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Abstract

This paper develops a dynamic macroeconomic model of monopolistic competition with overlapping generations, where real national income and the real interest rate are simultaneously determined in the differentiated good and capital good markets. Keynesian multipliers of government spending financed by lump-sum or income taxation are explored. In addition to the profit and diversity effects on aggregate demand in the static framework, it is shown that the interest effect crucially influences the short-run and long-run multipliers. The impact of spending on welfare of the representative agent is also examined.

Keywords: Keynesian multipliers, Monopolistic competition, Overlapping generations

JEL classification: E62, L13, L16

1. Introduction

Recently, Mankiw (1988), Molana and Moutos (1992), Heijdra and van der Ploeg (1996), and Heijdra, Lighthart, and van der Ploeg (1998) have examined Keynesian multipliers in macroeconomic models of monopolistic competition. Their models are characterized by strategic complementarity among firms: each firm has no incentive to increase output alone, but has such an incentive if other firms behave similarly. The government can step in to resolve this coordination failure. Expansive spending policy produces the multiplier process of national income, through the *profit* effect on aggregate demand in the short run or the *diversity* effect of product variety in the long run. However, since the models are *static*, they cannot analyze how an interaction between saving and investment in the capital market affects the multipliers.

In order to tackle this problem, in the present paper, we develop a *dynamic* model with overlapping generations.[1] This model consists of agents living for two periods and two production sectors: a perfectly competitive sector producing a capital good and a monopolistic competitive one producing differentiated goods. Aggregate demand is composed of consumption of the young and old generations, investment of the young, and government spending. Real national income and the real interest rate are simultaneously determined in the differentiated good and capital good markets. Concentrating on the *interest* effect on aggregate demand, we study Keynesian multipliers of government spending financed by lump-sum or income taxation.

In the short run, capital stock and the number of monopolistic firms are fixed. If a change in the real interest rate can be neglected, the short-run multiplier of government spending financed by lump-sum taxation has the same characteristics as those of the *static* one of Mankiw (1988) and others. That is, the multiplier is positive but is less than unity. If the real interest rate changes, the multiplier may exceed unity by the

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[1] Pagano (1990) and Heijdra (1998) developed dynamic models. In a *finite* horizon model with overlapping generations, the former studied how a demand expansion by fiscal policy is beneficial in a long-run equilibrium with *unemployment*. In an *infinite* horizon model, the latter extended the perfect competition model of Baxter and King (1993) to the framework of monopolistic competition.

interest effect, reinforcing the multiplier process through the profit effect. In the long run, capital stock and the number of firms change over time. Since profits of each firm are zero, the diversity and interest effects produce the multiplier process. Though the long-run multiplier is positive in the static model of Heijdra and van der Ploeg (1996), it is not always so in our dynamic one. If utility in the old period of each agent is hardly discounted, then the long-run multiplier is *negative*.

When government spending is financed by labour income taxation, the short-run multiplier is negative. This is the same as the results of Molana and Moutos (1992) and Heijdra *et al.* (1998). However, the long-run multiplier is not always so, depending on an effect of spending on aggregate demand through a change in the real interest rate. If this effect is positive, then it is possible that the long-run multiplier is *positive*.

We also investigate how an increase in government spending financed by lump-sum taxation affects welfare of each agent. In the short run, utility of an old agent is improved, while that of a young one is ambiguous. In the long run, whether or not spending increases the lifetime utility of the representative agent depends on the diversity effect. If this effect strongly works, then his lifetime utility rises.

This paper is organized as follows. Section 2 formulates the model. Section 3 describes a symmetric equilibrium. Section 4 analyzes the short-run and long-run multipliers of government spending financed by lump-sum taxation. Section 5 refers to the multipliers when spending is financed by income taxation. Section 6 examines the welfare effects of spending on each agent. Finally, Section 7 concludes the paper.

2. An Overlapping Generations Model

In this section, we develop an overlapping generations model consisting of homogeneous agents, two production sectors, and the government.[2] Each agent lives for two periods, working in the young period and enjoying retirement in the old. The

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[2] We combine the overlapping generations model of Diamond (1965) with the monopolistic competition model of Dixit and Stiglitz (1977).

production sectors are a sector where perfectly competitive firms produce a capital good and a sector where monopolistically competitive firms produce differentiated goods. The government imposes taxes on agents to provide a public good.[3]

2.1 Agent's Behaviour

A new generation of J_t agents is born at each period $t = 1,2,...\infty$. We assume that population does not grow, so that $J_t = J$ for all t. The representative agent born at period t has nominal disposable income, D_t, which is composed of the three components in the young period: (i) after-tax labour income, $(1-\tau_t)w_t l_t$, where τ_t, w_t, and l_t are a labour income tax rate, a nominal wage rate, and his labour supply at period t, respectively, (ii) his share of aggregate nominal profits of monopolistic firms, Π_t^{Σ}/J, and (iii) a lump-sum tax, T_t.[4] Thus, $D_t = w_t^{\tau} l_t + (\Pi_t^{\Sigma}/J) - T_t$, where w^{τ} is an after-tax wage rate, i.e., $w^{\tau} = (1-\tau)w$. This income is allocated to consumption expenditure, E_t^1, and savings, S_t. Hence, the budget constraint in the young period is $D_t = E_t^1 + S_t$. His nominal income in the old period is $(1+r_{t+1})S_t$, where r_{t+1} is the nominal rate of interest at period $t+1$. Since he spends this income on consumption expenditure, E_{t+1}^2, the budget constraint in the old period is $(1+r_{t+1})S_t = E_{t+1}^2$. Eliminating S_t from the single-period budget constraints, the lifetime budget constraint is $R_{t+1}E_t^1 + E_{t+1}^2 = R_{t+1}D_t$, where $R_{t+1} = 1 + r_{t+1}$.

The representative agent has a time endowment of unity and his lifetime utility function is given as $U_t = \ln u_t + \delta \ln v_{t+1}$, $0 < \delta < 1$, where δ is the discount factor. The sub-utility function for the young period, u_t, is a CES function of a *composite* good, C_t^1, and leisure, $1 - l_t$:

$$u_t = [\varepsilon^{\frac{1}{\sigma}}(C_t^1)^{\frac{\sigma-1}{\sigma}} + (1-\varepsilon)^{\frac{1}{\sigma}}(1-l_t)^{\frac{\sigma-1}{\sigma}}]^{\frac{\sigma}{\sigma-1}}, \quad \sigma > 0, \quad 0 < \varepsilon < 1,$$

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[3] The public good is treated as 'wasteful', entering neither utility nor production functions, in order to focus on multiplier effects of government spending on national income.

[4] For simplicity, we assume that income in the old period of each agent does not include a share of profits and a lump-sum tax. Though this changes magnitudes of the short-run and long-run multipliers, signs of the multipliers are not affected.

where σ is the elasticity of substitution between consumption and leisure. The composite good, C_t^1, is a CES-aggregation of varieties, $\mathbf{c}_t^1 = \{c_t^{1h}\}_{h=1,\dots,H_t}$, where c_t^{1h} is consumption of the variety produced by the h th monopolistic firm:

$$C_t^1 = \Phi(\mathbf{c}_t^1, H_t; \theta, \alpha) = (H_t)^{\alpha}[(H_t)^{-1} \sum_{h=1}^{H_t} (c_t^{1h})^{\frac{\theta-1}{\theta}}]^{\frac{\theta}{\theta-1}}, \ \theta > 1, \ \alpha \geq 1,$$

where θ is the elasticity of substitution between two varieties, $\alpha > 1$ implies a preference for a diversity of variety, and H_t is the number of monopolistic firms at period t. On the other hand, since he does not work in the old period, his sub-utility function, v_{t+1}, depends only on a composite good, C_{t+1}^2, which is a CES-aggregation of $\mathbf{c}_{t+1}^2 = \{c_{t+1}^{2h}\}_{h=1,\dots,H_{t+1}}$: $v_{t+1} = C_{t+1}^2 = \Phi(\mathbf{c}_{t+1}^2, H_{t+1}; \theta, \alpha)$.

The lifetime utility of the representative agent is maximized subject to the lifetime budget constraint. This problem is solved by the three stages as follows.[5] At first, given C_t^1 and a price vector: $\mathbf{p}_t = \{p_t^h\}_{h=1,\dots,H_t}$, minimize consumption expenditure in the young period, $E_t^1 = \mathbf{p}_t \mathbf{c}_t^1 = \sum_{h=1}^{H_t} p_t^h c_t^{1h}$, with respect to $\mathbf{c}_t^1$. Then, we have $\tilde{E}_t^1 = P_t^C C_t^1$, where the price index P_t^C is a CES-aggregation of $\mathbf{p}_t$:

$$P_t^C = \Psi(\mathbf{p}_t, H_t; \theta, \alpha) = (H_t)^{-\alpha}[(H_t)^{-\theta} \sum_{h=1}^{H_t} (p_t^h)^{1-\theta}]^{\frac{1}{1-\theta}}.$$

Similarly, in the old period, minimizing $E_{t+1}^2 = \mathbf{p}_{t+1}\mathbf{c}_{t+1}^2$ with respect to $\mathbf{c}_{t+1}^2$, we have $\tilde{E}_{t+1}^2 = P_{t+1}^C C_{t+1}^2$, where $P_{t+1}^C = \Psi(\mathbf{p}_{t+1}, H_{t+1}; \theta, \alpha)$. Secondary, given u_t, minimize total expenditure in the young period, $P_t^C C_t^1 + w_t^{\tau}(1 - l_t)$. Then, we obtain

$$P_t^C \tilde{C}_t^1 + w_t^{\tau}(1 - \tilde{l}_t) = P_t^u u_t, \ \tilde{C}_t^1 = \varepsilon(P_t^C / P_t^u)^{-\sigma} u_t, \ 1 - \tilde{l}_t = (1 - \varepsilon)(w_t^{\tau} / P_t^u)^{-\sigma} u_t.$$

The price index P_t^u of u_t is $P_t^u = [\varepsilon(P_t^C)^{1-\sigma} + (1-\varepsilon)(w_t^{\tau})^{1-\sigma}]^{\frac{1}{1-\sigma}}$. It follows from $v_{t+1} = C_{t+1}^2$ that $P_{t+1}^v = P_{t+1}^C$. Finally, maximize the lifetime utility U_t with respect to u_t and v_{t+1} subject to $R_{t+1}P_t^u u_t + P_{t+1}^v v_{t+1} = R_{t+1}F_t$, where F_t is nominal *full* income: $F_t = w_t^{\tau} + (\Pi_t^{\Sigma} / J) - T_t$. We obtain the indirect utility function:

$$V_t = (1+\delta)\ln F_t - \ln P_t^u - \delta \ln(P_{t+1}^v / R_{t+1}) - A, \text{ where } A = \ln(1+\delta)[(1+\delta)/\delta]^{\delta}.$$

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[5] In what follows, we mark the optimal level of each variable with a macron (~).

The sub-utilities are given by

$$\tilde{u}_t = F_t/(1+\delta)P_t^u \quad \text{and} \quad \tilde{v}_{t+1} = \delta R_{t+1}F_t/(1+\delta)P_{t+1}^v .$$

They determine the demand vector, $\{\tilde{C}_t^1, \tilde{C}_{t+1}^2, \tilde{l}_t\}$.

2.2 Two Production Sectors

The capital goods sector operates under constant returns to scale technology. At period $t-1$, each identical perfectly competitive firm issues private debt to purchase differentiated inputs, $\mathbf{i}_{t-1} = \{i_{t-1}^h\}_{h=1,\dots,H_{t-1}}$, from monopolistically competitive firms. The production function of the capital good, k_t, at period t is a CES-aggregation form of $\mathbf{i}_{t-1}$: $k_t = \Phi(\mathbf{i}_{t-1}, H_{t-1}; \lambda, \beta)$, where λ is the elasticity of substitution between two inputs. The diversity effect on productivity is shown by $\beta > 1$. Each firm maximizes its discounted nominal profits, $p_t^k k_t / R_t - \mathbf{p}_{t-1}\mathbf{i}_{t-1}$, where p_t^k is a nominal price of the capital good at period t. For this purpose, given k_t and $\mathbf{p}_{t-1}$, the cost, $\mathbf{p}_{t-1}\mathbf{i}_{t-1}$, is minimized. The cost function is $P_{t-1}^k k_t$, where P_{t-1}^k is the *unit* cost of the capital good. This is a CES-aggregation of $\mathbf{p}_{t-1}$: $P_{t-1}^k = \Psi(\mathbf{p}_{t-1}, H_{t-1}; \lambda, \beta)$. Since the firm's profit is zero in equilibrium, it holds that $p_t^k = R_t P_{t-1}^k$.

In the differentiated goods sector, using labour L_t^h and capital K_t^h, the h th monopolistic firm produces output Q_t^h. As in Pagano (1990), we assume a Cobb-Douglas function: $Q_t^h = (K_t^h)^a (L_t^h)^{1-a} - f$, where $0 < a < 1$ and $f\,(>0)$ is a constant setup cost. Given prices of other firms, the h th firm maximizes nominal profits, $\Pi_t^h = p_t^h Q_t^h - w_t L_t^h - p_t^k K_t^h$, with respect to L_t^h and K_t^h. The first-order conditions are given by

$$p_t^h[1-(1/\zeta_t^h)]a(K_t^h/L_t^h)^{a-1} = p_t^k \quad \text{and} \quad p_t^h[1-(1/\zeta_t^h)](1-a)(K_t^h/L_t^h)^a = w_t ,$$

where $\zeta_t^h\,(>1)$ is the price elasticity of demand. Thus, the mark-up, μ_t^h, is defined by $\mu_t^h = \zeta_t^h/(\zeta_t^h - 1)$.

2.3 The Government

The revenue of the government at period t consists of lump-sum and labour income taxes, JT_t and $\tau_t w_t J l_t$. This is devoted to public expenditure for providing a public good G_t. The production function of the public good is a CES-aggregation form of differentiated inputs, $\mathbf{e}_t = \{e_t^h\}_{h=1,\dots,H_t}$: $G_t = \Phi(\mathbf{e}_t, H_t; \eta, \gamma)$, where η is the elasticity of substitution between two inputs and $\gamma > 1$ shows a diversity effect. Given G_t, the government minimizes expenditure, $\mathbf{p}_t \mathbf{e}_t$, with respect to $\mathbf{e}_t$. The minimum expenditure function is $P_t^G G_t$, where P_t^G is the price index of the public good: $P_t^G = \Psi(\mathbf{p}_t, H_t; \eta, \gamma)$. Hence, the budget constraint of the government is given by $J(T_t + \tau_t w_t l_t) = P_t^G G_t$.

3. Symmetric Equilibrium

In this section, we describe a symmetric equilibrium where all monopolistic firms set the same price, $p_t^h = p_t$. For simplicity, we normalize the population size of each generation to unity (i.e., $J = 1$) and set the following three assumptions, which are the same as those in the benchmark case of Heijdra (1998): (a) the elasticity of substitution between consumption and leisure is unity: $\sigma = 1$, (b) the elasticities of substitution between two varieties in the CES form are the same: $\theta = \lambda = \eta$, and (c) the diversity effects of variety on consumption and production are the same: $\alpha = \beta = \gamma$.[6] It follows from these assumptions that

$$P_t^C = P_t^k = P_t^G = (H_t)^{1-\alpha} p_t \quad \text{and} \quad p_t^k = R_t P_{t-1}^k = R_t (H_{t-1})^{1-\alpha} p_{t-1}.$$

Since each firm uses the same amounts of labour and capital and produce the same amounts of output and profits, it holds that $L_t^h = L_t$, $K_t^h = K_t$, $Q_t^h = Q_t$, and $\Pi_t^h = \Pi_t$. Real national income, Y_t, is defined by

[6] The assumption (a) implies that composite consumption and leisure in the young period of each agent do not depend on the real interest rate. The assumption (b) means that the mark-up becomes constant, i.e., $\mu_t = \theta/(1-\theta)$, because the price elasticity ξ_t is a weighted average of θ, λ, and η.

$$Y_t = \sum_{h=1}^{H_t} p_t^h Q_t^h / P_t^C = (H_t)^\alpha Q_t . \tag{1}$$

The first-order conditions for the profit maximization of the representative monopolistic firm at period t reduce to the factor price frontier:

$$\omega_t = \omega(\rho_t, H_t) = a^{\frac{a}{1-a}}(1-a)\mu^{\frac{-1}{1-a}}(1+\rho_t)^{\frac{-a}{1-a}}(H_t)^{\frac{\alpha-1}{1-a}}, \tag{2}$$

where ω_t is the real wage rate and ρ_t is the real interest rate: $\omega_t = w_t / P_t^C$ and $\rho_t = p_t^k / P_t^C - 1$. The real interest rate is related to the nominal interest one r_t as follows: $1+\rho_t = (1+r_t)/(P_t^C / P_{t-1}^C)$. The function $\omega(\rho_t, H_t)$ has partial derivatives,

$$\omega_\rho = -K/L < 0 \text{ and } \omega_H = (\alpha - 1)\omega/(1-a)H \geq 0 . \tag{3}$$

Hence, the real wage rate is a decreasing function of the real interest rate, but is a non-decreasing function of the number of the firms. When the diversity effect exists, the factor price frontier in the $\rho - \omega$ space shifts upward with an increase in the number of the firms, because the relative price of differentiated goods to the price of composite goods rises. The aggregate capital demand function is given by

$$K_t^\Sigma = K^\Sigma(Y_t, \rho_t, H_t) = a[Y_t + (H_t)^\alpha f] / \mu(1+\rho_t), \tag{4}$$

where $K_t^\Sigma = H_t K_t$. This function has the following partial derivatives:

$$K_Y^\Sigma = a/\mu(1+\rho),\ K_\rho^\Sigma = -K^\Sigma/(1+\rho),\ K_H^\Sigma = \alpha a f H^{\alpha-1}/\mu(1+\rho). \tag{5}$$

Thus, aggregate capital demand is an increasing function of real national income and the number of the firms, but is a decreasing function of the real interest rate. The real aggregate profit function is represented as

$$\pi_t = \Pi_t^\Sigma / P_t^C = \pi(Y_t, H_t) = [(\mu-1)Y_t - (H_t)^\alpha f]/\mu, \tag{6}$$

where $\Pi_t^\Sigma = H_t \Pi_t$. Since this function has a partial derivative, $\pi_Y = 1/\zeta > 0$, real aggregate profits rise when real national income increases.

On the other hand, the results of the utility maximization of the representative agent born at period t reduce to the consumption demand functions:

$$C_t^1 = C^1(I_t) = \varepsilon I_t /(1+\delta), \tag{7}$$

$$C_{t+1}^2 = C^2(I_t, \rho_{t+1}) = \delta(1+\rho_{t+1})I_t /(1+\delta), \tag{8}$$

where $I_t = \omega_t^\tau + \pi_t - Z_t$ and $Z_t = T_t / P_t^C$. The marginal propensities to consume out of real full income I_t in the young and old periods, ϕ^1 and ϕ^2, are

$$\phi^1 = \varepsilon /(1+\delta) \text{ and } \phi^2 = \phi^2(\rho_{t+1}) = \delta(1+\rho_{t+1})/(1+\delta).$$

Note that the real interest rate affects ϕ^2. The labour supply function is given by

$$l_t = l(\omega_t^\tau, N_t) = l^* - (1-\varepsilon)N_t /(1+\delta)\omega_t^\tau, \tag{9}$$

where $l^* = (\varepsilon+\delta)/(1+\delta)$ and N_t is non-labour income: $N_t = \pi_t - Z_t$. The function $l(\omega^\tau, N)$ has the following partial derivatives,

$$l_{\omega^\tau} = (1-\varepsilon)N/(1+\delta)(\omega^\tau)^2 \text{ and } l_N = -(1-\varepsilon)/(1+\delta)\omega^\tau.$$

It will be useful to make two remarks on this function: (i) when N_t is zero, labour supply is constant over time: $l_t = l^*$ and (ii) according as N_t is positive (negative), labour supply is smaller (greater) than l^* and is an increasing (decreasing) function of the real after-tax wage rate.

Since savings of the young support the production of the capital good, it holds that $S_{t-1} = \mathbf{p}_{t-1}\mathbf{i}_{t-1} = p_t^k k_t / R_t$. Therefore, considering (8), $1+\rho_t = p_t^k / P_t^C = R_t /(P_t^C / P_{t-1}^C)$, and the budget constraint in the old period, $R_t S_{t-1} = P_t^C C_t^2$, then the aggregate capital supply function is written as

$$k_t = k(I_{t-1}) = \delta I_{t-1} /(1+\delta). \tag{10}$$

Using (1), (4), (7), (8) and (10), the equilibrium conditions in the differentiated good and capital good markets at period t can be represented in terms of real variables:[7]

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[7] Using $k_{t+1} = (H_t)^\alpha i_t$, we can derive the condition (11) from the original one:

$$Y_t = C^1(I_t) + k(I_t) + C^2(I_{t-1}, \rho_t) + G_t, \tag{11}$$

$$k(I_{t-1}) = K^{\Sigma}(Y_t, \rho_t, H_t), \tag{12}$$

where $I_t = (1-\tau_t)\omega(\rho_t, H_t) + \pi(Y_t, H_t) - Z_t$. It should now be noted that the second term on RHS of (11) shows *investment* demand for differentiated goods by the young generation at period t. It follows from (10) that the marginal propensity to invest out of full income, ψ, is $\psi = \delta/(1+\delta)$. Finally, the budget constraint of the government is rewritten as $\tau_t \omega_t l_t + Z_t = G_t$.

4. Lump-sum Taxation

In this section, we analyze the short-run and long-run multipliers of government spending financed by lump-sum taxation. In the short run, the number of monopolistic firms is fixed and capital stock is predetermined. On the other hand, in the long run, the number of firms is adjusted such that profits of each firm are zero, and capital stock changes over time.

4.1 The Short-run Multiplier

The budget constraint of the government is $Z_t = G_t$. The equilibrium condition (11) in the differentiated good market is represented by real national income, Y_t, and the real rate of interest, ρ_t:

$$Y_t = (\phi^1 + \psi)[\omega(\rho_t, H_t) + \pi(Y_t, H_t) - G_t] + (1+\rho_t)k(I_{t-1}) + G_t, \tag{13}$$

where $k(I_{t-1})$ and H_t are given. Totally differentiating (13) in the neighborhood of the short-run equilibrium point $(\hat{Y}_t, \hat{\rho}_t)$, we have

$$dY_t = [1-(\phi^1 + \psi)]dG + (\phi^1 + \psi)(\hat{\omega}_\rho d\rho_t + \hat{\pi}_Y dY_t) + k(I_{t-1})d\rho_t. \tag{14}$$

The first term on RHS of (14) shows the first-round effect of government spending in

$Q_t = c_t^1 + i_t + c_t^2 + e_t$.

the multiplier process. That is, a rise in spending boosts aggregate demand for differentiated goods, but the rise in aggregate demand is attenuated by the rise in lump-sum taxes on the young. The second and the third terms indicate changes in expenditure of the young and old generations caused by changes in real national income and the real interest rate, respectively.

Differentiating the equilibrium condition (12) in the capital good market, we obtain

$$d\rho_t = \hat{\rho}_Y dY_t \text{, where } \hat{\rho}_Y = -\hat{K}_Y^{\Sigma} / \hat{K}_{\rho}^{\Sigma}. \tag{15}$$

It follows from (5) and (12) that $\hat{\rho}_Y = a / \mu k(I_{t-1}) > 0$. Therefore, the real interest rate rises with an increase in real national income. The reason is that an increase in real national income boosts demand for capital, while the real interest rate must rise to attain a new equilibrium in the capital market.

Considering (15), we have

$$\hat{\omega}_{\rho} d\rho_t = \hat{\omega}_{\rho} \hat{\rho}_Y dY_t \text{ and } k(I_{t-1}) d\rho_t = k(I_{t-1}) \hat{\rho}_Y dY_t. \tag{16}$$

These equations imply that the first-round effect of government spending on real national income reduces real full *labour* income of the young generation, but raises *interest* income of the old. Together with an increase in real profit income of the young, these marginal incomes create potentials for the subsequent rounds of the multiplier process. Hence, an increase in interest income of the old strengthens this process, while a decrease in labour income of the young weakens it.

Substituting (16) into (14) and using $\hat{\pi}_Y = 1/\zeta$, we obtain the *short-run* multiplier:

$$\left(\frac{dY_t}{dG_t}\right)_{LT}^{SR} = \frac{1-(\phi^1+\psi)}{1-[(\phi^1+\psi)/\zeta]-\hat{D}_{\rho}\hat{\rho}_Y} > 0, \tag{17}$$

where

$$1-\phi^1-\psi = (1-\varepsilon)/(1+\delta) > 0 \text{ and } \hat{D}_{\rho} = (\phi^1+\psi)\hat{\omega}_{\rho} + k(I_{t-1}).$$

The numerator on RHS of (17) indicates the first-round effect of government spending.

The second term in the denominator on RHS shows an increase in expenditure of the young by an increment in real *profit* income. The third term in the denominator represents a change in aggregate demand caused by an increase in the real *interest* rate. When a change in the real interest rate can be neglected, we obtain the *simple* multiplier in the short run:

$$1 > \left(\frac{\partial Y_t}{\partial G_t}\right)_{LT}^{SR} = \frac{1-(\phi^1+\psi)}{1-[(\phi^1+\psi)/\xi]} > 1-(\phi^1+\psi) > 0 .$$

Hence, the multiplier has the same characteristics as those in the static models of Mankiw (1988), Heijdra and van der Ploeg (1996), and others.

Since it can be easily shown that $1 > [(\phi^1+\psi)\hat{\pi}_Y] + \hat{D}_\rho \hat{\rho}_Y$, the short-run multiplier is *positive* even when the real interest rate changes.[8] However, since the sign of $\hat{D}_\rho$ is ambiguous, the short-run multiplier has different features from the simple (static) one. Using (3), (10), and the equilibrium conditions in the labour and capital markets, we have $\hat{D}_\rho = \hat{k}_t(1 - l^* / \hat{l}_t)$. Thus, if $\hat{l}_t = l^*$ in equilibrium, then the short-run multiplier is equal to the simple one. Otherwise, the short-run multiplier differs from the simple one. The former is larger than the latter, when it holds that $\hat{l}_t > l^*$. It is because a decrease in the real wage rate raises labour supply. This reinforces the multiplier process by an increase in real profit income. Consequently, it is possible that the short-run multiplier *exceeds* unity. On the other hand, when $\hat{l}_t < l^*$, the short-run multiplier is less than the simple one and may be *below* the first-round effect.

Let us now illustrate the short-run and simple multipliers in the Y - ρ space of Figure 1. The equilibrium conditions in the differentiated good and capital good markets are represented as the CM and YM curves, respectively.[9] From (15), the CM curve is an upward straight line. On the other hand, the YM curve has three different shapes,

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[8] This inequality assures the stability condition of the short-run equilibrium, which implies that the positive slope of the CM line is larger than that of the YM.

[9] The YM and CM curves correspond to the IS and LM curves in the standard Keynesian model, respectively. While a slope of the IS curve is downward, that of the YM curve is not always so.

depending on the sign of $\hat{D}_\rho$. That is, (i) when $\hat{D}_\rho = 0$, the YM-curve is the horizontal line: $Y = Y^*$, (ii) when $\hat{D}_\rho > 0$, it is upward sloping, and (iii) when $\hat{D}_\rho < 0$, it is downward sloping. The intersection point of the YM and CM curves is the short-run equilibrium, which is unique and stable. The point E is the old equilibrium before an increase in spending and the point G is the new equilibrium. The movement from E to G corresponds to the short-run multiplier. We can decompose it into two effects: the *profit* and *interest* effects. Given a level of the real interest rate, an increase in spending shifts the YM curve upward, but leaves the CM line unchanged. The movement from E to F is the profit effect, which corresponds to the simple multiplier. The movement from F to G shows the interest effect. Whether real national income is raised by the interest effect depends on shapes of the YM curve.

4.2 The Long-run Multiplier

Under free entry and exit of firms in the long run, profits of each firm are driven to zero. Therefore, it follows from (6) that the number of the firms is given by[10]

$$H = H(Y) \equiv [(\mu - 1)Y / f]^{(1/\alpha)}. \tag{18}$$

Since this function has a derivative: $H_Y = H / \alpha Y > 0$, the number of monopolistic firms is an increasing function of real national income. Substituting (18) into the factor price frontier (2), we have $\omega = \omega[\rho, H(Y)]$. Thus, when the *diversity* effect works, i.e., $\omega_H > 0$, an increase in real national income raises the real wage rate through an increase in the number of the firms.

Since the aggregate capital supply function is represented as $k = k(\check{I}) = \psi(\omega - G)$ in a long-run stationary state, the equilibrium conditions in the differentiated good and capital good markets can be written as follows:

$$Y = (\phi^1 + \psi)\{\omega[\rho, H(Y)] - G\} + \phi^2(\rho)\{\omega[\rho, H(Y)] - G\} + G, \tag{19}$$

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[10] From (1) and (18), the scale of production for each firm is constant: $Q = f / (\mu - 1)$.

$$\psi\{\omega[\rho,H(Y)]-G\}=K^{T}[Y,\rho,H(Y)]. \tag{20}$$

Differentiating (19) in a neighborhood of an equilibrium point $(\overline{Y},\overline{\rho})$, we have.

$$\begin{aligned} dY &= \{1-[\phi^{1}+\psi+\phi^{2}(\overline{\rho})]\}dG+(\phi^{1}+\psi)(\overline{\omega}_{\rho}d\rho+\overline{\omega}_{H}\overline{H}_{Y}dY) \\ &\quad +\phi^{2}(\overline{\rho})(\overline{\omega}_{\rho}d\rho+\overline{\omega}_{H}\overline{H}_{Y}dY)+\phi^{2}_{\rho}(\overline{\omega}-G)d\rho \end{aligned}, \tag{21}$$

where $\phi^{2}_{\rho}=\delta/(1+\delta)$. The first term on RHS of (21) shows the first-round effect of government spending. In the long run, the rise in aggregate demand is attenuated by not only a reduction in labour income of the young but also that in interest income of the *old.* The second shows a change in expenditure of the young due to changes in national income and the interest rate, and the third and the fourth altogether show that of the old.

Differentiating (20), we obtain

$$d\rho=\overline{\rho}_{Y}dY+\overline{\rho}_{G}dG, \tag{22}$$

where

$$\overline{\rho}_{Y}=-[(\overline{K}^{\Sigma}_{Y}+\overline{K}^{\Sigma}_{H}\overline{H}_{Y})-\psi\overline{\omega}_{H}\overline{H}_{Y}]/(\overline{K}^{\Sigma}_{\rho}-\psi\overline{\omega}_{\rho}),$$

$$\overline{\rho}_{G}=-\psi/(\overline{K}^{\Sigma}_{\rho}-\psi\overline{\omega}_{\rho}).$$

Assuming that the market of the capital good is stable, the excess demand function of this good is a decreasing function of the real interest rate: $\overline{K}^{\Sigma}_{\rho}-\psi\overline{\omega}_{\rho}<0$. Therefore, it holds that $\overline{\rho}_{G}>0$. It is because an increase in government spending reduces supply of the capital good through financing by lump-sum taxation. On the other hand, $\overline{\rho}_{Y}$ is ambiguous because of the diversity effect. If this effect does not work, it holds that $\overline{\rho}_{Y}>0$. If it works strongly, $\overline{\rho}_{Y}$ is negative. Hence, the real interest rate reduces with an increase in real national income in order to balance the capital market.

Substituting (22) into (21), we obtain the *long-run* multiplier:

$$\left(\frac{dY}{dG}\right)^{LR}_{LT}=\frac{[1-\phi^{1}-\psi-\phi^{2}(\overline{\rho})]+\overline{D}_{\rho}\overline{\rho}_{G}}{1-[\phi^{1}+\psi+\phi^{2}(\overline{\rho})]\overline{\omega}_{H}\overline{H}_{Y}-\overline{D}_{\rho}\overline{\rho}_{Y}}, \tag{23}$$

where $\overline{D}_\rho = [\phi^1 + \psi + \phi^2(\overline{\rho})]\overline{\omega}_\rho + \phi^2_\rho(\overline{\omega} - G)$. Assuming that the equilibrium point $(\overline{Y}, \overline{\rho})$ is stable, the denominator on RHS of (23) is *positive*. The second and third terms indicate the diversity and interest effects, respectively, in the multiplier process. The numerator represents the first-round effect of government spending. The first term shows a *direct* impact on aggregate demand. The second shows an *indirect* impact through a change in the real interest rate.

When a change in the real interest rate can be neglected, the first-round effect of government spending consists of the direct impact alone. We have the following *simple* multiplier in the long-run:

$$\left(\frac{\partial Y}{\partial G}\right)^{LR}_{LT} = \frac{1-[\phi^1 + \psi + \phi^2(\overline{\rho})]}{1-[\phi^1 + \psi + \phi^2(\overline{\rho})]\overline{\omega}_H \overline{H}_Y} = \frac{[1-\varepsilon - \delta(1+\overline{\rho})]/(1+\delta)}{1-[\phi^1 + \psi + \phi^2(\overline{\rho})]\overline{\omega}_H \overline{H}_Y}.$$

Therefore, the sign of this multiplier is ambiguous because of the term $\phi^2(\overline{\rho})$. It is positive, zero, or negative according as δ is smaller than, equal to, or greater than $(1-\varepsilon)/(1+\overline{\rho})$. On the other hand, when the real interest rate changes, the first-round effect depends on the indirect impact, too. It can be easily shown that sign $\overline{D}_\rho$ = sign $(\overline{l} - l^{**})$, where $\overline{l} < 1$ and $l^{**} = [\varepsilon + \delta + \delta(1+\overline{\rho})]/(1+\delta)$. Considering that $l^{**} = \phi^1 + \psi + \phi^2$, the long-run multiplier is negative when the simple one is non-positive. That is, if utility in the old period of each agent is hardly discounted, the long-run multiplier is *negative*. It should be noted that the long-run multiplier is not always positive, even if the simple one is positive.

Let us now illustrate the above results by using Figure 2. If each agent hardly discounts his utility in the old period, the YM curve shift downward with an increase in government spending. This curve has a negative slope in the neighborhood of the equilibrium point. The long-run multiplier is negative in this case, since the CM curve usually shifts rightward with an increase in spending. Otherwise, the YM curve shifts upward, but its slope is ambiguous. As long as the YM curve has a positive slope, the long-run multiplier is positive. However, it is negative, if the YM curve has a negative slope and if the right shift of the CM curve is very large.

5. Income Taxation

In this section, we refer to the short-run and long-run multipliers of government spending financed by labour income taxation. Totally differentiating the budget constraint of the government in the short-run, $\tau_t \omega_t l(\omega_t^{\tau}, \pi_t) = G_t$, we obtain

$$\omega_t d\tau_t = \tau_G dG_t + \tau_\pi \pi_Y dY_t + \tau_\omega \omega_\rho d\rho_t, \tag{24}$$

where

$$\tau_G = 1/M, \quad \tau_\pi = -\tau\omega l_\pi / M, \quad \tau_\omega = -[l + \omega l_{\omega^\tau}(1-\tau_t)]\tau / M, \quad M = l - \tau\omega l_{\omega^\tau}.$$

We assume that $M > 0$, so that $\tau_G > 0$. Therefore, the government must raise the tax rate in order to increase its spending. This implies that the economy operates on the upward sloping part of the Laffer curve. Since the labour supply function is $l_t = l^* - (1-\varepsilon)\pi_t/(1+\delta)\omega_t^{\tau}$, it holds that $l_{\omega\tau} > 0$ and $l_\pi < 0$. It follows from these inequalities that $\tau_\pi \pi_Y > 0$ and $\tau_\omega \omega_\rho > 0$. Thus, the tax rate is an increasing function of real national income and the real interest rate.

The short-run equilibrium condition in the differentiated good market is

$$Y_t = (\phi^1 + \psi)[(1-\tau_t)\omega(\rho_t, H_t) + \pi(Y_t, H_t)] + (1+\rho_t)k(I_{t-1}) + G_t. \tag{25}$$

Totally differentiating (25) in the neighborhood of the short-run equilibrium point $(\hat{Y}_t', \hat{\rho}_t')$ and then using (16) and (24) to eliminate $d\rho_t$ and $d\tau_t$, we have

$$dY_t = [1 - (\phi^1 + \psi)\hat{\tau}_G']dG_t + (\phi^1 + \psi)(1 - \hat{\tau}_\pi')\hat{\pi}_Y' dY_t + \hat{D}_\rho' \hat{\rho}_Y' dY_t,$$

where $\hat{D}_\rho' = (\phi^1 + \psi)(1 - \hat{\tau}_t' - \hat{\tau}_\omega')\hat{\omega}_\rho' + k(I_{t-1})$. It follows from $\hat{l}_t' < l^*$, $\hat{l}_{\omega^\tau}' > 0$, and $\hat{M}_t' > 0$ that $1 - (\phi^1 + \psi)\hat{\tau}_G' < 0$. Therefore, assuming that the equilibrium point is stable, the *short-run* multiplier is *negative*:

$$\left(\frac{dY_t}{dG_t}\right)_{IT}^{SR} = \frac{1 - (\phi^1 + \psi)\hat{\tau}_G'}{1 - [(\phi^1 + \psi)(1 - \hat{\tau}_\pi')/\zeta] - \hat{D}_\rho' \hat{\rho}_Y'} < 0.$$

This result can be explained in Figure 3-(i). An increase in spending financed by labour income taxation shifts the YM curve downward, leaving the CM line with a positive slope unchanged. Thus, real national income decreases in the short-run.

On the other hand, since labour supply is constant in the long-run, $l = l^*$, the budget constraint of the government is $\tau\omega[\rho, H(Y)]l^* = G$. Using this to eliminate τ from the long-run equilibrium conditions, they can be represented as

$$Y = [\phi^1 + \psi + \phi^2(\rho)]\{\omega[\rho, H(Y)] - (G/l^*)\} + G,$$

$$\psi\{\omega[\rho, H(Y)] - (G/l^*)\} = K^T[Y, \rho, H(Y)].$$

Differentiating these in the neighborhood of the equilibrium point $(\overline{Y}', \overline{\rho}')$, we obtain

$$(1 - \overline{D}_H'\overline{H}_Y')dY - \overline{D}_\rho' d\rho = \{1 - [\phi^1 + \phi^2(\overline{\rho}) + \psi]/l^*\}dG, \tag{26}$$

$$d\rho = \overline{\rho}_Y' dY + (\overline{\rho}_G'/l^*)dG, \tag{27}$$

where $1 - \overline{D}_H'\overline{H}' > 0$ and $\overline{\rho}_G' > 0$. Substituting $d\rho$ in (26) into (27), the *long-run* multiplier can be derived:

$$\left(\frac{dY}{dG}\right)_{lT}^{LR} = \frac{1 - [\phi^1 + \phi^2(\overline{\rho}') + \psi]/l^* + \overline{D}_\rho'(\overline{\rho}_G'/l^*)}{1 - \overline{D}_H'\overline{H}_Y' - \overline{D}_\rho'\overline{\rho}_Y'}. \tag{28}$$

It follows from $\phi^1 + \psi = l^*$ that $(\phi^1 + \phi^2 + \psi)/l^* > 1$. Therefore, assuming that the equilibrium point is stable, the sign of the long-run multiplier depends on the third term in the numerator on RHS of (28). If this term is nonpositive, the multiplier is negative. Otherwise, it is possible to be *positive*. The positive multiplier can be illustrated in Figure 3-(ii). The YM curve shifts downward with an increase in government spending. This curve has a positive slope if the sign of $\overline{D}_\rho'$ is positive. The CM curve shifts rightward with an increase in spending. If the YM curve has a positive slope and if the right shift of the CM curve is very large, the long-run multiplier is positive.

Finally, we will briefly make a remark on the short-run multiplier when the government assesses not only labour income but also profit income at the same tax rate. In this case, labour supply does not depend on the tax rate: $l_t = l^* - (1-\varepsilon)\pi_t/(1+\delta)\omega_t$. Substituting l_t into the budget constraint of the government, $\tau_t(\omega_t l_t + \pi_t) = G_t$, it becomes $\tau_t l^*(\omega_t + \pi_t) = G_t$. Considering this and $l^* = \phi^1 + \psi$, the equilibrium condition in the differentiated good market becomes

$$Y_t = (\phi^1 + \psi)(1-\tau_t)(\omega_t + \pi_t) + (1+\rho_t)k_{t-1} + G_t = (\phi^1 + \psi)(\omega_t + \pi_t) + (1+\rho_t)k_{t-1}.$$

Hence, the short-run multiplier is zero.

6. Welfare Analysis

In this section, we examine how an increase in government spending affects the short-run and long-run utilities of the representative agent. It is assumed that the government finances spending by lump-sum taxation.

The short-run utilities of the young and old agents at period t, $\tilde{u}_t$ and $\tilde{v}_t$, are written in terms of real variables, (I, ω, ρ), as follows:

$$\tilde{u}_t = I_t/(1+\delta)\omega_t^{1-\varepsilon}, \tag{29}$$

$$\tilde{v}_t = \delta(1+\rho_t)I_{t-1}/(1+\delta). \tag{30}$$

It follows from $\rho_G > 0$ and (30) that utility of the old is improved. Differentiating (29) with respect to G_t, we have

$$(d\tilde{u}_t/dG_t)I_t/\tilde{u}_t = \pi_Y Y_G - 1 + [\varepsilon - (1-\varepsilon)(\pi_t - G_t)/\omega_t)]\omega_G, \text{ where } \omega_G < 0.$$

The first and second terms on RHS of this equation altogether show the effect of spending on utility of the young through a change in non-labour income, i.e., $\pi_t - G_t$. It follows from $\pi_Y = 1/\xi$ and $0 < Y_G < 1$ that $\pi_Y Y_G - 1 < 0$. The sign of the third term, which is an effect through a decrease in the real wage rate, is ambiguous. Since this term is negative when non-labour income is non-positive, utility of the young falls.

The lifetime utility of the representative agent in the long run is given by

$$V = (1+\delta)\ln\{\omega[\rho, H(Y)] - G\} + \delta\ln(1+\rho) - (1-\varepsilon)\ln\omega[\rho, H(Y)] - A.$$

Differentiating this with respect to G, we obtain

$$\omega I V_G = [(1+\delta)\omega - (1-\varepsilon)I]\omega_G + \omega I(\frac{\delta}{1+\rho})\rho_G - (1+\delta)\omega, \tag{31}$$

where $I = \omega - G < \omega$ and $\rho_G > 0$. The first, second, and third terms on RHS of (31) imply the wage, interest, and lump-sum tax effects, respectively, on the lifetime utility. Differentiating the factor price frontier (2) with respect to G, we derive

$$\omega_G = \omega_\rho \rho_G + \omega_H H_Y Y_G = \frac{[1-\alpha(1-a)](1-\phi^1-\psi)\bar{k}}{\alpha \bar{l}\,(\bar{K}_\rho^\Sigma - \psi\bar{\omega}_\rho)(1-\bar{D}_H \bar{H}_Y - \bar{D}_\rho \bar{\rho}_Y)}.$$

When the diversity effect does not work, it holds that $1-\alpha(1-a) < 0$.[11] Thus, the sign of ω_G is negative. In this case, since the signs of ω_G and ρ_G are different, the sign of V_G is likely to be ambiguous. However, when the diversity effect is strong, the wage and interest effects are both positive. If these effects dominate the negative tax effect, an increase in government spending raises the lifetime utility.

7. Conclusion

In a dynamic model of monopolistic competition with overlapping generations, where national income and the interest rate are simultaneously determined in the differentiated good and capital good markets, we have studied Keynesian multipliers of government spending financed by taxes. The multiplier process depends on the interest effect on aggregate demand as well as the profit and diversity effects in the static framework. It has been shown that the dynamic multipliers have the following features different from those of the static ones. First, when spending is financed by lump-sum taxation, the short-run multiplier may exceed unity and the long-run one may be negative. Second, when spending is financed by labour income taxation, the short-run multiplier is

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[11] This inequality implies the *diminishing* marginal productivity of aggregate labour in the *macro* production function: $Y = \mu^{-\alpha}[(\mu-1)/f]^{\alpha-1}(HK)^{\alpha a}(HL)^{\alpha(1-a)}$.

negative but the long-run one may be positive.

It has also been shown that an impact of spending financed by lump-sum taxation on welfare may contradict with that on national income. In particular, when the diversity effect strongly works, the lifetime utility of the representative agent increases. Then, an expansive policy of spending should be evaluated, even if the long-run multiplier is negative. This contrasts with the result derived by Pagano (1990), who showed that a balanced-budget increase in spending reduces the steady-state level of welfare.

Finally, it will be useful to compare our model with that of Heijdra (1998). In his model, the time preference rate of the representative agent determines the interest rate in the long-run equilibrium. Thus, instead of the interest effect, the intertemporal substitution effect in labour supply influences the long-run multiplier of spending. On the other hand, in our model, the interest rate determined by the saving-investment decisions plays an important role in the multiplier process.

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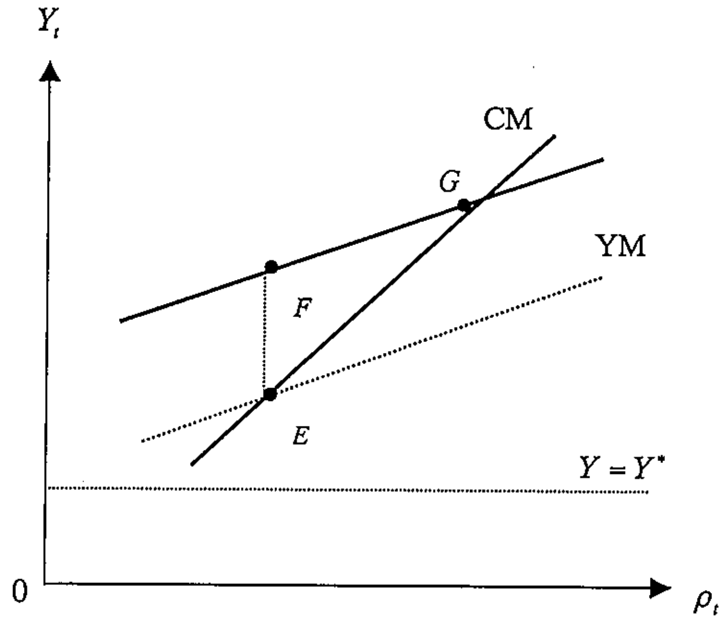
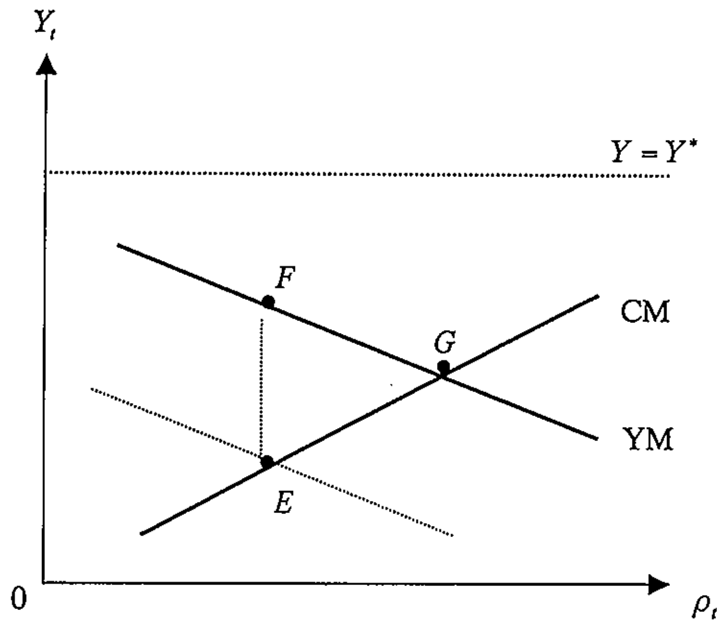
Figure 1

The short-run multiplier in the lump-sum tax case



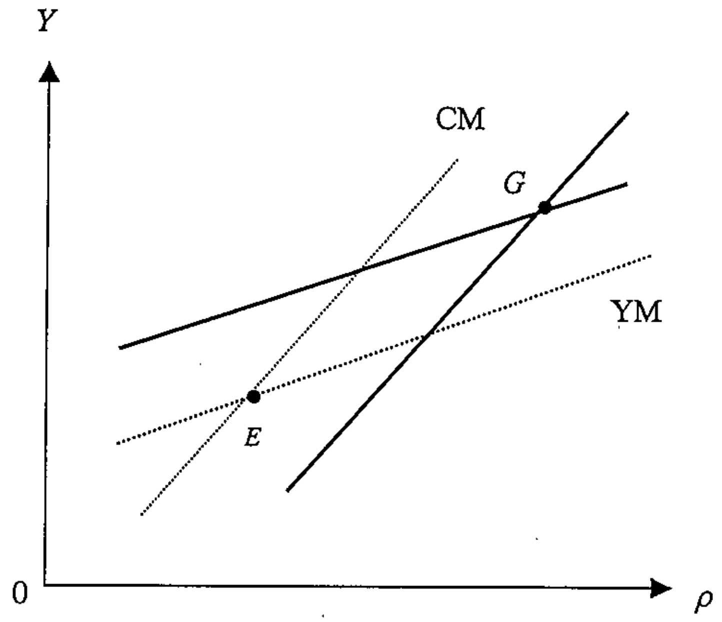
(i) The short-run multiplier ($E \rightarrow G$) > The simple one ($E \rightarrow F$)



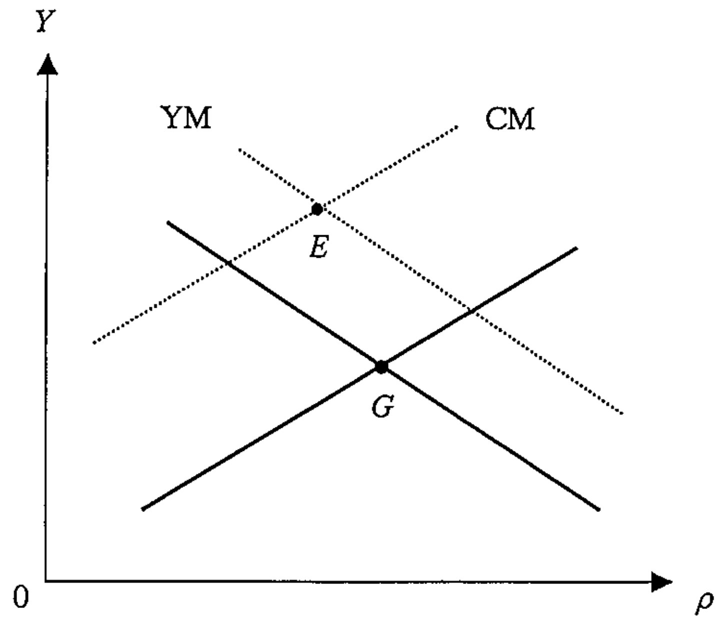
(ii) The short-run multiplier ($E \rightarrow G$) < The simple one ($E \rightarrow F$)

Figure 2

The long-run multiplier in the lump-sum tax case



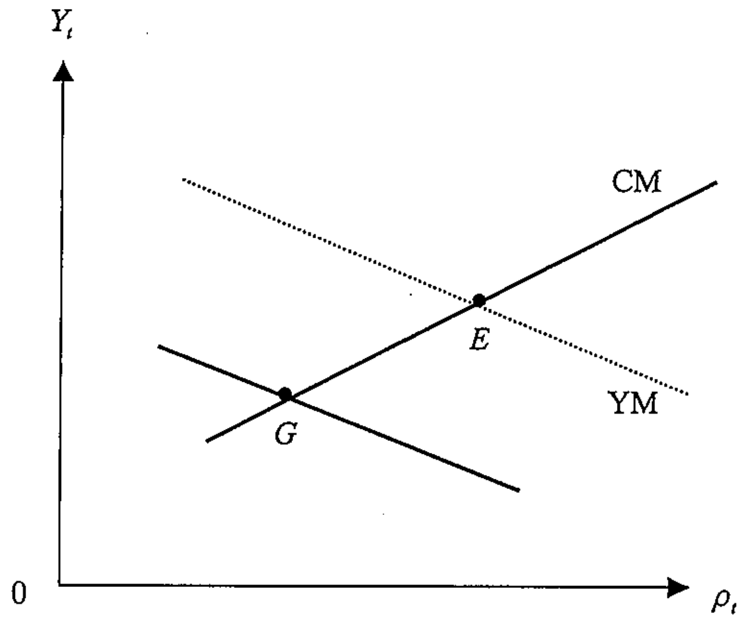
(i) The *positive* long-run multiplier ($E \rightarrow G$)



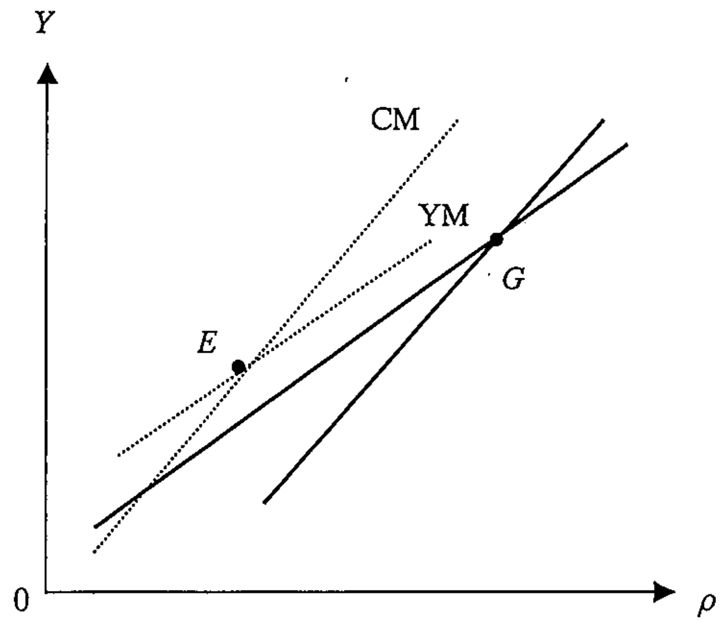
(ii) The *negative* long-run multiplier ($E \rightarrow G$)

Figure 3

The short-run and long-run multipliers in the labour-income tax case



(i) The *negative* short-run multiplier ($E \rightarrow G$)



(ii) The *positive* long-run multiplier ($E \rightarrow G$)