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COUNTING HANDOVERS IN A CELLULAR MOBILE COMMUNICATION NETWORK: DELAYED RENEWAL PROCESS APPROACH

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Abstract

Knowing the number of handovers that a user makes during a call session is particularly important in cellular mobile communication networks in order to make appropriate dimensioning of virtual circuits for wireless cells. In this paper, we study the probability distributions and statistical moments for the number of handovers per call for a variety of combinations of the call holding time (CHT) and cell residence time (CRT) distributions. We assume a mixed platform environment, which means that the first CRT in the originating cell has different statistics from the CRTs in the subsequent cells. In particular, we consider circular cells. Based on the formulation in terms of delayed renewal processes, we obtain analytical expressions for the probability mass functions and moments of the handover number distribution.

Keywords: Cellular mobile communication networks; handover; call holding time; cell residence time; renewal theory; delayed renewal process

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1. Introduction

In cellular wireless communication networks, the wireless cells have limited coverage range, which means that mobile users will be crossing several cell boundaries during his call duration. Each boundary crossing may need switching of a communication channel, whether it is a frequency band, a time slot or a code, which is necessary in order to maintain the connectivity with the network as well as the tracking of the location of the user. This cell switching, or *handover*, tends to cause connectivity disruption and/or extra transmission delay if it is not handled properly. In multimedia service environment, the connectivity and timely transmission are essential. For instance, real-time audio and video information is less tolerant to both overall delay and delay variation than traditional voice calls. Thus, the traffic disruption/delay, as a result of handover failure, can seriously affect the Quality of Service (QoS) of the network.

Wireless multimedia networks are intended to be direct extension of the fixed/wireline broadband ATM networks with uniform end-to-end QoS guarantee. The handover traffic in cellular wireless networks is a complex function of many factors such as the size of wireless cells, user's mobility and call patterns, etc. However it has a direct impact on the signaling traffic, the call admission policy for new users, and also the QoS for the admitted users. Therefore, the study of the handover process is a fundamental issue in the design of multimedia cellular wireless communication networks.

In this paper, we study the statistical characteristics of the number of handovers that a mobile user makes during a call session in a cellular network for a variety of combinations of the call holding time (CHT) and cell residence time (CRT) distributions. In the past, Nanda [9] considered the case of exponentially distributed CHT and CRT. The same case has been extensively studied by Lin et al. [7]. However, the CHT cannot be assumed as an exponentially distributed random variable in multimedia services which typically have long tail in distribution. The CRT is not exponentially distributed either. Recently, Fang et al. [3, 4] have derived a set of recursive equations for the Erlang distributed CHT. Rodríguez-Dagnino [11] has initiated an innovative method based on the renewal theory for obtaining the distribution and moments of the number of handovers explicitly for a mixture of Erlang (including exponential) distributions for the CHT; this work is elaborated later in [13, 14]. However, these studies only consider a single homogeneous platform where the same distribution is assumed for the CRTs in all the wireless cells. Recently, Orlik and Rappaport [10] have presented some results for a mixed platform where it is assumed that each CRT may have different distribution. In the present paper, we extend the methodology of [11, 13, 14] to the case of a mixture of two platforms, in which the first CRT and the subsequent CRTs may have two different distributions. We present closed-form expressions for the probability distribution and the moments of the number of handovers per call [12]. Our approach can be extended to the case of a mixture of three or more platforms such as in [10].

Our approach is based on the renewal theory. Let $N(t)$ be the number of renewals in a fixed time interval $[0, t]$, and the interrenewal times occur according to a sequence of random variables $\{X_1, X_2, \dots, X_i, \dots\}$, where X_1 is started at time 0. This represents the sequence of CRTs that a mobile user experiences during a call such that X_i is the CRT in the ith cell ($i = 1, 2, \dots$). Now, let T be a random variable representing a CHT; throughout the paper we do not take into account the forced termination of calls due to the blocking of handover process. Let us also assume that T is independent of $\{X_1, X_2, \dots, X_i, \dots\}$. Hence $N(T)$ is a random variable which represents the number of renewals (handovers) in a random interval $[0, T]$ (a CHT). The problem of finding the probability distribution of $N(T)$ has been solved in several specific cases by Cox in his monograph [2, sec. 3.4] under the title "The number of renewals in a random time." Most of the results presented by Cox are based on the *ordinary* renewal process, i.e., all the random variables $X_i, i = 1, 2, \dots$ come from the same distribution [2, p.25]. However, a common situation in cellular networks is that a mobile user begins his call somewhere inside a cell. Thus it is more appropriate to consider the case in which only $X_i, i = 2, 3, \dots$ come from the same distribution as a random variable X_2 while X_1 may come from a different distribution. Such a case is called the *modified* or *delayed* renewal process [2, p.28]. This is just the process that we will use as a model of the sequence of CRTs in this paper. As a special case of the delayed renewal process, if X_1 is a residual life of X_2, we have the *equilibrium* renewal process [2, p.28]. This case has been studied by Lin [6] and the present authors [11, 13, 14]. As a generalization of the delayed renewal process, we may assume that each CRT $X_1, X_2, \dots$ may have different distribution. See Figure 1 for the diagram of a CHT and CRTs associated

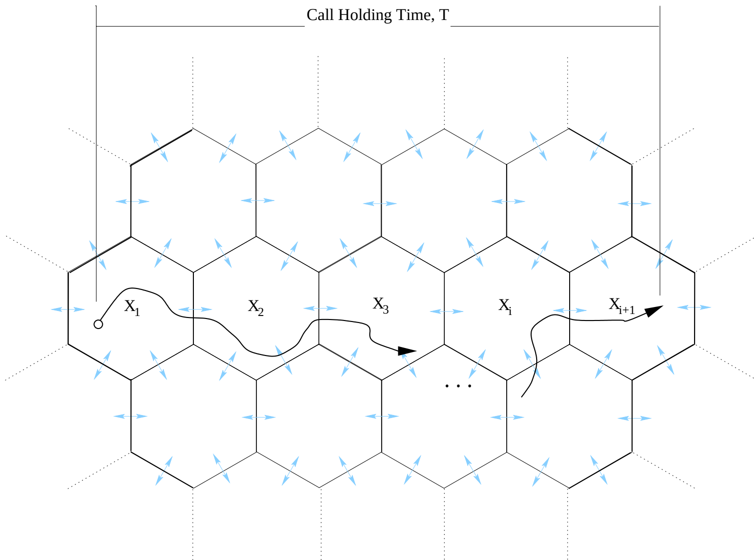

Figure 1: Mobility of a user in a cellular network.

with a mobile user in a cellular network.

The remainder of this paper is organized as follows. In Section 2 we present the basic methodology to calculate the probability generating function (*pgf*) for $N(T)$ for the CHT with Erlang distribution and for the CRT with any distribution. In Section 3, we consider special cases in which the CHT is exponentially distributed. We derive explicit expressions for the probability mass function (*pmf*) $P[N(T) = j]$ as well as the binomial moments $E[\binom{N(T)}{\ell}]$ of $N(T)$ in terms of the Laplace transforms of the probability density function (*pdf*) for X_1 and X_2. In particular, we give simple results for the case of exponentially distributed CRTs. In Section 4, we extend the method of Section 2 to the CHT with a mixture of several exponential distributions, and present closed-form expressions for the *pmf* and the binomial moments of $N(T)$ when the CRT has exponential, gamma, and generalized gamma distributions. We also analyze the case of circular cells such that the first CRT X_1 corresponds to the distance from an arbitrary point in a circle to its perimeter in an arbitrary direction and the subsequent CRTs $X_2, X_3, \ldots$ correspond to the length of a straight line segment cut by a circle. We call this case the circularly distributed CRT. In Section 5 we turn to the case in which the CRTs are exponentially distributed while the CHT has arbitrary distribution. In this case, the *pmf* and the binomial moments of $N(T)$ can be expressed in terms of the Laplace transform of the *pdf* for T. In Section 6, we treat the case in which each CRT $X_1, X_2, \ldots$ may have different distribution. Finally, concluding remarks are made in Section 7.

2. The *pgf* method for Erlang CHT

Let $G_{N(T)}(z)$ be the *pgf* for $N(T)$, the number of handovers in a *random* interval $[0,T]$, where T represents a CHT. It is given by

$$G_{N(T)}(z) = \int_{t=0}^{\infty} G_{N(T)}(t,z)\ f_T(t)\ dt, \tag{1}$$

where $f_T(t)$ is the *pdf* of the random variable T, and

$$G_{N(T)}(t,z) := \mathbf{E}\left[z^{N(t)}\right] = \sum_{j=0}^{\infty} P[N(T)=j|T=t]z^j \tag{2}$$

is the *pgf* of $N(t)$, the number of handovers in a *fixed* interval $[0,t]$. Once $G_{N(T)}(z)$ is obtained, the *pmf* of $N(T)$ is given by

$$P[N(T)=j] = \frac{1}{j!}\frac{d^j}{dz^j}G_{N(T)}(z)\bigg|_{z=0}; \quad j=0,1,2,\ldots\ . \tag{3}$$

The ℓth binomial moment of $N(T)$ is given by

$$\mathbf{E}\left[\binom{N(T)}{\ell}\right] = \frac{1}{\ell!}\frac{d^\ell}{dz^\ell}G_{N(T)}(z)\bigg|_{z=1}; \quad \ell=0,1,2,\ldots\ . \tag{4}$$

Let us consider a special case in which the CHT can be fitted by a k-stage Erlang *pdf*, say

$$f_T(t) = \frac{\lambda^k t^{k-1} e^{-\lambda t}}{(k-1)!}; \quad t \geq 0 \tag{5}$$

with mean $\mathbf{E}[T] = k/\lambda$. In this case the relation between $G_{N(T)}(z)$ and $G_{N(t)}(t,z)$ has been solved by Cox [2, p.43, eq.(4)] as

$$G_{N(T)}(z) = \frac{\lambda^k}{(k-1)!}\left(-\frac{\partial}{\partial s}\right)^{k-1}\left\{G^*_{N(T)}(s,z)\right\}\bigg|_{s=\lambda}, \tag{6}$$

where $G^*_{N(T)}(s,z)$ is the Laplace transform of $G_{N(T)}(t,z)$ defined by

$$G^*_{N(T)}(s,z) := \int_{t=0}^{\infty} e^{-st} G_{N(T)}(t,z) dt. \tag{7}$$

For the *delayed* renewal process $\{X_1, X_2, \ldots\}$ of CRTs, we have [2, p.38, eq.(5)]

$$G^*_{N(T)}(s,z) = \frac{1}{s} + \frac{(z-1)f^*_{X_1}(s)}{s[1-zf^*_{X_2}(s)]}, \tag{8}$$

where $f^*_X(s)$ is the Laplace transform of the *pdf* $f_X(x)$ for the random variable X. Hence we get

$$\begin{aligned} G_{N(T)}(z) &= \frac{\lambda^k}{(k-1)!}\left(-\frac{\partial}{\partial s}\right)^{k-1}\left\{\frac{1}{s} + \frac{(z-1)f^*_{X_1}(s)}{s[1-zf^*_{X_2}(s)]}\right\}\Bigg|_{s=\lambda} \\ &= 1 + \frac{\lambda^k(z-1)}{(k-1)!}\left(-\frac{\partial}{\partial s}\right)^{k-1}\frac{f^*_{X_1}(s)}{s[1-zf^*_{X_2}(s)]}\Bigg|_{s=\lambda}. \end{aligned} \tag{9}$$

From this we can express the *pmf* of $N(T)$ as

$$P[N(T)=j] = \begin{cases} 1 - \dfrac{\lambda^k}{(k-1)!}\left(-\dfrac{\partial}{\partial s}\right)^{k-1}\dfrac{f^*_{X_1}(s)}{s}\Bigg|_{s=\lambda} & ; \quad j=0 \\ \dfrac{\lambda^k}{(k-1)!}\left(-\dfrac{\partial}{\partial s}\right)^{k-1}\dfrac{f^*_{X_1}(s)}{s}[1-f^*_{X_2}(s)][f^*_{X_2}(s)]^{j-1}\Bigg|_{s=\lambda} & ; \quad j=1,2,\ldots \end{cases}. \tag{10}$$

We can also express the ℓth binomial moment of $N(T)$ as

$$\mathbf{E}\left[\binom{N(T)}{\ell}\right] = \frac{\lambda^k}{(k-1)!}\left(-\frac{\partial}{\partial s}\right)^{k-1}\frac{f^*_{X_1}(s)[f^*_{X_2}(s)]^{\ell-1}}{s[1-f^*_{X_2}(s)]^{\ell}}\Bigg|_{s=\lambda} \quad \ell=1,2,\ldots. \tag{11}$$

We note that if $f^*_{X_1}(s) \equiv f^*_{X_2}(s)$, we have an *ordinary* renewal process for the sequence of CRTs. On the other hand, if.

$$f^*_{X_1}(s) \equiv \frac{1-f^*_{X_2}(s)}{\mathbf{E}[X_2]s}, \tag{12}$$

we have the *equilibrium* renewal process. The latter case has been studied extensively by the authors in [14].

3. Exponentially distributed CHT and general CRT

Assume that the CHT T is modeled by an exponential *pdf*, say

$$f_T(t) = \lambda e^{-\lambda t}; \quad t \geq 0 \tag{13}$$

with mean $\mathbf{E}[T] = 1/\lambda$. Then the *pgf* of $N(T)$ is given by

$$G_{N(T)}(z) = \lambda \left\{ G^*_{N(T)}(s,z) \right\} \Big|_{s=\lambda} = 1 + \frac{(z-1)f^*_{X_1}(\lambda)}{1 - z f^*_{X_2}(\lambda)}. \tag{14}$$

The jth derivative of this *pgf* is given by

$$\frac{d^j}{dz^j} G_{N(T)}(z) = \frac{j! f^*_{X_1}(\lambda)[1 - f^*_{X_2}(\lambda)][f^*_{X_2}(\lambda)]^{j-1}}{[1 - z f^*_{X_2}(\lambda)]^{j+1}}; \quad j = 1, 2, \ldots . \tag{15}$$

Substituting (15) into (3), we obtain the *pmf* of $N(T)$ as

$$P[N(T) = j] = \begin{cases} 1 - f^*_{X_1}(\lambda) & ; \quad j = 0 \\ f^*_{X_1}(\lambda)[1 - f^*_{X_2}(\lambda)][f^*_{X_2}(\lambda)]^{j-1} & ; \quad j = 1, 2, \ldots \end{cases} . \tag{16}$$

Substituting (15) into (4), we obtain the ℓth binomial moment of $N(T)$ as

$$\mathbf{E}\left[\binom{N(T)}{\ell} \right] = \frac{f^*_{X_1}(\lambda)[f^*_{X_2}(\lambda)]^{\ell-1}}{[1 - f^*_{X_2}(\lambda)]^{\ell}} \quad \ell = 1, 2, \ldots . \tag{17}$$

In particular, we have the mean

$$\mathbf{E}[N(T)] = \frac{f^*_{X_1}(\lambda)}{1 - f^*_{X_2}(\lambda)}. \tag{18}$$

The variance is given by

$$\mathbf{Var}[N(T)] = \frac{2 f^*_{X_1}(\lambda) f^*_{X_2}(\lambda)}{[1 - f^*_{X_2}(\lambda)]^2} + \mathbf{E}[N(T)] - \mathbf{E}^2[N(T)]. \tag{19}$$

We can study several interesting cases by specifying the *pdf* for both types of CRTs. However, let us defer the most results until Section 4 where we deal with a mixture of exponential distributions for the CHT. In the following subsection, we only consider the case of exponentially distributed CRTs as it reduces to a particularly simple result.

3.1. Exponentially distributed CRT

Let us assume that the first CRT X_1 is exponentially distributed with mean $\mathbf{E}[X_1] = 1/\mu_1$, and the subsequent CRTs, each being represented by X_2, are also exponentially distributed with mean $\mathbf{E}[X_2] = 1/\mu_2$. The corresponding Laplace transforms are given by

$$f^*_{X_r}(s) = \frac{\mu_r}{s + \mu_r}; \quad r = 1, 2. \tag{20}$$

Hence, the *pmf* in (16) reduces to

$$P[N(T)=j]=\begin{cases}\dfrac{1}{1+\rho_1} & ; \quad j=0\\ \left(\dfrac{\rho_1}{1+\rho_1}\right)\left(\dfrac{1}{1+\rho_2}\right)\left(\dfrac{\rho_2}{1+\rho_2}\right)^{j-1} & ; \quad j=1,2,\ldots\end{cases}, \tag{21}$$

where $\rho_r = \mathbf{E}[T]/\mathbf{E}[X_r] = \mu_r/\lambda$ for $r=1,2$. The ℓth binomial moment of $N(T)$ is given by

$$\mathbf{E}\left[\binom{N(T)}{\ell}\right] = \rho_1\left(\frac{1+\rho_2}{1+\rho_1}\right){\rho_2}^{\ell-1}; \quad \ell=1,2,\ldots . \tag{22}$$

Therefore, we have

$$\mathbf{E}[N(T)] = \rho_1\left(\frac{1+\rho_2}{1+\rho_1}\right) \quad ; \quad \mathbf{Var}[N(T)] = \frac{\rho_1(1+\rho_2)(1+2\rho_2+\rho_1\rho_2)}{(1+\rho_1)^2}. \tag{23}$$

4. Hyperexponentially distributed CHT and general CRT

Let us first assume that the CHT T is well modeled by a mixture of M Erlang distributions, say

$$f_T(t) = \sum_{i=1}^{M} p_i \frac{\lambda_i^{k_i} t^{k_i-1} e^{-\lambda_i t}}{(k_i-1)!}; \quad t\geq 0, \tag{24}$$

where $\sum_{i=1}^{M} p_i = 1$, and $k_i; i=1,2,\ldots,M$ is a positive integer. This model may represent a situation such that there are several applications each user can choose from, such as the voice conversation, data transmission for making ticket reservation, and browsing www information on his mobile phone. Application i is chosen with probability $p_i; i=1,2,\ldots,M$. The *pgf* of $N(T)$ is then given by

$$G_{N(T)}(z) = \sum_{i=1}^{M} p_i \frac{\lambda_i^{k_i}}{(k_i-1)!}\left(-\frac{\partial}{\partial s}\right)^{k_i-1}\left\{G^*_{N(T)}(s,z)\right\}\Bigg|_{s=\lambda_i}, \tag{25}$$

where $G^*_{N(T)}(s,z)$ is given by (8). Thus we can obtain the expressions for the *pmf* and moments of $N(T)$ similar to (10) and (11).

If we consider a mixture of exponential distributions for the CHT, or $k_i=1$ for all i in (24), we have a hyperexponential *pdf*

$$f_T(t) = \sum_{i=1}^{M} p_i \lambda_i e^{-\lambda_i t}; \quad t\geq 0, \tag{26}$$

where $\sum_{i=1}^{M} p_i = 1$. In this case, the coefficient of variation for T is larger than unity, which is typical for data communication. Then the *pgf* of $N(T)$ is given by

$$G_{N(T)}(z) = \sum_{i=1}^{M} p_i \lambda_i \left\{ G^*_{N(T)}(s,z) \right\} \Big|_{s=\lambda_i}, \tag{27}$$

where $G^*_{N(T)}(s,z)$ is still given in (8). Thus we get

$$G_{N(T)}(z) = 1 + (z-1) \sum_{i=1}^{M} p_i \frac{f^*_{X_1}(\lambda_i)}{1 - z f^*_{X_2}(\lambda_i)}, \tag{28}$$

where $T = \sum_{i=1}^{M} p_i T_i$ is the mixture of M exponentially distributed random variables T_i with mean $\mathbf{E}[T_i] = 1/\lambda_i$. The jth derivative of this *pgf* is given by

$$\frac{d^j}{dz^j} G_{N(T)}(z) = j! \sum_{i=1}^{M} p_i \frac{f^*_{X_1}(\lambda_i)[1 - f^*_{X_2}(\lambda_i)][f^*_{X_2}(\lambda_i)]^{j-1}}{[1 - z f^*_{X_2}(\lambda_i)]^{j+1}}; \quad j = 1, 2, \ldots . \tag{29}$$

Hence, the *pmf* of $N(T)$ is given by

$$P[N(T) = j] = \begin{cases} 1 - \sum_{i=1}^{M} p_i f^*_{X_1}(\lambda_i) & ; \quad j = 0 \\ \sum_{i=1}^{M} p_i f^*_{X_1}(\lambda_i)[1 - f^*_{X_2}(\lambda_i)][f^*_{X_2}(\lambda_i)]^{j-1} & ; \quad j = 1, 2, \ldots \end{cases} . \tag{30}$$

The ℓth binomial moment of $N(T)$ is given by

$$\mathbf{E}\left[\binom{N(T)}{\ell} \right] = \sum_{i=1}^{M} p_i \frac{f^*_{X_1}(\lambda_i)[f^*_{X_2}(\lambda_i)]^{\ell-1}}{[1 - f^*_{X_2}(\lambda_i)]^{\ell}} \quad \ell = 1, 2, \ldots \tag{31}$$

Thus we have the mean

$$\mathbf{E}[N(T)] = \sum_{i=1}^{M} p_i \frac{f^*_{X_1}(\lambda_i)}{1 - f^*_{X_2}(\lambda_i)}. \tag{32}$$

The variance is given by

$$\mathbf{Var}[N(T)] = 2 \sum_{i=1}^{M} p_i \frac{f^*_{X_1}(\lambda_i) f^*_{X_2}(\lambda_i)}{[1 - f^*_{X_2}(\lambda_i)]^2} + \mathbf{E}[N(T)] - \mathbf{E}^2[N(T)]. \tag{33}$$

We can obtain several interesting cases by specifying the Laplace transforms of the *pdf* for both types of CRTs, as it is shown in the following subsections [12].

4.1. Exponentially distributed CRT

Let us assume that CRTs are exponentially distributed as in (20). Then the *pmf* in (30) reduces to

$$P[N(T)=j]=\begin{cases} 1-\sum_{i=1}^{M} p_i \frac{\rho_{1,i}}{1+\rho_{1,i}} & ; \quad j=0 \\ \sum_{i=1}^{M} p_i \left(\frac{\rho_{1,i}}{1+\rho_{1,i}}\right)\left(\frac{1}{1+\rho_{2,i}}\right)\left(\frac{\rho_{2,i}}{1+\rho_{2,i}}\right)^{j-1} & ; \quad j=1,2,\ldots \end{cases}, \tag{34}$$

where $\rho_{r,i}=\mathbf{E}[T_i]/\mathbf{E}[X_r]=\mu_r/\lambda_i$ for $r=1,2$. The ℓth binomial moment of $N(T)$ is given by

$$\mathbf{E}\left[\binom{N(T)}{\ell}\right]=\sum_{i=1}^{M} p_i\rho_{1,i}\left(\frac{1+\rho_{2,i}}{1+\rho_{1,i}}\right)\rho_{2,i}^{\ell-1}; \quad \ell=1,2,\ldots . \tag{35}$$

Therefore, the mean value is given by

$$\mathbf{E}[N(T)]=\sum_{i=1}^{M} p_i\rho_{1,i}\left(\frac{1+\rho_{2,i}}{1+\rho_{1,i}}\right). \tag{36}$$

and the variance is given by

$$\mathbf{Var}[N(T)]=2\sum_{i=1}^{M} p_i\rho_{1,i}\rho_{2,i}\left(\frac{1+\rho_{2,i}}{1+\rho_{1,i}}\right)+\mathbf{E}[N(T)]-\mathbf{E}^2[N(T)]. \tag{37}$$

4.2. Gamma distributed CRT

We can apply the above formulas to any distribution for X_1 and X_2 with closed-form Laplace transforms. For instance, we can assume that CRTs X_1 and X_2 are gamma distributed with different scale parameters, namely, X_1 is gamma distributed with parameters (μ_1,α_1), and X_2 has the parameters (μ_2,α_2). Their mean values are given by $\mathbf{E}[X_r]=\alpha_r/\mu_r$, $r=1,2$, and their Laplace transforms are as follows:

$$f^*_{X_r}(s)=\left(\frac{\mu_r}{s+\mu_r}\right)^{\alpha_r}; \quad r=1,2. \tag{38}$$

Hence the *pmf* for $N(T)$ is given by

$$P[N(T)=j]=\begin{cases} 1-\sum_{i=1}^{M} p_i \left(\frac{\rho_{1,i}}{{\alpha_1}^{-1}+\rho_{1,i}}\right)^{\alpha_1}; & j=0 \\ \sum_{i=1}^{M} p_i \left(\frac{\rho_{1,i}}{{\alpha_1}^{-1}+\rho_{1,i}}\right)^{\alpha_1} \left[1-\left(\frac{\rho_{2,i}}{{\alpha_2}^{-1}+\rho_{2,i}}\right)^{\alpha_2}\right] \left(\frac{\rho_{2,i}}{{\alpha_2}^{-1}+\rho_{2,i}}\right)^{\alpha_2(j-1)} & \\ & j=1,2,\dots\,. \end{cases} \tag{39}$$

The ℓth binomial moment of $N(T)$ is given by

$$\mathbf{E}\left[\binom{N(T)}{\ell}\right]=\sum_{i=1}^{M} p_i \frac{\left(\frac{\rho_{1,i}}{{\alpha_1}^{-1}+\rho_{1,i}}\right)^{\alpha_1}}{\left(\frac{\rho_{2,i}}{{\alpha_2}^{-1}+\rho_{2,i}}\right)^{\alpha_2}\left[\left(\frac{{\alpha_2}^{-1}+\rho_{2,i}}{\rho_{2,i}}\right)^{\alpha_2}-1\right]^{\ell}}; \quad \ell=1,2,\dots\,. \tag{40}$$

Therefore, the mean value is given by

$$\mathbf{E}[N(T)]=\sum_{i=1}^{M} p_i \frac{\left(\frac{\rho_{1,i}}{{\alpha_1}^{-1}+\rho_{1,i}}\right)^{\alpha_1}}{1-\left(\frac{\rho_{2,i}}{{\alpha_2}^{-1}+\rho_{2,i}}\right)^{\alpha_2}}. \tag{41}$$

4.3. Generalized gamma distributed CRT

According to Zonoozi and Dassanayake [16], the CRT for the first cell and that for the subsequent cells can be modeled by generalized gamma distributions with the *pdf*

$$f_{X_r}(x)=\frac{c_r x^{\alpha_r c_r-1}}{{b_r}^{\alpha_r c_r}\Gamma(\alpha_r)} e^{-(x/b_r)^{c_r}}; \quad x\geq 0,\ \alpha_r, b_r, c_r>0; \quad r=1,2, \tag{42}$$

where $\Gamma(\alpha)=\int_0^\infty u^{\alpha-1}e^{-u}du$, $\alpha>0$, is the gamma function. In fact, by simulations of mobile patterns, a good matching has been produced by assuming that $\alpha_1=0.62$, $b_1\approx 1.85R$, and $c_1=1.88$ for the first cell, and that $\alpha_2=2.31$, $b_2\approx 1.22R$, and $c_2=1.72$ for the subsequent cells, where R is the radius of circular cells. It is interesting to note that these values remain unchanged even for more general assumptions in the simulation such as the changes in velocity and direction of the user movement [16].

The *pdf* in (42) may not have a rational Laplace transform, but its kth moment about the origin is given by

$$\mathbf{E}[X_r^k]=\frac{{b_r}^k\Gamma(\alpha_r+k/c_r)}{\Gamma(\alpha_r)}; \quad k=0,1,2,\dots\,. \tag{43}$$

We can obtain several special cases of (42) by selecting the corresponding parameters. For instance, we can obtain the gamma distribution by choosing $c_r = 1$ and $b_r = 1/\mu_r$, and from here the exponential distribution by letting $\alpha_r = 1$.

We should remember that for a random variable X with *pdf* $f_X(x)$, its Laplace transform $f_X^*(s)$ can be expanded in the moments of X as follows:

$$f_X^*(s) = \sum_{k=0}^{\infty} (-1)^k \frac{s^k}{k!} \mathbf{E}[X^k]. \tag{44}$$

Hence, by using the moments in (43), we can expand $f_{X_r}^*(\lambda_i)$ as

$$f_{X_r}^*(\lambda_i) = 1 + \sum_{k=1}^{\infty} (-1)^k \frac{G_{k,r}}{k! {\rho_{r,i}}^k}; \quad r = 1, 2, \tag{45}$$

where

$$G_{k,r} = \frac{\Gamma^{k-1}(\alpha_r)\Gamma(\alpha_r + k/c_r)}{\Gamma^k(\alpha_r + 1/c_r)}; \quad r = 1, 2 \tag{46}$$

and

$$\rho_{r,i} = \frac{\mathbf{E}[T_i]}{\mathbf{E}[X_r]} = \frac{\Gamma(\alpha_r)}{\lambda_i b_r \Gamma(\alpha_r + 1/c_r)}; \quad r = 1, 2. \tag{47}$$

Note that the dependence on the parameters λ_i and b_r is concentrated only in the relative mobility ratio $\rho_{r,i}$ given in (47). The expansion in (45) can be substituted into (30) and (31) to obtain the *pmf* and the moments of $N(T)$, respectively. We also note that the series expansion for $f_X^*(s)$ in (44) is useful to obtain the *pmf* and the moments of $N(T)$ for those distributions of X that do not have closed-form expression for the Laplace transform.

4.4. Circularly Distributed CRT

The hexagonal geometry for the wireless cells has been approximated by circles by Hong and Rappaport [5] and Yeung and Nanda [15]. They have derived the CRT distributions under the assumptions that the mobile users are uniformly distributed in the system and that they move in straight lines with direction uniformly distributed over $[0, 2\pi)$. The *pdf* for the distance Z_1 from an arbitrary interior point of a circle with radius R to its boundary in an arbitrary direction is given by [5, eq.(46)]

$$f_{Z_1}(z) = \frac{2}{\pi R^2} \sqrt{R^2 - \left(\frac{z}{2}\right)^2}; \quad 0 \le z \le 2R. \tag{48}$$

Hence the CRT in the first cell is given by the random variable$X_1 := Z_1/V$, where V is the velocity of the user, which we will assume to be constant in the rest of this paper. In fact, this is a typical assumption also made in [5] and [15]. Under this assumption of constant velocity, the kth moment of X_1 about the origin is given by

$$\mathbf{E}[{X_1}^k] = \mathbf{E}\left[\frac{{Z_1}^k}{V^k}\right] = \frac{1}{V^k}\int_{z=0}^{2R} z^k f_{Z_1}(z)dz, \tag{49}$$

or equivalently

$$\mathbf{E}[{X_1}^k] = \frac{2}{\pi R^2 V^k}\int_{z=0}^{2R} z^k \sqrt{R^2-(z/2)^2}dz = \frac{\Gamma\left(\frac{k+1}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{k}{2}+2\right)}\left(\frac{2R}{V}\right)^k. \tag{50}$$

Substituting (50) into (44), we obtain

$$f^*_{X_1}(\lambda_i) = \sum_{k=0}^{\infty}(-1)^k \frac{M_{k,1}}{k!{\rho_{1,i}}^k}, \tag{51}$$

where

$$M_{k,1} = \pi^{k-\frac{1}{2}}\left(\frac{3}{4}\right)^k \frac{\Gamma\left(\frac{k+1}{2}\right)}{\Gamma\left(\frac{k}{2}+2\right)} \qquad ; \qquad \rho_{1,i} = \frac{\mathbf{E}[T_i]}{\mathbf{E}[X_1]} = \frac{3\pi V}{8R\lambda_i}. \tag{52}$$

Similarly, the *pdf* for the distance Z_2 from an arbitrary point on the boundary of a circle with radius R at which the mobile user enters the cell to another point on the boundary in a random direction at which he exits from the cell in a straight line is given by [5, eq.(51)]

$$f_{Z_2}(z) = \frac{2}{\pi\sqrt{4R^2-z^2}}; \quad 0 \le z < 2R. \tag{53}$$

We note that the path of a user in this case is equivalent to the *random chord* of a circle which has been studied in the field of geometrical probability [8, p.198]. There are several ways to define the *randomness* of the chord which lead to different probability measures. The most appropriate one in the modeling of user movement in cellular systems seems to be that the direction of the path is uniformly distributed over $[0, 2\pi)$, which is called *S-randomness* [1]. In fact, the *pdf* in (53) is exactly the same as the *pdf* for the length of a random chord of a circle with radius R in the sense of S-randomness [8, eq.(2.3.41), p.198]. Then the kth moment of the CRT $X_2 := Z_2/V$ for the second and subsequent cells is given by

$$\mathbf{E}[{X_2}^k] = \frac{\Gamma\left(\frac{k+1}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{k+2}{2}\right)}\left(\frac{2R}{V}\right)^k. \tag{54}$$

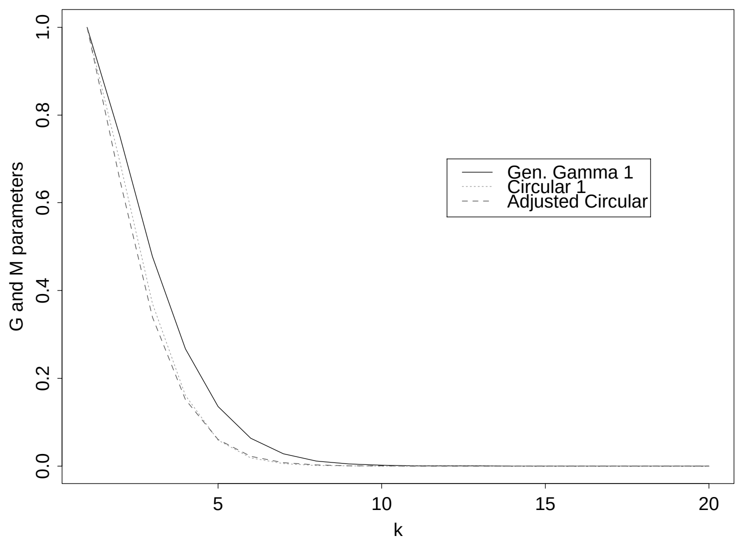

Figure 2: Plots of $G_{k,1}/k!$ and $M_{k,1}/k!$ as a function of k. The generalized gamma case assumes $\alpha_1 = 0.62$ and $c_1 = 1.88$, and the adjusted circular case $\alpha_1 = 1.0$ and $c_1 = 1.88$.

Substituting (54) into (44), we obtain

$$f^*_{X_2}(\lambda_i) = \sum_{k=0}^{\infty} (-1)^k \frac{M_{k,2}}{k! {\rho_{2,i}}^k}, \tag{55}$$

where

$$M_{k,2} = \frac{\pi^{k-\frac{1}{2}}}{2^k} \frac{\Gamma\left(\frac{k+1}{2}\right)}{\Gamma\left(\frac{k}{2}+1\right)} \qquad ; \qquad \rho_{2,i} = \frac{\mathbf{E}[T_i]}{\mathbf{E}[X_2]} = \frac{\pi V}{4R\lambda_i}. \tag{56}$$

Let us call the above distributions for X_1 and X_2 the circular distributions.

We note that the dependence of $f^*_{X_r}(\lambda_i)$ on the parameters λ_i, V, and R is all concentrated in the relative mobility ratios $\rho_{r,i}$ given in (52) and (56). We also observe that the series expansions in the above for the circular distributions are similar to the series expansion of the generalized gamma distribution in (45). In the above derivation, the parameter $M_{k,r}$ is a function of only k for $r = 1, 2$. Thus we can match the series expansion for the generalized gamma distribution in (45) with those in (51) and (55) for the circular distributions. We have found that the parameters reported in [16] for α_r and c_r produce reasonable matching, as shown in Figures 2 and 3. We can

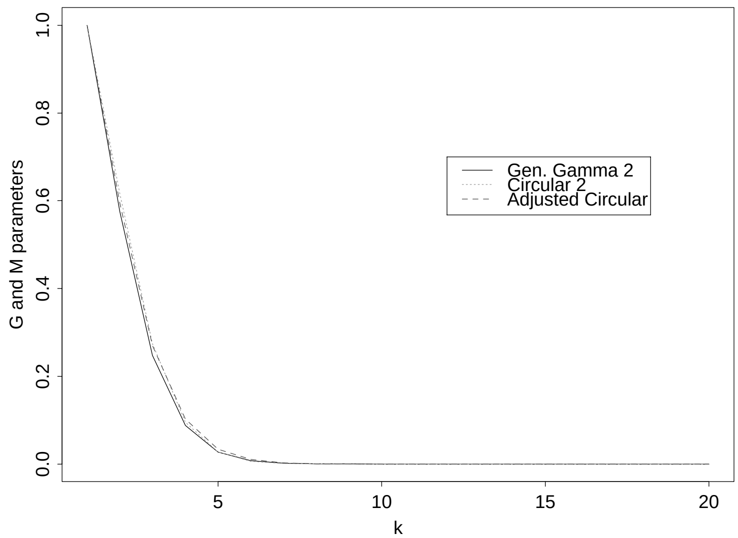

Figure 3: Plots of $G_{k,2}/k!$ and $M_{k,2}/k!$ as a function of k. The generalized gamma case assumes $\alpha_2 = 2.31$ and $c_2 = 1.72$, and the adjusted circular case $\alpha_2 = 1.85$ and $c_2 = 1.72$.

present better matching to the circularly distributed case by adjusting the parameters of the generalized gamma distribution. While many combinations of the values for α_r and c_r are possible, we show in Figure 2 only one of them by fixing $c_1 = 1.88$ and finding $\alpha_1 = 1.0$. Similarly, in Figure 3 we fix $c_2 = 1.72$ and find $\alpha_2 = 1.85$ for the best matching. We may call the generalized gamma distribution with these parameter values the adjusted circular distribution.

It may also be interesting to compare the *pdf* $f_{Z_1}(z)$ for Z_1 given in (48) and the *pdf* for $\hat{Z}_2$, the residual life of Z_2, which is given by

$$\begin{aligned} f_{\hat{Z}_2}(z) &= \frac{1}{\mathbf{E}[Z_2]}\left[1 - \int_0^z f_{Z_2}(x)dx\right] = \frac{\pi}{4R}\left[1 - \frac{2}{\pi}\arcsin\left(\frac{z}{2R}\right)\right] \\ &= \frac{1}{2R}\arccos\left(\frac{z}{2R}\right); \quad 0 \le z \le 2R. \end{aligned} \tag{57}$$

As shown in Figure 4, they are different. This fact may justify the analysis of the two platform system in this paper.

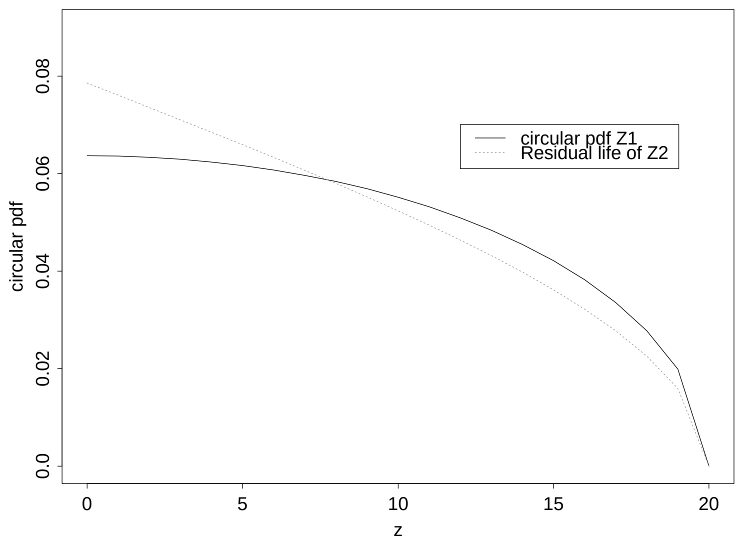

Figure 4: Plots of the *pdf* $f_{Z_1}(z)$ for Z_1 and the *pdf* for the residual life of Z_2.

5. General CHT and exponentially distributed CRT

In this section, we consider a general *pdf* for the CHT, say $f_T(t)$, and exponentially distributed CRTs as in (20).

In this case, the *pgf* for $N(t)$ is given by

$$G_{N(T)}(t,z) = \frac{(\mu_2-\mu_1)(z-1)}{\mu_1+\mu_2(z-1)}\, e^{-\mu_1 t} + \frac{\mu_1 z}{\mu_1+\mu_2(z-1)}\, e^{-\mu_2(1-z)t}. \tag{58}$$

This follows by substituting (20) into (8), expanding the resulting function in partial fractions in s, and then inverting the Laplace transform. We remark that the corresponding *pgf* for the ordinary and equilibrium renewal processes can be obtained by making $\mu_1 = \mu_2 = \mu$. In such a case, we get $G_{N(T)}(t,z) = e^{-\mu(1-z)t}$ [14].

The *pgf* for $N(T)$ is given by

$$G_{N(T)}(z) = \frac{(\mu_2-\mu_1)(z-1)}{\mu_1+\mu_2(z-1)}\, f_T^*(\mu_1) + \frac{\mu_1 z}{\mu_1+\mu_2(z-1)}\, f_T^*[\mu_2(1-z)], \tag{59}$$

which follows by substituting (58) into (1).

We need the jth derivative of $G_{N(T)}(z)$ in order to find the *pmf* and the moments of $N(T)$. It is given by

$$\begin{aligned}\frac{d^j G_{N(T)}(z)}{dz^j} &= \frac{(-1)^{j-1} j! \mu_1 (\mu_2 - \mu_1) {\mu_2}^{j-1}}{[\mu_1 + \mu_2(z-1)]^{j+1}} \, f_T^*(\mu_1) \\ &+ (-1)^j j! \mu_1 (\mu_2 - \mu_1) {\mu_2}^{j-1} \sum_{i=0}^{j-1} \frac{f_T^{*(i)}[\mu_2(1-z)]}{i![\mu_1 + \mu_2(z-1)]^{j-i+1}} \\ &+ \frac{(-1)^j \mu_1 {\mu_2}^j z}{\mu_1 + \mu_2(z-1)} \, f_T^{*(j)}[\mu_2(1-z)]; \qquad j = 1, 2, \dots ,\end{aligned} \tag{60}$$

where

$$f_T^{*(j)}(s) := \frac{d^j f_T^*(s)}{ds^j} = (-1)^j \int_{t=0}^{\infty} t^j e^{-st} f_T(t) dt; \quad j = 1, 2, \dots . \tag{61}$$

From (60), we find the *pmf* for $N(T)$ as

$$P[N(T) = j] = \begin{cases} f_T^*(\mu_1) & ; \quad j = 0 \\ \dfrac{\mu_1 {\mu_2}^{j-1}}{(\mu_2 - \mu_1)^j} \left[f_T^*(\mu_1) - \displaystyle\sum_{i=0}^{j-1} \frac{(\mu_1 - \mu_2)^i}{i!} f_T^{*(j)}(\mu_2) \right] & ; \quad j = 1, 2, \dots \end{cases} . \tag{62}$$

The ℓth binomial moment of $N(T)$ is given by

$$\begin{aligned}\mathbf{E}\left[\binom{N(T)}{\ell} \right] &= \left(1 - \frac{\mu_1}{\mu_2}\right) \left(\frac{\mu_2}{\mu_1}\right)^\ell \left\{ (-1)^{\ell-1} f_T^*(\mu_1) + \sum_{i=0}^{\ell-1} \frac{(-1)^{\ell-i}}{i!} {\mu_1}^i \mathbf{E}[T^i] \right\} \\ &+ \frac{{\mu_2}^\ell}{\ell!} \mathbf{E}[T^\ell]; \quad \ell = 1, 2, \dots .\end{aligned} \tag{63}$$

For example, the mean is given by

$$\mathbf{E}[N(T)] = \frac{\mu_2 - \mu_1}{\mu_1} [f_T^*(\mu_1) - 1] + \mu_2 \mathbf{E}[T]. \tag{64}$$

If the CHT is exponentially distributed as in (13), we recover the results in Section 3.1.

6. Generalized delayed renewal process for CRTs

Let us generalize the delayed renewal process of CRTs so that each CRT $X_1, X_2, \dots$ may have different distribution. A similar situation is considered in [10].

Suppose that the first R CRTs $X_1, X_2, \dots, X_R$ may have different distributions for which the Laplace transforms of the *pdf* are given by $f_{X_1}^*(s), f_{X_2}^*(s), \dots, f_{X_R}^*(s)$, respectively, and that the subsequent CRTs $X_{R+1}, X_{R+2}, \dots$ have the same distribution as X_R. As special cases of this process, we have an ordinary renewal process for $R = 1$, a delayed renewal process for $R = 2$, and the case in which all CRTs are distinct for $R = \infty$.

For this process, from [2, p.37,eq.(3)], we have

$$\begin{aligned} G^*_{N(T)}(s,z) &= \frac{1}{s} + \frac{z-1}{s}\left[\sum_{j=1}^{R-1} z^{j-1}\prod_{r=1}^{j} f^*_{X_r}(s) + \sum_{j=R}^{\infty} z^{j-1}F^*_R(s)\left[f^*_{X_R}(s)\right]^{j-R}\right] \\ &= \frac{1}{s} + \frac{z-1}{s}\sum_{j=1}^{R-1} z^{j-1}\prod_{r=1}^{j} f^*_{X_r}(s) + \frac{(z-1)z^{R-1}F^*_R(s)}{s\left[1-zf^*_{X_R}(s)\right]}, \end{aligned} \tag{65}$$

where we have introduced for notational convenience

$$F^*_R(s) := \prod_{r=1}^{R} f^*_{X_r}(s). \tag{66}$$

This result is a generalization of (8).

6.1. Exponentially distributed CHT

If the CHT T is exponentially distributed with mean $\mathbf{E}[T] = 1/\lambda$ as in (13), we have the *pgf* for $N(T)$ as

$$\begin{aligned} G_{N(T)}(z) &= \lambda G^*_{N(T)}(s,z)\Big|_{s=\lambda} \\ &= 1 + (z-1)\sum_{j=1}^{R-1} z^{j-1}\prod_{r=1}^{j} f^*_{X_r}(\lambda) + \frac{(z-1)z^{R-1}F^*_R(\lambda)}{1-zf^*_{X_R}(\lambda)}. \end{aligned} \tag{67}$$

It is straightforward as before to obtain the *pmf* and moments of $N(T)$ from (67). As the coefficient of z^j in the expansion of (67) in powers of z, the *pmf* of $N(T)$ is given by

$$P[N(T)=j] = \begin{cases} 1 - f^*_{X_1}(\lambda) & ; \quad j = 0 \\ \left[1 - f^*_{X_{j+1}}(\lambda)\right]\prod_{r=1}^{j} f^*_{X_r}(\lambda) & ; \quad 1 \le j \le R-1 \\ F^*_R(\lambda)\left[1 - f^*_{X_R}(\lambda)\right]\left[f^*_{X_R}(\lambda)\right]^{j-R} & ; \quad j \ge R \end{cases}. \tag{68}$$

As the coefficient of $(z-1)^\ell$ in the expansion of (67) in powers of $z-1$, the ℓth binomial moment of $N(T)$ is given by

$$\mathbf{E}\left[\binom{N(T)}{\ell}\right] = \begin{cases} \sum_{j=\ell}^{R-1} \binom{j-1}{\ell-1} \prod_{r=1}^{j} f^*_{X_r}(\lambda) + F^*_R(\lambda) \sum_{j=0}^{\ell-1} \binom{R-1}{\ell-j-1} \frac{[f^*_{X_R}(\lambda)]^j}{[1-f^*_{X_R}(\lambda)]^{j+1}} \; ; \\ \qquad\qquad 1 \le \ell \le R-1 \\ F^*_R(\lambda) \sum_{j=\ell-R}^{\ell-1} \binom{R-1}{\ell-j-1} \frac{[f^*_{X_R}(\lambda)]^j}{[1-f^*_{X_R}(\lambda)]^{j+1}} \; ; \quad \ell \ge R \end{cases} . \tag{69}$$

In particular, the mean is given by

$$\mathbf{E}[N(T)] = \sum_{j=1}^{R-1} \prod_{r=1}^{j} f^*_{X_r}(\lambda) + \frac{F^*_R(\lambda)}{1-f^*_{X_R}(\lambda)}. \tag{70}$$

All the above expressions reduce to those in Section 3 when $R = 2$.

For $R = \infty$, by assuming that $\lim_{R\to\infty} F^*_R(\lambda) = 0$ for $\lambda > 0$, the *pgf* of $N(T)$ is given by

$$G_{N(T)}(z) = 1 + (z-1) \sum_{j=1}^{\infty} z^{j-1} \prod_{r=1}^{j} f^*_{X_r}(\lambda). \tag{71}$$

Thus we have

$$P[N(T) = j] = \begin{cases} 1 - f^*_{X_1}(\lambda) & ; \quad j = 0 \\ \left[1 - f^*_{X_{j+1}}(\lambda)\right] \prod_{r=1}^{j} f^*_{X_r}(\lambda) & ; \quad j = 1, 2, \ldots \end{cases} \tag{72}$$

and

$$\mathbf{E}\left[\binom{N(T)}{\ell}\right] = \sum_{j=\ell}^{\infty} \binom{j-1}{\ell-1} \prod_{r=1}^{j} f^*_{X_r}(\lambda); \quad \ell = 1, 2, \ldots \; . \tag{73}$$

6.2. General CHT and exponentially distributed CRTs

As an extension to the case of Section 5, we can consider a general *pdf* for the CHT and exponentially distributed CRTs as

$$f^*_{X_r}(s) = \frac{\mu_r}{s + \mu_r}; \quad r = 1, 2, \ldots, R. \tag{74}$$

Let us assume for simplicity that all μ_r's are distinct. Substituting (74) into (65) and expanding in partial fractions in s yields

$$G^*_{N(T)}(s,z) = (1-z)\sum_{r=1}^{R-1} \frac{B_r(z)}{s+\mu_r} + \frac{C(z)}{s+\mu_R(1-z)}, \tag{75}$$

where

$$B_r(z) := \frac{(\mu_r - \mu_R + \mu_R z)\sum_{j=r}^{R-2} z^{j-1}\left[\prod_{i=1}^{j}\mu_i\right]\left[\prod_{i=j+1}^{R-1}(\mu_i-\mu_r)\right] + z^{R-2}(\mu_r-\mu_R)\prod_{j=1}^{R-1}\mu_j}{\mu_r(\mu_r-\mu_R+\mu_R z)\prod_{j=1(j\neq r)}^{R-1}(\mu_j-\mu_r)};\quad r=1,2,\ldots,R-1 \tag{76}$$

and

$$C(z) := z^{R-1}\prod_{j=1}^{R-1}\frac{\mu_j}{\mu_j-\mu_R+\mu_R z}. \tag{77}$$

Substituting the inversion of (75) into (1), we obtain the *pgf* of $N(T)$ as

$$G_{N(T)}(z) = (1-z)\sum_{r=1}^{R-1} B_r(z) f^*_T(\mu_r) + C(z) f^*_T[\mu_R(1-z)], \tag{78}$$

where $f^*_T(s)$ is the Laplace transform of the *pdf* for the generally distributed CHT T.

It is possible to derive the *pmf* and moments of $N(T)$ from (78). For example, the mean is given by

$$\mathbf{E}[N(T)] = -\sum_{r=1}^{R-1} B_r(1) f^*_T(\mu_r) + R + \mu_R\left(\mathbf{E}[T] - \sum_{r=1}^{R}\frac{1}{\mu_r}\right), \tag{79}$$

where

$$B_r(1) = \frac{\mu_r\sum_{j=r}^{R-2}\left[\prod_{i=1}^{j}\mu_i\right]\left[\prod_{i=j+1}^{R-1}(\mu_i-\mu_r)\right] + (\mu_r-\mu_R)\prod_{j=1}^{R-1}\mu_j}{{\mu_r}^2\prod_{j=1(j\neq r)}^{R-1}(\mu_j-\mu_r)};\quad r=1,2,\ldots,R-1. \tag{80}$$

These results reduce to those in Section 5 when $R=2$.

7. Conclusions

In this paper, we have derived explicit forms for the *pmf* and the statistical moments of the number of handovers during a random call holding time (CHT) when the CHT distribution is well-fitted by a mixture of exponential distributions while the cell residence time (CRT) is arbitrarily distributed. As specific distributions for the CRT, we have dealt with exponential, gamma, and generalized gamma distributions as well as the distribution that comes from the length of a random chord of a circle for a model of circular cells. We have provided good numerical matching between the generalized gamma distribution and the circular distribution, which improves the result in [16]. We have also considered the case of general CHT and exponentially distributed CRTs.

In the present study, we have first assumed two different platforms for cellular regions. This model may be interesting for situations where the call originates in a picocellular cell and the portable moves into the microcellular environment or vice versa. Our mathematical framework is the delayed renewal process [12] as a generalization of the equilibrium renewal process studied in [11, 13, 14] for the homogeneous platform. We have also presented the case of more than two platforms.

Many other interesting combinations of the CHT and CRT distributions can be handled in the same framework. We also remark that obtaining the *pmf* and the moments for the number of handovers during a call is an important step for obtaining other performance measures, including the probability of completing a call and the handover traffic rate, for mobile communication networks.

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