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Intergenerational Externalities and Optimal Taxation of Elderly Care Services

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Abstract

This paper explores the problem of optimal taxation on elderly care services. In an altruistic economy, the time allocated by children to caring for their parents in the home will be short of the socially optimal level. The first-best tax rates on the care services for internalizing such an intergenerational externality are characterized. Their second-best tax rates are considered in the case where the government cannot levy lump-sum taxes on the young and old generations.

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1. Introduction

Caring for the elderly is now an important policy issue in Japan where aging is rapidly progressing. Although households so far produce most elderly care services, private firms have also started to provide them as their demands increase. Since only the services supplied by the firms can be taxed, the following optimal taxation problem arises: what tax rates should be imposed on the care services relative to other services?

Sandmo (1990) and Kleven, Richter, and Sorensen (2000) examined the second-best taxation problem in an economy, where consumer services such as home repair, car maintenance, and garden care not only are supplied from the market but also are produced within the household sector.[1] In their models, consumers use their own labour as an input for home production of the services. Therefore, we cannot use the models to consider the optimal taxation problem of elderly care services, because those needing care cannot provide it themselves. They need the effort of the young to produce the care services in home.

In this paper, we study the optimal taxation problem on elderly care services in an altruistic economy, where children are willing to devote time to the home production of the service. Since they discount parent's utility relative to their own utilities, the allocated care time is short of the first-best level. The government would like to internalize this intergenerational externality by carrying out the Pigouvian policy. However, it will be difficult to subsidize the care time directly, since the value of home production is untaxable. Instead, it need consider an indirect policy that levies taxes on the service and the labour traded in the market. Although it seems to be intuitively desirable that they are both taxed to correct the externality, it is not always optimal to assess them due to the distortional effects on the service demand and the labour supply. We clarify the circumstances under which the government should subsidize either of them by comparing the opposite effects of each tax on allocation, that is, the positive

[1] They showed that since a decrease in leisure may not raise labour supply, the Corlett-Hague rule, which insists that a relatively high tax rate should be imposed on services that are complements to leisure, does not always hold.

externality-correcting effect and the negative distortional effect.

The government requires lump-sum taxes on the young and old generations to attain the first-best optimum. If these taxes are not available, it can pursue a second-best tax rule. In the second-best optimum, each tax consists of the sum of the internalization component and the Ramsey component. When children do not discount their parent's utility very much and the population growth rate is close to zero, the internalization component can be neglected. Then the tax rule is similar to that derived by Kleven et al. (2000) in a static model without externalities. Hence, if the care service and leisure are complements and a large fraction of the service demand is satisfied through home production, the service should be subsidized or imposed at a relatively low tax rate. This tax system is even more beneficial if they are substitutes.

This paper is organized as follows. In section 2, we develop a two-period overlapping generations model with home production of an elderly care service. In section 3, we characterize the first-best tax rule when the government can use intergenerational lump-sum taxes. In section 4, we characterize the second-best tax rule in the case where these taxes are not available. Finally, Section 5 concludes the paper.

2. An Overlapping Generations Model with Home Production

In this section, we develop an overlapping generations model extended to allow for the existence of a service, care for the elderly, and its market and home production. The closed economy considered here consists of homogeneous consumers and competitive firms. At first, we will formulate the behaviour of a representative consumer who lives for two periods, working in the young and being retired in the old.

The representative consumer born at period t has $1+n\ (n \geq 0)$ children at the beginning of the old period of his life.[2] He cares about the welfare of his parent, which we call the "gift motive". Following Carmichael (1982), this concern is modeled by

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[2] This follows the standard convention of ignoring the fact that it takes two people from different families to produce children. Thus, each consumer has $1+n$ children and has *one* parent.

including the maximum attainable utility of the parent as an argument in the consumer's utility function. The parent's behaviour is influenced by time that children allocate to look after their parent.[3] In order to simplify the model, we assume that consumers do not consume in youth. Then, his utility function is specified as

$$u_t = v(b_t, c_{t+1}, d_{t+1}) + \delta u^*_{t-1}, \tag{1}$$

where b_t is his young-period leisure, c_{t+1} is his old-period consumption, d_{t+1} is his demand for the care service in old age, and u^*_{t-1} is his parent's indirect utility function as perceived by him.[4] The parameter δ measures the strength of the "gift" motive. It is assumed that $0 < \delta < 1$, because the individual's utility function becomes $u = v(b, c, d)/(1-\delta)$ in a steady state in which each per worker variable is equal from generation to generation.[5]

The consumer has one unit of time endowment in the first period of his life. Denoting time devoted to looking after his parent by e_t, his labour supply l_t is equal to $1 - b_t - e_t$. His budget constraint in the young period is given by

$$s_t = (1-\gamma_t) w_t (1 - b_t - e_t) - T^y_t, \tag{2}$$

where s_t is his savings, w_t is a wage rate, γ_t is a tax rate on labour income, and T^y_t is a lump-sum tax. His income in the old period is $[1 + (1-\theta_{t+1}) r_{t+1}] s_t - T^o_{t+1}$, where r_{t+1} is an interest rate, θ_{t+1} is a tax rate on interest income, and T^o_{t+1} is a lump-sum tax. He uses this to purchase the consumption good c_{t+1} and the care service x_{t+1} at the

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[3] The literature since Barro (1974) has focused mainly on two patterns of intergenerational altruism: in the first, parents care about their children and leave them bequests; in the second, children care about parents and provide them with gifts in their old age. In this paper, however, we ignore bequests and consider only gifts so as to focus attention on the problem of caring for the old.

[4] It is assumed that the utility function $v(b,c,d)$ is twice differentiable and strictly quasi-concave with positive but diminishing marginal utilities (v_b, v_c, v_d), where $v_b(0,c,d) = v_c(b,0,d) = v_d(b,c,0) = \infty$ and $v_b(\infty,c,d) = v_c(b,\infty,d) = v_d(b,c,\infty) = 0$.

[5] The utility function $u_t = v_t + \delta u_{t-1}$ implies that $u_t = \sum_{j=0}^{\infty} \delta^j v_{t-j}$. Therefore, if $\delta > 1$, then the utility u_t is unbounded as t approaches infinity.

prices 1 and p_{t+1} from the good and service markets, respectively. Thus, denoting *ad valorem* tax rates on c_{t+1} and x_{t+1} by τ^c_{t+1} and τ^x_{t+1}, respectively, his budget constraint in the old period is written as

$$(1+\tau^c_{t+1})c_{t+1} + (1+\tau^x_{t+1})p_{t+1}x_{t+1} = [1+(1-\theta_{t+1})r_{t+1}]s_t - T^o_{t+1}. \tag{3}$$

The consumer's service demand d_{t+1} is the sum of the service x_{t+1} bought in the market and that produced in the household. The home-produced service y_{t+1} is produced by using the care time e_{t+1} from each child through the home production function $y_{t+1} = H[(1+n)e_{t+1}]$, where $H' > 0$ and $H'' < 0$.[6] Thus, we have

$$x_{t+1} = d_{t+1} - y_{t+1} = d_{t+1} - H[(1+n)e_{t+1}]. \tag{4}$$

Eliminating s_t and x_{t+1} from (2)-(4), then the life-time budget constraint of the representative consumer born at period t can be summarized as

$$q^1_{t+1}c_{t+1} + q^2_{t+1}d_{t+1} + q^3_t b_t = q^3_t(1-e_t) + q^2_{t+1}H[(1+n)e_{t+1}] - T_t, \tag{5}$$

where

$$q^1_{t+1} \equiv (1+\tau^c_{t+1})/(1+\rho_{t+1}), \quad q^2_{t+1} \equiv (1+\tau^x_{t+1})p_{t+1}/(1+\rho_{t+1}), \quad q^3_t \equiv (1-\gamma_t)w_t,$$

$$T_t \equiv T^y_t + T^o_{t+1}/(1+\rho_{t+1}), \quad \text{and} \quad \rho_{t+1} \equiv (1-\theta_{t+1})r_{t+1}.$$

The consumer price vector and the present value of lump-sum taxes are given by $q_t \equiv (q^1_{t+1}, q^2_{t+1}, q^3_t)$ and T_t, respectively.

Given q_t, T_t, and e_{t+1}, the consumer maximizes the utility function (1) by choosing $(c_{t+1}, d_{t+1}, b_t, e_t)$ subject to the budget constraint (5). This problem is solved by the two stages as follows. At first, minimize $q^1_{t+1}c_{t+1} + q^2_{t+1}d_{t+1} + q^3_t b_t$ with respect to c_{t+1}, d_{t+1},

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[6] Like Sandmo (1990), we assume that home production function is subject to *diminishing* returns although the production function in the market-produced service is constant returns to scale. The returns to home production are restricted by the fixity of the physical infrastructure in the household.

and b_t subject to $v_t = v(b_t, c_{t+1}, d_{t+1})$, where v_t is given. Then, we have the expenditure function, $E(q^1_{t+1}, q^2_{t+1}, q^3_t, v_t)$, which has the well-known properties:

$$E = \sum_{i=1}^{3} q^i E_i \text{ and } \sum_{i=1}^{3} q^i E_{ji} = 0 \text{ for } j = 1,2,3, \tag{6}$$

where

$$E_1 \equiv \partial E / \partial q^1 = c^*, \; E_2 = d^*, \; E_3 = b^*,$$

$$E_{11} = c_1^* < 0, \; E_{22} = d_2^* < 0, \; E_{33} = b_3^* < 0, \text{ and } E_{ij} = E_{ji}, \; i \neq j.$$

Next, noting that u^*_{t-1} depends on e_t, maximize $u_t = v_t + \delta u^*_{t-1}$ with respect to v_t and e_t subject to[7]

$$E(q^1_{t+1}, q^2_{t+1}, q^3_t, v_t) = q^3_t(1-e_t) + q^2_{t+1} H[(1+n)e_{t+1}] - T_t. \tag{7}$$

The first-order conditions for an internal maximum are given by $1 = \alpha_t(\partial E / \partial v_t)$ and

$$\delta \frac{\partial u^*_{t-1}(e_t)}{\partial e_t} = \alpha_t q^3_t, \tag{8}$$

where α_t is the Lagrange multiplier for the constraint (7). Now, using the envelope theorem, we can derive

$$\frac{\partial u^*_t(e_{t+1})}{\partial e_{t+1}} = \alpha_t q^2_{t+1} \frac{\partial H}{\partial e_{t+1}}. \tag{9}$$

Eliminating α_t from (8) and (9), we have

$$\delta \frac{\partial u^*_{t-1}(e_t)}{\partial e_t} - \frac{q^3_t}{(1+n) q^2_{t+1} H'} \frac{\partial u^*_t(e_{t+1})}{\partial e_{t+1}} = 0.$$

[7] Since we treat each generation as a representative agent, we do not deal with the strategic issues that arise when $1+n$ children are *separately* looking after the same parent.

In the steady state, this equation becomes

$$\delta(1+n)q^2H'[(1+n)e]-q^3=0. \tag{10}$$

Therefore, we obtain the *supply* function of the care time, $e=e^*(q^2,q^3)$, where $e_1^*\equiv\partial e^*/\partial q^2=-H'/q^2H''>0$ and $e_2^*=1/\delta(1+n)^2q^2H''<0$. There is the following relationship between e_1^* and e_2^*:

$$e_1^*=-\delta(1+n)^2H'e_2^*. \tag{11}$$

Substituting $e=e^*$ into (7), the lifetime budget constraint in the steady state reduces to

$$E=q^3(1-e^*)+q^2y^*-T, \tag{12}$$

where $y^*=H[(1+n)e^*]$. This budget constraint implicitly defines the steady state level of the utility v as a function of q and T.[8] Since the compensated labour supply function is given by $l=l^*(q^1,q^2,q^3,v)=1-E_3-e^*$, we obtain $l_1^*=-E_{31}$, $l_2^*=-E_{32}-e_1^*$, and $l_3^*=-E_{33}-e_2^*>0$. The compensated demand function of the market-produced care service is given by $x=x^*(q^1,q^2,q^3,q^4,v)=E_2-y^*$. Therefore, we have $x_1^*=E_{21}$, $x_2^*=E_{22}-y_1^*>0$, and $x_3^*=E_{23}-y_2^*$.

Next, we formulate the behaviours of competitive firms. In period t, using labour L_t^X as the only input, they produce the output X_t of the care service subject to a linear production function with the input-output ratio equal to 1, i.e., $X_t=L_t^X$. Since they maximize profits and markets are assumed to be competitive, the equilibrium profits are zero. Thus, the market price p_t of the service is equal to the wage rate w_t. They also produce the output Y_t of the good, which is homogeneous and can be either be consumed or used as capital, subject to a neoclassical linearly homogeneous production function: $Y_t=F(K_t,L_t^Y)$, where K_t is the stock of capital and L_t^Y is the

[8] The steady state level of the utility u is given by $u=v/(1-\delta)$.

labour input. This function can be written in intensive form as $Y_t / L_t^Y = f(k_t)$, where k_t is the capital-labour ratio, $k_t \equiv K_t / L_t^Y$, $f' > 0$, and $f'' < 0$. Since each factor is paid its marginal product in competitive factor markets, it holds that $r_t = f'(k_t)$ and $w_t = f(k_t) - k_t f'(k_t)$. From these relations, we have the factor price frontier, $w_t = w(r_t)$, where $w'(r_t) = -k_t$.

Finally, the equilibrium conditions in the good, service, labour, and capital markets are written as, $Y_t + K_t = N_{t-1}c_t + K_{t+1} + N_t g$, $X_t = N_{t-1}x_t$, $N_t l_t = L_t^Y + L_t^X$, and $N_t s_t = K_{t+1}$, respectively, where N_t is the size of the working age population in period t and g is the constant government's expenditure per worker. These conditions can be rewritten in terms of *per worker* variables:

$$l_t^Y [f(k_t) + k_t] = \frac{c_t}{1+n} + (1+n)k_{t+1}l_{t+1}^Y + g, \tag{13.1}$$

$$x_t = (1+n)l_t^X, \tag{13.2}$$

$$l_t = l_t^Y + l_t^X, \tag{13.3}$$

$$s_t = (1+n)k_{t+1}l_{t+1}^Y, \tag{13.4}$$

where $l_t^Y \equiv L_t^Y / N_t$ and $l_t^X \equiv L_t^X / N_t$. Eliminating l^Y and l^X from (13.1)-(13.4) and using (3) and (4), then we obtain the following equations in the steady state:

$$[(1+n)l^* - E_2 + y^*][w(r) + (n-r)w'(r)] = E_1 + (1+n)g, \tag{14.1}$$

$$\sum_{i=1}^{2} q^i E_i - q^2 y^* + T - T^y = -w'(r)[(1+n)l^* - E_2 + y^*]. \tag{14.2}$$

From $p = w(r)$ and $\rho = 1 + (1-\theta)r$, the consumer price vector q and the present value of lump-sum taxes T depend on the rate of interest r and six policy variables

$(\tau^c,\tau^x,\gamma,\theta,T^y,T^o)$.[9] Hence, given the independent five policy variables, (13.1) and (13.2) together with (12) determine the steady-state levels of r, the utility v, and the remaining policy variable.

3. The First-Best Tax Rule

In this section, we derive the first-best tax rule where lump-sum taxes may be levied in both the young and old periods. The government objective in a steady state is to choose the policy variables $(T^y,T^o,\tau^c,\tau^x,\gamma,\theta)$ so as to maximize the utility v. We can solve this maximization problem in terms of the lump-sum taxes (T^y,T), the consumer price vector $q=(q^1,q^2,q^3)$, and the interest rate r.[10] Let ψ, ϕ, and λ be the Lagrange multipliers for the constraints (12), (14.1), and (14.2), respectively. Since the first-order conditions with respect to T^y and T are given by $\lambda=0$ and $\psi+\lambda=0$, respectively, we have $\psi=\lambda=0$. Thus, we need not consider the constraints (12) and (14.2) to derive the first-order conditions with respect to (r,q^1,q^2,q^3). From the condition with respect to r, we obtain $r=n$. Hence, the *golden rule* holds, when the lump-sum taxes are available and all the consumer prices can be controlled. Considering (14.1) and $r=n$, the conditions with respect to q^1, q^2, and q^3 are given by

$$\frac{E_{11}}{1+n}+(\frac{E_{21}}{1+n}+E_{31})w(n)=0, \tag{15.1}$$

$$\frac{E_{12}}{1+n}+(E_{32}+e_1^*+\frac{E_{22}-y_1^*}{1+n})w(n)=0, \tag{15.2}$$

$$\frac{E_{13}}{1+n}+(E_{33}+e_2^*+\frac{E_{23}-y_2^*}{1+n})w(n)=0, \tag{15.3}$$

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[9] Only five of the six policy variables are in fact independent. Once the values of any six have been selected, the value of the sixth is automatically determined from the government's budget constraint, $(\tau^c c+\tau^x x+T^o)/(1+n)+\theta rkl+\gamma wl+T^y=g$. This constraint is not considered in the following analyses, since it can be derived from the equilibrium conditions (13.1)-(13.4).

[10] For the expenditure function approach, see Ihori (1996). It should be noted that the distortional tax rates $(\tau^c,\tau^x,\gamma,\theta)$ affect the problem only through the consumer price vector and that the lump-sum tax in old age, T^0, is replaced with the presented value of lump-sum taxes, T.

where $y_1^* = H'(1+n)e_1^*$ and $y_2^* = H'(1+n)e_2^*$.

Now, define the tax wedge t^i, which is the difference between the consumer price q^i and the producer price:

$$t^1 \equiv q^1 - \frac{1}{1+r}, \quad t^2 \equiv q^2 - \frac{w}{1+r}, \quad t^3 \equiv q^3 - w.$$

Using these definitions and (6), the first-order conditions (15.1) -(15.3) are rewritten as

$$t^1 E_{11} + t^2 E_{12} + t^3 E_{13} = 0 \ , \tag{16.1}$$

$$t^1 E_{12} + t^2 (E_{22} - y_1^*) + t^3 (E_{32} + e_1^*) = -\Phi(q^2, q^3) e_1^*, \tag{16.2}$$

$$t^1 E_{13} + t^2 (E_{23} - y_2^*) + t^3 (E_{33} + e_2^*) = -\Phi(q^2, q^3) e_2^*, \tag{16.3}$$

where

$$\Phi(q^2, q^3) \equiv (1+n) q^2 H'[(1+n)e^*] - q^3.$$

These conditions are regarded as simultaneous equations with respect to the tax wedges (t^1, t^2, t^3). Considering (10), the common coefficient $\Phi(q^2, q^3)$ of e_1^* and e_2^* in (17.2) and (17.3), respectively, is positive: $\Phi > 0$. This implies that zero tax wedges, $t^i = 0$, are not optimal.

The socially desirable level of e is given by $\tilde{e}$ which satisfies

$$(1+n) q^2 H'[(1+n)e] - q^3 = 0.$$

Comparing this equation with (10), it is clear that $e^* < \tilde{e}$. The *private* marginal benefit of the care time is smaller than its *social* marginal benefit, since children discount the parent's utility by δ in a steady state.[11] We have, therefore, an *intergenerational externality* from children to their parents: children receive only part of the social benefit from looking after their parents, but have to bear the full social cost of the care time. In order to internalize this externality, it is necessary to subsidize the care time e. This is the so-called Pigouvian policy. As pointed out by Sandmo (1990), however, this policy will be not feasible, since the value of home production in a dynastic family is *untaxable*. Hence, instead of subsidizing the care time directly, we need consider a policy that imposes taxes indirectly on the care service x and the labour supply l.[12]

The care time e gives rise to a positive externality, but a subsidy to e is not feasible. Instead, from $e_1^* > 0$, a tax on x might do the trick. As well as the corrective effect, however, such a tax on x would also give rise to a distortion in the use of x. In that case, the use of an additional instrument may improve on the allocation. Does the optimal policy involve a tax on l because of $e_2^* < 0$? The answer to this question is not necessary yes. This is because a tax on l may not only correct e but also affect the distortion in x. If a tax on l further reduces the consumption of x, the case is not clear. Then, it is important to determine whether it is the tax on x or the tax on l which reduces externalities more in relation to the distortional effect on the consumption of x. Only if a tax on l is more effective in the above sense than a tax on x, should a tax accrue to l.[13] Could it be the case that optimality requires subsidies to both x and l? The answer is no, as will be seen in the subsequent analyses.

Eliminating t^1 from (16.1)-(16.3), then we have the simultaneous equations:

$$\hat{x}_2^* t^2 - \hat{l}_2^* t^3 = -\Phi e_1^*, \tag{17.1}$$

$$\hat{x}_3^* t^2 - \hat{l}_3^* t^3 = -\Phi e_2^*, \tag{17.2}$$

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[11] On the other hand, for externalities from parents to their children in altruistic economies with endogenous fertility, see Cigno (1983).

[12] For corrective taxation on goods that are related to consumption externalities, see Wijkander (1985).

[13] The question of whether it is more efficient to impose a tax on x has to be subjected to the same considerations.

where

$$\hat{x}_2^* \equiv x_2^* + x_1^* \frac{\partial t^1}{\partial t^2} = E_{22} - y_1^* - \frac{E_{21}E_{12}}{E_{11}}, \quad \hat{l}_2^* \equiv l_2^* + l_1^* \frac{\partial t^1}{\partial t^2} = -E_{32} - e_1^* + \frac{E_{31}E_{12}}{E_{11}},$$

$$\hat{x}_3^* \equiv x_3^* + x_1^* \frac{\partial t^1}{\partial t^3} = E_{23} - y_2^* - \frac{E_{21}E_{13}}{E_{11}}, \quad \hat{l}_3^* \equiv l_3^* + l_1^* \frac{\partial t^1}{\partial t^3} = -E_{33} - e_2^* + \frac{E_{31}E_{13}}{E_{11}}.$$

It follows from $e_1^* > 0$, $e_2^* < 0$, and the concavity of the expenditure function that $\hat{x}_2^* < 0$ and $\hat{l}_3^* > 0$. However, since the sign of $E_{11}E_{23} - E_{12}E_{13}$ is ambiguous, so are those of $\hat{l}_2^*$ and $\hat{x}_3^*$. Considering (11), there is the relationship between $\hat{l}_2^*$ and $\hat{x}_3^*$ as follows: $\hat{l}_2^* = -\hat{x}_3^* - [1 - 1/\delta(1+n)]e_1^*$. If we assume that $\delta(1+n) = 1$, we obtain $\hat{l}_2^* = -\hat{x}_3^*$.[14] We also assume that the determinant of the coefficient matrix is positive, i.e., $D \equiv \hat{l}_2^* \hat{x}_3^* - \hat{x}_2^* \hat{l}_3^* > 0$. Then, (17.1) and (17.2) can be solved for the first-best corrective tax wedges t^2 and t^3:

$$t^2 = \frac{\Phi(e_1^* \hat{l}_3^* - e_2^* \hat{l}_2^*)}{D}, \tag{18.1}$$

$$t^3 = \frac{\Phi(e_1^* \hat{x}_3^* - e_2^* \hat{x}_2^*)}{D}. \tag{18.2}$$

If any two goods of b, c, and d are *complements*, i.e., $E_{12} < 0$, $E_{13} < 0$, or $E_{23} < 0$, it is possible that $\hat{l}_2^* \geq 0$ and $\hat{x}_3^* \leq 0$. In this case, since it holds that $t^2 > 0$ and $t^3 < 0$, x and l should be taxed. In particular, when $\hat{l}_2^* = \hat{x}_3^* = 0$, we obtain

$$t^2 = -\frac{\Phi e_1^*}{\hat{x}_2^*} > 0 \text{ and } t^3 = \frac{\Phi e_2^*}{\hat{l}_3^*} < 0.$$

Namely, the optimal levels of the tax wedges, t^2 and t^3, are equal to the values of

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[14] This assumption is justified by specifying the utility function as $u_t = v_t + (1+a)v_{t-1}/(1+n)$, where the interpersonal discount rate a is zero. Burbidge (1983) used this type of the utility function in the gift economy. He argued that since each generation is $1+n$ times larger than the previous generation, parental welfare should be weighted by the factor $1/(1+n)$ in the descendant's utility function.

raised externalities per unit of reduced quantities of the care service x bought from the market and the labour supply l, respectively.

On the other hand, if b, c, and d are all *substitutes*, i.e., $E_{12} > 0$, $E_{13} > 0$, and $E_{23} > 0$, it holds that $\hat{l}_2^* < 0$ and $\hat{x}_3^* > 0$. In this case, subsidies to x or l would be conceivable. The positive determinant D is equivalent to the inequality:

$$\frac{\hat{l}_2^*}{\hat{l}_3^*} > \frac{\hat{x}_2^*}{\hat{x}_3^*}. \tag{19}$$

It follows from (18.1) and (18.2) that

$$t^2 \gtrless 0 \quad \text{as} \quad \frac{\hat{l}_2^*}{\hat{l}_3^*} \gtrless \frac{e_1^*}{e_2^*}, \tag{20.1}$$

$$t^3 \gtrless 0 \quad \text{as} \quad \frac{\hat{x}_2^*}{\hat{x}_3^*} \gtrless \frac{e_1^*}{e_2^*}. \tag{20.2}$$

Now, combining (19), (20.1), and (20.2), the following three possibilities arise:

$$\frac{\hat{l}_2^*}{\hat{l}_3^*} > \frac{\hat{x}_2^*}{\hat{x}_3} > \frac{e_1^*}{e_2^*} \Rightarrow t^2 > 0,\ t^3 > 0,$$

$$\frac{\hat{l}_2^*}{\hat{l}_3^*} > \frac{e_1^*}{e_2^*} > \frac{\hat{x}_2^*}{\hat{x}_3^*} \Rightarrow t^2 > 0,\ t^3 < 0,$$

$$\frac{e_1^*}{e_2^*} > \frac{\hat{l}_2^*}{\hat{l}_3^*} > \frac{\hat{x}_2^*}{\hat{x}_3^*} \Rightarrow t^2 < 0,\ t^3 < 0.$$

Thus, at least of x and l should be taxed.

Under what circumstances is it optimal to tax or subsidize l? We can answer this problem in the following intuitive way: suppose that only one tax instrument is used, a tax wedge t^2 on x.[15] From $e_1^* > 0$, t^2 should be set at a positive level to internalize externalities: $t^2 = \bar{t}^2 > 0$. However, this tax wedge causes a distortion in the consumption of x by $\hat{x}_2^* < 0$. The use of an additional instrument, a tax wedge t^3 on l, would improve on the allocation. Consider a marginal change Δt^3 in t^3 from $\bar{t}^3 = 0$. At first, assume $\Delta t^3 < 0$. Under this assumption, it follows from $e_2^* < 0$ that e increases. If $\hat{x}_3^* \leq 0$, then x increases or is unchanged. Hence, because a marginal decrease in t^3 raises externalities, reducing or not changing the distortion in x caused by $\bar{t}^2$, it is desirable that l is taxed, i.e., $t^3 < 0$. However, if $\hat{x}_3^* > 0$, then x decreases. This implies that the distortion is aggravated. In this case, we need compare the *external* and *distortional* effects of $\Delta t^3 < 0$ on e and x, respectively, with those of a marginal *increase* in t^2 from $\bar{t}^2$, $\Delta t^2 > 0$. The *benefit-cost* ratios of $\Delta t^3 < 0$ and $\Delta t^2 > 0$ are given by $-e_2^*/\hat{x}_3^* (> 0)$ and $-e_1^*/\hat{x}_2^* (> 0)$, respectively. If the former is greater than the latter, i.e., $e_1^*/e_2^* > \hat{x}_2^*/\hat{x}_3^*$, then $\Delta t^3 < 0$ is justified so that $t^3 < 0$. This result is consistent with the sign judgment from (20.2). Next, assume $\Delta t^3 > 0$. A reduction in e occurs under this alternative assumption. However, if $\hat{x}_3^* > 0$, then x increases.[16] Thus, the distortion of x caused by $\hat{x}_2^* < 0$ is alleviated. We now need compare the effects of $\Delta t^3 > 0$ on e and x with those of a marginal *decrease* in t^2 from $\bar{t}^2$, i.e., $\Delta t^2 < 0$. The benefit-cost ratios of $\Delta t^3 > 0$ and $\Delta t^2 < 0$ are given by $-\hat{x}_3^*/e_2^* (> 0)$ and $-\hat{x}_2^*/e_1^* (> 0)$, respectively. If the former is greater than the latter, i.e., $e_1^*/e_2^* < \hat{x}_2^*/\hat{x}_3^*$, then $\Delta t^3 > 0$ is justified so that $t^3 > 0$.

4. The Second-Best Tax Rule

In this section, we derive the second-best tax rule where two types of lump-sum taxes, T^y and T^o are not available. In this case, (12) and (14.1) are homogeneous of degree zero with respect to the consumer price vector q, but (14.2) is not. Following Ihori

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[15] Using a similar method, we can answer the problem whether it is optimal to tax or subsidize x under what circumstances.

[16] If $\hat{x}_3^* \leq 0$, then x decreases or is unchanged. Therefore, since $\Delta t^3 > 0$ is not justified, l should never be subsidized in this case.

(1996), if we consider the problem: $\max v$ subject to (12) and (14.1), q is uniquely determined up a proportionality. Then, since (14.2) gives the level of q^1, we obtain $\lambda = 0$. Since (12) does not explicitly involve r, we obtain $r = n$ from the first-order condition with respect to r. Hence, the *golden rule* holds as long as all the consumer prices are controlled. Using $\hat{l}_2^* = -\hat{x}_3^*$, the first-order conditions with respect to q^1, q^2, and q^3 can be written as[17]

$$\frac{t^1 E_{11} + t^2 E_{12} + t^3 E_{13}}{E_1} = \frac{\psi}{\phi}, \tag{21.1}$$

$$\frac{t^1 E_{21} + t^2 (E_{22} - y_1^*) + t^3 (E_{23} - y_2^*)}{E_2 - y^*} = \frac{\psi}{\phi} - (1 + \frac{\psi}{\phi}) \frac{\Phi e_1^*}{E_2 - y^*}, \tag{21.2}$$

$$\frac{-t^1 E_{31} - t^2 (E_{32} + e_1^*) - t^3 (E_{33} + e_2^*)}{1 - E_3 - e^*} = \frac{\psi}{\phi} + (1 + \frac{\psi}{\phi}) \frac{\Phi e_2^*}{1 - E_3 - e^*}, \tag{21.3}$$

These conditions are a variation on the Ramsey rule familiar from the theory of optimal taxation, since the left-hand sides are the relative reductions in the compensated demand or supply for taxed goods, c, x, and l. In the standard version of the conditions, these rates of relative reductions should be the same. In this case, however, they differ because of the last terms on the right-hand sides of (21.2) and (21.3). These terms show the *external* effects of the care time e on the relative reductions of x and l. From $\Phi > 0$, $e_1^* > 0$, and $e_2^* < 0$, these reduction rates are greater than that of c.

Now, eliminating t^1 from (21.1)-(21.3) and then the solving for t^2 and t^3, we have the following solutions:

[17] The Lagrange multiplier ψ corresponds to the marginal *utility* of lump-sum transfer T^y for each individual of the young generation financed by distortional taxes. Therefore, ψ is negative on the golden-rule path: $\psi < 0$. On the other hand, the multiplier ϕ is positive: $\phi > 0$, because it corresponds to the social marginal utility of government revenue. Thus, it holds that $\mu \equiv \psi / \phi < 0$, which implies the marginal *valuation* of the lump-sum *transfer* financed by distortional taxes in terms of government revenue. It should be now noted that since $1 + \mu$ it shows the *social* marginal valuation of *income* in terms of government revenue, it is *positive*: $1 + \mu > 0$.

$$t^2 = (1+\frac{\psi}{\phi})\frac{\Phi(e_1^*\hat{l}_3^* - e_2^*\hat{l}_2^*)}{D} - \frac{\psi}{\phi}\frac{(x^* - \frac{E_1E_{21}}{E_{11}})\,\hat{l}_3^* + (l^* + \frac{E_1E_{31}}{E_{11}})\,\hat{l}_2^*}{D}, \tag{22.1}$$

$$t^3 = (1+\frac{\psi}{\phi})\frac{\Phi(e_1^*\hat{x}_3^* - e_2^*\hat{x}_2^*)}{D} - \frac{\psi}{\phi}\frac{(x^* - \frac{E_1E_{21}}{E_{11}})\,\hat{x}_3^* + (l^* + \frac{E_1E_{31}}{E_{11}})\,\hat{x}_2^*}{D}. \tag{22.2}$$

The first-terms on the right-hand sides of (22.1) and (22.2) represent the components of the taxes that the government has to impose in order to exactly internalize the external effects. They will be called the *second-best internalization* component.[18] The second-terms may be called the Ramsey components, since they minimize the excess burden of the tax system. Each tax t^i, $i = 1,\dots,3$, consists of the sum of the second-best internalization component and the Ramsey component.[19] Since we have analyzed the characteristics of the internalization component in the previous section, we now concentrate our attention on those of the Ramsey component.

Let us assume that the population growth rate is close to zero, i.e., $n \cong 0$. Using $\delta(1+n) = 1$ under this assumption, we obtain $\delta \cong 1$ so that $\Phi \cong 0$. Thus, after some manipulations, (22.1) and (22.2) can be summarized in terms of elasticities, that is, $\sigma_i^c \equiv q^i E_{1i}/E_1$, $\sigma_i^d \equiv q^i E_{2i}/E_2$, $\sigma_i^b \equiv q^i E_{3i}/E_3$ for $i = 1,2,3$, and $\varepsilon_j^e \equiv q^{j+1} e_j^*/e^*$ for $j = 1,2$:

$$-\frac{\frac{t^2}{q^2}}{\frac{t^3}{q^3}} = \frac{A\sigma_3^d + B(\frac{b^*}{l^*})(\sigma_3^b - \frac{\sigma_3^c\sigma_1^b}{\sigma_1^c}) - C(\frac{e^*}{l^*})\varepsilon_2^e}{A\sigma_2^d - \frac{\sigma_1^d\sigma_2^c}{\sigma_1^c} + B(\frac{b^*}{l^*})\sigma_2^b - C(\frac{e^*}{l^*})\varepsilon_1^e}, \tag{23}$$

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[18] For the second-best internalization component in the presence of consumption externalities, see Orosel and Schob (1996).

[19] Sandmo (1975) called this the *additivity* property of a second-best tax system. When taxes can be imposed directly on externality-creating goods, the internalization component enters only the tax formulae for these goods. However, in cases where only related goods can be taxed, it should be noted that this component enters the tax formulae for all related goods.

where

$$A \equiv 1 + \frac{\sigma_1^b}{\sigma_1^c}\frac{b^*}{l^*}, \quad B \equiv \frac{x^*}{d^*} - \frac{q^1}{q^2}\frac{\sigma_1^d}{\sigma_1^c}, \quad C \equiv \frac{q^1 c^*}{q^2 d^*} + \frac{\sigma_1^b}{\sigma_1^c}\frac{q^3 b^*}{q^2 d^*} - \frac{q^1}{q^2}\frac{\sigma_1^d}{\sigma_1^c}.$$

If the cross elasticities, σ_1^b and σ_1^d are numerically very small relative to the own elasticity σ_1^c so that $\sigma_1^b/\sigma_1^c \cong 0$ and $\sigma_1^d/\sigma_1^c \cong 0$, then it holds that $A \cong 1$, $B \cong x^*/d^*$, and $C \cong q^1 c^*/q^2 d^*$. In this case, (23) approximately reduces to

$$-\frac{\frac{t^2}{q^2}}{\frac{t^3}{q^3}} = \frac{\sigma_3^d + (\frac{x^*}{d^*})(\frac{b^*}{l^*})\sigma_3^b - (\frac{e^*}{l^*})(\frac{q^1 c^*}{q^2 d^*})\varepsilon_2^e}{\sigma_2^d + (\frac{x^*}{d^*})(\frac{b^*}{l^*})\sigma_2^b - (\frac{e^*}{l^*})(\frac{q^1 c^*}{q^2 d^*})\varepsilon_1^e},$$

where $\sigma_3^b < 0$, $\sigma_2^d < 0$, $\varepsilon_2^e < 0$, and $\varepsilon_1^e > 0$. This tax rule is the same as the one derived by Kleven et al. (2000) in the static setting.[20] Hence, under the realistic assumption that the tax rate on labour income is positive, i.e., $t^3 < 0$, we have the following results: (i) if care for the elderly and leisure are complements: $\sigma_3^d > 0$ and $\sigma_2^b > 0$, the service should be heavily taxed when home production is insignificant; (ii) however, when a large fraction of the service demand is satisfied through home production, it may well be optimal to subsidize to the service or to impose a relatively low tax rate on it; (iii) if they are substitutes, it seems even more likely that the second-best tax system will subsidize to the service or to impose a low tax rate on it.

5. Conclusion

In an overlapping generations model where altruistic children devote a part of their time endowment to producing the service of care for their parents, we have showed that the allocated effort is short of the socially optimal level and have characterized the

[20] This is because the time that a child devotes to looking after his parent in the young period is just like that for taking care of him in the old period, if the size of population is almost constant over time.

first-best taxes on the market-produced service and the labour supply that internalize such an intergenerational externality. Whether the tax on the market-produced service or the tax on the labour supply reduces the external effects more in relation to the distortional effects on the service demand and the labour supply determines the appropriate tax policy. We have also examined the tax rule in the second-best optimum where lump-sum taxes are not available and have verified the well-known result that each tax consists of the sum of the internalization component and the Ramsey component.

Finally, it will be useful to refer to some basic assumptions used in this paper. We have assumed that (i) consumers do not consume in youth, (ii) the old do not employ the young helpers from the labour market for the home production of the care service, and (iii) parents do not leave children bequests. Since the assumptions (i) and (ii) do not affect the results, the consumption-saving choice and the employment of home-helpers are secondary to our main concerns. On the other hand, the assumption (iii) will be not appropriate in the case where children are not altruistic. In this case, we need develop a model of strategic bequests in which parents influence the decisions of children by conditioning the division of bequests on the care services provided from children.[21] An exploration of the optimal taxation problem in an overlapping generations economy with the strategic bequest motive will be interesting but it is left for future scrutiny.

[21] For the strategic bequest motive, see Bernheim, Schleifer, and Summers (1985).

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