No. 95 (80-33)

Some Findings on an Empirical Aggregate Production Function with Government Capital*/

by

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October 1980

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I. Introduction

The role played by government capital (public goods or in much wider concept social overhead capital) in the private production processes has seldomly been an issue of central concern in the past empirical, if not theoretical, research.[1/] Even in a series of well-known studies on productivity-growth sources of Denison, for instance, this item is not seen in the extensive list of contributors. He neglects government capital partly because he presupposes that it yields, if any, but a minimal contribution to output-growth and partly because it is impossible for his approach to correctly assess the role of government capital since it is freely used and receives no rewards.[2/]

However, as all of us observe, most private production activities rely more or less on the services from government capital such as highways and industrial ports. Thus, despite Denison's conjecture above, we think it important for someone to conduct an empirical study to properly assess the role of government capital. We have tried it for the manufacturing sector for two countries: the United States and Japan. The present paper is a report from this study.

At the outset we make it clear that we do not pretend to have a solid theoretical basis for the present empirical study. Moreover, we have to admit that the employed approach is simple and just preliminary. Nonetheless, we think that the obtained findings are too surprising and significant to be abandoned without warning the traditional view as represented by Denison. The main findings are as follows.

In both countries, the contribution of government capital in the

private production processes turns out to be very significant and similar, although there is an apparent structural difference in the endowments of private capital and labor. The shadow returns to government capital are likely to have been imputed wholly to labor, implying that the accumulation of government capital has resulted in a trend rise in labor productivity. This is in clear contrast with the theoretical analysis of Negishi (1973) that, in a competitive situation, the contribution of government capital is imputed wholly to private capital when goverment capital is the unpaid factor of production.

Section II discusses the approach taken in our empirical study, and Section III reports the findings and discusses the implication of them. The fourth and final section concludes the paper by pointing out the shortcomings of our study and suggesting the direction for the future studies on this subject.

II. Production Function with Government Capital

The employed approach is a simple one; we try to estimate a production function including government capital as the factor of production in addition to private capital and labor. We consider *a priori* the Cobb-Douglas type production function:[3/]

$$y_t = A_t K_t^{\alpha} G_t^{\beta} L_t^{\gamma}, \qquad (1)$$

where y_t = output, K_t = services of private capital, G_t = services of government capital, L_t = laobr, and A_t = adjusting factor of demensions which may also embody technical progresses as a function of time t.

The elasticities α, β, and γ are nonnegative constants.

Meade (1952) and Negishi classify that there are two types of public goods or, to use our terminology, the services derived from government capital. One is that of "unpaid factors of production" such as the free transportation services of highways; and the other is that of "creation of atmosphere" such as government research activities and the administrative services that promote private production activities. Accordingly, in a competitive situation, production function (1) will be homogeneous of degree one with respect to K_t, G_t, and L_t,

$$\alpha + \beta + \gamma = 1, \tag{2}$$

when government capital is in the case of the unpaid factor; and it is so with respect to K_t and L_t,

$$\alpha + \gamma = 1, \tag{3}$$

in the case of creation of atmosphere.

Although the above is a crucial theoretical difference, we do not a priori presume which of the two, i.e., (2) and (3), holds for the manufacturing-sector production functions of the United States and Japan. Instead we shall leave it unsettled until the empirical analysis indicates one, rather than the other, or none.

Without government capital, production functions are usually estimated under the assumption of the marginal productivity principle (hereafter MPP); the demand for each input is determined at that level which equates the marginal productivity to cost or the price of factor input. If one faithfully follows this approach, a production

function has to be estimated as but one equation comprising a simultaneous-equations system. Otherwise, the estimators are generally biased.

In the case of equation (1), however, we shall not follow this approach. This is because we cannot simply assume that the MPP holds when government capital enters the private production function and when it is of the unpaid factor of production.

Without the MPP, then we lack for a mechanism which simultaneously determines the employment of each input and the production of output. This prevents us from estimating (1) within a simultaneous-equations framework since we have for the moment no alternative theory. Although we shall propose a mechanism of the simultaneous determination of y_t, K_t, G_t, L_t, and some other endogenous variables after the empirical study, equation (1) will have to be estimated at this stage by applying a single-equation estimation technique.

In order to justify the single-equation formulation of production function, we shall regard equation (1) as describing a purely technical relationship or a so-called engineering production function. This allows us to interprete K_t, G_t, and L_t as exogenous variables; and then the estimators will be unbiased and consistent even by a single-equation estimation technique.

The actual estimation is based on the natural-logarithmic equivalent of (1):

$$\ln y_t = \ln A_t + \alpha \ln K_t + \beta \ln G_t + \gamma \ln L_t,$$

where

$$\ln A_t = \text{const.} + \delta t + u_t;$$

and the error term, u_t, follows the first-order autocorrelation,

$$u_t = \rho u_{t-1} + v_t$$

with $|\rho| < 1$, $Ev_t = 0$, and $Ev_t^2 = \sigma^2$.

III. The Results and Implication

As we have already mentioned, our study is based on the manufacturing sector in two countries: the United States and Japan. We utilized annual data and the sample period was chosen to be 1948-77 for the United States and 1957-77 for Japan. These sample periods were constrained by the availability of the data on capacity-utilization rate, which is necessary to construct the data on the services of capital from the stocks of them, in both countries. The time series data on both private and government capitals were not available and had to be constructed either partially or for the entire period for both countries. The procedure of the construction of these date, together with the choice and explanation of the other data, are summarized in appendix. All the variables measured in prices are in real terms.

The estimation was executed by employing the maximum-likelihood iterative regression technique in order to increase efficiency in the presence of serial correlation in the error term. The estimation results are summarized in table 1.

Table 1. Estimation Results

	USA	JAPAN
$\hat{\alpha}$ (K)	.186 (.233)	.470 (.145)
$\hat{\beta}$ (G)	.302 (.163)	.301 (.149)
$\hat{\gamma}$ (L)	.542 (.271)	.269 (.169)
$\hat{\delta}$ (t)	.012 (.006)	-.012 (.012)
$\hat{\alpha}+\hat{\beta}+\hat{\gamma}$	1.030 (.073)	1.041 (.134)
period	1948 - 77	1957 - 77
R^2	.997	.989
$\hat{\rho}$	.686 (.018)	.733 (.022)

a/. The numbers in parentheses are standard errors.

b/. See appendix for data sources.

The regressions indicate quite high R^2's for both the United States and Japan, implying that omitted variables, if any, are not so important. Although the time trend estimate indicates the counter-intuitive negative sign in the case of Japan, it is not significant in any way.[4/] All the other estimates show the correct sign as expected in the formulation of production function (1), and the standard errors are reasonably small in comparison to the corresponding mean estimates except for the estimate of the elasticity of private capital, α, for the United States.

From the obtained results, we can observe three surprising findings.

First, the contribution of government capital in the private production processes is by no means negligible; the estimates of β amount to 0.3 with reasonable standard errors for both the United States and Japan. It is interesting to note that the similarity of the estimates of β in the two countries is seen when the estimates of α and γ are quite different. However, it is not clear form the present analysis alone whether this similarity is just coincidental or not. The relative magnitudes of the estimates of α and γ seem to be plausible for both countries in the sense that they may reflect the relative scarcity in the endowments of private capital and labor.

Second, the sum of α, β, and γ almost equals one both in the United States and in Japan. This implies that the production function of each country exhibits homogeneity of degree one or constant returns to scale with respect to <u>three</u> factors of production: services of private capital, services of government capital, and labor. In fact, the null hypothesis of (3) that $\alpha + \gamma = 1$ can be rejected with

considerably small significance levels for the two countries while that of (2) that $\alpha + \beta + \gamma = 1$ cannot be rejected at a standard significance level.[5/] This indicates that the services of government capital to the manufacturing sector is of the unpaid factor type both in the United States and in Japan.[6/]

Third and what is most interesting, the estimates of α in both countries are almost equal to the actual share of private capital, s_K, in manufacturing sector.[7/] For the United States, the computed s_K, for the period 1948-76 is almost stable and equals on average 0.2. And for Japan, the manufacturing-sector s_K is reported to be on average 0.64 for the period 1952-62 [Watanabe and Egaitsu (1967)] and 0.57 for 1960-71 [Shinohara and Asakawa (1974)].[8/] Moreover, there is a tendency that s_K has been decreasing. These results for the two countries suggest that private capital receives its own contribution in production processes as indicated by the MPP. This is because the MPP yields $s_K = \alpha$ for the Cobb-Douglas type production function (1) with constant returns to scale. This in turn implies that almost all the contribution by government capital is paid to labor. In other words, the accumulation of government capital has resulted in an observed trend rise in labor productivity, keeping the productivity of private capital unaffected.

This last implication is the opposite of the theoretical analysis of Negishi that, in a competitive situation, all the contribution of government capital is imputed to private capital when government capital is the unpaid factor of production. Then, there needs some explanation to reconcile the theory and empirical results. Although one can argue that the fact that the present results are obtained by a single-equation regression technique should reduce such necessity and the mere abandonment of the assumption on the perfect competition should reconcile the theory and reality, we shall not take such interpretation. We shall instead offer an alternative mechanism which simulatneously determines the employment of each input and the production of output along the line suggested by our empirical study.

The alternative theory we propose is the following. Let the production function be written more generally than (1) as

$$y_t = F(K_t, G_t, L_t). \tag{4}$$

Let p_t, r_t, and w_t denote the prices of, respectively, output, private capital, and labor. Then the MPP for private capital yields

$$p_t F_K(K_t, G_t, L_t) = r_t. \tag{5}$$

And the determination of the labor share yields

$$w_t L_t = p_t y_t - r_t K_t. \tag{6}$$

To begin with, we suppose that the demand for government capital is constrained by the predetermined supply of it at each period, i.e., we have

$$G_t = \bar{G}_t.$$

Then, equations (4), (5), and (6) are sufficient to simultaneously determine y_t, K_t, and L_t as functions of p_t, r_t, w_t, and $\bar{G}_t$. Once one moves, as the next step, to the general equilibrium framework in order to determine the prices of output and factor inputs (and possibly the supply of government capital) relying on the equilibrium conditions of relevant markets, one is now situated in a simultaneous-equations system.

IV. Concluding Remarks

In this paper, we have reported the empirical findings on the role of government capital in the private production activities. The obtained results indicate that, on the contrary to the traditional presupposition, the contribution of government capital--which is likely to have been imputed wholly to labor--amounts to about one third of the manufacturing-sector GNP or GDP both in the United States and in Japan. This may shed a new light on the sources of productivity growth left in large part unexplained by Denison (1979) for the United States and by Denison and Chung (1976) for Japan.

The employed approach may be subject to various criticisms: that it lacks for solid theoretical basis and motivation; that it suffers from the statistical problems of multicollinearity of regressors and of simultaneous-equations bias; that it is vulnerable to possible errors in data mining; and so on. However, we think that these shortcomings

are admittable for the first, heuristic investigation especially when we put much empahsis on the reassessment of the role of government capital. Therefore, we shall be content when a series of more satisfactory studies on this subject are carried out, no matter whether they endorse our findings reported here or they end up with contradicting them.

There are suggestions for the future research. First, instead of running regressions without theory, a nested model such as the one suggested in the last part of Seciton III should first be presented and the findings of this paper should be tested by utilizing the most satisfactory statistical techniques. Although the present paper incidentally conducted statistical tests against the theory of Negishi, they are not satisfactory owing to the reliance on the single-equation estimation technique. Second, similar studies should be pursued for countries other than the United States and Japan. Then the puzzle left unanswered that the value of the elasticity of government capital, β, or the shadow income share of government capital equals about 0.3 across countries can be checked. This investigation is important because it offers policy recommendations not only on the problems of functional income distribution between private capital and labor but also on productivity growth in developing as well as developed countries.

Footnotes

1/. There have been a number of theoretical studies on the role of public goods or government capital on the production activities. They have mainly dealt with the efficient resource-allocation problems of public goods. Negishi (1973) discusses the theoretical issues to which we shall frequently refer in this paper.

2/. See, for example, Denison (1967), pp. 135-7.

3/. Recent empirical studies on production functions take the approach that does not a priori specify the fucntional forms but begins with a general specification such as the translog production function [Christensen et al (1971)]. However, we cast doubt on the desirability of such a step when the available sample data are relatively small, as is the case with the present study.

4/. See footnote 6 below.

5/. The F-statistics to the null hypothesis (3) are $F(1,25) = 2.00$ (16.6%) for the United States and $F(1,16) = 4.93$ (3.9%) for Japan; and to the null hypothesis (2) are $F(1,25) = .17$ (68.7%) for the United States and $F(1,16) = .09$ (75.9%) for Japan. The percentage values in parentheses are significance levels.

6/. That the null hypothesis (2) is hardly rejected weakens the following criticism: "Government capital turns out to be significant simply because the time series of it substitutes for that of some other variables, such as the technical progresses, which are important in the production function. For instance, the insignificant estimate of the time-trend term in Japan reflects the problem of multicollinearity

between government capital and time trend." We shall answer to this by only pointing out that a mere chance can hardly bring about (2).

7/. The following s_K, which is usually computed as the remainder of labor share, is before-tax based. Therefore, the criticism which may arise that the services of government capital are paid in the form of taxes is irrelevant.

8/. We could not directly compute the manufacturing-sector s_K in the case of Japan, since some necessary data in doing so were available only in the form of index.

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Data Appendix

All the data except for the stock of government capital refer to the manufacturing sector both in the United States and in Japan. The stock of government capital applies to the whole economy. We shall define the following relations which will soon become obvious of themselves:

$$K_t = KS_t \times CU_t,$$

$$G_t = GS_t \times CU_t,$$

$$L_t = N_t \times H_t.$$

The United States

All the variables measured in prices are or converted to be at 1972 billion $. The data are collected from tables in Economic Report of the President (Washington D. C., 1979) unless otherwise noted.

y_t = real GNP (table B-5);

CU_t = Federal Reserve measures of capacity-utilization rate (table B-42);

KS_t = net stocks of fixed nonresidential business capital (Survey of Current Business, September 1978 and other issues cited therein);

GS_t = net nonresidential government structures [Goldsmith (A138 : 1947-51) and Tice and Duff (A139 : 1952-68) estimates in Long Term Economic Growth, 1860-1970

(Washington D. C.: US Bureau of Economic Analysis, 1973) are converted to 1972 prices. Other years (1967-77) are constructed by the formula $GS_t = (1 - \mu)GS_{t-1} +$ (gross public construction: table B-43)$_{t-1}$ with the depreciation rate $\mu = .02$. This particular μ is chosen because with it the known past data are best extrapolated.];

N_t = number of wage and salary workers (table B-34);

H_t = average yearly working hour [from average weekly working hour (table B-35)].

Japan

All the variables measured in prices are or converted to be at 1970 prices, then they are in turn converted to indices (1970 = 100) in order to maintain consistency since some data are available only in the form of index. The data are collected from various issues of cited data sources.

y_t = real GDP (National Income Statistics : Economic Planning Agency);

CU_t = capacity-utilization rate (Tsūsan Tōkei : Ministry of International Trade and Industry);

KS_t = net private capital stock [The formula $KS_t = (1 - \mu)KS_{t-1}$ + (gross investment)$_{t-1}$ is applied by making the net fixed capital stock at 1970 (National Wealth Survey: Statistics Bureau Prime Minister's Office) as the bench mark. The time series on gross investment is constructed from that

of gross capital stock estimated by Economic Planning Agency. The particular depreciation rate $\mu = .115$ is chosen to meet the other bench-mark net fixed capital stock by construction and by National Wealth Survey at 1955];

GS_t = net nonresidential government capital stock [This is constructed in the same way as KS_t except that $\mu = .05$ is used by the same reason as above and that nonresidential gross government investment from National Income Statistics is used.];

N_t = number of regular workers (Maigetsu Kinrō Tōkei Chōsa Sōgō Hōkoku Sho: Ministry of Labor);

H_t = monthly hours worked of regular workers [The same source as N_t.].

Notes

Both private and government capitals are net stocks, although output is in gross terms. This seems to be a correct choice of variables in production functions.

The capacity-utilization rate of private capital is employed to construct the services of government capital from the stock of it. Although this may be a problematic procedure, we could find no alternative way.