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A Model of Labor Quality, Wage Differentials and Unemployment

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1. Introduction

Concerning about wage differentials among occupations or industries, we can broadly observe two distinct phenomena in several European countries, the United States and Japan.[1] One is that wage differentials tend to widen in business booms and to narrow in recessions, and the other that there have been general tendencies of wage differentials to narrow since the beginning of this century. Reder (1955) provided an penetrating explanation for these findings by observing that firms vary not only wage rates but also minimum hiring standards in response to business fluctuations or the improvement in the ease of substitution between the skilled and the unskilled. It is a wonder that, although his hypothesis has prevailed among many economists for more than two decades, it has not been developed satisfactorily, specifically, in the framework of general equilibrium.

This paper attempts to extend Reder's model mainly in the following three directions. First, we make it clear how a firm determines the combination of wage rate and hiring standard through the optimization of its profit under a labor supply constraint. It appears that there still remains some room to make up in Reder's discussion concerning about the microeconomic behavior of a firm.

Second, it is necessary to explain the coexistence of cyclical variations in wage differentials and unemployment. Reder insists that his hypothesis holds only when labor demand increases enough to absorb the labor reserve,[2] that is, unemployment and underemployment, and wage rates for unskilled jobs are sufficiently high above

SM(social minimum) and elastic to the labor market condition. But this insistence appears awkward because we can easily find unemployed work-seekers, low-income farm youth, oldstars, juveniles and etc, to whom Reder refered as components of a labor reserve, in the real economy even when cyclical variations in wage differentials can be seen.

Thirdly, we will analyze how the size of the labor reserve is determined and affected by business fluctuations in the framework of general equilibrium. Reder did not discuss this problem at all since his main concern was with the cyclical behavior of wage differentials. According to the job search theory which has been developed after Reder's work (1955),[3] wage differentials affect the decision of a worker on the choice between unemployment (job-seek) and underemployment. On the other hand, the level of unemployment is one of the determinants of the labor supply condition for a firm which manipulates the relative wage and the hiring standard. Because of this interaction between unemployment and wage differentials we require a general equilibrium framework to analyse the determination of the labor reserve.

The structure of our model is essentially in line with the two-sector models developed by Harris and Todaro (1970), Stiglitz (1974) and Eaton and Neher (1975) although the way of the determination of the wage in the high-wage sector is different. In the works of Harris and Todaro, and Eaton and Neher the wage in the high-wage sector is fixed by the institutional factor, such as the government or trade unions, while in the model of Stiglitz it is an endogenous variable such that the firm determines it so as to minimize the labor costs, wage payment plus turnover cost (hiring and training cost), equilibrating quits and new hires.[4] In our model the wage in the high-wage sector is also endogenous, but it is determined by the firm considering the feasibility of its employment plan under the current hiring standard.

This paper is organized as follows: In Section 2 the basic model for analysing wage differentials and unemployment will be presented. The nature of unemployment created in our model will be discussed comparing to those in other models in Section 3. In Section 4 we will operate comparative statics on the wage differential and unemployment with respect to the product prices, in order to examine Reder's hypothesis and to know the effects of aggregate demand policy. In Section 5 we also do comparative statics with respect to the parameters in the training cost function, which are considered to be determined by the general level of the education of workers and mechanization of equipments.

2. The Basic Model

The model is composed of three sectors: the I sector characterized by high wage, the II sector marked by low wages and an unemployment sector. The I sector is the industry where firms produce output with the skilled, who are trained at the firms' costs. Since labor is assumed to differ in ability to acquire skills from training, the firms have an incentive to hire more qualified labor in order to reduce training costs. However, such behavior requires them to pay higher wages than those in the II sector because they must allure many applicants to sort out. Thus they come to face the problem of how the combination of the wage rate and the quality level of the employees should be determined under a labor supply constraint.

In sorting out labor, the firms take the policy that they first set its minimum hiring standard, check labor quality of each applicant at costs, and hire him if his quality is proved to pass the standard. This hiring policy to fill job vacancies is optimal for the firms in the labor market where matching with a job seeker is random and information concerning about the labor quality of each arrival is imperfect although they know its objective distribution. That is, we can reason the rationality of minimum hiring standards as well as "the reservation wage" is done in the job search theory.[5]

Since it is assumed that there is an agreement among employers against "labor piracy" in the I sector, a firm facing an increase in the product demand can not expand the skilled by drawing from the others, but by training employees of lower quality. Reder stresses this point in the following sentences: "Oligopsonistic agreements in restraint of labor market competition are rarely of a formal nature, but their operation is frequently reported and there is good reason why employers should adhere to them. For if an employer attempts to secure more workers in a 'tight' labor market by raising wages, competing employers are likely to match his increases. And, as knowledge of the wage increase is likely to reach rival employers before it reaches many prospective job applicants, it is unlikely to give the aggressive firm even a temporary advantage" (Reder 1955, p.835).

Thus, where an agreements against labor piracy is operative, it is likely to occur that a wage rate offered by a leading firm becomes to prevail in the industry and as a consequence all the firms quote the same wage rate.

We assume that a job in the II sector does not require any special skill and can be accomplished by any labor in the same level of performance. This assumption menas that it is easy for labor to enter this sector and to find a job since the firms do not discriminate among labor. Thus labor receives the wage equal to his marginal product in this sector.

The unemployment sector consists of a group of job seekers who are attempting to find a job in the I sector. It is assumed that labor does not apply for a job while working at a firm because the cost of search on the job is higher than that on unmeployment. As is well known, this assumption is the core of the models which purpose to analyze search unemployment. It is also likely that the firm takes so much time to check labor quality that a job seeker can not apply for a high-wage job without becoming umemployed.

A. The Production condition of The Firm in The I Sector

Since all the firms always quote the same wage as that of a leading firm, we deal with the problem of production and wage determination as if only a firm exists in this sector. The representative firm is assumed to produce its output (Q) with the skilled employees (L_1) according to a decreasing returns production function,

$$Y_1 = Q(L_1), \qquad Q' > 0, \qquad Q'' < 0 \tag{1}$$

The unskilled are trained at the outset of employment, and the training costs per employee (T) depend on his ability (x) in the following manner,

$$T = t(x), \qquad t' < 0 \tag{2}$$

where T is expressed in money terms. It is assumed in its background that the firm can ascertain a labor's ability by examining him when he applies, and can express it in some index.

There is no economic incentive for the employed in this sector to quit for another job because all the firms quote the same wage. Therefore, the quit rate of the employees (q) is assumed to be given exogenously. That is, q is independent of a level of the wage. If an employee separates from the firm, this is because of old age retirement, bad health or death

B. The Labor Supply Condition for The Firm

Since labor is assumed to search for a high-wage job while unemployed, the total number of job seekers is equal to the number unemployed. An individual working in the II sector makes a decision of whether he becomes unemployed or remains there according to the following decision rule, which is essentially the same as those utilized by Harris and Todaro (1970), Stiglitz (1974) and Eaton and Neher (1975): Given q, that is, the percentage of the employees quitting the firm in the

I sector at any time, the expected duration on a high-wage job is $\frac{1}{q}$. If the individual expects the wage differential to persist at least for the period of $\frac{1}{q}$, he will expect to gain

$$(w_1 - w_2) \frac{1}{q}$$

from being employed in a high-wage job, where w_1 is the wage rate in the I sector and w_2 is the wage rate in the II sector. Let U and N be the unemployed and new hires in any period, respectively. Then the probability that an unemployed worker can find a job in the I sector is given by $\frac{N}{U}$, assuming that the individual does not know how this ability will be evaluated by the firm or what scale is used in measuring ability there. On the other hand, note that the search cost per period is w_2, which is the foregone earnings in the II sector. Thus we have the decision rule for the individual to choose unemployment like this,

$$(w_1 - w_2) \frac{1}{q} \frac{N}{U} = w_2 \tag{3}$$

If the left hand side is greater than the right hand side in equation (3), it means that, since the expected net returns from being unemployed for one period is larger than the search cost, the workers in the I sector will enter the unemployment pool to search for a high-wage job. In the adverse case, the unemployed will flow out of the pool and find a job in the II sector. This discussion shows that equality in equation (3) is maintained by adjustment in the number of the unemployed. That is, unemployment is determined by equation (3).

Letting L_1 be the employment level in the I secotr, we have $N = qL_1$ since the firm must hire new workers by qL_1 in each period in order to maintain its level. Substituting $N = qL_1$ into equation (3) and rearranging it, the unemployment function is given by

$$U = \left(\frac{w_1}{w_2} - 1\right)L_1 \qquad (4)$$

which means that unemployment is a increasing function of the realtive wage and the job opportunities in the I sector. From point of view of the firm in the I sector an employed labor is an applicant. Therefore, the total number of the applicants for the firm is equal to the unemployed and can be considered to be a function of w_1 relative to w_2 and L_1 in the same manner as the unemployment function.

As discussed earlier, the firm discriminates among the applicants because they differ in quality. Let x_0 be the minimum hiring standard and $F(x)$ be the distribution function of x of a labor, which is assumed to be objectively known to the firm, then the qualified applicants for employment under x_0 is given by

$$A' = [1 - F(x_0)]U$$
$$= [1 - F(x_0)]\left(\frac{w_1}{w_2} - 1\right)L_1 \qquad (5)$$

where A' is the number of the qualified applicants.

Since the conditional probability of x when $x \geq x_0$ is $f(x)/1 - F(x_0)$, the expected training costs of a worker employed under x_0 is

$$T^* = \frac{\int_{x_0}^{\infty} T(x)f(x)dx}{1 - F(x_0)} \tag{6}$$

where $f(x)$ is a density function of x. And the total labor costs per employee for a period (w) is

$$w = w_1 + qT^* \tag{7}$$

Note here that the training costs are amortized over the expected duration of employment.

C. The Firm's Problem

Let us assume that the firm faces a perfect competitive product market at a price equal to P_1. Then, the optimization problem of the firm can be written as follows.

$$\underset{\{w_1,\ x_0,\ L_1\}}{\text{Max } R} = P_1Y_1 - wL_1$$

$$w = w_1 + qT^*$$

subject to

$$qL_1 \le [1 - F(x_0)](\frac{w_1}{w_2} - 1)L_1$$

Letting λ be the multiplier, we have the Lagrangian Π

$$\Pi = P_1Q(L_1) - wL_1 + \lambda\{[1 - F(x_0)](\frac{w_1}{w_2} - 1) - q\}$$

Since the firm maximizing its profit does not pay unnecessarily high wages, we can neglect the case of inequality in the constranint. The first-order conditions at an interior solution are given as follows:

$$\frac{\partial \Pi}{\partial L_1} = P_1 Q' - w = 0 \tag{8}$$

$$\frac{\partial \Pi}{\partial w_1} = - L_1 + \lambda[1 - F(x_0)] \frac{1}{w_2} = 0 \tag{9}$$

$$\frac{\partial \Pi}{\partial x_0} = f(x_0) \{ [\frac{T(x_0)}{1 - F(x_0)} - \frac{T^*}{1 - F(x_0)}]qL_1 - \lambda (\frac{w_1}{w_2} - 1)\} = 0 \tag{10}$$

$$\frac{\partial \Pi}{\partial \lambda} = [1 - F(x_0)](\frac{w_1}{w_2} - 1) - q = 0 \tag{11}$$

Equation (8) means that in determining employment the firm equates the marginal value product of an additional worker to its marginal cost, that is, the wage payment plus the training cost amortized for a single period. Substituting (9) into (10), we have

$$\frac{f(x_0)[T(x_0) - T^*]}{1 - F(x_0)} qL_1 = \frac{w_2 f(x_0)}{1 - F(x_0)} (\frac{w_1}{w_2} - 1)L_1 \tag{12}$$

The left-hand side is the marginal decrease of the expected training costs which is brought about by a unit of rise in the hiring standard, and the right-hand side is the additional wage increase required to attract more applicants in order ot make that higher standard feasible in the Labor market. The firm equates both by operating x_0.

Using equation (11), we have the simplist form of equation (12) like this,

$$T(x_0) - T^* = \frac{w_2}{1 - F(x_0)} \tag{13}$$

Thus, given w_2, equations (7), (8), (11) and (13) can be solved for L_1, w, w_1 and x_0. The system is very simple because w_1 and x_0 are determined independently of L_1 by minimizing w under the constraint and L_1 is determined by w. Each variable is a function of the single exogenous variable w_2. It is easy to show that

$$x_0 = x_0(w_2) \qquad x_0{}' = \frac{1}{[1 - F(x_0)]T'} < 0$$

$$w_1 = w_1(w_2) \qquad w_1{}' = \frac{w_1}{w_2} + \frac{f(x_0)(w_1 - w_2)x_0{}'}{1 - F(x_0)} \quad ?$$

$$w = w(w_2) \qquad w' = \frac{w_1}{w_2} + \frac{f(x_0)}{1 - F(x_0)}[w_1 - w_2 + q(-T(x_0) + T^*)]x_0{}'$$

$$= \frac{w_1}{w_2} > 0$$

$$L_1 = L_1(w_2, P_1) \qquad \frac{\partial L_1}{\partial w_2} = \frac{1}{P_1 Q''}\frac{1}{w'} < 0, \qquad \frac{\partial L_1}{\partial P_1} = -\frac{Q}{P_1 Q''} > 0$$

Further, as for the wage differential we have

$$\frac{w_1}{w_2} = d(w_2) \qquad d' = \frac{f(x_0)}{1 - F(x_0)}\left(\frac{w_1}{w_2} - 1\right)x_0{}' < 0$$

Thus, as the wage rate in the II sector rises, the firm finds it difficult to fill job vacancies under the current hiring standard. Hence it adjusts the wage rate and the hiring standard to the new state, minimizing the total labor costs. Its hiring policy is unambiguously to lower the minimum standard, but its wage policy is ambiguous because the lowering of the standard mitigates the difficulty in filling job vacancies to some extent. What we can predict about it is that the firm allows the wage differential to fall. At all events these adjustments lead to an increase in the total labor costs, which makes the firm reduce employment.

D. The II Sector

There is no incentive for the firms in the II sector to attract more applicants than they desire to employ by paying higher wages. Therefore, the wage in this sector is flexible, so that labor is paid to the value of its marginal product. Let the production function in the II sector be such that,

$$Y_2 = G(L_2) \qquad G' > 0,\ G'' < 0 \tag{14}$$

where Y_2 and L_2 are output and the labor input. Then, the profit-maximizing condition is given by

$$w_2 = P_2 G' \tag{15}$$

where w_2 and P_2 are the wage and the product price, respectively, in the II sector.

E. Labor Endowment

Finally we have the labor supply condition for the whole economy There is a labor constraint that the sum of workers actually employed in both sectors plus the number of the unemployed must be equal to the total labor endowment (L):

$$L = L_1 + L_2 + U \tag{16}$$

where L is given exogeneously.

3. The Market Equilibrium and Unemployment

We have the complete system of structural equations (4), (7), (8), (11), (13), (15) and (16), where the endogeneous variables are w_1, w_2, w, x_0, L_1, L_2 and U. Here it should be stressed that unemployment does not necessarily disapper at equilibrium. As far as a wage differential exists and the possibility of being hired in a high-wage job is not zero, it is rational for workers to apply it while unemployed.

Is this type of unemployment voluntary or involuntary? According to the discussion by Eaton and Neher (1975), who consider that union power gives rise to a wage differential by restricting entry of labor or by setting the wage high relatively to that of the free market, unemployment created by wage dispersion is voluntary since there is no one in this economy who can not find a job at some wage, that is, the unemployed are what they are in order to improve their economic conditions. This discussion means that involuntary unemployment can not logically coexist with underemployment in a model.

Search unemployment discussed in the current paper is not identical to those formulated by Mortensen (1970), Phelps (1970) and Lucas and Prescott (1974) in the sense that our unemployment is created by some wage rigidity in the high-wage sector. In their models, since wages are determined so as to clear the labor market in each island (spatially distinct market), a worker can take a job if he desire so at the wage before his eyes. And the distribution of wages over markets is brought about by stochastical variations in product demands and by imperfect information about wages across islands. Therefore, if a worker becomes unemployed, this is because, since he can not satisfy with the wage prevailing in his island, he leaves that place to seek for a good luck.

On the other hand, in our model it can occur that, even if he fortunately finds a job vacancy with a high wage, a worker can not be employed unless he is qualified for it. This nature of our model is one of the necessary conditions to produce involuntary unemployment. To show this, following Salop(1979), lt us assume that the II sector does not exist in this economy and, instead, a labor supply function is given by

$$L = L(z) \qquad L' > 0, \quad L'' < 0$$

$$z = (1 - \frac{U}{L})w_1$$

where z is the expected wage. And the firm determines x_0, w_1 and L_1 by maximizing its profit under the following labor supply constraint,

$$qL_1 \geq [1 - F(x_0)]L$$

In this economy[6] there exists involuntary unemployment at equilibrium. This is because two elements which are necessary to produce it are introduced in this model. One is that the unemployed do not have a job of last resort when they are cast out of the I sector. The other is that this model embodies wage rigidity which rejects some applicants from jobs.

Salop introduced involuntary unemployment into his model by considering a situation where a firm faces two labor markets, an internal market for trained employees and an external market for new applicants, but can use only a single wage rate to clear both. Therefore, if collection of application fees is not permitted and the firm uses its wage only to clear the internal market, then the external market can not be ensured to be cleared and as a result involuntary unemployment comes to exist in his model.

4. Cyclical Variations in Wage Differentials and Unemployment

In this section we will consider the effects of changes in the product prices on the wage differential and unemployment by operating comparative statics. Our main concern is to analyze how cyclical variations in wage differentials are produced in response to business fluctuations and what effets aggregate demand policies can have on unemployment through affecting product prices.

From Appendix A we get the comparative static results, as set out in the first and second columns of table 1. It reports the algebraic sign of the changes in the endogeneous variables. As is shown there, an increase in P_1 or P_2 will lead to an decrease in the wage differential and a downgrading in the hiring standard. This is because, when the wage in the II sector is forced up by labor shortages or demand pressures caused by an increase in P_1 or P_2, the firm in the I sector must adjust its employment policy according to the minimization

Table 1

Endogenous \ Exogneous	w_2 = flexible		w_2 = fixed			
	P_1	P_2	P_1	P_2	a	b
$\frac{w_1}{w_2}$	−	−	0	0	+	?
w_1	?	?	0	0	?	?
w_2	+	+	0	0	−	+
w	+	+	0	0	+	?
x_0	−	−	0	0	+	?
L_1	+	−	+	0	−	?
L_2	−	+	−	0	+	−
U	?	−	+	0	?	?

of the labor costs, w. In so doing, as we have seen before, it is rational for the firm to lower the hiring standard and to maintain its wage stable relative to that in the II sector, allowing the wage differential to decline. Thus, without depending on institutional factors such as the minimum wage law, we can explain cyclical variations in wage differentials with the existence of unemployment.

When we consider the comparative statics in the labor surplus economy where the II sector is nonindustrial (agricultural and self-employed)[7] and the wage is fixed at the subsistence level, then the results change drastically. That is, as the third and forth columns in table 1 show, cyclical variations in wage differentials do not occur. The reason is the following[8]: Since in our system the wage differential directly depends on only the wage rate in the II sector which functions as a link to connect both sectors in our economy, fixing w_2 essentially divides our system into two dependent parts. More specifically, under the unlimited supply of labor the firm in the I sector can freely shift its level of employment without changing the hiring standard and/or the wage. Therefore, an increase in P_1 does not affect the wage differential. But note that it makes the level of search unemployment higher by increasing job opportunities in the I sector. An increase in P_2 has no influence on the other variables simply because there is no root to affect in the labor surplus economy.

Based on the findings that in Japan the wage differential between both sectors has cyclically behaved corresponding to the long-run swings of business, Taira (1970) discussed that since the beginning of this century the Japanese labor market has worked as the neo-classical theory predicts. This view is opposed to the traditional one[9] that the unlimited supply of labor has dominated and characterized the Japanese labor market before the turning point from the labor surplus economy to the neo-classical one came in about 1960. Our model implys that Taira's discussion is correct if his findings are approved true.

We now consider the effects of aggregate demand policies on unemployment. First of all, it is worth mentioning that there is a difference between P_1 and P_2 in their effects on unemployment. That is, while an increase in P_2 forces unemployment to certainly decline, the effect of P_1 is ambiguous. We can explain it as follows: The labor demand pressure caused by the business boom in the I sector not only reduces the wage differential but also raises the possibility of being hired there via increasing L_1. The reduction in the wage differential works to decline unemployment, while the increase in job opportunities with high wages has the adverse effect. Thus the total effect is indeterminate. In contrast, an increase in P_2 lowers the possibility of being hired in a high-wage job because it raises the labor costs by increasing w_2 and reduces employment in the I sector. This analysis indicates that the effectiveness of aggregate demand policy depends on the structure of the product demands. That is, the more elastic the product demand for the low-wage sector, the more effective the aggregate demand policy in reducing unemployment.

From another point of view the negative relation between the wage differential and the employment level of the I sector is interesting.

As is well known, the existence of wage differentials is not sufficient to induce labor mobility. Job opportunities are also required for that, and at times by the existence of job opportunities labor is induced to move among sectors[10] in spite of reduced wage differentials. Clearly this situation occurs in our model when the labor demand increases in the I sector.

5. Wage Differentials and Labor Quality

In the United States and several European countries there have been general tendencies of occupational wage differentials to narrow since the begining of this century. Reder (1955) ascribes these tendencies to the secular improvement in the education of working class children and mechanization and specialization of equipment by the reason that they have increased the ease of substituting less-for more-skilled workers. In other words, workers become more able to acquire new skills the higher the level of their education, and technical changes tend to reduce the need for broadly skilled workers and to facilitate the utilization of partially skilled operatives. This ease of substitution can be considered to bring about a decline in wage differentials by changing hiring standards for skilled jobs and the demand structure between the skilled and the unskilled.

The purpose of this section is to consider the effects of the improved level of workers' education and mechanization of equipment in our framework in order to make Reder's discussion more clear and to develop it further. To begin with, we specifiy the training cost function as follows,

$$T = a - bx \tag{2)'$$

where $a > 0$ and $b > 0$. Linearity is assumed not only for simplicity but also for the sake of no a priori reason to impose concavity or convexity on the function.

We first examine the effects of a, which is reduced by the general improvement in the education of workers and/or mechanization of equipment because the ease of interskill substitution means the curtailment of the training costs for the firm.[11] As can be seen in the fifth column of table 1,[12] a decrease in a increases the demand for labor in the I sector by directly reducing the labor costs. This puts demand pressure upon the entire labor market, and leads to an increase in w_2 and a decline in the wage differential. In our model the firm does not lower the hiring standard by the change in a itself, but indirectly by the increased difficulty in hiring labor at the current standard. It is interesting to note that manpower policy of which main purpose is to lower unemployment by improving labor quality is not always successful. This is because, while the improvement in labor quality reduces the wage differential, it also increases job opportunities in the high-wage sector, which make workers more "selective" on their jobs. That is, more workers become to apply for a high-wage job even if the wage differential is reduced.

How differences in labor quality are important for the firm in training workers depends on its production technique, organizational form, training method, job design and so on. In our model this importance of labor quality is represented by b in equation (2)' because it means how much a worker whose quality is better than others by a unit can save the training costs. Mechanization or specialization of equipment which Reder pointed out as a cause of wage differentials to narrow can be considered to reduce b by making labor quality more irrelevant for the firm.

Substituting equation (2)' into (13) and differentiating it with respect to b, we have

$$\frac{dx_0}{db} = \frac{x^* - x_0}{b} > 0$$

$$x^* \equiv \frac{\int_{x_0}^{\infty} xf(x)dx}{1 - F(x_0)}$$

which means that the firm sets the hiring standard higher the more important labor quality is. Further, differentiating equation (11) with respect to b, we obtain.

$$\frac{\left(\frac{w_1}{w_2}\right)}{db} = \frac{1}{w_2}\frac{dw_1}{db} = \frac{\left(\frac{w_1}{w_2} - 1\right)f(x_0)}{1 - F(x_0)}\frac{dx_0}{db} > 0$$

which tells us that the relative wage is also raised with the increase in x_0. In addition, the increase in b reduces the labor costs and leads to an increase in employment as the following calculation shows us:

$$\frac{dw}{db} = \frac{dw_1}{db} - \frac{dqT^*}{db}$$

$$= \frac{f(x_0)}{1 - F(x_0)} \{w_1 - w_2 - bq(x_0 - x^*)\} \frac{dx_0}{db} - qx^*$$

$$= - qx^* < 0$$

Note here that the indirect effect of b on w through increasing x_0 is zero because the increase in w_1 offsets the reduction in costs brought about by the increased quality level of the employees. Thus, we know the direct effects of b on the behavior of the firm.

As for the total effects of b on the wage differential and the hiring standard, however, the results are ambiguous, as can be seen in the sixth column of table 1. For both increases in job opportunities and the wage differential allure more applicants to the I sector, reduce employment in the II sector, and increase w_2 which affects the behavior of the firm. This means that, as far as the importance of labor quality concerns, Reder's discussion does not necessarily hold.

Appendix A

The effects of the product prices

Let us define D as follows,

$$D \equiv \begin{vmatrix} P_1 Q'' & -w' P_2 G'' \\ \frac{w_1}{w_2} & P_2 L_1 d' G'' + 1 \end{vmatrix} < 0$$

Then, we have the following results concerning about the effects of P_1 and P_2, respectively;

The effects of P_1

$$\frac{dL_1}{dP_1} = \frac{-Q'(P_2 L_1 d' G'' + 1)}{D} > 0$$

$$\frac{dL_2}{dP_1} = \frac{\frac{w_1}{w_2} Q'}{D} < 0$$

$$\frac{dw_2}{dP_1} = P_2 G'' \frac{dL_2}{dP_1} > 0$$

$$\frac{dw}{dP_1} = w' \frac{dw_2}{dP_1} > 0$$

$$\frac{dx_0}{dP_1} = x_0' \frac{dw_2}{dP_1} < 0$$

$$\frac{d \frac{w_1}{w_2}}{dP_1} = d' \frac{dw_2}{dP_1} < 0$$

$$\frac{dw_1}{dP_1} = w_h' \frac{dw_2}{dP_1} \qquad ?$$

$$\frac{dU}{dP_1} = \frac{d\frac{w_1}{w_2}}{dP_1} L_1 + \left(\frac{w_1}{w_2} - 1\right) \frac{dL_1}{dP_1} \qquad ?$$

<u>The effects of P_2</u>

$$\frac{dL_1}{dP_2} = \frac{w'G'(P_2L_1d'G''+1) - P_2L_1w'd'G'G''}{D} = \frac{w'G'}{D} < 0$$

$$\frac{dL_2}{dP_2} = \frac{-P_1L_1d'GQ'' - \frac{w_1}{w_2}w'G'}{} > 0$$

$$\frac{dw}{dP_2} = P_1Q'' \frac{dL_1}{dP_2} > 0$$

$$\frac{dw_2}{dP_2} = \frac{1}{w'} \frac{dw}{dP_2} > 0$$

$$\frac{dw_1}{dP_2} = w_h' \frac{dw_2}{dP_2} \qquad ?$$

$$\frac{dx_0}{dP_2} = x_0' \frac{dw_2}{dP_2} < 0$$

$$\frac{d\frac{w_1}{w_2}}{dP_2} = d'\frac{dw_2}{dP_2} < 0$$

$$\frac{dU}{dP_2} = \frac{d\frac{w_1}{w_2}}{dP_2} L_1 + \left(\frac{w_1}{w_2} - 1\right) \frac{dL_1}{dP_2} < 0$$

Appendix B

The effects of a

$$\frac{dL_1}{da} = \frac{q(d'P_2L_1G'' + 1)}{D} < 0$$

$$\frac{dL_2}{da} = -\frac{q\,\frac{w_1}{w_2}}{D} > 0$$

$$\frac{dw_2}{da} = P_2G''\,\frac{dL_2}{da} < 0$$

$$\frac{dw}{da} = P_1Q''\,\frac{dL_1}{da} > 0$$

$$\frac{d(\frac{w_1}{w_2})}{da} = d'\,\frac{dw_2}{da} > 0$$

$$\frac{dw_1}{da} = w_1'\,\frac{dw_2}{da} \qquad ?$$

$$\frac{dx_0}{da} = x_0'\,\frac{dw_2}{da} > 0$$

$$\frac{dU}{da} = \frac{d(\frac{w_1}{w_2})}{da}\,L_1 + (\frac{w_1}{w_2} - 1)\,\frac{dL_1}{da} \qquad ?$$

The effects of b

$$\frac{dL_2}{db} = \frac{-P_1L_1Q''\,\frac{(\frac{w_1}{w_2} - 1)f(x_0)}{1 - F(x_0)}\,\frac{x^* - x_0}{b} + qx^*\frac{w_1}{w_2}}{D} < 0$$

$$\frac{dw_2}{db} = P_2G''\,\frac{dL_2}{db} > 0$$

$$\frac{dL_1}{db} = \frac{P_2 L_1 G'' \frac{1}{b} \frac{(\frac{w_1}{w_2} - 1) f(x_0)}{1 - F(x_0)} [-x^* + \frac{w_1}{w_2} x_0] - qx^*}{D} \quad ?$$

Since the calculations of the other effects are tedious and the results are ambiguous, we omit them here.

FOOTNOTES

1 See Brown (1977, Chap. 3) for a survey. As for Japan, see Taira (1970) and Minami (1971).

2 Reder defines a labor reserve as a group of "unemployed work-seekers, low-income farm youths, oldsters, jeveniles, housewives, etc., who will accept jobs in the business or government sectors of the economy of going wage rates whenever such jobs are avilable," (Reder 1955, p.839).

3 See Lippman and McCall (1976) for a survey.

4 As we see in Section 3, this mechanism of wage determination is also introduced by Salop (1979) in a different context.

5 This kind of problem is named as "beauty contest" or "secretary" problem. See Gilbert and Mosteller (1966), or Chow, Robbins and Siegmund (1971).

6 The system of this economy is as follows:

The behavior of the firm

$$x_0 = x_0(P_1, u)$$

$$w_1 = w_1(P_1, u)$$

$$L_1 = L_1(P_1, u)$$

The labor market conditions

$$L = L(z)$$

$$z \equiv (1 - u)w_1$$

$$u \equiv \frac{U}{L}$$

$$L = L_1 + U$$

where the endogenous variables are x_0, w_1, L_1, L, z, u and U.

7 Lewis (1954) called the II sector as the subsistence sector and the I sector as the capitalist sector in his model with unlimited supplies of labor.

8 We omit the mathematical derivation of these results here because they are intuitively clear by the following discussion.

9 For example, see Minami (1971).

10 See O. E. C. D. (1965) for a empirical survey on job opportunity hypothesis.

11 If every workers improve their educational levels in the same amount, i, the training costs for a worker whose ability was x before will be such that

$$T = a - b(x + i) = (a - bi) - bx$$

This means that the general improvement in the education has the same effect as the change in a. As for the mechanization of equipment, its effect is clear.

12 See Appendix B for the derivation of the results.

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