Performance Analysis of the IEEE 1394 Serial Bus

Takashi Norimatsu,* Hideaki Takagi,† and H. Richard Gail††

*Doctoral Program in Policy and Planning Sciences, University of Tsukuba

†Institute of Policy and Planning Sciences, University of Tsukuba
1-1-1 Tennoudai, Tsukuba-shi, Ibaraki 305-8573 Japan
E-mail: takagi@shako.sk.tsukuba.ac.jp

††IBM Thomas J. Watson Research Center

Abstract

IEEE 1394 is a standard for a high performance serial bus interface. It encompasses both isochronous transfer mode, which is suitable for real-time applications, and asynchronous transfer mode, which is appropriate for delay-insensitive applications. This standard can be used as a basis for constructing a small-size local area network. Two queueing models are proposed for a network operating under the IEEE 1394 specification. The average waiting time in steady state of an asynchronous packet is calculated, and the effect on it due to isochronous traffic is studied. Numerical results from both analysis and simulation are presented in order to evaluate the performance of such a system.

Keywords: IEEE 1394; Isochronous transfer mode; Asynchronous transfer mode; Performance evaluation; Markov chain; Waiting time

1 Introduction

The IEEE 1394 specification for a high performance serial bus interface describes a method by which computers and I/O equipment can be interconnected [1, 3, 12, 13, 17, 18]. This was standardized by the IEEE in 1995 based on the specification of a bus called *FireWire* that had been developed by Apple as a promising alternative to the Small Computer System Interface (SCSI). It can be used for a bridge bus, but also when interconnecting personal computers, peripheral devices, video decks and digital video cameras. When the IEEE 1394 standard is used to interconnect several devices, the topology of the resulting network may be in the form of a daisy-chain or a tree. However, it can be viewed as a bus on a transport layer. Such a system will be called an IEEE 1394 network.

The IEEE 1394 specification has some useful features. An IEEE 1394 network can be reconfigured automatically by its attached devices without any intervention by the users each time the network topology changes. Thus users that are unfamiliar with communication networks can operate an IEEE 1394 network easily. In addition, IP over IEEE 1394, a technology that enables IP transmission over an IEEE 1394 network, has already been released. This technology allows the use of Internet applications over an IEEE 1394 network. However, there is a limit on the size of the network both in terms of the number of attached devices and the distance involved. An IEEE 1394 network can accommodate up to 62 attached devices, with the devices being separated by up to 72 meters. These capabilities and limitations can be used to construct a small-size local area network environment such as Small Office Home Office (SOHO) [14].

When it was standardized in 1995, the maximum transmission speed of the bus was 400 Mbps with copper wire or optical fiber. Then IEEE planned to specify a higher speed version called P1394b [11], which supports transmission speeds of 800 Mbps, 1.6 Gbps, and 3.2 Gbps with optical fiber. This version enhances the current IEEE 1394 standard to a large-size local area network for the office and other environments.

The IEEE 1394 standard has several characteristics that differ from other LAN protocols such as Ethernet and token-ring. Specifically, the following two properties related to the performance of this bus are of particular interest. The first is that the IEEE 1394 specification uses an *arbitration method* for medium access control. This arbitration method is of centralized type, and there exists a special node that controls the access to the bus by all nodes in the network.

The second is that the IEEE 1394 standard specifies two kinds of data transfer modes, namely, *isochronous transfer mode* (ITM) and *asynchronous transfer mode* (ATM). The IEEE 1394 standard essentially guarantees periodic data transmissions in ITM, and so ITM is suitable for real-time applications. On the other hand, a node with permission to send an ATM packet can do so when the bus is free. However, the node that wants to transmit a packet in ATM must defer to other nodes transmitting ITM packets. Thus the transmission of an ITM packet has priority over that in ATM. In the IEEE 1394 specification, time is divided into fixed-size frames called *cycles*, each of which has a duration of 125 μseconds. At most 80% of a cycle is available for transmission of packets in ITM, while the rest of the cycle is available for ATM packets. During the ATM part of the cycle, any node that wants to transmit a packet in ITM must defer to the nodes transmitting packets in ATM. The transmission capacity for ITM packets during a cycle is independent of the ATM traffic load, but that of transmitting ATM packets in a cycle depends on the ITM traffic load. Because of this asymmetry, the performance of the bus while in ATM is affected by the traffic conditions in ITM.

The system we analyze in this paper has some similarity to an integrated circuit and packet switching facility, since its transmission capacity is also shared by two types of traffic. To implement this facility, a synchronous time-division multiplexed frame structure is used. Time is divided into frames of fixed duration, and each frame is subdivided into slots. The frame is partitioned into two regions by a boundary, with one region assigned to circuit switching type data such as voice calls, while the other region is devoted to packet switching type data. The boundary may be fixed or movable, and the use of a movable boundary leads to dynamic resource allocation for the two different types of traffic in terms of slots. Such an integrated transmission system with a movable boundary has been studied in [2, 4, 8, 9, 16, 19, 20, 21, 22, 24].

From a theoretical point of view, the transmission system analyzed in this paper can be thought of as one with both random inputs and scheduled periodic inputs. A system with scheduled secondary inputs was analyzed in [20]. However, the approach proposed in [20] cannot deal with dynamic ITM traffic, because it does not distinguish between ITM packets and ATM packets in the queue. An approximate analysis of such a system was presented in [16] and [8] by using fluid models. The model studied in [9] considers every slot in the frame as homogeneous, by assuming that the average fraction of slots available for packet transmission per frame is the probability that packets are transmitted in the current slot. The analytical models in [4] and [22] assume that the packets arriving in a frame are not transmitted in the current frame. In [2] and [24] extensive results for the hybrid model are presented based on simulation, with no analytical results. An analytical model considered in [19] assumes variable length frames.

The dynamic ITM traffic model analyzed in Section 4 yields a Markov chain that takes the same form as one studied in [15]. However, as pointed out in [5], the analysis of the chain in [15] is incorrect. Furthermore, in [5] a model that does not allow arrivals in a frame to be transmitted in the same frame was considered. The analysis in this paper of a model that does allow such transmissions follows the approach of [5].

The purpose of this paper is to study the performance of the bus in terms of the average waiting time of an arbitrary asynchronous packet in steady state. Here the "waiting time" of a packet means the interval beginning at the instant when a packet arrives and ending at the instant when it leaves the system, including the time spent during transmission. The probability generating function for the number of asynchronous packets in the buffer is derived using a Markov chain that includes the overhead due to isochronous traffic. The average waiting time of an asynchronous packet is then calculated and used as a performance index of the bus. Numerical results are presented based on our analysis as well as from simulation.

The remainder of the paper is organized as follows. In Section 2 we describe how the isochronous and asynchronous transmissions are carried out. Two analytical models are then proposed in Sections 3 and 4. The level of ITM traffic is assumed to be constant in the model of Section 3, while the ITM traffic is modeled by a Markov chain in the model of Section 4. Numerical results based on these analytical models are presented in Section 5 together with simulation results. In Section 6 we summarize the results of this paper and discuss future work.

2 Media Access Control

Since the IEEE 1394 standard is a bus specification, at most one node can transmit data at a time. Therefore, as is the case for Ethernet, an IEEE 1394 network needs some form of media access control. However, unlike Ethernet, an IEEE 1394 network employs an access method called *arbitration*. Two types of arbitration are used, namely, *isochronous arbitration* and *asynchronous arbitration*. Figure 1 depicts a typical cycle involving the two data transfer modes of the IEEE 1394 specification.

2.1 Isochronous Transfer Mode (ITM)

The transmission of a packet in ITM is divided into three phases:

(1) Isochronous arbitration (arb phase);

(2) Data transfer (data phase);

(3) Isochronous gap (gap phase).

According to [18, p. 117], data transmission in ITM proceeds as follows.

(a) All nodes attached to the bus set their clock by receiving a *cycle start* (CS) packet sent by the *root*, a node that controls and manages access to the bus. The root sends a CS packet every 125 μseconds. The interval between consecutive CS packets is a *cycle*. An overall network cycle synchronization must be maintained.

(b) A node that wishes to transmit a packet sends the root a request for transmission after it detects an *isochronous gap* (IG). The IG is a state during which no signal propagates on the bus for a short time.

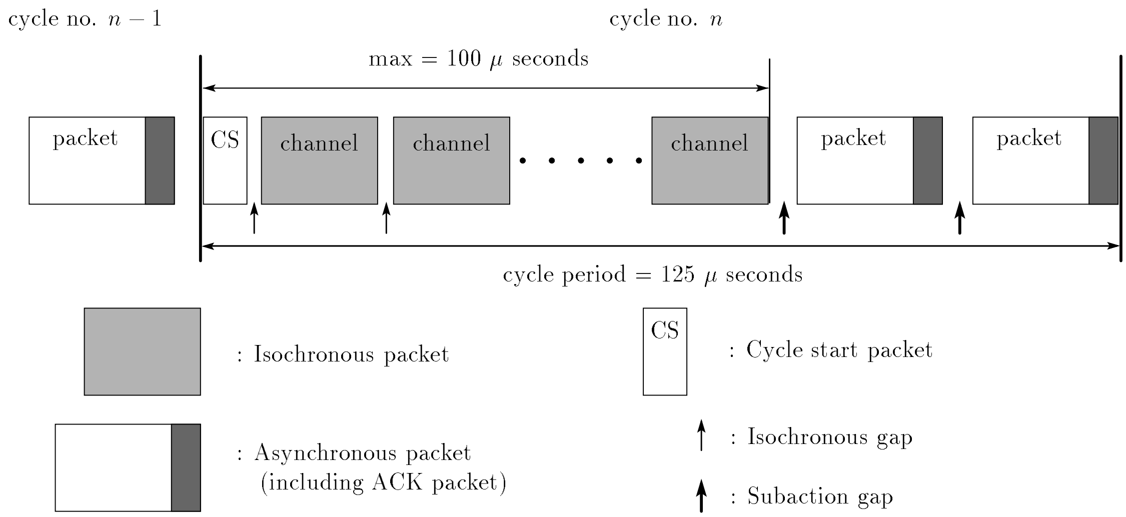

Figure 1: The organization of a cycle in IEEE 1394 [3].

(c) The root assigns a certain channel to the node that has first sent a request, and this node transmits a packet. After sending the packet it is then prohibited from transmitting again until the next cycle. Since at most 100 μseconds are available to transmit ITM packets in a cycle, the transmission request is refused if the time taken for transmitting a packet exceeds 100 μseconds.

(d) Steps (a), (b) and (c) are repeated until no nodes want to transmit data in ITM, or until 100 μseconds elapse, whichever occurs first. In ITM, an ACK is not returned. Each node releases the bus when no ITM traffic is left in its buffer. After a subaction gap (see Section 2.2), the nodes start to transmit ATM packets, if any.

2.2 Asynchronous Transfer Mode (ATM)

The transmission of a packet in ATM is divided into four phases:

(1) Asynchronous arbitration (arb phase);

(2) Data transfer (data phase);

(3) Acknowledgment (ack phase);

(4) Subaction gap or arbitration reset gap (gap phase).

According to [18, p. 116], data transmission in ATM proceeds as follows.

(a) A node that wishes to transmit a packet sends a request for transmission to the root after it detects a *subaction gap* (SG) or an *arbitration reset gap* (ARG). The SG is a state during which no signal propagates on the bus for a short time. The ARG is a state during which no signal propagates on the bus for a period that is much longer than the SG.

(b) The root allows the node that has first sent a request to transmit a single packet. This node is then prohibited from transmitting again until it detects an ARG.

(c) The node that receives a packet returns an ACK. The bus then enters the state in which no signal propagates (SG). If all nodes that want to transmit packets are prohibited from sending requests, an ARG occurs instead of an SG. Then every node can again send the root a request for another transmission.

(d) Steps (a), (b) and (c) are repeated until the end of the cycle.

The mechanism by which the transmission of a packet in ITM has priority over that in ATM is as follows. Assume that one node wishes to transmit a packet in ITM, and another node wishes to transmit a packet in ATM. At the beginning of a cycle, the root sends a CS packet, and then the bus enters a state in which no signal propagates. After a period of time both nodes identify this state as an IG, since it is shorter than an SG and an ARG. Thus the node that has a packet in ITM will transmit first.

3 Static ITM Traffic Model

Suppose that ITM traffic is generated by a real-time application such as streaming video or online meeting. Then the interarrival times and holding times for ITM traffic are much longer than those for ATM traffic. As a result, the number of slots used to transmit ITM traffic in each cycle remains constant over a number of consecutive cycles, while the ATM packet queue quickly reaches steady state. In this section we assume that it takes a fixed duration to transmit ITM traffic in every cycle and propose a so-called static ITM traffic model to study this case. For example, as discussed above this model can be used when the ITM traffic is generated by long real-time applications.

3.1 Model Description

A discrete-time queueing system is used to model the case of static ITM traffic. The modeling assumptions include the following.

(1) All ATM packets have the same constant size.

(2) A cycle is divided into N fixed-length intervals called *slots*. A slot is the time to transmit an ATM packet. All of the arbitration, data transfer, and SG or ARG occur in one slot in ATM.

(3) The number of slots used to transmit ITM traffic in a cycle is fixed at K. According to the IEEE 1394 specification, this number must satisfy $K \leq \lfloor 0.8 \times N \rfloor$, where $\lfloor x \rfloor$ is the integer part of the real number x (in particular, $K < N$).

(4) The ATM packets generated from all nodes of the network reside in a single infinite-capacity FIFO queue until transmission.

(5) The arrival stream of ATM packets in each node is assumed to be Poisson. The total number of ATM packets that arrive in a slot from all nodes has a Poisson distribution with mean λ_{ATM}. Arrivals and departures of ATM packets occur immediately before slot boundaries.

The time to transmit a CS packet at the beginning of a cycle is much smaller than a slot and is assumed to be negligible. The model does not distinguish among the nodes generating ATM packets, so the effect of arbitration for fair access is not taken into consideration. To do so would require a very large multi-dimensional Markov chain to account for individual node characteristics.

A discrete-time Markov chain will be used to analyze the number of ATM packets present in the network. We first introduce random variables to describe their queueing behavior. Let Y_n be the number of ATM packets in the buffer, including the transmitter, at the beginning of the nth cycle, and let Y_n^h, $h = 1, \ldots, N$, be the number of ATM packets in the buffer at the end of the hth slot of the nth cycle. Also, let A_n^h, $h = 1, \ldots, N$, be the number of arrivals of ATM packets from all nodes in the network in the hth slot of the nth cycle. By assumption, A_n^h are iid with a Poisson distribution with mean λ_{ATM}. A diagram illustrating Y_n and Y_n^h, $h = 1, \ldots, N$, is shown in Figure 2.

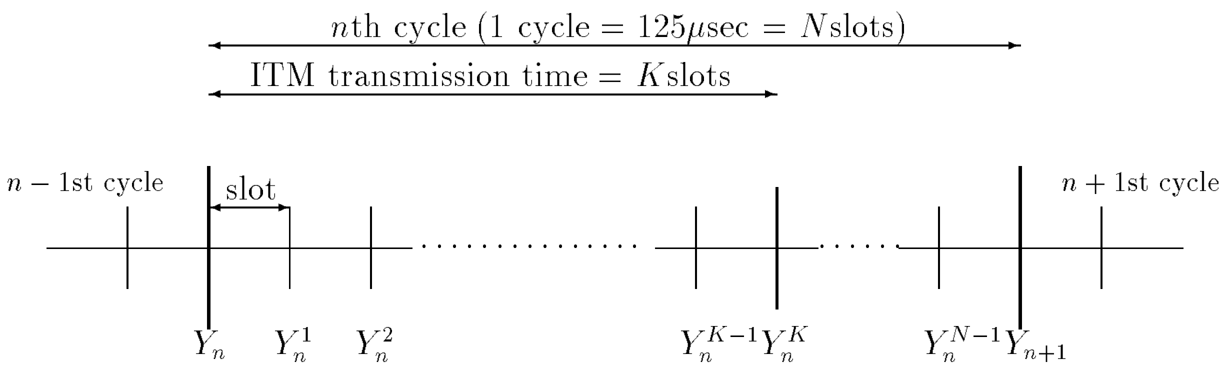

Figure 2: Random variables Y_n and Y_n^h.

These random variables satisfy

$$Y_n^h = \begin{cases} A_n^h + Y_n^{h-1}; & h = 1, \ldots, K \\ A_n^h + [Y_n^{h-1} - 1]^+; & h = K+1, \ldots, N, \end{cases} \tag{3.1}$$

where $[y]^+ := \max(0, y)$ and $Y_n^0 := Y_n$ (for the case $h = 1$). We also have

$$Y_{n+1} = Y_n^N; \qquad n = 0, 1, 2, \ldots. \tag{3.2}$$

Note that Y_n^h in (3.1) is the sum of two terms. The first term represents the ATM packets arriving in the hth slot of the nth cycle, while the second term represents the ATM packets that were in the buffer at the beginning of the hth slot and remain there at the end of this slot. The sequence $\{Y_n;\ n = 1, 2, \ldots\}$ constitutes a discrete-time Markov chain.

3.2 Analysis of the Model

We first derive the probability generating function (PGF) for the number of ATM packets in the buffer in steady state, and then we obtain the average waiting time of an ATM packet as a performance measure. Define the generating functions

$$\begin{aligned} Y_n^h(z) &:= \sum_{k=0}^{\infty} P(Y_n^h = k) z^k; \qquad h = 1, \ldots, N, \\ Y_n(z) &:= \sum_{k=0}^{\infty} P(Y_n = k) z^k = Y_n^0(z). \end{aligned} \tag{3.3}$$

Transforming (3.1) yields

$$Y_n^h(z) = \begin{cases} e^{\lambda_{\mathrm{ATM}}(z-1)} Y_n^{h-1}(z); & h = 1, \ldots, K \\ \dfrac{e^{\lambda_{\mathrm{ATM}}(z-1)}}{z} \left[Y_n^{h-1}(z) + (z-1) P(Y_n^{h-1} = 0) \right]; & h = K+1, \ldots, N. \end{cases} \tag{3.4}$$

Iterating on (3.4), we claim that $Y_n^h(z)$ can be expressed in terms of $Y_n(z)$ as

$$Y_n^h(z) = \begin{cases} e^{h\lambda_{\mathrm{ATM}}(z-1)} Y_n(z); & h = 1, \ldots, K, \\ \dfrac{1}{z^{h-K}} [e^{h\lambda_{\mathrm{ATM}}(z-1)} Y_n(z) & \\ \qquad + (z-1) \sum_{i=K}^{h-1} P(Y_n^i = 0) e^{(h-i)\lambda_{\mathrm{ATM}}(z-1)} z^{i-K}]; & h = K+1, \ldots, N. \end{cases} \tag{3.5}$$

To see this, note that the result is obvious from (3.4) for $h = 1, \ldots, K$. It remains to show (3.5) for $h = K+1, \ldots, N$, which can be done by induction on h. First, substituting the expression for $Y_n^K(z)$ from (3.5) into (3.4) for $h = K+1$ gives

$$Y_n^{K+1}(z) = \frac{1}{z} \left[e^{(K+1)\lambda_{\mathrm{ATM}}(z-1)} Y_n(z) + (z-1) P(Y_n^K = 0) e^{\lambda_{\mathrm{ATM}}(z-1)} \right],$$

which is the desired result for $Y_n^{K+1}(z)$. The general case is shown in the same manner. Using the induction hypothesis, substitute the expression for $Y_n^h(z)$ from (3.5) into (3.4). Then a straightforward calculation shows that $Y_n^{h+1}(z)$ also satisfies (3.5).

Using (3.5) with $h = N$ and (3.2), the PGF for Y_{n+1} is given by

$$Y_{n+1}(z) = \frac{1}{z^{N-K}} \left[e^{N\lambda_{\mathrm{ATM}}(z-1)} Y_n(z) + (z-1) \sum_{i=K}^{N-1} P(Y_n^i = 0) e^{(N-i)\lambda_{\mathrm{ATM}}(z-1)} z^{i-K} \right]. \tag{3.6}$$

Assume that the system enters steady state as $n \to \infty$. Then $Y(z) := \lim_{n\to\infty} Y_n(z)$ is the PGF for the number of ATM packets in the buffer at the beginning of a cycle in steady state, while $P^i(k) := \lim_{n\to\infty} P(Y_n^i = k)$, $k = 0, 1, 2, \ldots$, is the steady state probability that there are k ATM packets in the buffer at the end of the ith slot. With these definitions, letting $n \to \infty$ in (3.6) yields

$$Y(z) = \frac{(z-1) e^{(N-K)\lambda_{\mathrm{ATM}}(z-1)} \sum_{i=K}^{N-1} P^i(0) \left(z e^{-\lambda_{\mathrm{ATM}}(z-1)} \right)^{i-K}}{z^{N-K} - e^{N\lambda_{\mathrm{ATM}}(z-1)}}. \tag{3.7}$$

The numerator of $Y(z)$ in (3.7) includes $N-K$ unknowns $\{P^i(0); i = K, \ldots, N-1\}$, which can be determined by solving a set of $N-K-1$ linear equations obtained from zeros of the denominator in the open unit disk together with the condition $Y(1) = 1$. Using L'Hospital's rule, this normalizing condition is simply

$$\sum_{i=K}^{N-1} P^i(0) = N - K - N\lambda_{\mathrm{ATM}}, \tag{3.8}$$

which equates two expressions for the mean number of idle slots in a cycle. Since the sum in (3.8) must be positive (otherwise $Y(z) \equiv 0$), we must have

$$N\lambda_{\mathrm{ATM}} < N - K. \tag{3.9}$$

A sufficient condition for the stability of the ATM packet queue is given by (3.9), which states that the mean number of ATM packets that arrive in a cycle is smaller than the number of slots available for transmitting them in a cycle.

Now let us consider the zeros of the denominator of $Y(z)$ in (3.7), or, equivalently, the zeros of the function

$$F(z) := z^{N-K} - e^{N\lambda_{\mathrm{ATM}}(z-1)}. \tag{3.10}$$

We apply Rouché's theorem to find the number of zeros of $F(z)$ in the closed unit disk and then apply Lagrange's theorem to compute the zeros explicitly.

Theorem 1 *If $N\lambda_{\mathrm{ATM}} < N - K$, then $F(z)$ has $N-K$ zeros in the closed unit disk, and all of them are simple zeros. Specifically, $F(z)$ has $N-K-1$ (simple) zeros in the punctured disk $\{z : 0 < |z| < 1\}$ and one zero on the unit circle (a simple zero at $z = 1$).*

Proof. We write $F(z) = f(z) + g(z)$, where

$$f(z) = z^{N-K}; \qquad g(z) = -e^{N\lambda_{\mathrm{ATM}}(z-1)}.$$

Note that $f(z)$ and $g(z)$ are entire functions (analytic in the entire complex plane). For $\epsilon > 0$, on the circle $|z| = 1+\epsilon$ we have

$$|f(z)| = (1+\epsilon)^{N-K}; \qquad |g(z)| \le e^{N\lambda_{\mathrm{ATM}}\epsilon}.$$

The function $h(\epsilon) := (1+\epsilon)^{N-K} - e^{N\lambda_{\mathrm{ATM}}\epsilon}$ satisfies $h(0) = 0$ and $h'(0) = N - K - N\lambda_{\mathrm{ATM}}$. Since (3.9) holds, $h'(0) > 0$ and so $|f(z)| - |g(z)| \ge h(\epsilon) > 0$ for small $\epsilon > 0$. Clearly, $f(z)$ has $N-K$ zeros in $|z| < 1+\epsilon$. By Rouché's Theorem [23, p. 116], $F(z)$ also has $N-K$ zeros in $|z| < 1+\epsilon$. Letting $\epsilon \to 0$, we see that $F(z)$ has $N-K$ zeros in $|z| \le 1$.

Let $\widehat{z}$ be a zero of $F(z)$ in the closed unit disk, so that

$$\widehat{z}^{N-K} = e^{N\lambda_{\mathrm{ATM}}(\widehat{z}-1)}. \tag{3.11}$$

Clearly, this implies that $\widehat{z} \ne 0$. If $\widehat{z}$ is not a simple zero, then it also satisfies the equation

$$(N-K)\widehat{z}^{N-K-1} = N\lambda_{\mathrm{ATM}} e^{N\lambda_{\mathrm{ATM}}(\widehat{z}-1)}. \tag{3.12}$$

Since $\widehat{z} \ne 0$, the only solution of (3.11) and (3.12) is $\widehat{z} = (N-K)/N\lambda_{\mathrm{ATM}} > 1$, a contradiction. Thus all zeros in the closed unit disk are simple.

We now show that $z = 1$ is the only zero of $F(z)$ in (3.10) on $|z| = 1$. To do so, suppose that $\widehat{z}$ is a zero of $F(z)$ such that $|\widehat{z}| = 1$. Then $\widehat{z}$ satisfies (3.11), and taking absolute values gives $1 = |e^{N\lambda_{\mathrm{ATM}}(\widehat{z}-1)}| = e^{N\lambda_{\mathrm{ATM}}(\widehat{x}-1)}$, where $\widehat{x} := \Re(\widehat{z})$. This implies that $\widehat{x} = 1$, and so $\widehat{z} = 1$. $\square$

Let $\omega = e^{2\pi \mathrm{i}/(N-K)}$ be a primitive $(N-K)$th root of unity, where $\mathrm{i} := \sqrt{-1}$. For $k = 0, \ldots, N-K-1$, define the entire function

$$F_k(z) := z - \omega^k e^{N\lambda_{\mathrm{ATM}}(z-1)/(N-K)}.$$

Using the argument in the proof of Theorem 1, each $F_k(z)$ has exactly one zero in the closed unit disk, which we denote by z_k. In particular, we have $z_0 = 1$. Note that each z_k is also a zero of $F(z)$. To see that these zeros are distinct, suppose that $z_k = z_l$ where $l \le k$. Then

$$\omega^k = z_k e^{-N\lambda_{\mathrm{ATM}}(z_k-1)/(N-K)} = z_l e^{-N\lambda_{\mathrm{ATM}}(z_l-1)/(N-K)} = \omega^l,$$

and so $\omega^{k-l} = 1$. It follows that $k - l$ is a multiple of $N - K$, and so we must have $k = l$. Thus the $N - K$ points z_k, $k = 0, \ldots, N - K - 1$, are the complete set of simple zeros of $F(z)$ in the closed unit disk.

From the numerator of $Y(z)$ in (3.7), the equation that results from the zero z_k in the open unit disk is

$$\sum_{i=K}^{N-1} P^i(0)\left(z_k e^{-\lambda_{\mathrm{ATM}}(z_k-1)}\right)^{i-K} = 0; \qquad k = 1, \ldots, N-K-1. \tag{3.13}$$

The $N - K$ equations from (3.8) and (3.13) may be written in matrix form as

$$\mathbf{pV} = \mathbf{u},$$

where $\mathbf{p} := [P^K(0), \ldots, P^{N-1}(0)]$ and $\mathbf{u} := [N - K - N\lambda_{\mathrm{ATM}}, 0, \ldots, 0]$ are row vectors, and

$$\mathbf{V} := \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 1 & z_1 e^{-\lambda_{\mathrm{ATM}}(z_1-1)} & \cdots & z_{N-K-1} e^{-\lambda_{\mathrm{ATM}}(z_{N-K-1}-1)} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \left(z_1 e^{-\lambda_{\mathrm{ATM}}(z_1-1)}\right)^{N-K-1} & \cdots & \left(z_{N-K-1} e^{-\lambda_{\mathrm{ATM}}(z_{N-K-1}-1)}\right)^{N-K-1} \end{bmatrix}. \tag{3.14}$$

Note that $\mathbf{V}$ is a Vandermonde matrix [10, p. 29]. If $z_k e^{-\lambda_{\mathrm{ATM}}(z_k-1)} = z_l e^{-\lambda_{\mathrm{ATM}}(z_l-1)}$, then raising this equation to the $(N-K)$th power and using the fact that both z_k and z_l are zeros of $F(z)$ yields $e^{K\lambda_{\mathrm{ATM}}(z_k-1)} = e^{K\lambda_{\mathrm{ATM}}(z_l-1)}$. From this it follows that $z_k = z_l$, and so $k = l$. Thus the quantities $z_k e^{-\lambda_{\mathrm{ATM}}(z_k-1)}$ in $\mathbf{V}$ are distinct, which implies that the determinant of this Vandermonde matrix is not zero, i.e., $\mathbf{V}$ is nonsingular. In particular, this gives a direct proof that the $N - K$ equations from (3.8) and (3.13) are linearly independent and the $N-K$ unknowns $\{P^i(0); i = K, \ldots, N-1\}$ are uniquely determined.

Recalling that $z_k = \omega^k e^{N\lambda_{\mathrm{ATM}}(z_k-1)/(N-K)}$, an explicit expression for z_k may be obtained from Lagrange's Theorem [25, p. 132]. In our case, we have

$$z_k = \sum_{n=1}^{\infty} \frac{(\omega^k)^n}{n!} \frac{d^{n-1}}{dz^{n-1}} \left(e^{N\lambda_{\mathrm{ATM}}(z-1)/(N-K)}\right)^n \Bigg|_{z=0},$$

or, explicitly,

$$z_k = \sum_{n=1}^{\infty} \frac{\omega^{kn}}{n!} \left(\frac{nN\lambda_{\mathrm{ATM}}}{N-K}\right)^{n-1} e^{-nN\lambda_{\mathrm{ATM}}/(N-K)}; \qquad k = 1, \ldots, N-K-1. \tag{3.15}$$

We are now able to calculate the average waiting time of an arbitrary ATM packet in steady state. Let $\overline{Y}$ be the mean number of ATM packets in the buffer at the beginning of a cycle in steady state. Differentiating $Y(z)$ in (3.7) with respect to z and then evaluating the result at $z = 1$ yields

$$\overline{Y} = \frac{\sum_{i=K}^{N-1} P^i(0)\left[(N-i)\lambda_{\mathrm{ATM}} + i - K\right]}{N - K - N\lambda_{\mathrm{ATM}}} - \frac{\left[(N-K)(N-K-1) - N^2\lambda_{\mathrm{ATM}}^2\right]}{2(N-K-N\lambda_{\mathrm{ATM}})}. \tag{3.16}$$

Let $\overline{Y^h}$, $h = 1, \ldots, N$, be the mean number of ATM packets in the buffer at the end of the hth slot of a cycle in steady state, and set $\overline{Y^0} = \overline{Y}$. From (3.1), the expectations $\overline{Y^h}$ satisfy the recurrence relations

$$\overline{Y^h} = \begin{cases} \lambda_{\mathrm{ATM}} + \overline{Y^{h-1}}; & h = 1, \ldots, K \\ \lambda_{\mathrm{ATM}} + \overline{Y^{h-1}} - [1 - P^{h-1}(0)]; & h = K+1, \ldots, N. \end{cases} \tag{3.17}$$

Iterating on (3.17) yields

$$\overline{Y^h} = \begin{cases} h\lambda_{\text{ATM}} + \overline{Y}; & h = 1, \ldots, K \\ h\lambda_{\text{ATM}} + \overline{Y} - \sum_{i=K}^{h-1} [1 - P^i(0)]; & h = K+1, \ldots, N. \end{cases} \tag{3.18}$$

Setting $h = N$ in (3.18) and using $\overline{Y^N} = \overline{Y}$ gives another way of obtaining the normalizing condition (3.8).

Let ξ be the number of ATM packets in the buffer at the end of (or, equivalently, the beginning of) an arbitrary slot, and let $\overline{\xi}$ be its expectation, i.e., $\overline{\xi}$ is the mean number of ATM packets over all slot boundaries. Using renewal theoretic reasoning, we see that

$$P(\xi = k) = \frac{1}{N} \sum_{h=1}^{N} P^h(k).$$

Since $\overline{Y^h}$ is the expectation of the distribution $\{P^h(k); k = 0, 1, 2, \ldots\}$ with $\overline{Y^N} = \overline{Y}$, we have

$$\overline{\xi} = \frac{1}{N} \sum_{h=1}^{N} \overline{Y^h} = \frac{1}{N} \left[\overline{Y} + \sum_{h=1}^{N-1} \overline{Y^h} \right].$$

It thus follows from Little's theorem that the average waiting time $\overline{W}$ of an ATM packet in steady state is given by

$$\overline{W} = \frac{\overline{\xi}}{\lambda_{\text{ATM}}} = \frac{1}{N\lambda_{\text{ATM}}} \left[\overline{Y} + \sum_{h=1}^{N-1} \overline{Y^h} \right]. \tag{3.19}$$

Using (3.18) for $\overline{Y^h}$, we obtain

$$\overline{W} = \frac{N-1}{2} + \frac{\overline{Y}}{\lambda_{\text{ATM}}} - \frac{1}{N\lambda_{\text{ATM}}} \sum_{i=K}^{N-2} (N-1-i)[1 - P^i(0)], \tag{3.20}$$

where $\overline{Y}$ is given by (3.16).

4 Dynamic ITM Traffic Model

We now proceed to present an analytical model that incorporates dynamic ITM traffic. That is, we assume that the time devoted to transmitting ITM traffic varies probabilistically from cycle to cycle. This model may be applied to the case for which both ITM and ATM traffic requests have similar time scales.

4.1 ITM Modeling and Analysis

We first consider a Markov chain that describes the number of slots used by ITM traffic in successive cycles. Another Markov chain is then constructed that corresponds to the queueing behavior of ATM packets. Modeling assumptions for the dynamic ITM traffic model are the same as for the static ITM traffic model described in Section 3, except for the following ITM traffic assumptions.

(1) The arrival stream of transmission requests for ITM traffic from each node is assumed to be Poisson. Thus the aggregated stream of requests over the entire network forms a Poisson process with rate, say, λ_{ITM} (/slot).

(2) The holding time of each ITM transmission process is assumed to be exponentially distributed with mean $1/\mu_{\mathrm{ITM}}$ slots.

(3) At most $M = \lfloor 0.8 \times N \rfloor$ slots in a cycle are reserved for ITM traffic (in particular, $M < N$). Any ITM requests that arrive when all of the M slots are already occupied by ITM traffic are lost (*loss model*).

(4) The number of slots occupied by ITM traffic in each cycle is determined at the beginning of that cycle, and it only depends on the events that occur in the immediately preceding cycle.

We begin by studying the behavior of ITM traffic in successive cycles. Let X_n be the number of slots used to send ITM traffic in the nth cycle. Since the holding time for an ITM transmission process is exponentially distributed and all ITM sources have the same characteristics, then the probability that an individual ITM transmission terminates is identical for each cycle and (due to the assumption that the ITM traffic arrives at constant rate over the cycle) is given by

$$p = 1 - e^{-N\mu_{\mathrm{ITM}}}.$$

It is clear that $\{X_n; n = 1, 2, \ldots\}$ forms a Markov chain.

According to the assumptions of the loss model described above, the state transition probabilities $q_{ij} := P(X_n = j | X_{n-1} = i)$, $i = 0, \ldots, M$, $j = 0, \ldots, M$, are found to be

$$q_{ij} = \begin{cases} \displaystyle\sum_{k=[i-j]^+}^{i} B(i,k)C(j+k-i); & j = 0, \ldots, M-1 \\ \displaystyle\sum_{k=0}^{i} B(i,k) \sum_{l=M+k-i}^{\infty} C(l); & j = M. \end{cases} \tag{4.1}$$

Here we have defined the Bernoulli probabilities

$$B(i,k) := \binom{i}{k} p^k (1-p)^{i-k} \tag{4.2}$$

and the Poisson probabilities

$$C(k) := \frac{(N\lambda_{\mathrm{ITM}})^k}{k!} e^{-N\lambda_{\mathrm{ITM}}}. \tag{4.3}$$

To see this when $j = 0, \ldots, M-1$, suppose that k transmission processes of ITM traffic terminate in the nth cycle. For j transmission processes of ITM traffic to be in progress in the $(n+1)$st cycle, it is necessary that $j + k - i$ ITM traffic requests arrive in the nth cycle. Thus q_{ij} is obtained by summing up these probabilities over k. In the case of $j = M$, q_{iM} can be obtained in the same manner except that at least $M + k - i$ ITM traffic requests must arrive in the nth cycle.

Making the change of variable $i - k$ for k, we may rewrite q_{ij} as

$$q_{ij} = \begin{cases} \displaystyle\sum_{k=0}^{\min(i,j)} B(i,i-k)C(j-k); & j = 0, \ldots, M-1 \\ \displaystyle\sum_{k=0}^{i} B(i,i-k) \sum_{l=M-k}^{\infty} C(l); & j = M. \end{cases} \tag{4.4}$$

The $(M+1) \times (M+1)$ matrix $\mathbf{Q} = [q_{ij}]$ gives the transitions of the chain $\{X_n\}$. We now list some of its properties that will be useful later in our analysis.

Theorem 2 *The stochastic matrix* $\mathbf{Q}$ *is positive, irreducible, aperiodic and nonsingular.*

Proof. Since $0 < p < 1$, the probabilities q_{ij} are positive, and thus $\mathbf{Q}$ is irreducible and aperiodic. We claim that $\mathbf{Q}$ has rank $M+1$, i.e., it is nonsingular. For $i = 0, \ldots, M$, let $\mathbf{q}_i = [q_{i0}, \ldots, q_{iM}]$ be the ith row of $\mathbf{Q}$. We will show that the vectors $\mathbf{q}_i$ are linearly independent. Assume that there are (complex) scalars α_i, $i = 0, \ldots, M$, such that $\sum_{i=0}^{M} \alpha_i \mathbf{q}_i = \mathbf{0}$, and we wish to show that all $\alpha_i = 0$. Using $\sum_{i=0}^{M} \sum_{k=0}^{\min(i,j)} = \sum_{k=0}^{j} \sum_{i=k}^{M}$, we obtain after interchanging the order of summation

$$\sum_{i=0}^{M} \alpha_i q_{ij} = \sum_{k=0}^{j} C(j-k) \sum_{i=k}^{M} \alpha_i B(i, i-k) = 0; \qquad j = 0, \ldots, M-1 \tag{4.5}$$

$$\sum_{i=0}^{M} \alpha_i q_{iM} = \sum_{k=0}^{M} \left[\sum_{l=M-k}^{\infty} C(l) \right] \sum_{i=k}^{M} \alpha_i B(i, i-k) = 0; \qquad j = M. \tag{4.6}$$

We next claim that

$$\sum_{i=k}^{M} \alpha_i B(i, i-k) = 0; \qquad k = 0, \ldots, M. \tag{4.7}$$

This can be shown by induction on k. Equation (4.5) for $j = 0$ shows (4.7) for $k = 0$, since $C(0) \neq 0$. If (4.7) holds for $k = 0, \ldots, l-1$, then (4.5) for $j = l$ (or (4.6) in case $l = M$) shows that (4.7) also holds for $k = l$.

We can now show that $\alpha_l = 0$ for $l = 1, \ldots, M$ by induction on l. Using (4.7) for $k = M$, we obtain $\alpha_M = 0$ since $B(M, 0) \neq 0$. If we know that $\alpha_M = \cdots = \alpha_{l+1} = 0$, then from (4.7) for $k = l$ it follows that $\alpha_l B(l, 0) = 0$. Since $B(l, 0) \neq 0$, we have $\alpha_l = 0$. Continuing in this manner we see that all $\alpha_l = 0$. $\square$

Let $\boldsymbol{\pi} = [\pi_0, \ldots, \pi_M]$ denote the vector of steady state probabilities:

$$\pi_i := \lim_{n \to \infty} P(X_n = i); \qquad i = 0, \ldots, M. \tag{4.8}$$

These probabilities can be obtained by solving the equations

$$\boldsymbol{\pi} = \boldsymbol{\pi} \mathbf{Q} \quad \text{and} \quad \boldsymbol{\pi} \mathbf{1}_{M+1} = 1, \tag{4.9}$$

where $\mathbf{1}_{M+1} = [1, \ldots, 1]^t$.

4.2 ATM Modeling and Analysis

A two-dimensional discrete-time Markov chain is considered for modeling the queueing process of ATM traffic. Notation for the ATM traffic is the same as that in Section 3, namely, Y_n is the number of ATM packets in the buffer at the beginning of the nth cycle, and Y_n^h, $h = 1, \ldots, N$, is the number of ATM packets in the buffer at the end of the hth slot of the nth cycle (also set $Y_n^0 := Y_n$). Further, A_n^h, $h = 1, \ldots, N$, denotes the total number of ATM packets arriving in the hth slot of the nth cycle. Similar to (3.1) and (3.2), we obtain

$$Y_n^h = \begin{cases} A_n^h + Y_n^{h-1}; & h = 1, \ldots, X_n \\ A_n^h + \left[Y_n^{h-1} - 1 \right]^+; & h = X_n + 1, \ldots, N. \end{cases} \tag{4.10}$$

We also have

$$Y_{n+1} = Y_n^N; \qquad n = 0, 1, 2, \ldots, \tag{4.11}$$

i.e., the ATM packets in the system at the beginning of the $(n+1)$st cycle are those present at the end of the nth cycle. Note that X_{n+1}, the number of slots to send the ITM traffic in the $(n+1)$st cycle, depends only on X_n. Thus, the sequence $\{(X_n, Y_n)\,;\ n = 1, 2, \ldots\}$ forms a two-dimensional discrete-time Markov chain.

We first find the PGF for the number of ATM packets in the buffer in steady state, and then we obtain the average waiting time of an ATM packet as a performance measure of the IEEE 1394 network. Let us derive the PGF for the number of ATM packets in the buffer at the beginning of a cycle in steady state. Define the vector generating functions

$$\begin{aligned} \mathbf{Y}_n^h(z) &:= \left[Y_n^h(z;0), \ldots, Y_n^h(z;M)\right]; \qquad h = 0, \ldots, N \\ \mathbf{Y}_n(z) &:= \left[Y_n(z;0), \ldots, Y_n(z;M)\right], \end{aligned} \tag{4.12}$$

where the entries are given by

$$\begin{aligned} Y_n^h(z;i) &:= \sum_{k=0}^{\infty} P_n^h(k;i) z^k; \qquad i = 0, \ldots, M,\ h = 1, \ldots, N \\ Y_n(z;i) &:= \sum_{k=0}^{\infty} P_n(k;i) z^k; \qquad i = 0, \ldots, M. \end{aligned} \tag{4.13}$$

Here we have defined

$$\begin{aligned} P_n^h(k;i) &:= P(X_n = i, Y_n^h = k); \qquad k = 0, 1, \ldots,\ i = 0, \ldots, M,\ h = 1, \ldots, N \\ P_n(k;i) &:= P(X_n = i, Y_n = k); \qquad k = 0, 1, \ldots,\ i = 0, \ldots, M. \end{aligned} \tag{4.14}$$

Note that $Y_n^0(z;i) = Y_n(z;i)$, $i = 0, \ldots, M$, from the above definitions. Recalling (4.11) and the transition from X_n at the end of the nth cycle to X_{n+1} at the beginning of the $(n+1)$st cycle, we have

$$\mathbf{Y}_{n+1}(z) = \mathbf{Y}_n^N(z)\mathbf{Q}; \qquad n = 0, 1, 2, \ldots. \tag{4.15}$$

Similar to Section 3, transforming (4.10) yields (for $i = 0, \ldots, M$)

$$Y_n^h(z;i) = \begin{cases} e^{\lambda_{\mathrm{ATM}}(z-1)} Y_n^{h-1}(z;i); & h = 1, \ldots, i \\ \dfrac{e^{\lambda_{\mathrm{ATM}}(z-1)}}{z} \left[Y_n^{h-1}(z;i) + (z-1)P_n^{h-1}(0;i)\right]; & h = i+1, \ldots, N. \end{cases} \tag{4.16}$$

Iterating on (4.16), similar to the proof of (3.5), we see that

$$Y_n^h(z;i) = \begin{cases} e^{h\lambda_{\mathrm{ATM}}(z-1)} Y_n(z;i); & h = 1, \ldots, i \\ \dfrac{1}{z^{h-i}}[e^{h\lambda_{\mathrm{ATM}}(z-1)} Y_n(z;i) & \\ \qquad + (z-1)\displaystyle\sum_{k=i}^{h-1} P_n^k(0;i) e^{(h-k)\lambda_{\mathrm{ATM}}(z-1)} z^{k-i}]; & h = i+1, \ldots, N. \end{cases} \tag{4.17}$$

Finally, by using (4.17) with $h = N$ and (4.15) we have (for $j = 0, \ldots, M$)

$$\begin{aligned} Y_{n+1}(z;j) \quad = \quad & \frac{e^{N\lambda_{\mathrm{ATM}}(z-1)}}{z^N} \sum_{i=0}^{M} Y_n(z;i) z^i q_{ij} \\ & + \frac{z-1}{z^N} \sum_{i=0}^{M} \sum_{k=i}^{N-1} P_n^k(0;i) e^{(N-k)\lambda_{\mathrm{ATM}}(z-1)} z^k q_{ij}. \end{aligned} \tag{4.18}$$

We now assume that the system enters steady state as $n \to \infty$ and set

$$\mathbf{Y}(z) := \lim_{n\to\infty} \mathbf{Y}_n(z).$$

This vector has entries $Y(z;i) := \lim_{n\to\infty} Y_n(z;i)$, $i = 0, \ldots, M$. We also define

$$P^j(k;i) := \lim_{n\to\infty} P_n^j(k;i); \qquad i = 0,\ldots,M,\ j = i,\ldots,N-1,\ k = 0,1,2,\ldots .$$

Letting $n \to \infty$ in (4.18), we obtain the matrix equation

$$\mathbf{Y}(z)\mathbf{F}(z) = (z-1)\mathbf{V}(z)\mathbf{A}(z), \tag{4.19}$$

where

$$\mathbf{F}(z) := z^N \mathbf{I}_{M+1} - \mathbf{A}(z) \tag{4.20}$$

(here $\mathbf{I}_{M+1}$ is the $(M+1)\times(M+1)$ identity matrix) and

$$\mathbf{A}(z) := e^{N\lambda_{\mathrm{ATM}}(z-1)}\,\mathrm{diag}\{1, z, \ldots, z^M\}\mathbf{Q} \tag{4.21}$$

with entries

$$A_{ij}(z) = e^{N\lambda_{\mathrm{ATM}}(z-1)} z^i q_{ij}; \qquad i = 0,\ldots,M,\ j = 0,\ldots,M.$$

The vector $\mathbf{V}(z) = [V_0(z), \ldots, V_M(z)]$ has entries

$$V_i(z) := \sum_{j=i}^{N-1} P^j(0;i) e^{-j\lambda_{\mathrm{ATM}}(z-1)} z^{j-i}; \qquad i = 0,\ldots,M, \tag{4.22}$$

Therefore there are

$$S := N(M+1) - \frac{M(M+1)}{2}$$

unknown constants to be determined in (4.19) to find $\mathbf{Y}(z)$.

Let $\mathrm{adj}\,\mathbf{F}(z)$ denote the adjoint matrix of $\mathbf{F}(z)$. Multiplying (4.19) on the right by $\mathrm{adj}\,\mathbf{F}(z)$, we obtain

$$\mathbf{Y}(z) \;=\; \frac{(z-1)\mathbf{V}(z)\mathbf{A}(z)\,\mathrm{adj}\,\mathbf{F}(z)}{\det \mathbf{F}(z)}. \tag{4.23}$$

The (vector) generating function $\mathbf{Y}(z)$ is analytic in the open unit disk and continuous in the closure. However, since $\mathbf{V}(z)$, $\mathbf{A}(z)$ and $\mathbf{F}(z)$ are all entire functions (i.e., analytic in the entire complex plane), then $\mathbf{Y}(z)$ is in fact analytic in a neighborhood of the closed unit disk. We will determine the S unknowns $\{P^j(0;i); i = 0,\ldots,M,\ j = i,\ldots,N-1\}$ by solving a set of $S-1$ linear equations obtained from zeros of $\det \mathbf{F}(z)$ in the open unit disk together with an additional equation corresponding to the zero at $z = 1$.

An equation can be obtained from the normalizing condition $\mathbf{Y}(1)\mathbf{1}_{M+1} = 1$. However, note from the definitions that $\mathbf{Y}(1) = \boldsymbol{\pi}$, and so this condition is simply $\boldsymbol{\pi}\mathbf{1}_{M+1} = 1$, which is not immediately useful. Instead we differentiate (4.19) and evaluate the result at $z = 1$ to get

$$\mathbf{Y}'(1)\mathbf{F}(1) + \boldsymbol{\pi}\mathbf{F}'(1) = \mathbf{V}(1)\mathbf{Q},$$

where we have used $\mathbf{Y}(1) = \boldsymbol{\pi}$ and $\mathbf{A}(1) = \mathbf{Q}$. We next multiply this equation on the right by $\mathbf{1}_{M+1}$ and note that $\mathbf{F}(1)\mathbf{1}_{M+1} = [\mathbf{I}_{M+1} - \mathbf{Q}]\mathbf{1}_{M+1} = \mathbf{0}$ using (4.20) and $\mathbf{Q}\mathbf{1}_{M+1} = \mathbf{1}_{M+1}$. Thus we obtain

$$\boldsymbol{\pi}\mathbf{F}'(1)\mathbf{1}_{M+1} = \mathbf{V}(1)\mathbf{1}_{M+1}. \tag{4.24}$$

To determine the right-hand side of this equation, from the definition (4.22) we see that

$$\mathbf{V}(1)\mathbf{1}_{M+1} = \sum_{i=0}^{M}\sum_{j=i}^{N-1} P^j(0;i). \tag{4.25}$$

To determine the left-hand side of (4.24), we differentiate (4.20), evaluate the result at $z = 1$ and then multiply on the right by $\mathbf{1}_{M+1}$. This yields

$$\mathbf{F}'(1)\mathbf{1}_{M+1} = N\mathbf{1}_{M+1} - \mathbf{A}'(1)\mathbf{1}_{M+1}.$$

A similar sequence of operations on (4.21) gives

$$\mathbf{A}'(1)\mathbf{1}_{M+1} = N\lambda_{\mathrm{ATM}}\mathbf{1}_{M+1} + \mathbf{u}, \tag{4.26}$$

where $\mathbf{u} := [0, 1, 2, \ldots, M]^t$. Thus we obtain

$$\mathbf{F}'(1)\mathbf{1}_{M+1} = N\mathbf{1}_{M+1} - N\lambda_{\mathrm{ATM}}\mathbf{1}_{M+1} - \mathbf{u}. \tag{4.27}$$

Note that $\boldsymbol{\pi}\mathbf{u} = \sum_{i=0}^{M} i\pi_i = E[X]$, where this expectation represents the mean number of slots occupied by the ITM traffic per cycle. Multiplying (4.27) on the left by $\boldsymbol{\pi}$ and substituting the result into (4.24) yields the final equation for the unknown constants as

$$\sum_{i=0}^{M}\sum_{j=i}^{N-1} P^j(0;i) = N - N\lambda_{\mathrm{ATM}} - E[X]. \tag{4.28}$$

This equation can be viewed as simply identifying two different expressions for the mean number of idle slots per cycle.

The sum in (4.28) must be positive, since otherwise $\mathbf{V}(z)$ and hence $\mathbf{Y}(z)$ would be identically zero. Therefore,

$$N\lambda_{\mathrm{ATM}} < N - E[X] \tag{4.29}$$

must hold. That is, the average ATM traffic load per cycle must be less than the average number of slots available to support it. This is a sufficient condition for the stability of the ATM packet queue.

To obtain the additional S equations for the unknown constants, we investigate the number of zeros of $\det \mathbf{F}(z)$ (from the denominator of $\mathbf{Y}(z)$ in (4.23)) in the closed unit disk under the stability condition (4.29).

Theorem 3 *If $N\lambda_{\mathrm{ATM}} < N - E[X]$, then $\det \mathbf{F}(z)$ has $N(M+1)$ zeros (counting multiplicities) in the closed unit disk. Specifically, there are*

(a) *$S - 1$ zeros in the punctured disk $\{z;\ 0 < |z| < 1\}$;*

(b) *one zero on the unit circle, namely, a simple zero at $z = 1$; and*

(c) *a zero of multiplicity $M(M+1)/2$ at $z = 0$.*

Proof. Recalling that $\mathbf{A}(1) = \mathbf{Q}$ is an irreducible stochastic matrix, we wish to apply Theorem 4 of [6]. As described in that theorem, the number and location of the zeros of $\det \mathbf{F}(z)$ in the closed unit disk depend on the quantity

$$\gamma := \frac{d}{dz}\det \mathbf{F}(z)\bigg|_{z=1}. \tag{4.30}$$

To determine γ, we use the well-known relations

$$\mathbf{F}(z)\,\mathrm{adj}\,\mathbf{F}(z) = \det\mathbf{F}(z)\mathbf{I}_{M+1} = \mathrm{adj}\,\mathbf{F}(z)\mathbf{F}(z). \tag{4.31}$$

Differentiating the second equality, evaluating the result at $z = 1$, multiplying on the right by $\mathbf{1}_{M+1}$ and using $\mathbf{F}(1)\mathbf{1}_{M+1} = \mathbf{0}$ yields

$$\gamma\mathbf{1}_{M+1} = \mathrm{adj}\,\mathbf{F}(1)\mathbf{F}'(1)\mathbf{1}_{M+1}. \tag{4.32}$$

An expression for $\mathrm{adj}\,\mathbf{F}(1)$ may be found as follows. By evaluating (4.31) at $z = 1$, where $\mathbf{F}(1) = \mathbf{I}_{M+1} - \mathbf{Q}$, and using $\det\mathbf{F}(1) = 0$ (since $\mathbf{F}(1)$ is singular), we obtain

$$\mathbf{Q}\,\mathrm{adj}\,\mathbf{F}(1) = \mathrm{adj}\,\mathbf{F}(1) = \mathrm{adj}\,\mathbf{F}(1)\mathbf{Q}.$$

Since $\mathbf{Q}$ is an irreducible stochastic matrix, the first equality implies that each column of $\mathrm{adj}\,\mathbf{F}(1)$ is a multiple of $\mathbf{1}_{M+1}$ (recall that $\mathbf{Q}\mathbf{1}_{M+1} = \mathbf{1}_{M+1}$). Similarly, the second equality implies that each row of $\mathrm{adj}\,\mathbf{F}(1)$ is a multiple of $\boldsymbol{\pi}$ (recall that $\boldsymbol{\pi} = \boldsymbol{\pi}\mathbf{Q}$). It follows that there is a (real) constant c such that

$$\mathrm{adj}\,\mathbf{F}(1) = c\begin{bmatrix}\boldsymbol{\pi}\\ \vdots \\ \boldsymbol{\pi}\end{bmatrix}. \tag{4.33}$$

We claim that $\mathrm{adj}\,\mathbf{F}(1)$ is a positive matrix [7, p. 75]. From the form (4.33), it is enough to show that the diagonal elements, say, $\eta_{i,i}(1)$, $i = 0, \ldots, M$, of $\mathrm{adj}\,\mathbf{F}(1)$ are positive. To see this, note that

$$\eta_{i,i}(1) = (-1)^{i+i}\det[\mathbf{F}_{(i,i)}(1)] = \det[\mathbf{I}_M - \mathbf{Q}_{(i,i)}],$$

where $\mathbf{Q}_{(i,i)}$ is the matrix $\mathbf{Q}$ with its ith row and ith column removed. Since $\mathbf{Q}$ is irreducible, the spectral radius of $\mathbf{Q}_{(i,i)}$ is strictly less than unity. This implies that $\det[\mathbf{I}_M - t\mathbf{Q}_{(i,i)}] \neq 0$ for real t satisfying $0 \leq t \leq 1$. Since this determinant function of t is positive for $t = 0$ and never zero, by continuity it is also positive for $t = 1$, i.e., $\eta_{i,i}(1) > 0$. Thus $\mathrm{adj}\,\mathbf{F}(1)$ is positive, and we conclude that $c > 0$ in (4.33).

From (4.27) and (4.33), each row of (4.32) gives the equation

$$\frac{\gamma}{c} = N - N\lambda_{\mathrm{ATM}} - E[X].$$

Using $c > 0$ and the condition (4.29), we see that γ is positive.

According to Theorem 4 of [6], when $\gamma > 0$ then $\det\mathbf{F}(z)$ has exactly $N(M+1) - \kappa$ zeros (counting multiplicities) in the open unit disk and simple zeros at the κth roots of unity on the unit circle, where κ denotes the number of zeros of $\det\mathbf{F}(z)$ on the unit circle. Therefore, if we can show that parts (b) and (c) are valid, then part (a) will also hold automatically.

We first prove part (b), i.e., $\kappa = 1$. Suppose that z_0 is a zero of $\det\mathbf{F}(z)$ on the unit circle, that is, $|z_0| = 1$. It follows from (4.20) with (4.21) that there exists a vector $\mathbf{v} = [v_0, \ldots, v_M]^t \neq \mathbf{0}$ such that

$$z_0^N\mathbf{v} \;=\; e^{N\lambda_{\mathrm{ATM}}(z_0-1)}\,\mathrm{diag}\left\{1, z_0, \ldots, z_0^M\right\}\mathbf{Q}\mathbf{v}. \tag{4.34}$$

This means that

$$z_0^N v_i = e^{N\lambda_{\mathrm{ATM}}(z_0-1)} z_0^i \sum_{j=0}^{M} q_{ij} v_j; \qquad i = 0, \ldots, M. \tag{4.35}$$

We pick l such that $|v_l| := \max\{|v_i|; i = 0, \ldots, M\}$. Since $\mathbf{v} \neq \mathbf{0}$, then $v_l \neq 0$. For this l, (4.35) becomes

$$1 = e^{N\lambda_{\mathrm{ATM}}(z_0-1)} z_0^{l-N} \sum_{j=0}^{M} q_{lj} \frac{v_j}{v_l}.$$

Since $|z_0| = 1$ it follows that $x_0 := \Re(z_0) \leq 1$. Hence $|e^{N\lambda_{\mathrm{ATM}}(z_0-1)}| = e^{N\lambda_{\mathrm{ATM}}(x_0-1)} \leq 1$. Thus, using $|v_j| \leq |v_l|$, $j = 0, \ldots, M$, we have

$$1 \leq \sum_{j=0}^{M} q_{lj} \frac{|v_j|}{|v_l|} \leq \sum_{j=0}^{M} q_{lj} = 1.$$

Therefore

$$\sum_{j=0}^{M} q_{lj} \left[1 - \frac{|v_j|}{|v_l|}\right] = 0. \tag{4.36}$$

Since $q_{lj} > 0$ and $1 - |v_j|/|v_l| \geq 0$ for all j, (4.36) implies that $|v_j| = |v_l|$, $j = 0, \ldots, M$. This further implies that

$$1 \leq |e^{N\lambda_{\mathrm{ATM}}(z_0-1)}| \sum_{j=0}^{M} q_{lj} \frac{|v_j|}{|v_l|} = |e^{N\lambda_{\mathrm{ATM}}(z_0-1)}| \leq 1.$$

Then we obtain $1 = |e^{N\lambda_{\mathrm{ATM}}(z_0-1)}| = e^{N\lambda_{\mathrm{ATM}}(x_0-1)}$. It follows that $x_0 = 1$, and so $z_0 = 1$. Thus, part (b) has been proved.

Finally, we prove part (c). To consider terms of the lowest degree in $\det \mathbf{F}(z)$, we use the factorization

$$\mathbf{F}(z) = \mathrm{diag}\{1, z, \ldots, z^M\} \mathbf{G}(z), \tag{4.37}$$

where, from (4.20) and (4.21), we have introduced

$$\mathbf{G}(z) := \mathrm{diag}\{z^N, z^{N-1}, \ldots, z^{N-M}\} - e^{N\lambda_{\mathrm{ATM}}(z-1)} \mathbf{Q}, \tag{4.38}$$

which is an entire function of z. This gives the determinant of $\mathbf{F}(z)$ from (4.37) as

$$\det \mathbf{F}(z) = z^{M(M+1)/2} \det \mathbf{G}(z). \tag{4.39}$$

We claim that $z = 0$ is not a zero of $\det \mathbf{G}(z)$. Since $\mathbf{G}(0) = -e^{-N\lambda_{\mathrm{ATM}}} \mathbf{Q}$ is nonsingular because $\mathbf{Q}$ is from Theorem 2, we see that $\det \mathbf{G}(0) \neq 0$. Thus $\det \mathbf{F}(z)$ has a zero of multiplicity exactly $M(M+1)/2$ at $z = 0$. Thus, part (c) has been proved. This completes the proof of Theorem 3. $\square$

The next theorem shows that $z^{M(M+1)/2}$ cancels from both the numerator and the denominator of $\mathbf{Y}(z)$ in (4.23).

Theorem 4 *The numerator of $\mathbf{Y}(z)$ in (4.23) has a factor $z^{M(M+1)/2}$.*

Proof. From (4.31) and the definition of $\mathbf{F}(z)$ in (4.20), we have

$$\mathbf{A}(z) \operatorname{adj} \mathbf{F}(z) = z^N \operatorname{adj} \mathbf{F}(z) - \det \mathbf{F}(z) \mathbf{I}_{M+1}. \tag{4.40}$$

We claim that this term in the numerator of $\mathbf{Y}(z)$ in (4.23) has a factor of $z^{M(M+1)/2}$. This has already been shown for $\det \mathbf{F}(z)$ in (4.39), and we now show it also holds for $z^N \operatorname{adj} \mathbf{F}(z)$. Let $\eta_{ij}(z)$ be the (i,j) element of $\operatorname{adj} \mathbf{F}(z)$. From the definition of the adjoint matrix, recall that

$$\eta_{ij}(z) = (-1)^{i+j} \det \mathbf{F}_{(j,i)}(z); \qquad i = 0, \ldots, M,\ j = 0, \ldots, M, \tag{4.41}$$

where the notation $\mathbf{M}_{(j,i)}$ represents a matrix $\mathbf{M}$ without its jth row and ith column. Using the factorization (4.37), it follows that

$$\mathbf{F}_{(j,i)}(z) = \operatorname{diag}\{1, z, \ldots, z^{j-1}, z^{j+1}, \ldots, z^M\} \mathbf{G}_{(j,i)}(z).$$

Substituting this into (4.41) yields

$$\eta_{ij}(z) = (-1)^{i+j} z^{[M(M+1)/2]-j} \det \mathbf{G}_{(j,i)}(z).$$

Therefore we see that

$$z^N \eta_{ij}(z) = z^{M(M+1)/2} (-1)^{i+j} \det \mathbf{G}_{(j,i)}(z) z^{N-j},$$

which in matrix terms means that

$$z^N \operatorname{adj} \mathbf{F}(z) = z^{M(M+1)/2} \operatorname{adj} \mathbf{G}(z) \operatorname{diag}\{z^N, z^{N-1}, \ldots, z^{N-M}\}. \tag{4.42}$$

From (4.39) and (4.42), we conclude that the term $\mathbf{A}(z) \operatorname{adj} \mathbf{F}(z)$ in the numerator of $\mathbf{Y}(z)$ in (4.23) has a factor $z^{M(M+1)/2}$. □

Substituting (4.39) and (4.42) into (4.23) gives

$$\mathbf{Y}(z) = \frac{(z-1)\mathbf{V}(z) \left[\operatorname{adj} \mathbf{G}(z) \operatorname{diag}\{z^N, z^{N-1}, \ldots, z^{N-M}\} - \det \mathbf{G}(z) \mathbf{I}_{M+1}\right]}{\det \mathbf{G}(z)}. \tag{4.43}$$

From Theorem 3, the denominator $\det \mathbf{G}(z)$ has $S-1$ zeros in the open unit disk and a simple zero on the unit circle at $z = 1$. Furthermore, $z = 0$ is not a zero of $\det \mathbf{G}(z)$. For a zero, say $\widehat{z}$, of $\det \mathbf{G}(z)$ in the punctuated open unit disk $\{z; 0 < |z| < 1\}$, we have $\det \mathbf{G}(\widehat{z}) = 0$, $\widehat{z} \neq 1$, and $\operatorname{diag}\{\widehat{z}^N, \ldots, \widehat{z}^{N-M}\}$ is invertible (since $\widehat{z} \neq 0$). Therefore, using the analyticity of $\mathbf{Y}(z)$ in the closed unit disk, the zero $\widehat{z}$ yields

$$\mathbf{V}(\widehat{z}) \operatorname{adj} \mathbf{G}(\widehat{z}) = \mathbf{0} \tag{4.44}$$

from the numerator of (4.43).

We now discuss the scalar equation(s) for the unknown probabilities that correspond to the matrix equation (4.44). First suppose that $\widehat{z}$ is a simple zero of $\det \mathbf{G}(z)$. Then

$$\mathbf{G}(\widehat{z}) \operatorname{adj} \mathbf{G}(\widehat{z}) = \mathbf{0}$$

and

$$\mathbf{G}(\widehat{z}) \frac{d}{dz} \operatorname{adj} \mathbf{G}(z) \bigg|_{z=\widehat{z}} + \mathbf{G}'(\widehat{z}) \operatorname{adj} \mathbf{G}(\widehat{z}) = \gamma_{\widehat{z}} \mathbf{I}_{M+1}, \tag{4.45}$$

where

$$\gamma_{\widehat{z}} := \frac{d}{dz} \det \mathbf{G}(z) \bigg|_{z=\widehat{z}} \neq 0.$$

Since $\det \mathbf{G}(\widehat{z}) = 0$, then $\operatorname{rank} \mathbf{G}(\widehat{z}) \leq M$. Now note that if $\operatorname{adj} \mathbf{G}(\widehat{z}) = \mathbf{0}$, then from (4.45) it follows that $\mathbf{G}(\widehat{z})$ is invertible. This contradiction shows that some element of $\operatorname{adj} \mathbf{G}(\widehat{z})$ is nonzero, which means that some $M \times M$ submatrix of $\mathbf{G}(\widehat{z})$ has a nonzero determinant. We conclude immediately that $\operatorname{rank} \mathbf{G}(\widehat{z}) = M$.

Since $\operatorname{adj} \mathbf{G}(\widehat{z})$ is a nonzero matrix, it has a nonzero column $\mathbf{w} = [w_0, \ldots, w_M]^t$. Letting $\mathbf{g}_0, \ldots, \mathbf{g}_M$ be the columns of $\mathbf{G}(\widehat{z})$, there is an index l so that the M vectors $\mathbf{g}_i$, $i \neq l$, constitute a basis for the column space of $\mathbf{G}(\widehat{z})$. Since $\mathbf{G}(\widehat{z})\mathbf{w} = \mathbf{0}$, we have $\sum_{i=0}^{M} w_i \mathbf{g}_i = \mathbf{0}$. If $w_l = 0$, then $\sum_{i \neq l} w_i \mathbf{g}_i = \mathbf{0}$ which implies that all $w_i = 0$ by linear independence, i.e., $\mathbf{w} = \mathbf{0}$. This contradiction shows that $w_l \neq 0$. Now let $\mathbf{u} = [u_0, \ldots, u_M]^t$ be any column of $\operatorname{adj} \mathbf{G}(\widehat{z})$, and consider the vector

$$\mathbf{v} = [v_0, \ldots, v_M]^t := \mathbf{u} - \frac{u_l}{w_l}\mathbf{w}$$

for which $v_l = 0$. But $\mathbf{G}(\widehat{z})\mathbf{v} = \mathbf{0}$, and so $\mathbf{v} = \mathbf{0}$ as shown above. Then $\mathbf{u} = (u_l/w_l)\mathbf{w}$, and we conclude that $\mathbf{w}$ is a basis for the column space of $\operatorname{adj} \mathbf{G}(\widehat{z})$. Thus the rank of $\operatorname{adj} \mathbf{G}(\widehat{z})$ is 1, and so (4.44) gives exactly one linearly independent (scalar) equation. In fact, any non-zero column of $\operatorname{adj} \mathbf{G}(\widehat{z})$ can be used to obtain the equation. If $\widehat{z}$ is a multiple zero, then additional equations are obtained from (4.43) by differentiation.

Recalling that $\widehat{z} \neq 0$, we obtain from (4.42) and (4.44) the equivalent equation

$$\mathbf{V}(\widehat{z}) \operatorname{adj} \mathbf{F}(\widehat{z}) = \mathbf{0}. \tag{4.46}$$

Since $\det \mathbf{F}(\widehat{z}) = 0$, then $\widehat{z}^N \mathbf{I}_{M+1} - \mathbf{A}(\widehat{z})$ is singular, which means that there exists a vector $\mathbf{v} \neq \mathbf{0}$ such that $\widehat{z}^N \mathbf{v} - \mathbf{A}(\widehat{z})\mathbf{v} = \mathbf{0}$. It follows that $\widehat{z}^N$ is an eigenvalue of the matrix $\mathbf{A}(\widehat{z})$. Thus eigenvalue packages can be used in numerical calculations, instead of solving for zeros of the (possibly poorly behaved) determinant function directly.

Once the zeros in the open unit disk are found, a set of $S-1$ linear simultaneous equations results from (4.44), with suitable modifications in the case of multiple zeros. Together with the additional equation (4.28), this linear system is solved to determine the S unknowns $\{P^j(0;i); i = 0, \ldots, M,\ j = i, \ldots, N-1\}$ uniquely. Hence we can obtain $\mathbf{Y}(z) = [Y(z;0), \ldots, Y(z;M)]$.

We are now in a position to calculate the average waiting time of an arbitrary ATM packet in steady state. Let $\overline{Y}(i)$ be the mean number of ATM packets in the buffer at the beginning of a cycle in which i slots are occupied by ITM traffic, and let $\overline{Y}$ be the mean number of ATM packets in the buffer at the beginning of a cycle, both in steady state. These expectations are given by

$$\begin{aligned} \overline{Y}(i) &= \frac{d}{dz} Y(z;i) \bigg|_{z=1}; \qquad i = 0, \ldots, M, \\ \overline{Y} &= \sum_{i=0}^{M} \overline{Y}(i). \end{aligned} \tag{4.47}$$

Furthermore, let $\overline{Y^h}(i)$, $i = 0, \ldots, M$, $h = 1, \ldots, N$, be the mean number of ATM packets in the buffer at the end of the hth slot of a cycle in steady state in which i slots are occupied by ITM traffic in that cycle. Similar to (3.18), these quantities are given by

$$\overline{Y^h}(i) = \begin{cases} h\lambda_{\mathrm{ATM}}\pi_i + \overline{Y}(i); & h = 1, \ldots, i, \\ h\lambda_{\mathrm{ATM}}\pi_i + \overline{Y}(i) - \sum_{j=i}^{h-1}[\pi_i - P^j(0;i)]; & h = i+1, \ldots, N, \end{cases} \tag{4.48}$$

where π_i, $i = 0, \ldots, M$, is the solution to (4.9). Note that evaluating (4.48) at $h = N$, using $\overline{Y^N}(i) \equiv \overline{Y}(i)$, and summing over $i = 0, \ldots, M$ recovers the condition (4.28).

Let ξ be the number of ATM packets in the buffer at the end of an arbitrary slot, and let $\overline{\xi}$ be its expectation. Then we have

$$P(\xi = k) = \frac{1}{N}\sum_{i=0}^{M}\sum_{h=1}^{N} P^h(k;i).$$

Noting that $\overline{Y^h}(i)$ is the expectation of the distribution $\{P^h(k;i); k = 0, 1, 2, \ldots\}$ and that $\overline{Y^N}(i) \equiv \overline{Y}(i)$, by using (4.47) we have

$$\overline{\xi} = \frac{1}{N}\sum_{i=0}^{M}\sum_{h=1}^{N}\overline{Y^h}(i) = \frac{1}{N}\left[\overline{Y} + \sum_{i=0}^{M}\sum_{h=1}^{N-1}\overline{Y^h}(i)\right].$$

It follows from Little's theorem that the average waiting time $\overline{W}$ of an ATM packet in steady state is given by

$$\overline{W} = \frac{\overline{\xi}}{\lambda_{\mathrm{ATM}}} = \frac{1}{N\lambda_{\mathrm{ATM}}}\left[\overline{Y} + \sum_{i=0}^{M}\sum_{h=1}^{N-1}\overline{Y^h}(i)\right]. \tag{4.49}$$

Using (4.48) for $\overline{Y^h}(i)$, we obtain

$$\overline{W} = \frac{N-1}{2} + \frac{\overline{Y}}{\lambda_{\mathrm{ATM}}} - \frac{1}{N\lambda_{\mathrm{ATM}}}\sum_{i=0}^{M}\sum_{j=i}^{N-2}(N-1-j)[\pi_i - P^j(0;i)]. \tag{4.50}$$

5 Numerical and Simulation Results

In this section, some numerical results from the analysis of the models are presented. Simulation results are also provided for an extended model which also includes the effect of arbitration among the users.

5.1 Static ITM Traffic Model

We first present numerical and simulation results for the average waiting time of an ATM packet in steady state based on the static ITM traffic model of Section 3.

The following parameter values are used:

- Transmission speed : $C = 400$ Mbps.
- Speed of signal : $V = 2.0 \times 10^8$ m/sec.
- Duration of a slot (unit of time) : $\tau = 25.0 \times 10^{-6}$ sec.
- Number of slots in a cycle : $N = 5$.

- Number of slots for ITM in a cycle : $K = 1, 2, 3, 4$.
- Maximum distance between nodes : $d = 500$ m.
- Duration of an SG : $D_{\text{SG}} = 2.5 \times 10^{-6}$ sec.
- Duration of an ARG : $D_{\text{ARG}} = 5 \times 10^{-6}$ sec.
- Duration of an IG : Negligible.
- Size of an ATM packet : $L_{\text{ATM}} = 1,000$ bytes.
- Amount of ITM traffic in a slot : $L_{\text{ITM}} = 1,000$ bytes.
- Size of an acknowledgment packet : $L_{\text{ACK}} = 1$ byte.

We assume that

$$\tau = \max(D_{\text{SG}}, D_{\text{ARG}}) + \frac{(L_{\text{ATM}} + L_{\text{ACK}}) \times 8}{C} + \frac{2d}{V} = \frac{125 \times 10^{-6}}{N} \quad \text{sec} \tag{5.1}$$

$$B_{\text{ITM}} \approx \frac{C \cdot K}{N} \quad \text{bps}, \tag{5.2}$$

where B_{ITM} indicates the bandwidth required to transmit ITM traffic. For reference, Table 1 shows the number of MPEG-2 streams (with a peak rate of 6 Mbps) that can be carried by ITM depending on the bandwidth for ITM.

Table 1: Number of MPEG-2 streams carried by ITM.

K	B_{ITM} (Mbps)	Number of MPEG-2 streams
1	80	13
2	160	26
3	240	40
4	320	53

We have also simulated the IEEE 1394 network under the following conditions:

- The simulation is carried out in discrete time.
- The number of nodes is 10.
- The arrival rate of ATM packets is the same at all nodes.
- The arbitration method is taken into consideration.

Inclusion of the last condition is the difference between the simulation model and the analytical model.

The average waiting time of an ATM packet in the presence of static ITM traffic is plotted in Figure 3. The quantity K represents the number of slots used for ITM traffic, with ITM(A) corresponding to the analytical model and ITM(S) corresponding to the simulation model (with 95% confidence interval). The figure indicates that the numerical results are in good agreement with the simulation results, which serves to validate the analysis in the paper that ignores arbitration among the users. We observe that, as the arrival rate of ATM packets approaches the limit given by the stability condition, the average waiting time grows arbitrarily large.

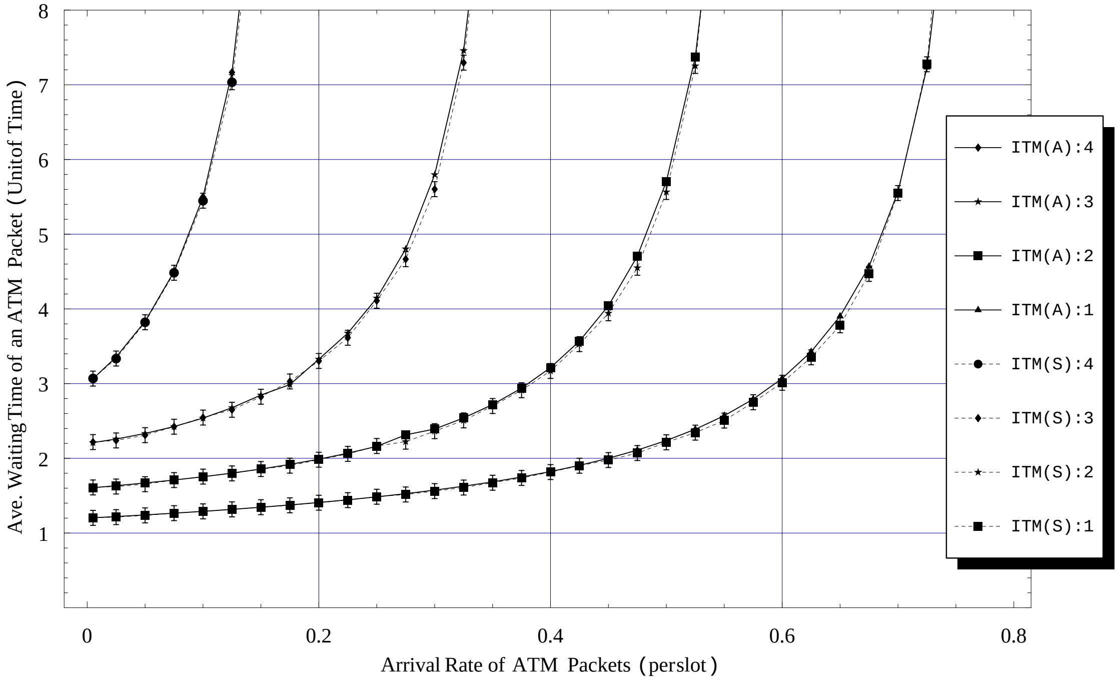

Figure 3: Average waiting time of an ATM packet for the static ITM traffic model.

5.2 Dynamic ITM Traffic Model

We next present the numerical and simulation results for the average waiting time of an ATM packet in steady state based on the dynamic ITM traffic model of Section 4. The parameters are the same as those for the static ITM traffic model, except that a dynamic number of slots for ITM traffic in each cycle is used. Instead of the constant K from the static ITM traffic model, we assume that the maximum number M of such slots is 4.

The average waiting time of an ATM packet in the presence of dynamic ITM traffic is plotted in Figure 4. In the case of Analysis/Simulation 1, the arrival rate of requests for ITM transmission is $\lambda_{\mathrm{ITM}} = 8.0 \times 10^3$ (/sec), and the average holding time for each ITM transmission is $1/\mu_{\mathrm{ITM}} = 1.25 \times 10^{-3}$ (sec). In the case of Analysis/Simulation 2, $\lambda_{\mathrm{ITM}} = 2.0 \times 10^3$ (/sec) and $\mu_{\mathrm{ITM}} = 1.25 \times 10^{-3}$ (sec). The figure again shows that the numerical results are in good agreement with the simulation results (with 95% confidence interval). We again observe that, as the arrival rate of ATM packets approaches the limit given by the stability condition, the average waiting time grows arbitrarily large.

Some comments may be in order on the above choice of parameter values. The holding times of ITM traffic generated by some real-time application are usually on the order of seconds or minutes. When we used such parameters for ITM traffic, the numerical calculations for the analytical model failed for the following reason. As we mentioned at the beginning of Section 3, the interarrival and holding times for ITM traffic are much longer than those for ATM traffic. It follows that some of the elements of the transition probability matrix for ITM traffic are on the order of 10^{-6}. This may cause a serious numerical problems when we try to find the zeros of an analytic function to obtain the unknown probabilities included in (4.23). On the other hand, the simulation time grows rapidly for such a case because of the difference in rates for ITM traffic and ATM traffic (i.e., the model is *stiff*). The parameter values listed in the paper represent cases for which

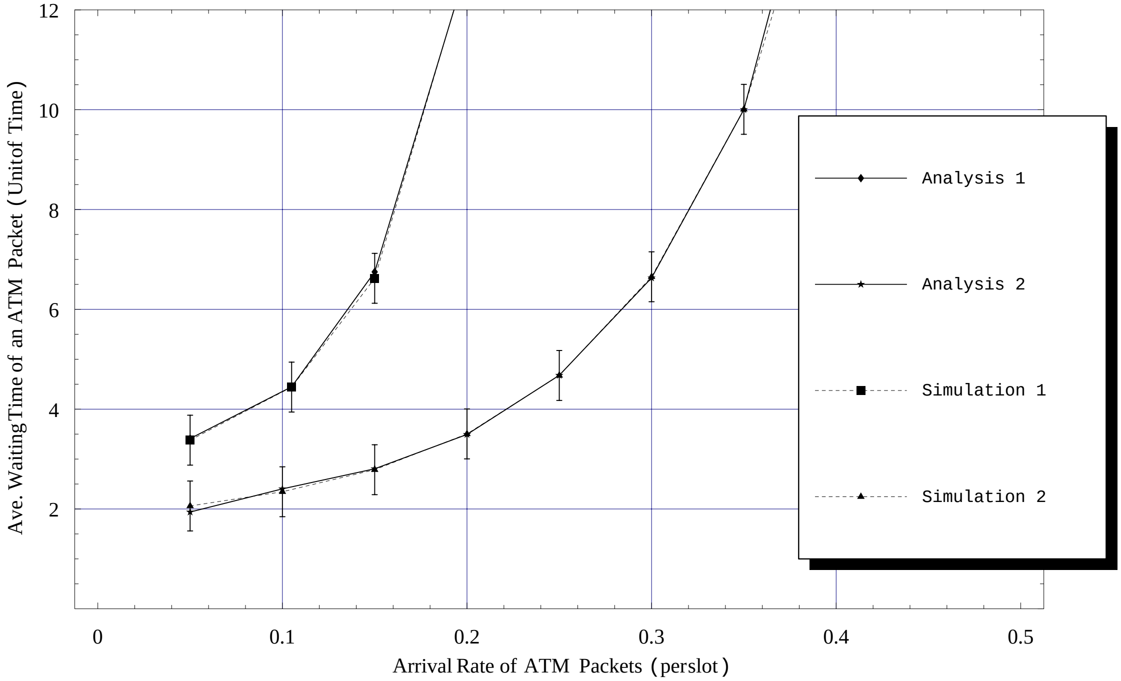

Figure 4: Average waiting time of an ATM packet for the dynamic ITM traffic model.

both the numerical calculations as well as the simulations were carried out successfully.

6 Concluding Remarks

As the demand for delay-sensitive applications such as online conference systems and MPEG video streams increases, so does the demand for networks with an advantage of dealing with such applications. One of the candidates for such a network is the IEEE 1394 network, and thus it seems useful to analyze its performance. In this paper, we have evaluated certain performance measures of that network by several queueing models. The numerical results based on our analysis have shown good agreement with simulation.

However, current analytical models do not take into account certain factors of the real system, including the arbitration scheme, the variation in size of ATM packets, the distinction among the nodes, and the network topology that affects the duration of an IG, SG or ARG. In future work, the present models should be extended to take these factors into consideration. Furthermore, it would be of interest to obtain additional performance measures analytically, such as the delay variation of an ATM packet among the nodes in the network.

Acknowledgments

We wish to thank Dr. Kotikalapudi Sriram of Lucent Technologies for providing a copy of his Ph.D. dissertation. This work has been carried out with the support of the Grant-in-Aid for Scientific Research (C), Project No. 11650407, from the Japan Society for the Promotion of Science (JSPS).

References

[1] D. Anderson, *FireWire System Architecture*, Second Edition, Addison-Wesley, 1999.

[2] T. Bially, A. J. McLaughlin, and C. J. Weinstein, "Voice communication in integrated digital voice and data networks," *IEEE Transactions on Communications*, Vol.COM-28, No.9, pp.1478–1490, September 1980.

[3] Draft: P1394 Standard for a High Performance Serial Bus, The Institute of Electrical and Electronics Engineers, Inc. (IEEE), 1995.

[4] M. J. Fischer and T. C. Harris, "A model for evaluating the performance of an integrated circuit-and packet-switched multiplex structure," *IEEE Transactions on Communications*, Vol.COM-24, No.2, pp.195–202, February 1976.

[5] H. R. Gail, S. L. Hantler, A. G. Konheim, and B. A. Taylor, "An analysis of a class of telecommunications models," *Performance Evaluation*, Vol.21, Nos. 1–2, pp.151–161, November 1994.

[6] H. R. Gail, S. L. Hantler, and B. A. Taylor, "Spectral analysis of $M/G/1$ and $G/M/1$ type Markov chains," *Advances in Applied Probability*, Vol.28, No.1, pp.114–165, March 1996.

[7] F. R. Gantmacher, *Applications of the Theory of Matrices*. Interscience, New York, 1959.

[8] D. P. Gaver and J. P. Lehoczky, "Channels that cooperatively service a data stream and voice messages," *IEEE Transactions on Communications*, Vol.COM-30, No.5, pp.1153–1162, May 1982.

[9] I. Gitman, W.-N. Hsieh, and B. J. Occhiogrosso, "Analysis and design of hybrid switching networks," *IEEE Transactions on Communications*, Vol.COM-29, No.9, pp.1290–1300, September 1981.

[10] R. A. Horn and C. A. Johnson, *Matrix Analysis*. Cambridge University Press, Cambridge, 1985.

[11] http://www.zayante.com/p1394b/

[12] IEEE1394 Trade Association's web site, http://www.1394ta.org

[13] M. Inada, *An Introduction to the IEEE 1394*. Gijyutsu Hyouron Sha, Tokyo, 1998 (in Japanese).

[14] J.-W. Jang, S. Choi, and E. K. Park, "The design of resource assignment algorithm using multiple queues FIFO over residential broadband network," *Proceedings of SPIE All-Optical Networking 1999*, Vol.3843, pp.162–172, Boston, Massachusetts, September 19–21, 1999.

[15] A. G. Konheim and M. Reiser, "The movable-boundary multiplexor stability and decomposability," *Teletraffic Analysis and Computer Performance Evaluation*, pp.375–394. Elsevier Science Publishers B. V., Amsterdam, 1986.

[16] A. Leon-Garcia, R. H. Kwong, and G. F. Williams, "Performance evaluation methods for an integrated voice/data link," *IEEE Transactions on Communications*, Vol.COM-30, No.8, pp.1848–1858, August 1982.

[17] Nikkei Business Publications, Inc., *IEEE1394 The High Speed Interface in the Digitization Era*. Nikkei Business Publications, Tokyo, 1998 (in Japanese).

[18] Nikkei Business Publications, Inc., *Nikkei Communication*, No.277, pp.110–119, Nikkei Business Publications, Tokyo, September 1998 (in Japanese).

[19] M. J. Ross and O. A. Mowafi, "Performance analysis of hybrid switching concepts for integrated voice/data communications," *IEEE Transactions on Communications*, Vol.COM-30, No.5, pp.1073–1087, May 1982.

[20] I. Sahin, U. N. Bhat, "A stochastic system with scheduled secondary inputs," *Operations Research*, Vol.19, No.2, pp.436–446, March-April 1971.

[21] K. Sriram, *A Study of Multiplexing Schemes for Digitized Voice and Data*, Ph.D. dissertation, Syracuse University, Syracuse, New York, August 1983.

[22] K. Sriram, P. K. Varshney, and J. G. Shanthikumar, "Discrete-time analysis of integrated voice/data multiplexers with and without speech activity detectors," *IEEE Journal on Selected Areas in Communications*, Vol.SAC-1, No.6, pp.1124–1132, December 1983.

[23] E. C. Titchmarsh, *The Theory of Functions*, 2nd ed. Oxford University Press, London, 1939.

[24] C. J. Weinstein, M. L. Malpass, and M. J. Fischer, "Data traffic performance of an integrated circuit-and packet switched multiplex structure," *IEEE Transactions on Communications*, Vol.COM-28, No.6, pp.873–878, June 1980.

[25] E. T. Whittaker and G. N. Watson, *A Course of Modern Analysis*, 4th ed. Cambridge University Press, Cambridge, 1927.