{X_1}^1 - q_1' = 0$,

(4-18) $P_A^1 F_{X_i}^1 - q_i' = 0, \; i = 3, 4, \ldots, m,$

(4-19) $\lambda P_A^1 F_{X_2^1}^1 + \delta = 0$,

(4-20) $\lambda P_A^2 F_{X_2^2}^2 + \delta = 0$,

(4-21) $Z_2 - a_2 \bar{Z}_2 + X_2^1 + X_2^2 = 0$,

(4-22) $(P_A^1 Q^1 - q_1' X_1 - \sum_{i=3}^{m} q_i' X_i) + P_A^2 Q^2 + q_1'(a_1 \bar{Z}_1 - Z_1) + I_A$

$- P_A^1 A^1 - P_A^2 A^2 - P_C C = 0,$

where

(4-23) $Q^1 = F^1(X_1, X_2^1, X_i; K_j), \; i = 3, 4, \ldots, m, \quad j = 1, 2, \ldots, n,$

(4-24) $Q^2 = F^2(X_2^2),$

(4-25) $X_2 = X_2^1 + X_2^2.$

From (4-12), (4-19), and (4-20), we have,

(4-26) $P_A^1 F_{X_2^1}^1 = P_A^2 F_{X_2^2}^2 = U_{Z_2} / \lambda$.

The first equal sign implies that the marginal productivity of child labor in the Q^1 production is equal to that in the Q^2 production. The second equal sign implies that the marginal productivity of child labor equals the marginal utility of leisure of child members of the household deflated by the marginal utility of income.

As is clear from the set of equations given in (4-17), (4-18), (4-19), (4-20), (4-23), and (4-24), the agricultural household in this model may be considered as maximizing profits neither in the (adult-specific) Q^1 production nor in the (child-specific) Q^2 production. That is, equations (4-19) and (4-20) indicate associations of the production plans with the consumption plan of the agricultural household. This is clearly shown in (4-26). This implies that variations in the household composition will have an influence on the agricultural production. Thus, the behavioral patterns indicated by this model are substantially different from the model where competitive markets exist for both adult and child labor.

Because of this feature of inseparability between the production and consumption behavior of the agricultural household, one may have to estimate the complete set of equations simultaneously when it comes to empirical implementation. This procedure, however, may encounter some practical difficulties. For example, unless the mathematical specifications for both the utility and the two production functions be kept simple, the estimating reduced-form equations will be very complex and (computer) time consuming.

A practical procedure to overcome this problem may be to specify the system of reduced-form equations on an ad hoc basis. In such a case, however, the household's behavior cannot be based on utility maximization considerations.

4.2 Case II

For this case we assume complete division of labor along adult-specific and child-specific production, with neither adult nor child labor crossing over production functions. The adult labor market is assumed to be competitive, but no market is assumed to exist for child labor as in Case I. Furthermore, family, hired, and exchange labor are assumed to be perfect substitutes in the adult labor category.

It is assumed that the agricultural household consumes part of both farm outputs and sells the rest in the separate markets for the different prices, P_A^1 and P_A^2.

The complete formal model of the agricultural household for this case may be written as follows.

$$U = U(Z_1, Z_2, Z_3, A^1, A^2, C; a_1, a_2, a_3), \tag{4-27}$$

$$Q^1 = F^1(X_1, X_i; K_j), \; i = 3, 4, \ldots, m, \;\; j = 1, 2, \ldots, n, \tag{4-28}$$

$$Q^2 = F^2(X_2), \tag{4-29}$$

$$Z_1 = a_1\bar{Z}_1 - X_1^f, \tag{4-30}$$

$$Z_2 = a_2\bar{Z}_2 - X_2^f, \tag{4-31}$$

$$Z_3 = a_3\bar{Z}_3, \tag{4-32}$$

$$(P_A^1Q^1 - q_1'X_1 - \sum_{i=3}^{m} q_i'X_i) + P_A^2Q^2 + q_1'(a_1\bar{Z}_1 - Z_1) + I_A = P_A^1A^1 + P_A^2A^2 + P_CC, \tag{4-33}$$

where

$$X_1 = X_1^f + X_1^h + X_1^e, \tag{4-34}$$

$$X_2 = X_2^f. \tag{4-35}$$

All variables are as defined earlier.

The Lagrangean equation for the maximization in the present model can be written as,

$$U^* = U + \delta[Z_2 - a_2\bar{Z}_2 + X_2] + \lambda[(P_A^1Q^1 - q_1'X_1 - \sum_{i=3}^{m} q_i'X_i) + P_A^2Q^2 + q_1'(a_1\bar{Z}_1 - Z_1) + I_A - P_A^1A^1 - P_A^2A^2 - P_CC]. \tag{4-36}$$

The first-order conditions are given as:

(4-37) $U_{Z_1} - \lambda q_1' = 0,$

(4-38) $U_{Z_2} + \delta = 0,$

(4-39) $U_{Z_3} - 0 = 0,$

(4-40) $U_A 1 - \lambda P_A^1 = 0,$

(4-41) $U_A 2 - \lambda P_A^2 = 0,$

(4-42) $U_C - \lambda P_C = 0,$

(4-43) $P_A^1 F_{X_1}^1 - q_1' = 0,$

(4-44) $P_A^1 F_{X_i}^1 - q_i' = 0,$

(4-45) $\lambda P_A^2 F_{X_2}^2 + \delta = 0,$

(4-46) $Z_2 - a_2 \bar{Z}_2 + X_2 = 0,$

(4-47) $(P_A^1 Q^1 - q_1' X_1 - \sum_{i=3}^{m} q_i' X_i) + P_A^2 Q^2 + q_1'(a_1 \bar{Z}_1 - Z_1) + I_A$

$- P_A^1 A^1 - P_A^2 A^2 - P_C C = 0,$

where

(4-48) $Q^1 = F^1(X_1, X_i; K_j),\ i = 3, 4, \ldots, m,\ \ j = 1, 2, \ldots, n,$

(4-49) $Q^2 = F^2(X_2).$

From (4-38) and (4-45), we have,

(4-50) $P_A^2 F_{X_2}^2 = U_{Z_2}/\lambda,$

indicating that the marginal productivity of child labor in the Q^2 production equals the marginal utility of leisure of child members deflated by the marginal utility of income. Through this relationship, one can see that the Q^2 production plan and the consumption plan of the agricultural household are interrelated. Thus, the agricultural household may not be considered to maximize profits in the child-specific Q^2 production.

However, equations (4-43), (4-44), and (4-48) may be solved simultaneously for the equilibrium quantities of X_1, X_i, and Q^i (i = 3, 4, ..., m) without reference to the other equations in the complete system. This implies that through the duality theorem we may estimate the profit and factor demand functions for this sub-system of the adult-specific Q^1 production of the agricultural household. The maximized profits, denoted as Π_1^*, may be assigned as agricultural non-labor income to the budget constraint for the utility maximization.

However, on this stage one faces a difficulty for the empirical estimation because of the inseparability between the child-specific Q^2 production plan and the consumption plan of the agricultural household. As in Case I, an ad hoc specifications of the system of reduced-form simultaneous equations may be introduced in order to obtain the empirical estimates.

4.3 Case III

The formal model for this case is similar to the one for Case II except for the introduction of sub-division of child labor into two categories, i.e., labor of children aged between 11 and 15 and labor of children aged under eleven. In the present case, the first type of child labor is assumed to be utilized only for the Q^1 production and a competitive market is assumed to exist for this type of child labor, parallel to the also competitive market for adult labor. These two types of labor are assumed to be non-substitutable for each other so that there exist two wage rates, q_1' and q_2', for adult and the first type child labor. However, within each category of labor, family, hired, and exchange labor is assumed to be perfectly substitutable. On the other hand, the second type of child labor is assumed to be utilized only for the Q^2 production and no market is assumed to exist for this category of labor.

It is assumed that there exist separate competitive markets for the two farm outputs, Q^1 and Q^2, and the agricultural household consumes part of both outputs and sells the rest in the respective markets at the respective prices, P_A^1 and P_A^2.

The complete model for the present case may be written as follows.

$$U = U(Z_1, Z_2, Z_3, Z_4, A^1, A^2, C; a_1, a_2, a_3, a_4), \tag{4-51}$$

$$Q^1 = F^1(X_1, X_2, X_i; K_j),\ i = 4, 5, \ldots, m,\ \ j = 1, 2, \ldots, n, \tag{4-52}$$

$$Q^2 = F^2(X_3), \tag{4-53}$$

$$Z_1 = a_1\bar{Z}_1 - X_1^f, \tag{4-54}$$

$$Z_2 = a_2\bar{Z}_2 - X_2^f, \tag{4-55}$$

$$Z_3 = a_3\bar{Z}_3 - X_3, \tag{4-56}$$

$$Z_4 = a_4\bar{Z}_4, \tag{4-57}$$

$$(P_A^1Q^1 - q_1'X_2 - q_2'X_2 - \sum_{i=4}^{m} q_i'X_i) + P_A^2Q^2 + q_1'(a_1\bar{Z}_1 - Z_1) + q_2'(a_2\bar{Z}_2 - Z_2) + I_A = P_A^1A^1 + P_A^2A^2 + P_CC, \tag{4-58}$$

where

$$X_1 = X_1^f + X_1^h + X_1^e, \tag{4-59}$$

$$X_2 = X_2^f + X_2^h + X_2^e, \tag{4-60}$$

$$X_3 = X_3^f. \tag{4-61}$$

The variables not already defined are as follows:

a_2 : the number of children aged between 11 and 15,

a_3 : the number of children aged under 11,

a_4 : the number of dependents,

Z_2 : the leisure time of children aged between 11 and 15,

Z_3 : the leisure time of children aged under 11,

Z_4 : the leisure time of dependents,

$\bar{Z}_2$: the maximum leisure time per child aged between 11 and 15,

$\bar{Z}_3$: the maximum leisure time per child aged under 11,

$\bar{Z}_4$: the maximum leisure time per dependent,

X_2^k : the labor time of children aged between 11 and 15, k = f, h, e for family, hired, and exchange labor,

X_3 : the labor time of children aged under 11.

The agricultural household is assumed to maximize its utility given in (4-51) subject to (4-56) and (4-58). By making use of a Lagrangean multiplier method for the maximization, we have the following first-order conditions.

(4-62) $U_{Z_1} - \lambda q_1' = 0$

(4-63) $U_{Z_2} - \lambda q_2' = 0,$

(4-64) $U_{Z_3} + \delta = 0,$

(4-65) $U_{Z_4} - 0 = 0,$

(4-66) $U_A 1 - \lambda P_A^1 = 0,$

(4-67) $U_A 2 - \lambda P_A^2 = 0,$

(4-68) $U_C - \lambda P_C = 0,$

(4-69) $P_A^1 F_{X_1}^1 - q_1' = 0,$

(4-70) $P_A^1 F_{X_2}^1 - q_2' = 0,$

(4-71) $P_A^1 F_{X_i}^1 - q_i' = 0, \; i = 4, 5, \ldots, m,$

(4-72) $\lambda P_A^2 F_{X_3}^2 + \delta = 0,$

(4-73) $Z_3 - a_3\bar{Z}_3 + X_3 = 0,$

(4-74) $(P_A^1 Q^1 - q_1'X_1 - q_2'X_2 - \sum_{i=4}^{m} q_i'X_i) + P_A^2 Q^2 + q_1'(a_1\bar{Z}_1 - Z_1)$

$+ q_2'(a_2\bar{Z}_2 - Z_2) + I_A - P_A^1 A^1 - P_A^2 A^2 - P_C C = 0,$

where

(4-75) $Q^1 = F^1(X_1, X_2, X_i; K_j), \; i = 4, 5, \ldots, m, \;\; j = 1, 2, \ldots, n,$

(4-76) $Q^2 = F^2(X_3).$

From equations (4-64) and (4-72), we have,

(4-77) $P_A^2 F_{X_3}^2 = U_{Z_3}/\lambda.$

This implies that the marginal productivity of labor in the Q^2 production (deflated by the marginal utility of income) equals the marginal utility of leisure of children aged under 11. Furthermore, this indicates that the plan of the child-specific Q^2 production and the consumption plan of the agricultural household cannot be separated from each other. The household consumption decision always influences the child-specific production decision, and vice versa.

However, equations (4-69), (4-70), (4-71), and (4-75) may be solved for the equilibrium quantities of X_1, X_2, X_i, and Q^1 ($i = 4, 5, \ldots, m$) separately from the other equations in the complete system of the first-order conditions. This implies that one may estimate the profit and factor demand functions by making use of the duality system. The maximized profits can then be assigned to the budget constraint for the household utility maximization.

However, as in Case II, one faces a complex problem for the empirical estimation because the agricultural household does not behave to maximize profits in the child-specific Q^2 production and hence this production plan cannot be separated from the consumption decision of the household as seen above. Therefore, one may have to estimate the system of reduced-form simultaneous equations based on ad-hoc specifications.

4.4 An Overview of the Three Cases

In all three cases, we have found that the agricultural household does not maximize profits in the child-specific Q^2 production, which in turn implied inseparability between the Q^2 production plan and the consumption plan of the household. It will be of great interest to estimate empirically each of the systems of reduced-form simultaneous equations and to compare the estimates related especially with such functions as the demand for and the supply of child labor not only in the adult-specific production but also in the child-specific production. By doing so, one may gain a better understanding of the effects of fertility decisions of the agricultural household on both the production and the consumption plans.

Cast as above, the household utility function is broader than the traditional "Z-goods" approach. The "Z-good" in the traditional approach is both produced and consumed, but not traded. The agricultural good, on the other hand is produced, consumed, and traded. As a result of being deprived of the objective valuation of a market price, the "Z-good" literature has not been empirically implementable.[4] In our case the assumption of market participation in both goods, the adult-produced commodity and the child-produced commodity, or Z-good, enhances both the realism of the model and makes it also operational. The important characteristic of the Z-good literature is however still maintained, viz. the Z-good production will eventually cease. This will be due to the improvement in the terms of trade between A^1 and C and also to the substitution of child labor for adult labor in the adult production function as the opportunity cost of child labor increases. In other words, both economic development and demographic development (decrease of the size of the family) will lead to the decrease in the Z-good production.

5. Summary and Conclusion

This paper has presented alternative specifications of the model of the subjective equilibrium of the agricultural household. The motivation was to cast the model in a way that could accommodate alternative demographic assumptions in the interest of studying the interactions between the economic and the demographic behavior of the household.

The incorporation of child labor into the production side of the household is the major contribution of the model. The resulting supply functions of child labor (and demand functions for leisure) make possible the unambiguous measurement of the benefits and costs of children.

An interesting model is the one that includes Z-good production which is carried out exclusively by children. There is evidence that the small animal production and the home garden-plot cultivation in the Mindanao Survey of the Philippines are precisely this type of activities. Since there exists a market for such output, although not necessarily for child labor also employed in the Z-good production function, the model can be empirically estimated.

Besides the contribution of children in production and consumption activities of the household, another demographic factor that enters the model is the composition of the family as between workers and dependents. This, however, enters as an exogenous variable, and its effects upon the equilibrium of the household can be studied only through parametric variation. The next step in the development of this project will be making the household composition an endogenous variable.

FOOTNOTES

1) See Cain and Mazumder [1980], for example . The Mindanao Survey provides additional evidence on that.

2) Unlike the "Chicago School" models of household production functions where abstract "commodities" are assumed to be produced by combining time and purchased goods and not to be sold in markets, we assume in the present model a production function in which a concrete commodity is related to the inputs and can be exchanged in market. Refer for the "Chicago School" household production models to Becker [1965], Willis [1973], Michael [1973], De Tray [1973], and Becker and Lewis [1973] to name only a few.

3) The distinction of inputs between variable and fixed is usually arbitrary depending on the time period one chooses for analysis. Considering the time period we are going to choose for the empirical estimation (one year), inputs such as labor, fertilizer, and the like are defined as variable inputs and inputs such as capital equipment and land as fixed inputs.

4) Hymer and Resnick [1969] and Barnum and Squire [1980] introduced Z-goods into the agricultural sector model and the agricultural household model, respectively. However, they did not estimate empirically the production functions of Z-goods in their models mainly because of the lack of any information on their valuation.

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