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Dynamic Mobility Management for Cellular Networks: A Delayed Renewal Process Approach

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Abstract

Tracking mobile users in cellular wireless networks involves two basic functions: location update and paging. Location update refers to the process of tracking the location of mobile users that are not in conversation. Three basic algorithms have been proposed in the literature, namely the distance-based, time-based, and movement-based algorithms. The problem of minimizing the location update and paging costs has been solved in the literature by considering exponentially distributed Cell Residence Times (CRT) and Inter-Call Time (ICT), which is the time interval between two consecutive phone calls.

In this paper we have selected the movement-based scheme since it is effective and easy to implement. Applying the theory for the delayed renewal process, we find the distribution of the number of cell crossings when the ICT is a mixture of exponentially distributed r.v.'s and the CRT comes from any distribution with Laplace-Stieltjes transform. In particular, we consider the case in which the first CRT may have a different distribution from the remaining CRT's, which includes the case of circular cells. We aim at the total cost minimization in this case.

1 Introduction

In personal communication networks (PCN), the wireless cells have a limited coverage range, which means that a user will be crossing through several wireless cells during a call duration as well as during an Inter-Call Time (ICT), the time interval between two consecutive phone calls. Each wireless cell crossing may need switching network facilities, which are necessary in order to maintain connectivity with the network and the location tracking of the mobile. The performance of PCN is affected by the manner the network manages the movements of the mobile users not only for handover management during a call duration but also to track the location of the mobile users during an ICT so that incoming calls can be delivered to the mobile subscribers.

Typically, for location tracking purposes a mobile service area is partitioned into several subareas, so-called registration areas (RA) or location areas (LA). One of the

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main challenges of mobility management is to update the registration (or location) of a mobile in an efficient manner when it moves from one RA to another. The registration and the call-delivery procedure take place by using the information in the Home Location Register (HLR) and the Visitor Location Register (VLR) mobility databases. The HLR is a database used to store all permanent subscriber data, whereas VLR is the database of the service area visited by a mobile user. The VLR database can overflow when many users move into the VLR-controlled area in a short period of time. Under this phenomenon a new mobile user entering this RA fails to register in the database, so it cannot receive the service, and the subsequent calls to this user may be misrouted.

Three basic schemes for location update are described in [1], namely the *movement-based* which updates the location as a function of the number of movements performed or crosses through different RA's, the *time-based* which performs location updates based on the time elapsed, and *distance-based* which uses the distance traveled since the last location update. Akyildiz et al. [2] justify the use of the movement-based location update policy due to its simplicity, since it does not require each mobile terminal to store information regarding distances or times, and each mobile terminal only needs to keep a counter of the number of cells visited. A registration is performed when this counter exceeds a predefined threshold value.

Recently, Li et al. have considered Cell Residence Times (CRT) with any probability distribution and exponentially distributed ICT [3] for a dynamic mobility management scheme with the movement-based strategy for the location update. They consider the joint minimization of the cost of paging ($C_p(d)$) and the cost of location update ($C_u(d)$) for the hexagonal and mesh configurations as a function of the optimal movement threshold d. A fundamental point in thier analysis is to find the probability mass function (*pmf*) of the number of cell boundary crossings during an ICT. This problem is mathematically equivalent to the problem of finding the *pmf* of the number of handovers in a Call Holding Time (CHT), as shown in [2] and [3]. The *pmf* for the number of cell boundary crossings in a random interval, namely CHT or ICT, have been derived for many special cases in several papers for a regular configuration or a single homogeneous wireless platform, where it is natural to assume the same distribution for the CRT in all the wireless cells; see for instance [4], [5], [6], [7], [8]. In our approach to finding such a *pmf*, we have followed several arguments from renewal theory, in particular that for the *equilibrium* renewal process defined by Cox [9]. In [10], we have extended our previous results to the case of a *delayed* renewal process, i.e., a renewal process where the first wireless cell has a different CRT probability distribution from the remaining cells. Our approach allows us to extend the Li et al.'s results [3] to the case of circular cell configurations.

2 Probability Distribution for the Number of Cell Crossings

Let $N(t)$ denote the number of renewals (or cell crossings) in a fixed interval $(0, t)$, and let us suppose that the interrenewal times occur according to a sequence of random variables $\{T_1, T_2, \ldots, T_i, \ldots\}$ with probability density functions (*pdf*) $f_{T_i}(t)$ whose Laplace-Stieltjes Transform (L.-S.T.) is denoted by $f^*_{T_i}(s)$. Now, let T_c be a random variable representing the ICT. Let us assume that T_c is independent of the random variables $T_i; i = 1, 2, \ldots$. In fact, the sequence of random variables $\{T_1, T_2, \ldots, T_i, \ldots\}$ represents the CRT's of a call in wireless cells. Hence, $N(T_c)$ is a random variable which gives us the number of cell crossings in the random interval $(0, T_c)$. As defined by Cox [9], we have an *ordinary* renewal process when all the random variables $T_i; i = 1, 2, \ldots$ come from the same distribution. We have an *equilibrium* renewal process when $\{T_2, T_3, \ldots, T_i, \ldots\}$ come from the same distribution $F_T(t)$, and T_1 is the excess life (or residual life) of the random variable T. This case seems to be the appropriate model for a single homogeneous wireless platform, as it has been established in [4] and considered in [3] as well. On the other hand, when the random variable T_1 comes from a different distribution, say $F_{T_1}(t)$, and $\{T_2, T_3, \ldots, T_i, \ldots\}$ come from the same distribution $F_{T_2}(t)$, we have the *modified* or *delayed* renewal process [9]. This is just the process that we will use to model the circular configuration in this paper.

We will present our analysis by considering the hyperexponential distribution for the ICT, and the CRT for T_1 corresponds to the distance from an arbitrary point in a circle to its perimeter in an arbitrary direction if the user moves at a constant speed, and the CRT for $T_2, T_3, \ldots$ corresponds to the length of a segment of a straight line that crosses the circle in an arbitrary direction [11], [12]. We will call this case the circularly distributed CRT.

Let us denote by $N(T_c)$ the number of cell crossings during a random ICT. Then, the probability generating function (*pgf*) of the number of cell crossings during the random interval T_c is given by

$$G_{N(T_c)}(z) = \int_{t=0}^{\infty} G_{N(t)}(t, z)\ f_{T_c}(t)\ dt \tag{1}$$

where $f_{T_c}(t)$ is the *pdf* of the random variable T_c, and $G_{N(t)}(t, z)$ is the *pgf* of $N(t)$.

An interesting relationship does exist between the *pgf*s of $N(t)$ and $N(T_c)$ in the special case of k-Erlang(λ) *pdf* for the ICT, which is expressed as follows:

$$G_{N(T_c)}(z) = \frac{\lambda^k}{(k-1)!} \left(-\frac{\partial}{\partial s} \right)^{k-1} \left\{ G^*_{N(t)}(s, z) \right\} \Big|_{s=\lambda} \tag{2}$$

where

$$G^*_{N(t)}(s, z) = \frac{1 - z f^*_{T_2}(s) + (z-1) f^*_{T_1}(s)}{s[1 - z f^*_{T_2}(s)]}$$

is the L.-S.T. of $G_{N(t)}(t, z)$ for the *modified* renewal process, as shown by Cox [9], page 38.

We will assume that the ICT random variable T_c is well modeled by a mixture of exponential *pdf*s, say

$$f_{T_c}(t) = \sum_{i=1}^{M} p_i \lambda_i e^{-\lambda_i t} \tag{3}$$

where $\sum_{i=1}^{M} p_i = 1$. Then the generating function for the number of cell crossings in the random interval T_c is given by

$$G_{N(T_c)}(z) = \sum_{i=1}^{M} p_i \lambda_i \left\{ G^*_{N(t)}(s, z) \right\} \Big|_{s=\lambda_i} = \sum_{i=1}^{M} p_i \left[\frac{1 - z f^*_{T_2}(\lambda_i) + (z-1) f^*_{T_1}(\lambda_i)}{1 - z f^*_{T_2}(\lambda_i)} \right] \tag{4}$$

where $T_c = \sum_{i=1}^{M} p_i T_{ci}$ is the mixture of M exponentially distributed random variables T_{ci} with mean value $\mathbf{E}[T_{ci}] = 1/\lambda_i$ $(i = 1, 2, \ldots, M)$. It follows from equation (4) that the *pmf* of the number of cell crossings during an ICT is given by

$$P[N(T_c) = \ell] = \begin{cases} 1 - \sum_{i=1}^{M} p_i f_{T_1}^*(\lambda_i) & ; \quad \ell = 0 \\ \sum_{i=1}^{M} p_i f_{T_1}^*(\lambda_i)[1 - f_{T_2}^*(\lambda_i)][f_{T_2}^*(\lambda_i)]^{\ell-1} & ; \quad \ell = 1, 2, \ldots \end{cases} \tag{5}$$

We can obtain several interesting cases by specifying the L.-S.T. of the *pdf* for both types of CRT's, as it is shown in the following subsections.

2.1 Exponentially Distributed CRT

Let us assume that the time of residence in the first cell is exponentially distributed with mean value $\mathbf{E}[T_1] = 1/\mu_1$, and that the CRT for the remaining cells is also exponentially distributed with mean value $\mathbf{E}[T_2] = 1/\mu_2$. Then the corresponding L.-S.T.'s are given by

$$f_{T_r}^*(s) = \frac{\mu_r}{s + \mu_r}; \quad r = 1, 2 \tag{6}$$

Hence, the *pmf* in equation (5) reduces to

$$P[N(T_c) = \ell] = \begin{cases} 1 - \sum_{i=1}^{M} p_i \frac{\rho_{1,i}}{1+\rho_{1,i}} & ; \quad \ell = 0 \\ \sum_{i=1}^{M} p_i \left(\frac{\rho_{1,i}}{1+\rho_{1,i}}\right) \left(\frac{1}{1+\rho_{2,i}}\right) \left(\frac{\rho_{2,i}}{1+\rho_{2,i}}\right)^{\ell-1} & ; \quad \ell = 1, 2, \ldots \end{cases} \tag{7}$$

where $\rho_{r,i} = \mathbf{E}[T_{ci}]/\mathbf{E}[T_r] = \mu_r/\lambda_i$ for $r = 1, 2$ and $i = 1, 2, \ldots, M$.

2.2 Circularly Distributed CRT

The hexagonal geometry for the wireless cells has been approximated by circles with radius R by Hong and Rappaport [11] and by Yeung and Nanda [12]. They have derived the CRT distributions under the assumptions that the mobiles are uniformly distributed in the system and that the mobiles move in straight lines with direction uniformly distributed over $[0, 2\pi)$. The *pdf* of the random variable Z_1, the distance from an arbitrary interior point to the boundary of the circle, is given by equation (46) in Hong and Rappaport [11] as

$$f_{Z_1}(z) = \frac{2}{\pi R^2}\sqrt{R^2 - \left(\frac{z}{2}\right)^2}; \quad 0 \le z \le 2R$$

where R is the radius of the equivalent circle. Hence, the CRT in the first wireless cell is given by the random variable $T_1 = Z_1/V$, where V is the velocity of the mobile, which we will assume to be a constant in the rest of this paper. In fact, this is a typical assumption also made in [11] and [12]. Under this assumption of constant velocity, we can obtain the kth moment of T_1 about the origin as follows:

$$\mathbf{E}[T_1^k] = \frac{2}{\pi R^2 V^k}\int_{z=0}^{2R} z^k \sqrt{R^2 - (z/2)^2}\,dz = \frac{\Gamma\left(\frac{k+1}{2}\right)}{\sqrt{\pi}\,\Gamma\left(\frac{k}{2}+2\right)}\left(\frac{2R}{V}\right)^k; \qquad k = 1, 2, \ldots \tag{8}$$

where $\Gamma(z)$ is the gamma function.

We should remember that if we have a random variable X with *pdf* $f_X(x)$, then its L.-S.T. $f_X^*(s)$ can be expanded as a function of the moments of X as follows:

$$f_X^*(s) = \sum_{k=0}^{\infty}(-1)^i \frac{s^k}{k!}\mathbf{E}[X^k] \tag{9}$$

This formula is also discussed in Cox [9], page 9. Now, by using equations (8) and (9), we can obtain

$$f_{T_1}^*(\lambda_i) = \sum_{k=0}^{\infty}(-1)^k \frac{M_{k,1}}{k!\rho_{1,i}^k} \tag{10}$$

where

$$M_{k,1} = \pi^{k-\frac{1}{2}}\left(\frac{3}{4}\right)^k \frac{\Gamma\left(\frac{k+1}{2}\right)}{\Gamma\left(\frac{k}{2}+2\right)} \quad ; \quad \rho_{1,i} = \frac{\mathbf{E}[T_{ci}]}{\mathbf{E}[T_1]} = \frac{3\pi V}{8R\lambda_i} \tag{11}$$

Similarly, the *pdf* for the random variable Z_2, the distance from an arbitrary point on the boundary of the circle where the mobile enters into a cell to another point on the boundary where the mobile exits from the cell in a straight line, is given by equation (51) in Hong and Rappaport [11] as follows:

$$f_{Z_2}(z) = \frac{2}{\pi\sqrt{4R^2 - z^2}}; \quad 0 \le z \le 2R$$

Then the CRT for this cell is given by $T_2 = Z_2/V$, and its kth moment is given by

$$\mathbf{E}[T_2^k] = \frac{\Gamma\left(\frac{k+1}{2}\right)}{\sqrt{\pi}\Gamma\left(\frac{k+2}{2}\right)}\left(\frac{2R}{V}\right)^k; \qquad k = 1, 2, \ldots \tag{12}$$

Hence, by using equations (9) and (12), we obtain the following expansion:

$$f_{T_2}^*(\lambda_i) = \sum_{k=0}^{\infty}(-1)^k \frac{M_{k,2}}{k!\rho_{2,i}^k} \tag{13}$$

where

$$M_{k,2} = \frac{\pi^{k-\frac{1}{2}}}{2^k}\frac{\Gamma\left(\frac{k+1}{2}\right)}{\Gamma\left(\frac{k}{2}+1\right)} \quad ; \quad \rho_{2,i} = \frac{\mathbf{E}[T_{ci}]}{\mathbf{E}[T_2]} = \frac{\pi V}{4R\lambda_i} \tag{14}$$

3 Costs of Location Update and Paging

The expected cost $C_u(d)$ of location update per call arrival is assumed to be

$$C_u(d) = U\sum_{k=1}^{\infty} k \sum_{\ell=kd}^{(k+1)d-1} P[N(T_c) = \ell] \tag{15}$$

where d is the movement threshold for the location update, and U (> 0) is the cost for performing an update which takes into account the wireline bandwidth utilization and the computational cost for updating. Substituting equation (5) into equation (15) we can obtain

$$C_u(d) = U \sum_{i=1}^{M} p_i f_{T_1}^*(\lambda_i) \frac{[f_{T_2}^*(\lambda_i)]^{d-1}}{1 - [f_{T_2}^*(\lambda_i)]^d} \tag{16}$$

Now, let us assume that the cost for polling a cell is P^o (> 0). Since the circular configuration has the same number of neighbors as the hexagonal configuration, then the expected paging cost $C_p(d)$ per call arrival is given by

$$C_p(d) = P^o[1 + 3d(d-1)]$$

The total cost, say $CT(d)$, is given by

$$CT(d) = C_u(d) + C_p(d)$$

This cost function can be optimized with respect to d by taking $\partial CT(d)/\partial d = 0$, which reduces to the following equation for d:

$$U \sum_{i=1}^{M} p_i f_{T_1}^*(\lambda_i) \frac{[f_{T_2}^*(\lambda_i)]^{d-1} \ln[f_{T_2}^*(\lambda_i)]}{[1 - [f_{T_2}^*(\lambda_i)]^d]^2} + 3P^o(2d-1) = 0 \tag{17}$$

Similarly to the analysis of Li et al. [3], it can be shown from the second derivative $\partial^2 CT(d)/\partial d^2 = 0$ that $CT(d)$ is a convex function.

4 Numerical Results

For numerical examples, we will consider the following three cases: (i) exponentially distributed ICT and exponentially distributed CRT as in [3] (EE), (ii) hyperexponentially distributed ICT and exponentially distributed CRT (HE), and (iii) hyperexponentially distributed ICT and circularly distributed CRT (HC). A number of parameters characterize the behavior of the optimal threshold. We will compare this behavior when each parameter is changed while the others remain fixed. We will take as a reference the following parameters: $U = 100$, $p_1 = p_2 = 0.5$, $\rho_{1,2} = 3\rho_{1,1}$. In order to observe the effect produced over the optimal threshold, we will change one by one these parameters, and P^o will be always fixed to 1.

In Figures 1 and 2, we show the behavior of the optimal threshold d as a function of the mobility ratio for location update cost $U = 100$ and 15, respectively. The EE case (dashed line in Figures 1 and 2) is the one studied in [3]; however, our definition of the mobility ratio

$$\rho := \frac{\text{Expected value of ICT}}{\text{Expected value of CRT}} \tag{18}$$

is the inverse of theirs. Thus, in our plots we have better resolution for large values of ρ. In addition, we can observe that the optimal threshold d becomes larger as ρ is increased. We should also mention that the HE and the HC cases depend on four relative mobility ratios ($\rho_{1,1}, \rho_{1,2}, \rho_{2,1}$ and, $\rho_{2,2}$), while the EE case depends only on the mobility ratio ρ defined in equation (18). It is rather complicated to extract a global mobility ratio for the HE and the HC cases; therefore, we have decided to use $\rho_{1,1}$ as our independent variable on the plots. This is obviously not a global mobility ratio as the one defined in equation (18); however, our definition of the realtive mobility ratios helps to understand the behavior of the optimal threshold d when different parameters of the model are changed.

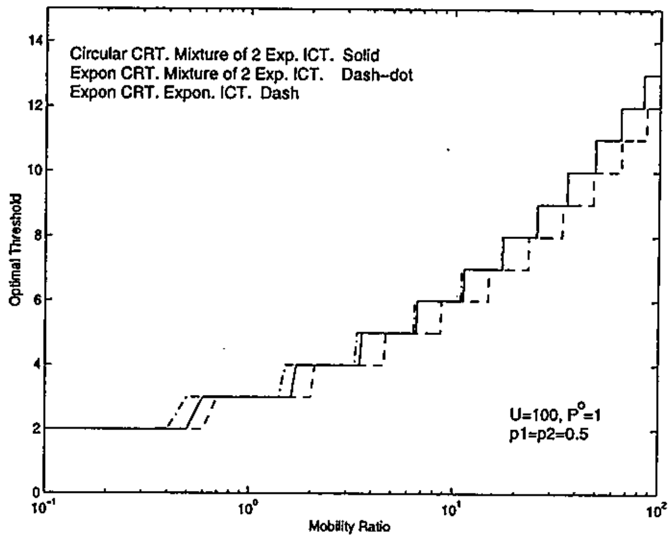

Figure 1: Optimal threshold as a function of the mobility ratio for $U = 100$.

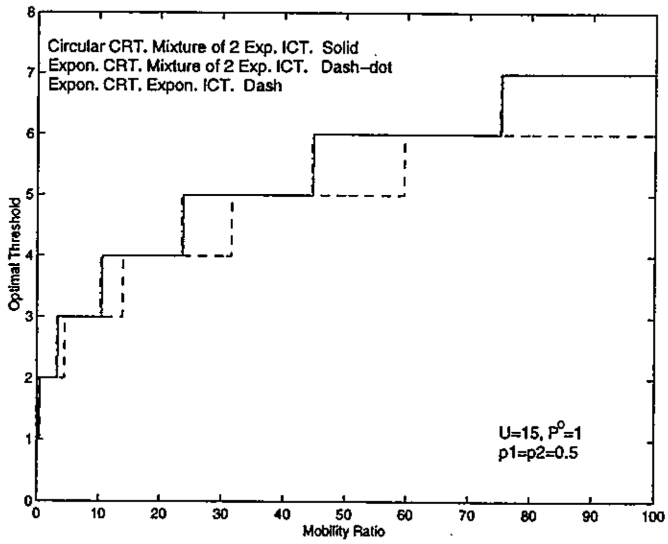

Figure 2: Optimal threshold as a function of the mobility ratio for $U = 15$.

Apparently in Figure 1, the HC (solid line) and the HE (dash-doted line) cases seem to merge. However, this is only a plotting effect caused by the semi-logarithmic scale. In fact, the difference between these two lines remains almost constant in a linear scale. This can be observed in Figure 2.

We now study the HC case in which the total cost equation depends on the following parameters: the mixing probabilities ($p_1 = p, p_2 = 1 - p$), the location update cost U, and the four mobility ratios ($\rho_{1,1}, \rho_{1,2}, \rho_{2,1}, \rho_{2,2}$). It can be seen that $\rho_{1,i} = \frac{3}{2}\rho_{2,i}$ in this case.

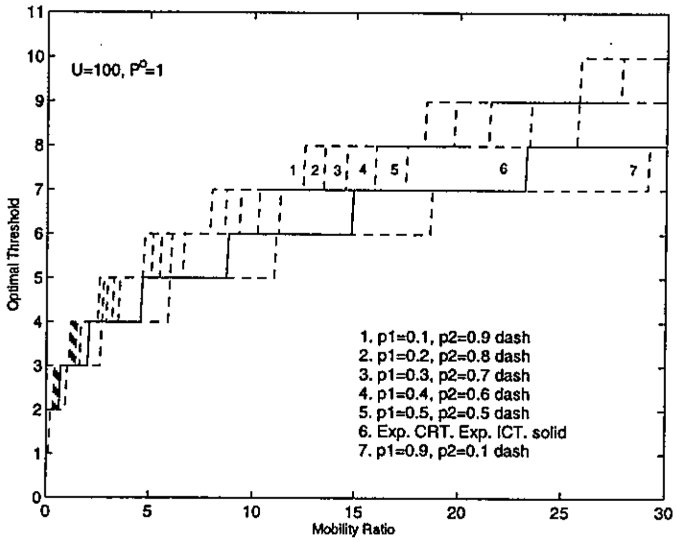

Figure 3: Optimal threshold in the HC case as a function of the mobility ratio and mixing probabilities p.

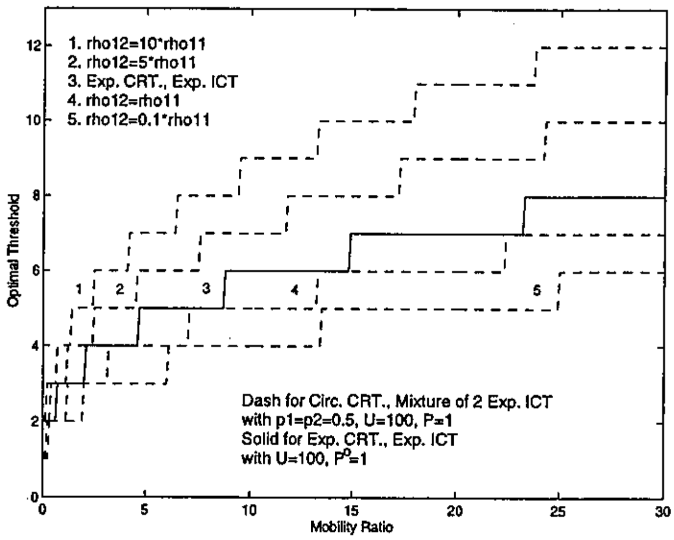

Figure 4: Optimal threshold in the HC case for several values of λ_1 and λ_2.

The behavior of the optimal threshold d, for the HC case, when the mixing probabilities are changed while all the other parameters remain fixed, is shown in Figure 3. It can be observed that the optimal threshold grows faster when $p_2 > p_1$, and that the growing speed of the optimal threshold depends on the mixing probability p. We also note that the size of the *steps* is slightly increased. In Figure 4 we observe the behavior of the optimal threshold when the ratio $\delta := \rho_{1,2}/\rho_{1,1}$ is changed whereas all the other parameters remain fixed. This is equivalent to changing the values of λ_1 and λ_2.

The effect produced by changing the ratio δ, affects the growing speed of the optimal threshold as well. A comparison between Figures 3 and 4 shows that the effect produced by changing δ is much stronger than that produced by changing p. Not only is the growing speed of the optimal threshold affected by δ, but also the length of the *steps* is changed. For instance, a small δ ratio results in very long *steps*, whereas large values of δ yield relatively short *steps*.

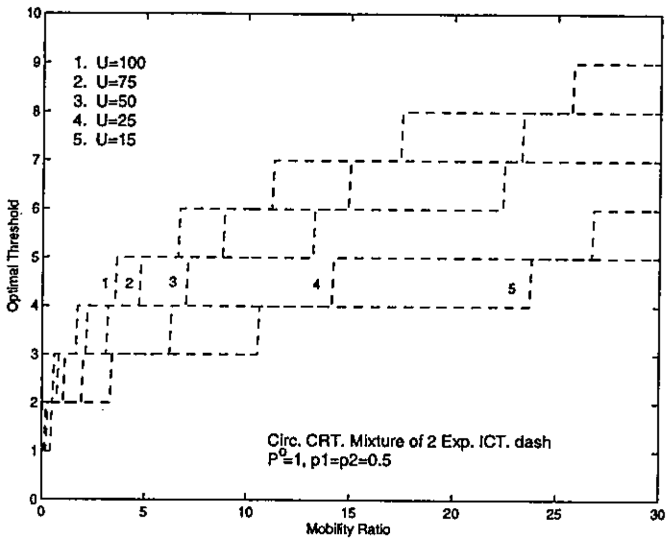

Figure 5: Optimal threshold in the HC case for several values of U.

In Figure 5, we show the behavior of the optimal threshold when different update costs U are used, whereas all the other parameters remain fixed. The effect produced by changing U is similar to that produced when δ is changed. We can also observe that the size of the *steps* is changed as a function of U, but this effect is not so strong as the one produced by changing δ.

5 Conclusions

Our definition of mobility ratio describes the mobility a user has, thus a large value of ρ means high user mobility. Hence, the results obtained above are in accord with intuition, since a higher mobility ratio will require a higher value of the optimal threshold. We have shown that the EE model of [3] has some limitation to capture the total cost behavior of the HC case. By making the comparison with the HE case, we have shown that not only the ICT distribution but also the CRT distribution is important to be specified in order to have a correct measure of the cost as a function of the mobility ratio.

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References

[1] A. Bar-Noy, I. Kessler, and M. Sidi. Mobile users: To update or not to update? *ACM-Baltzer J. Wireless Networks*, 1(2):175–186, July 1994.

[2] Ian F. Akyildiz, Joseph S. M. Ho, and Yi-Bing Lin. Movement-based location update and selective paging for pcs networks. *IEEE/ACM Transactions on Networking*, 4(4):629–638, August 1996.

[3] Jie Li, Hisao Kameda, and Keqin Li. Optimal dynamic mobility management for pcs networks. *IEEE/ACM Transactions on Networking*, 8(3):319–327, June 2000.

[4] Ramón M. Rodríguez-Dagnino. Handoff analysis in wireless multimedia networks. *SPIE, Conference on Performance and Control of Network Systems II*, 3530:76–84, November 1998.

[5] Ramón M. Rodríguez-Dagnino and C. A. Leyva-Valenzuela. Performance analysis in broadband wireless networks. *SPIE, Conference on Performance and Control of Network Systems III*, 3841:220–228, September 1999.

[6] S. Nanda. Teletraffic models for urban and suburban microcells: Cell sizes and handhoff rates. *IEEE Transactions on Vehicular Technology*, 42(4):673–682, November 1993.

[7] Y.-B. Lin, S. Mohan, and A. Noerpel. Queueing priority channel assignment strategies for pcs handoff and initial access. *IEEE Transactions on Vehicular Technology*, 43(3):704–712, August 1994.

[8] Y. Fang, I. Chlamtac, and Y.-B. Lin. Call performance for a pcs network. *IEEE Journal on Selected Areas in Communications*, 15(8):1568–1581, October 1997.

[9] D.R. Cox. *Renewal Theory*. Mathuen, London, 1st edition, 1962.

[10] Ramón M. Rodríguez-Dagnino, Gonzalo Hernández-Lozano, and Hideaki Takagi. Wireless handover distributions in mixed platforms with multimedia services. *SPIE, Conference on Internet Quality and Performance and Control of Network Systems*, 4211:59–69, November 2000.

[11] D. Hong and S. S. Rappaport. Traffic model and performance analysis for cellular mobile radio telephone systems with prioritized and nonprioritized handoff procedures. *IEEE Transactions on Vehicular Technology*, 35(3):77–92, August 1986.

[12] K.L. Yeung and S. Nanda. Channel management in microcell/macrocell cellular radio systems. *IEEE Transactions on Vehicular Technology*, 45(4):601–612, November 1996.