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The Evolution of Regional Income Distribution in Japan

by

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Abstract

Using data on per capita income of the 47 prefectures in Japan between 1955 and 1997, this paper shows that one of the most popular measurement of convergence, $\sigma-$ convergence, is not invariant with both transformation and definition of dispersion. Measured by the standard deviation, the actual per capita income has diverged whereas measured by the coefficients of variation, it has converged since 1955. The log transformed per capita income and relative income of all prefectures to Tokyo, however, have converged regardless of the measurement of dispersion. Furthermore, through the estimation of density function and transition matrix, the movements of each prefecture's per capita income relative to Tokyo prefecture are identified.

Keywords: convergence, density distribution, transition matrix.

JEL classification: O40, O50, R10

1 Introduction

Convergence, catching up of the poor with the rich, has been one of the controversial issues in economic growth literature. The most popular definitions are $\beta-$ and $\sigma-$convergence. The β-convergence holds that the growth rate, $(y_1 - y_0)/y_0$; y_0 as the base and y_1 as the reference period of income per capita, is a decreasing function of y_0. Despite of its popularity, this definition has a weakness of involving only one country over time whereas the verbal description of convergence involves many countries (or regions) over time.

On the other hand, the "σ-convergence" holds that the standard deviation (SD) of y_1 is smaller that the SD of y_0, i.e., $SD(y_1) < SD(y_0)$. The coefficients of variation is used as well which will be discussed in the next section. By looking at the SD for each period, this definition does involve many countries over time, unlike the $\beta-$convergence. The $\sigma-$convergence, however, does not capture the distribution dynamics that the $\beta-$convergence does. In addition, while β -convergence does not necessarily imply σ -convergence, while β -divergence does imply σ -divergence. See Barro and Sala-i-Martin (1995), Durlauf and Quah (1999), and the references therein for more on β- and σ -convergence.

Another strand of convergence definition is based upon "classification" or "clustering", motivated by the dichotomy of the poor and the rich: there is only one cluster at period 1 while there are two at period 0, which is noted as twin-peaks by Quah (1996a, 1996b). Although this approach also involves many countries over time, it is inadequate because the notion of dispersion is not necessarily reflected in the clusters. For instance, Bianchi (1997) compares the number of modes of the GDP distribution at period 0 and 1 and conclude that convergence is present if the number of modes declines ("modal convergence"). This approach, however, does not necessarily show a declining SD. Kang and Lee (2001) introduce a new convergence concept "Q-convergence", which defines convergence as a shrinking interquartile range (IQR) of the national income distribution, and confirms the findings of Bianchi (1997). The empirical results are not invariant to monotonic transformation of income per capita. Even with different methodologies, they find that the actual GDP per capita shows divergence whereas the log transformation of income per capita show convergence across countries.

Based on these definitions of convergence mentioned, particularly, β- and σ -convergence, this paper observes the evolution of income distribution across 47 prefectures of Japan between 1955 and 1997. We can compared the evolution of the evolution of actual income distribution and the transformation of income per capita. Three different definitions of real per capita income are used: actual income (KI), log of income (LKI), and income relative to that of Tokyo prefecture (RKI). The empirical findings for each of these definitions are compared. For $\sigma-$ convergence, two measurements of income dispersion - the Coefficients of variation (CV) and standard deviation (SD) are compared.

This paper shows that $\sigma-$convergence is not invariant to the monotonic transformation and measurement of dispersion whereas $\beta-$convergence is invariant to the monotonic transformation. For example, the actual income per capita has diverged while the log of actual income and relative income have converged regardless of the measurements of dispersion- standard deviation and the coefficients of variation since 1955. Also, through the density distribution estimation and transition matrix, the income movement of each prefecture between 1955 and 1997 is identified.

Section 2 summarizes the empirical results of β and $\sigma-$convergence with different variables and definitions. Section 3 identifies the intra- distribution dynamics of real income per capita across prefectures.

2 β and $\sigma-$ convergence

This section investigates the $\beta-$convergence and $\sigma-$convergence by using transformation of real income per capita. The $\beta-$convergence holds that poor economies tend to grow faster than rich economies. This is clearly illustrated by a cross-section analysis of all economies showing a negative relation between the growth rate of income per capita and its initial level. As Quah (1993) argues, this definition, however, fails to explain whether income distribution across the world becomes equitable over time.

In regard to the σ -convergence, Barro and Sala-i-Martin (1992, 1995) introduce a notion arguing that convergence is said to be present if the unequal distribution of income per capita across countries or regions, by using some convenient measures of dispersion, shows a tendency to decline over time. σ -convergence has two disadvantages. First is the lack of invariance (or equivariance) to monotonic transformations. In this paper, however, the log of income and income relative to the largest income of the same year that are popular in practice are used. Second is the perplexity of determining the measures of dispersion. In this paper, the two measures of dispersion: the coefficients of variation, CV, and (ii) the standard deviation, SD, are used.[1]

Assuming $\bar{y}_t \equiv (1/N)\sum_{i=1}^{N} y_{it}$ for year t where N is sample size,

$$CV_t(y_i) = \sqrt{\frac{1}{N}\sum_{i=1}^{n}\left(\frac{y_{it}-\bar{y}_t}{\bar{y}_t}\right)^2}, \text{ and } SD_t(y_i) = \sqrt{\frac{1}{N}\sum_{i=1}^{n}(y_{it}-\bar{y}_t)^2} \quad (1)$$

From equation (1), the CV is just the SD divided by the mean, $CV_t(y_i) = \frac{SD_t(y_i)}{\bar{y}_t}$. Rewriting the equation, we get

$$CV_t(y_i) = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\frac{y_{it}}{\bar{y}_t}-1\right)^2} = SD_t(\frac{y_{it}}{\bar{y}_t}) \quad (2)$$

where the mean of $\frac{y_i}{\bar{y}}$ is 1. The subscript t will be omitted throughout the discussion.

Here, CV in year t is just the SD of income per capita relative to its mean, emphasizing the importance of relative income rather than the deviation of actual income itself. Furthermore, from equation (2), since the CV of y_i in year t is just the same as the SD of $\frac{y_i}{\bar{y}}$ in year t, these two different measurements cannot be directly compared in deciding whether the income per capita has converged or not. Thus, this suggest that the CV of LKI is the same as the SD of LKI relative to the mean of LKI, and the CV of RKI is the same as the SD of RKI relative to the mean of RKI.

[1] In comparing the SD and CV to trace the convergence, convergence literature uses log of actual income for SD but actual income itself for CV. Dalgaard and Vastrup (2000) compare the two different measurememts with different transformation - raw income and log income per capita where they find that SD of log of income has increased, divergence, while the CV of actual income has decreased, suggesting convergence between 1970 and 1989.

There are three types of transformation of income per capita in this paper. First is the LKI, which is from a popular estimation of convergence that used the log of initial income rather instead of initial income. Then the coefficient for the log of initial income implies elasticity. Second is the RKI that is the ratio of income of the ith prefecture (y_i) to Tokyo (y_{TK}), $RKI = \frac{y_i}{y_{TK}}$. Income per capita across 47 prefectures of Japan are used from 1955 to 1997. The data on income per capita income is converted into real value by using national CPI. The income data are from the Annual Report on Prefectural Accounts (1991, 2000) while the National CPI is from International Financial Statistics (2000).[2]

As Salai-i-Martin (1996) found, the $\beta-$convergence has been approved even with log transformed and actual initial income in 1955. The coefficient for the log of initial income in 1955 is -0.0118 (t-value=-7.40). Figure 1 plots income growth rate against the log of initial income (in 1955) along the regression line. From this figure, the presence of $\beta-$convergence is observed that is consistent with the findings of Salai-i-Martin and other researchers. The income growth rate of prefectures with high initial income like Tokyo (TK), Osaka (OS), and Hyogo (HY) is lower than the growth rate of prefectures with low initial income like Okinawa (ON), Kagoshima (KS), and Yamanashi (YH). In addition, the regression result is not affected even without logging the initial income. The coefficient for initial income is -0.0258 with a t-value of -7.08.[3]

[2] The base year of the deflator between 1955 and 1974 is different from that between 1975 and 1997, so that it would be possible to estimate the all series of regional deflator based on the same year.

[3] Actual real income per capita is in thousands of yen.

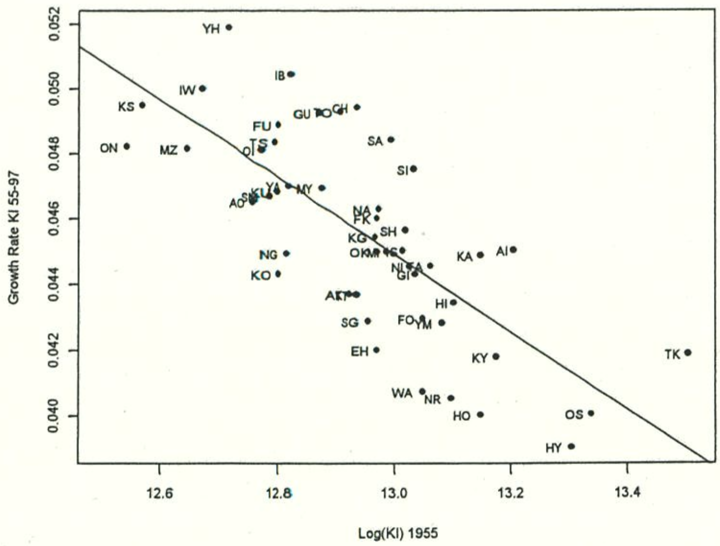

Figure 1: Income growth (1955-1997) and initial income (1955).

The results of estimation for σ -convergence is striking. As discussed above, the $\sigma-$convergence is affected by the transformation of income as well as by the definition of dispersion. The upper panel of Figure 2 shows the standard deviation (SD) of three different variables relative to 1955 while the bottom panel of Figure 2 shows the coefficients of variation (CV) at constant 1995 values. Although the CV of the three different variables show the same decreasing trend, the SD of KI increases while the SDs of two other variables decrease. This can be compared with the results of equation (2). Through the CV and SD at 1955 values, $\sigma-$convergence can be observed. The increase in SD of KI suggests that the actual income per capita has diverged while the other transformation of income per capita has converged since 1955.[4]

[4]This conclusion is the same as that of Bianchi (1997) and Kang and Lee (2001) on the

On the other hand, the CV of KI has decreased since 1955 because the CV itself controls mean of actual income. In addition, the dispersion of log transformed income per capita has narrowed compared to the dispersion of other variables. This suggests that conclusions on the dispersion of actual income per capita varies depending on the measurement of income dispersion. Furthermore, the decrease in the SDs and CVs of LKI and RKI shows that convergence has taken place regardless of the definition of dispersion. As we see from equation (1), the definition of CV controls the mean of income whereas the SD does not.

As shown above, while SD(KI) shows clear $\sigma-$divergence, SD(LKI) and SD(RKI) show a $\sigma-$convergence. To test this formally, let V_N denote the sample variance for year t: $V_N \equiv (1/N)\sum_{i=1}^{N}(y_i - \bar{y})^2$ where $\bar{y}_t \equiv (1/N)\sum_{i=1}^{N} y_i$. Then assume that 0 and 1 are the base and reference year, respectively,

$$\begin{aligned}\sqrt{N}(V_{N1} - V_{N0}) &= (1/\sqrt{N})\sum_{i=1}^{N}\{(y_{i1} - \bar{y}_1)^2 - (y_{i0} - \bar{y}_0)^2\} \\ &\Rightarrow N(0, C_v) \quad where \quad C_v \equiv E\{(y_{i1} - E(y_{i1}))^2 - (y_{i0} - E(y_{i0}))^2\}^2.\end{aligned}$$

C_v can be estimated consistently by

$$C_v \equiv (1/N)\sum_{i}\{(y_{i1} - \bar{y}_1)^2 - (y_{i0} - \bar{y}_0)^2\}^2;$$

real per capita income across countries.

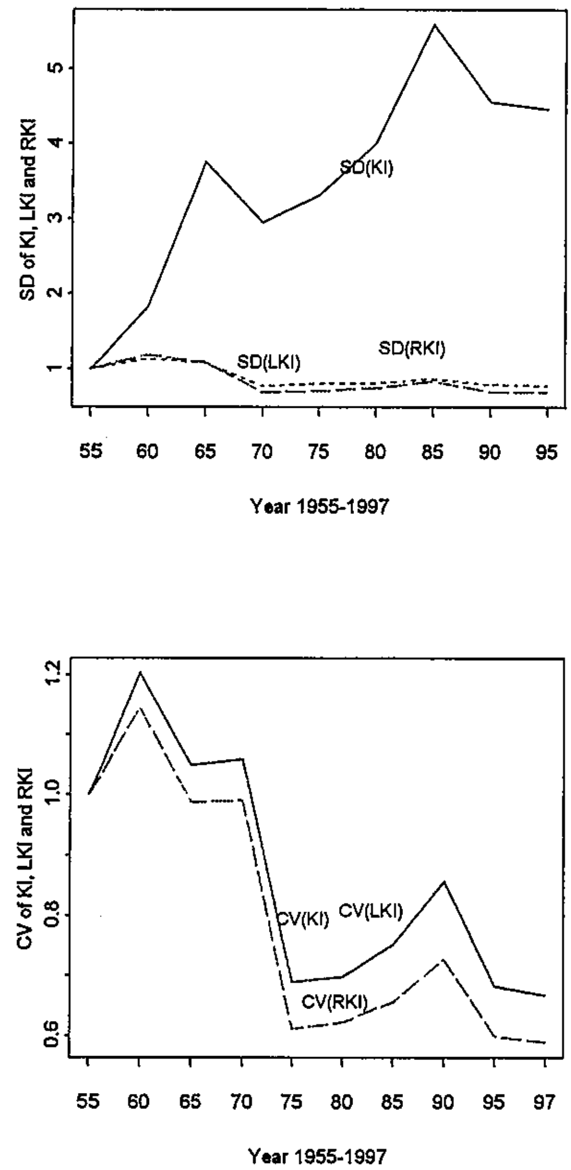

Figure 2: Standard deviation (top panel) and coefficients of variation (bottom panel) of KI, LKI, and RKI relative to 1955.

Table 1: Estimated t-values of KI, LKI, and RKI

	KI		LKI		RKI	
period	SD	CV	SD	CV	SD	CV
55-60	3.15	2.63	2.87	0.94	2.20	2.63
55-65	2.96	0.92	0.74	0.01	0.77	0.92
55-70	3.37	1.12	1.36	0.43	1.37	1.12
55-75	2.63	-3.24	-3.56	-3.65	-3.17	-3.24
55-80	3.05	-2.83	-3.25	-3.42	-2.52	-2.83
55-85	2.72	-2.72	-2.88	-3.22	-2.43	-2.72
55-90	2.62	-1.99	-2.16	-2.90	-1.94	-1.99
55-95	3.08	-2.72	-3.19	-3.42	-2.36	-2.72
55-97	3.09	-2.72	-3.18	-3.41	-2.36	-2.72

Although $(y_{i1} - \bar{y}_1)^2 - (y_{i0} - \bar{y}_0)^2$ are related to one another through $\bar{y}_0$ and $\bar{y}_1$, it can be shown that, for the asymptotic distribution, we just have to look at $(y_{i1} - E(y_{i1}))^2 - (y_{i0} - E(y_{i0}))^2$.

Here y_{i1} can be KI, LKI, or RKI depending on the transformation used. Table 1 reports the estimated t-values. First, for KI, the SD shows $\sigma-$convergence is statistically significant at 5% level while the CV shows strong $\sigma-$divergence except in the periods of 55-65 and 55-70. Recalling the results of equation (2) that the CV of y_i is the same as the SD of $y_i/\bar{y}_i$, the $y_i/\bar{y}_i$ can be interpreted as $\sigma-$convergence of actual income relative to the mean. For LKI and RKI, there has been strong $\sigma-$convergence regardless of the definition of dispersion except in the periods of 55-65 and 55-70.

The estimation results show that LKI and RKI experience both β and

$\sigma-$convergence. Although the actual income, KI, also shows $\beta-$convergence, is shows varying results on $\sigma-$convergence depending on the definition of dispersion. The CV of KI shows $\sigma-$convergence while the SD of KI shows $\sigma-$divergence. These results, however, cannot identify the intra- distribution dynamics of each prefecture over entire periods. As a remedy, the density distribution dynamics and transition matrix are used to observe the evolution of income distribution of each prefecture between 1955 and 1997.

3 Distribution Dynamics

As discussed in the introduction, the $\beta-$convergence considers the income distribution of each prefecture independently. The $\sigma-$convergence, in contrast, considers the distribution of income of all prefectures together. To see how these two convergence work together, the density distribution is used. Furthermore, to clarify the income movement of each prefecture over time, transition matrix is estimated.

3.1 Density Distribution

Consider a random variable y with realizations y_i, $i = 1, 2, 3, ..., N$. In our application, denoting by y_i the real income per capita across prefectures, and denote by $f(y)$ the density distribution of y.

Given a sample $\{y_i\}_i^N$ of independent and identically distributed observations, a kernel density estimator of $f(y)$ is constructed as (see Silveraman, 1986)

$$f(y) = (\frac{1}{Nh}) * \sum_{i=1}^{N} K(\frac{y - y_i}{h}) \tag{3}$$

where $h > 0$ is the bandwidth and $K(u) = (1/\sqrt{2\pi}) * \exp(-1/2u^2)$ where $u = \frac{y-y_i}{h}$. Bandwidth h governs the degree of smoothness of the density estimate, the larger the values of h is, the smoother the density estimate is.

In Figure 3, the four boxes of kernel density estimates are presented for year 1955 and 1997. The left two figures are findings for KI whereas the right two figures are findings for LKI. In the top two boxes, the bandwidth is $h = 0.9\sigma_y N^{-1/5}$, where $\sigma_y \equiv SD(y)$ for the given year.[5] In the bottom two boxes, the bandwidth is a "cross-validation" bandwidth, hcv.There are several ways to do a cross-validation, but here we use the one minimizing

$$hcv = \int \hat{f}(y : h)^2 dy - \frac{2}{N} \sum_{i=1}^{N} \hat{f}(y_i : h)_{-i}$$

where $\hat{f}(y : h)$ is the kernel density estimate evaluated at y, $\hat{f}(y_i : h)_{-1}$ is the "leave-one-out" estimate (i.e., i-th observation is not used for $\hat{f}(y_i : h)$), and hcv is the minimized.

What is common with these estimations is that their densities of income per capita all moved to the right, suggesting that the income of all prefectures has increased since 1955. For example, the minimum income in 1955 was Okinawa's income, 280 thousand yen and the maximum income was Tokyo's income, 733 thousand yen. In 1997, the Okinawa's income and Tokyo's income had increased to 2,119 thousand yen and 4,260 thousand yen,

[5] h has been used e.g. by Jones (1997) and shown to be optimal under certain conditions by Silverman (1986).

respectively.

As seen in Figure 3, the standard deviations of KI and LKI show different patterns of dispersion. The standard deviation of KI is increasing while the standard deviation of LKI is decreasing from 1955 to 1997. Here, the $\sigma-$convergence is interpreted by the time trend of the standard deviation of LKI and not of KI. If we assume that people's interests is their actual income, not the log of their income, then we should say that income distribution across prefectures has diverged since 1955.

On the other hand, one might say that the relative income to Tokyo is of more interests, compared to the actual income and log of income. In this case, since we control income with the highest income in 1955 and 1997, the density estimation shown in Figure 4 does not shift to right unlike in Figure 3. Thus, the income distribution across prefectures shows convergence to the income of Tokyo. The changing trend of income distribution across prefectures is shown in detail later.[6]

3.2 Transition Matrix

While Figure 3 fails to identify the income changes of each prefecture over time, Figure 4 helps to identify the growth pattern of each prefecture's relative income by plotting KI relative to Tokyo in 1955 and 1997. Changes in the distribution of income across prefectures are then illustrated by departures from the 45-degree line. Except 5 prefectures (OS, HY, HO, NR and

[6]From the density estimation, because all density distributions of KI, LKI, and RKI has only one mode, the modal convergence by Bianchi (1997) might lead to convergence of the three variables between 1955 and 1997.

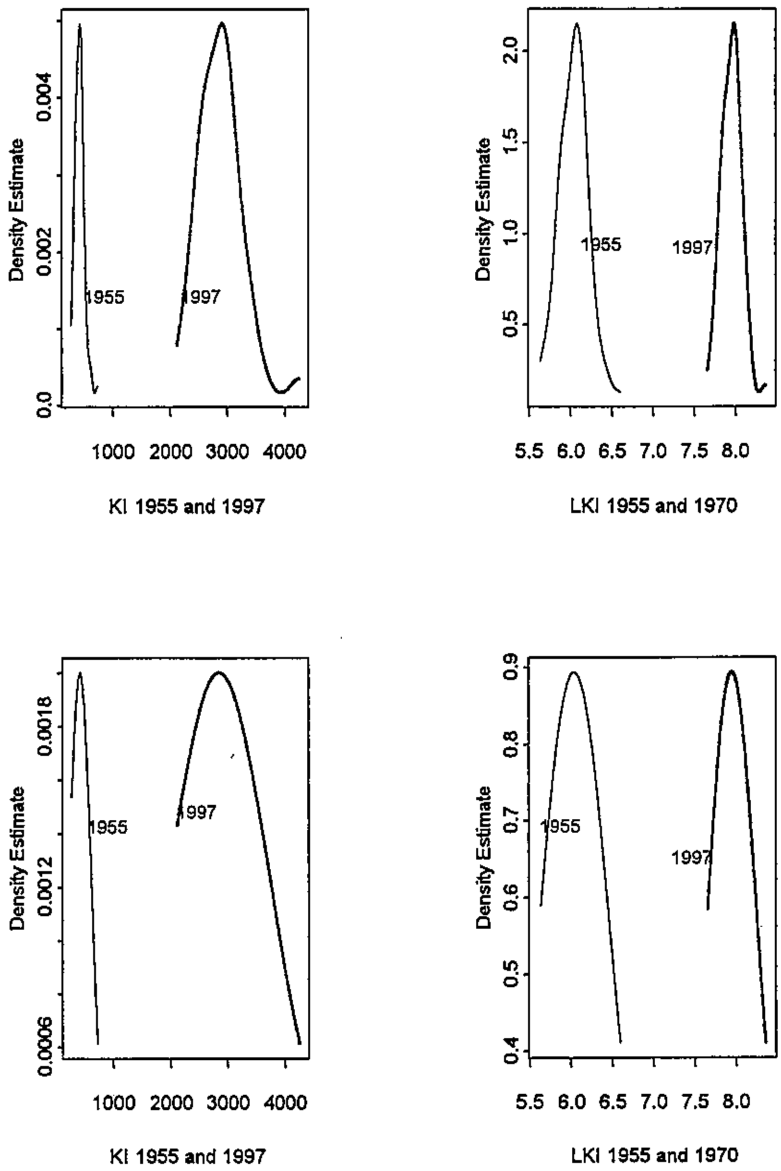

Figure 3: Density estimation of KI and LKI, with bandwidth h (top panel) and hcv (bottom panel).

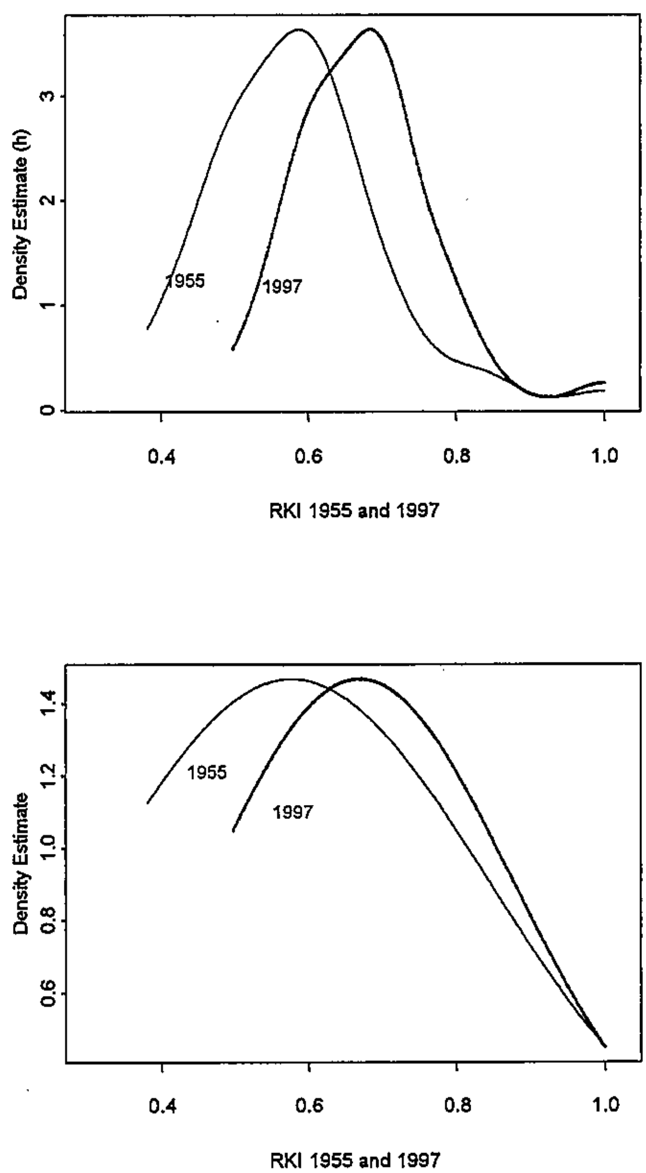

Figure 4: Density estimation of KI relative to Tokyo with bandwidth h (top panel) and hcv (bottom panel).

WA), the income of all prefectures has increased over the period 1955-1997.

In addition, transition matrix of Table 1 indicates the upward or downward movement of income of each prefecture. The prefectures along the diagonal shows that the quartiles they belong to remain the same over periods. I and IV represent the lower quartile (25% quantile) and the upper quartile (75% quantile), respectively. For example, TK (Tokyo) prefecture was in the highest quartile (IV) both in 1955 and 1997. However, the movement along off-diagonal shows that there is shifting of quartiles between 1955 and 1997. For example, YH (Yamanashi) prefecture moved from the lowest quartile in 1955 to the third quartile (III) in 1997, implying that its income per capita has increased, compared to other prefectures. WA (Kagoshima) prefecture moved from the highest quartile (IV) in 1955 to the lowest quartile I in 1997.

From the transition matrix by quartile, 23 prefectures out of 47 prefectures remains in the same quartile between 1955 and 1997. The number of prefectures that move by one quartile is 17 with 7 (upward) and 10 (downward) and the number of prefectures that move more than 2 quartile is 7 with 5 (upward) and 2 (downward). Thus, in 1997, 11 prefectures move upward and 13 prefectures move downward relative to the income position in 1955.

4 Conclusion

This paper investigates the evolution of income distribution across prefectures in Japan. Regardless of the transformation of actual income per capita, $\beta-$convergence is shown to be statistically significant. However, the conclusion on $\sigma-$convergence is not invariant with transformation and measure-

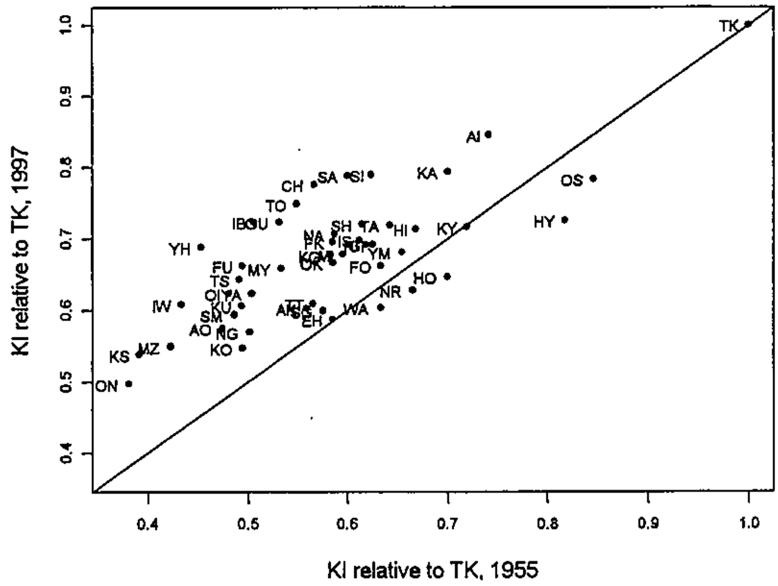

Figure 5: KI relative to TK, 1955 vs. 1997

Table 2: Transition matrix of KI from 1955 to 1997

		1997				
		I	II	III	IV	Total
	I	AO,SM KO,KU,MZ, KS,ON	IW, FU TS, OI	YH		12
1955	II	AK, EH SG, NG	MY, YA TT, KG		IB, TO GU, CH	12
	III		OK, FO	NT, NA IS, GI, MI, FK	SA, SH ST	11
	IV	WA	HO, NR	TA, KY HI, YM	TK, KA AT, OS, HY	12
	Total	12	12	12	11	47

ment. In terms of standard deviation, actual income is shown to be divergent as SD has increased since 1955 whereas the log of actual income and relative income have converged. The other measurement, CV, has shown convergence among prefectures. The CV, as a measurement of distribution, has shown the income relative to average income to be convergent regardless of the definition of per capita income.

In considering the actual income itself, the compared SDs of actual income of all prefectures show $\sigma-$divergence. On the other hand, in considering the relative income to Tokyo, the presence of $\sigma-$convergence is observed. Although the log transformation of actual income is shown to be convergent regardless of measurement of convergence, it cannot be justified in terms of interests.

These popular criteria of convergence, however, fails to show intra-distribution dynamics of each prefecture. Using the Gaussian density distribution estimation and transition matrix of actual income, 22 prefectures out of 47 prefectures are shown to remain in the same quartile between 1955 and 1997. However, 12 prefectures move upward and 13 prefectures move downward from their original quartile.

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Appendix

Table A: Prefectures and their abbreviation

	prefecture			prefecture			prefecture	
1	Hokkaido	HO	17	Nagano	NA	33	Okayama	OK
2	Aomori	AO	18	Shizuoka	SH	34	Hirosima	HI
3	Iwate	IW	19	Toyama	TA	35	Yamaguchi	YM
4	Miyagi	MY	20	Ishikawa	IS	36	Tokushima	TS
5	Akita	AK	21	Gifu	GI	37	Kagawa	KG
6	Yamagata	YA	22	Aichi	AI	38	Ehime	EH
7	Fukushima	FU	23	Mie	MI	39	Kochi	KO
8	Niigata	NI	24	Fukui	FK	40	Fukuoka	FO
9	Ibaraki	IB	25	Shiga	SI	41	Saga	SG
10	Tochigi	TO	26	Kyoto	KY	42	Nagasaki	NG
11	Gumma	GU	27	Osaka	OS	43	Kumamoto	KU
12	Saitama	SA	28	Hyogo	HY	44	Oita	OI
13	Chiba	CH	29	Nara	NR	45	Miyazaki	MZ
14	Tokyo	TK	30	Wakayama	WA	46	Kagoshima	KS
15	Kanagawa	KA	31	Tottori	TT	47	Okinawa	ON
16	Yamanashi	YH	32	Shimane	SM			