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EVALUATION OF THE REFORMED DIVISION SYSTEM WITH ENFORCEMENT OF SHORT-RANGE CORPORATE STRATEGY

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ABSTRACT

The traditional division system was reformed variously in the recent economic difficulty in the late 1970's. One of the major reformations was to enforce corporate strategy to overcome shortly the economic difficulty in harmony with the existing division system. This reformed division system with corporate strategy enforced for the immediate effect is herein modeled in terms of the partition of mixed-integer programming. The properties of this system is analyzed by exploiting the special structure of this system. Considering these properties, this system is evaluated to satisfy the intended goals on the criteria of autonomy, agreeability, enforcement of corporate strategy and the resource efficiency.

1. INTRODUCTION

Many literatures which were devoted to the theory of decentralized decision have been classified into the price-direction [3, 5] and resource-direction [6, 7] types according to the means of control. The control of these two types by the central authority is a routine and tactical job in that it is concerned with the efficiency of the resources which are consumed in production process continuously by an infinitesimal bit every day. The resources of this kind are therefore allocatable continuously by any amount within a certain limit. Accordingly the control is also continuous in that its infinitesimal variation is (almost) always feasible and meaningful.

The functional separation between the central authority and the operational divisions is primarily the separation between strategy and tactics. The strategic decision is hardly expressed by a continuous term because its variation is often infinitesimal to no division (for example, commercialization of a new product, installment of a new plant and so on.) It is, hence, desirably expressed by a discrete variable.

In the past such a discretized strategy was formed only in the long-range planning and was decided exogeneously to the short-range tactics planning at the divisional level. Recently, however, the life cycle of production plants and of new products became much shorter. For example, a new type of integrated circuits is put into mass production almost yearly and its produciton plant is accordingly replaced almost yearly. In this way the discretized strategy is now often formed in the short-range planning with deep interaction with the tactics planning of the divisions. Nevertheless it is decided at a strategic level by the central authority because it requires resources beyond their availability to the divisions. Furthermore the latest economic difficulty since mid 1970's required the immediate or short-range effect of strategic investment and this requirement leads to the same planning system.

This paper examines the validity and effectiveness of incorporating the discretized strategy formation at a corporate level with the single-period tactics planning at a divisional level. Accordingly our model proposed in the following sections diffiers from the existing ones

[3, 5, 6, 7, 8] in that some of the variables controlled directly by the central authority are discrete.

2. GLOBAL MODEL

Strategic decision is often discretized as yes-no selection, choice of some among severals, incompatibility, association and so on. These are usually expressed by using integer variables (for their expressions, see APPENDIX).

Strategy is constrained in two terms; (logical) intrarelation among strategies themselves and resource-augmenting interrelation between corporate strategy and divisional tactics. The former intra-relation whose detail is stated in APPENDIX is altogether expressed as in (1) to follow.

(1) $$A_0^1 x_0 \leqq g_0^1$$

with A_0^1 : $m_0^1 \times n_0^1$-matrix of logical coefficients as in APPENDIX; x_0 : $n_0^1 \times 1$-vector of nonnegative integer vaiables for discretized strategies; g_0^1 : $m_0^1 \times 1$-vector of logical constraints as in APPENDIX. The latter interaction is various but the major one is that a strategy (*e.g.*, installment of a new plant) increases the divisional resources. This relation is expressed:

$$B_k^1 y_k \leqq g_k + H_k x_0 \qquad \forall k \in \{1, \ldots, N\}$$

or equivalently with $A_k^1 = -H_k$,

(2) $$A_k^1 x_0 + B_k^1 y_k \leqq g_k^1 \qquad \forall k \in \{1, \ldots, N\}$$

with $B_k^1 : m_k \times n_k^2$-matrix of k-th divisional technology coefficients for its own resources; $y_k : n_k^2 \times 1$-vector of nonnegative continuous variables for k-th divisional activities; $y_k : m_k \times 1$ -vector of k-th divisional resources; H_k(or A_k^1) $: m_k \times n_k^1$-matrix of increase (or decrease) coefficients of the k-th divisional resources.

The corporate resources which commonly constraint the divisional activities are also consumed and increased by strategy as in (3) to follow.

(3) $$F^1 x_0 + \sum_{k=1}^{N} B_0^k y_k \leqq g_0^2 + F^2 x_0$$

with $F^1 : m_0^2 \times n_0^1$-matrix of consumption coefficients of the corporate resources; $B_0^k : m_0^2 \times n_k^2$-matrix of technology coefficients for the corporate resources; $g_0^2 : m_0^2 \times 1$-vector of the corporate resources; $F^2 : m_0^2 \times n_0^1$-matrix of increase coefficients of the corporate resources. Letting $A_0^2 = F^1 - F^2$ and $B_k^2 = B_0^k$, the constraint (3) is broken down into (3.0) and (3.k) as follows.

(3.0) $$A_0^2 x_0 + \sum_{k=1}^{N} x_k \leqq g_0^2$$

(3.k) $$-x_k + B_k^2 y_k \leqq 0 \qquad \forall k \in \{1, \ldots, N\}$$

with $x_k : m_0^2 \times 1$-vector of nonnegative continuous variables for the corporate resources allocated to k-th division. The constraint (3.0) is for allocating the corporate resources between divisions and (3.k) is for their consumption within the k-th division.

Rearranging these constraints in the order of (1), (3.0), (2.k) and (3.k), the whole coefficient matrix is structured as in Figure 1. A simpler expression may be,

$$(4)\qquad \begin{aligned} & A_0\, x \leqq g_0 \\ & A_k\, x + B_k\, y_k \leqq g_k \qquad \forall k \in \{1, \ldots, N\} \end{aligned}$$

with

$$A_0 = \begin{matrix} A_0^1, & 0, & \ldots, & 0 \\ A_0^2, & I, & \ldots, & I \end{matrix}\ , \qquad g_0 = \begin{matrix} g_0^1 \\ g_0^2 \end{matrix}$$

$$A_k = \begin{matrix} A_k^1, & 0, & \ldots\ldots\ldots\ldots, & 0 \\ 0, & 0, .., -I, 0, & \ldots\ldots \end{matrix}\ , \qquad B_k = \begin{matrix} B_k^1 \\ B_k^2 \end{matrix}\ , \qquad g_k = \begin{matrix} g_k^1 \\ 0 \end{matrix}\ .$$

Alternatively,

$$(5)\qquad A_x + B_y \leqq g$$

with

$$A = \begin{matrix} A_0 \\ A_1 \\ \vdots \\ A_k \\ \vdots \\ A_N \end{matrix}\ , \qquad B = \begin{matrix} 0, & \ldots\ldots\ldots\ldots, & 0 \\ B_1, & 0, \ldots\ldots, & 0 \\ \cdots & \cdots & \cdots \\ 0, & \ldots, B_k, \ldots, & 0 \\ \cdots & \cdots & \cdots \\ 0, & \ldots\ldots\ldots\ldots, & B_N \end{matrix}\ , \qquad g = \begin{matrix} g_0 \\ g_1 \\ \vdots \\ g_k \\ \vdots \\ g_N \end{matrix}\ .$$

The objective function may be of the conventional form in that only the divisional operations y_k, $\forall k \in \{1, \ldots, N\}$ yield the profit, *i.e.*,

(6) $$w = \sum_{k=1}^{N} c_k y_k \rightarrow \max$$

or equivalently with $c = (c_k)_{k=1}^{N}$, and y as above,

(7) $$w = cy \rightarrow \max$$

with w : scalar of the objective value; $c_k : 1 \times n_k^2$-vector of k-th divisional profit coefficients; $c : 1 \times (N \times n_k 2)$-vector.

The corporate strategy is assumed here to contribute no profit directly to the objective function as in the conventional decentralization and to consume only the existing resources, incurring no direct cost. This latter assumption is quite realistic in days of economic depression when a new business for exploring a new business area is often launched by exploiting the existing idle resources. This is in contrast with the situation in economic growth when a new business for expanding the existing business area is launched by procuring the new resources.

For the purpose of later reference, the problem of maximizing (6) or (7) as the objective function subject to (4) or (5) as the constraints is called *(G)* which means the global problem.

This global model as such reflects only the "static" aspect of finished decision rather than the "dynamic" aspect of successive decision. The latter aspect will be developed in the following section.

3. PROCEDUAL MODEL

The decision of the whole firm was modeled in the preceding section. As a firm is structured with vertical hierarchy and horizontal divisions, the decision is actually made via a successive and interactive procedure. The real procedure is very complicated but we first simplify it and then we model it in mathematical terms.

The desired decision procedure may conceptually be stated for iteration t starting with $t = 1$.

Step 0. Each division k proposes the divisional efficiency target u_k^t to the central authority.

Step 1. The central authority proposes the corporate strategy $x(u^t)$ and the associated profit target $w_k(u^t)$ in response to the proposed u^t.

Step 2. Each division k revises u_k^t to u_k^{t+1} and obtains its associated profit w_k^{t+1} in response to the proposed corporate strategy $x(u^t)$. It compares the proposed profit target w_k^t with its attainable profit $w_k(u^t)$. If they agree to each other, then the procedure terminates with the optimum attained. If else, then go to Step 1 and repeat the procedure for the reset iteration $t = t + 1$.

The procedure stated just above will mathematically be modeled in terms of the partition as will be developed hereafter.

The naïve models of the corporate strategy formation and of the

k-th divisional tactics plannig may be represented by the problems $(S(y))$ for given tactics y and $(T_k(x))$ for given corporate strategy respectively.

$$(S(y)) \qquad \begin{array}{l} w_S = 0\,x \rightarrow \max \\ \text{s.t.} \qquad Ax \leqq g - By \end{array}$$

$$(T_k(x)) \qquad \begin{array}{l} w_k = c_k\,y_k \rightarrow \max \\ \text{s.t.} \qquad B_k\,y_k \leqq g - A_k\,x \\ k \in \{1, \ldots, N\} \end{array}$$

As is easily seen, problems $(S(y))$ and $(T_k(x))$, $k \in \{1, \ldots, N\}$ altogether constitute the original global problem (G).

Aggregating $(T_k(x))$ into a single problem, we can have the problem $(T(x))$ to follow.

$$(T(x)) \qquad \begin{array}{l} w_T = c\,y \rightarrow \max \\ \text{s.t.} \qquad By \leqq g - Ax \end{array}$$

Note that problems $(S(y))$ and $(T(x))$ are the Benders' partition of (G) and yield the optimum to (G) [4].

For the simplicity the bounded unique optimum is assumed to exist for each of (G), $(S(y))$, $(T(x))$ and $(T_k(x))$, $\forall k \in \{1, \ldots, N\}$. Let (w^*, x^*, y^*), $(\bar{w}_S(y), \bar{x}(y))$, $(\bar{w}_T(x), \bar{y}(x))$ and $(\bar{w}_k(x), \bar{y}_k(x))$ denote respectively the optimum to (G), $(S(y))$, $(T(x))$ and $(T_k(x))$, $\forall k \in \{1, \ldots, N\}$. Let $\bar{u}_k(x)$ denote the dual optimum to $(T_k(x))$,

$\forall k \in \{1, \ldots, N\}$ under the assumption of the unique existence of the dual optimum. Also let y_k^* denote the k-part of y^*, that is, $y^* = (y_k^*)_{k=1}^N$.

As $(T(x))$ is the aggregation of $(T_k(x))$, $\forall k \in \{1, \ldots, N\}$, $\bar{\bar{w}}_T(x) = \sum_{k=1}^N \bar{w}_k(x)$, $\bar{y}(x) = (\bar{y}_k(x))_{k=1}^N$.

By the duality, solving the problem $(T_k(x))$ is equivalent to solving the problem $(\hat{T}_k(x))$ with u_k as a $1 \times m_k$-nonnegative variable vector, $\forall k \in \{1, \ldots, N\}$.

$$(\hat{T}_k(x)) \qquad \begin{array}{ll} & w_k \rightarrow \max \\ \text{s.t.} & w_k - u_k(g_k - A_k x) \leqq 0 \\ & -u_k B_k \leqq -c_k \end{array}$$

Futhermore let the problem $(\hat{S}(U))$ be defined for given U with $U = \times_{k=1}^N U_k$ where U_k is the set of all the feasible basic solutions to $(T_k(x))$. Note that U_k is independent of x (only its optimum depends on x). It is seen by way of $(\hat{T}_k(x))$ that solving $(\hat{S}(U))$ is equivalent to solving (G) [4].

$$\hat{S}(U)) \qquad \begin{array}{ll} & w \rightarrow \max \\ \text{s.t.} & w - \sum_{k=1}^N w_k \leqq 0 \\ & w_k + u_k^\ell A_k x \leqq u_k^\ell g_k, \ \forall u_k^\ell \in U_k, \ \forall k \in \{1, \ldots, N\} \end{array}$$

As $|U_k|$ is large, only its small subset U_k^t is generated at iteration t and then, if necessary, it is augmented. Hence, actually problem $(\hat{S}(U^t))$ is solved at iteration t with $U^t \supset U^{t-1}$ and $U^t = \overset{N}{\underset{k=1}{\times}} U_k{}^t$. Algorithmically, U^t starts with the singleton and is augmented one by one. Hence $|U^t| = t$, $\forall t$. Accrodingly U_k which is augmented at t will be denoted by $U_k{}^t$. The computational experience [9] shows that the optimum is usually attained long before U is enumerated. Moreover, some algorithmic improvement [2, 9] is proposed for faster convergence. In any method the optimality stopping criterion is the equality between the upper and lower bounds of the objective value. In our method the upper bound is provided by $(\hat{S}(U^t))$ for U^t and the lower bound is provided by $(T(x))$ or $(T_k(x))$ always for feasible x.

Let $(\hat{w}^t, \hat{w}_k{}^t, \hat{x}^t)$ denote the optimum to $(\hat{S}(U^t))$ and $\bar{U}_k{}^t$, $\bar{y}_k{}^t$ and $\bar{w}_k{}^t$ be the abbreviations for $\bar{u}_k(\hat{x}^{t-1})$, $\bar{y}_k(\hat{x}^{t-1})$ and $\bar{w}_k(\hat{x}^{t-1})$ respectively. Further let $v^\tau = \max_{\nu \leq \tau}\{\sum_{k=1}^{N} \bar{w}_k{}^t\}$. Verbally, v^τ is the best objective value up to iteraction τ and provides the lower bound of the objective value. By the duality, $\bar{w}_k{}^t = \bar{u}_k{}^t(g_k - A_k\,\hat{x}^{t-1}) = c_k\,\bar{y}_k{}^t$, $\forall k \in \{1, \ldots, N\}$ and $\forall t$. Then the optimality stopping criterion is

$$(8) \qquad \hat{w}^\tau = v^\tau$$

Lemma 1. (Monotonicity of upper bound) $\hat{w}_k{}^t$ and $\hat{w}^t$ decrease monotoniously in t, $\forall k \in \{1, \ldots, N\}$.

Proof. One of the second-type constraints of $(\hat{S}(U^t))$ holds with equality at the optimum at iteration t, $\forall t$. More precisely, the equality holds for the last augmented constraint (*i.e.*, the t-th constraint at t) for each k; that is, at iteration t,

(9) $$w_k + u_k^t A_k x = u_k^t g_k \qquad \forall k \in \{1, \ldots, N\}$$

Verbally, the last augmented constraint is always binding for every k. This means w_k is bounded at t more tightly than at $t-1$, $\forall k \in \{1, \ldots, N\}$. Hence $\hat{w}_k^t$ decreases monotoniously in t, $\forall k \in \{1, \ldots, N\}$. This implies $\hat{w}^t$ also decreases monotonously in t. Q.E.D.

Lemma 2. (Uniqueness of upper bound)

$\hat{w}_k^t$ is unique at t, $\forall k \in \{1, \ldots, N\}$, $\forall t$.

Proof. By the equation (9), w_k is tightly constrained, $\forall k \in \{1, \ldots, N\}$. Furthermore, by the uniqueness assumption of the optimal values of w and x, w_k is uniquely bounded from the both sides at iteration t, $\forall k \in \{1, \ldots, N\}$. This completes the proof. Q.E.D.

Theorem 1. (Componentwise optimality criterion)

Equation (8) holds if and only if

(10) $$\hat{w}_k^\tau = \bar{w}_k^{t(\tau)} \qquad \forall k \in \{1, \ldots, N\}$$

where $t(\tau)$ denotes the iteration at which the maximum is taken for v^τ, *i.e.*, $v^\tau = \sum_{k=1}^{N} \bar{w}_k^{t(\tau)}$.

Proof. As the first constraint of $(\hat{S}(U^t))$ holds with the equality at the optimum for $\forall t$, it holds that $\hat{w}^t = \sum_{k=1}^{N} \hat{w}_k^t$, $\forall t$. Hence by the definition of v^τ, summing up of the both sides of (10) over all

$k \in \{1, \ldots, N\}$ yields (8). This proves the if-part. Now let the only-if part be proved. In view of the equivalence between the second constraints of $(\hat{S}(U))$ and the first constraint of $(\hat{T}_k(X))$ and the equivalence between $(\hat{T}_k(x))$ and $(T_k(x))$, $\forall k \in \{1, \ldots, N\}$, there exist $t(k)$ and $t'(k)$ such that $\hat{w}_k^{t(k)} = \bar{w}_k^{t'(k)}$, $\forall k \in \{1, \ldots, N\}$. Now, by Lemma 1, $\hat{w}_k^t$ decreases monotonously in t and by Lemma 2, it is unique. Hence $\hat{w}_k^t$ can not repeat for different t before the optimum iteration, $\forall k \in \{1, \ldots, N\}$. Thus, since $v^\tau = (\bar{w}_k^{t(\tau)})_{k=1}^N$ by definition,

$$\hat{w}_k^t = \bar{w}_k^{t(\tau)}$$

holds only if (8) holds for $t = \tau$, $\forall k \in \{1, \ldots, N\}$. Q.E.D.

Remark on Theorem 1. Theorem 1 means two things: (1) it breaks down the optimality criterion (8) into the component criterion on which each divisional solution is separately judged its optimality, (ii) as the target of the objective value $\hat{w}_k^t$ decreases in t for every division k, it can not happen that, say, $\hat{w}_1^t$ decreases while $\hat{w}_2^t$ increases. If such a situation should occur, such a tradeoff between division 1 and division 2 would cause hostile between the divisions. Fortunately the monotonicity of the decrease of the target values for all the divisions prevents such hostile.

Let (8) hold at iteration τ.

Lemma 3. (Validity of partition) [4].

$$\hat{w}^{\tau} = w^* \quad \text{and} \quad \hat{x}^{\tau} = x^*$$

Lemma 4. (Divisional solution of the global optimum)

$$\bar{w}_k(x^{\tau}) = w_k^* \quad \text{and} \quad \bar{y}_k(x^{\tau}) = y_k^*, \ \forall k \in \{1, \ldots, N\}$$

Proof. Immediate from Lemma 3. Q.E.D.

Let $u^* = (u_k^*)_{k=1}^{N}$ be the dual optimum to (G) and assume its unique existence.

Lemma 5. (Basicness of dual optimum)

$$u^* \in U \quad \text{and} \quad u_k^* \in U_k, \ \forall k \in \{1, \ldots, N\}$$

proof. By Lemma 4, u^* and u_k^* are the dual optimum to $(T(x^{\tau}))$ and $(T_k(x^{\tau}))$, $\forall k \in \{1, \ldots, N\}$ respectively. That is, $u^* = \bar{u}(x^{\tau})$ and $u_k^* = \tilde{u}_k(x^{\tau})$. Hence they are basic under the uniqueness assumption.

Q.E.D.

Remark on Lemma 5. In the price-directive method the dual optimum is obtained by combining the foregoing optima (the same with the primal optimum). On the other hand, in the decomposition of our type, it is obtained as a basic solution by optimizimg a subproblem . Q.E.D.

Lemma 6. (Componentwise optimality)

$$\hat{w}_k^{\tau} = w_k^* \qquad \forall k \in \{1, \ldots, N\} \tag{11}$$

Proof. By Lemma 3, the summation of the both sides of (11) over $k \in \{1, \ldots, N\}$ are equal each other. By Lemma 1 and 2, w_k^t is unique and decreasing in t. Hence the equality holds only once as $\hat{w}_k^t$ decreases in t. That is, the equality holds only at $t = \tau$.

Q.E.D.

Theorem 2. (Divisional objective at the optimum iteration)

$$\bar{w}_k^{t(\tau)} = w_k^* \qquad \forall k \in \{1, \ldots, N\} \tag{12}$$

Proof. By Theorem 1 and Lemma 6. Q.E.D.

Theorem 3. (Divisional solution at the optimum iteration)

$$\bar{y}_k^{t(\tau)} = y_k^* \qquad \forall k \in \{1, \ldots, N\} \tag{13}$$

Proof. Since $\bar{w}_k^{t(\tau)}$ and w_k^* are associated with $\bar{y}_k^{t(\tau)}$ and y_k^* respectively, $\forall k \in \{1, \ldots, N\}$, Theorem 2 yields Theorem 3. Q.E.D.

Remark on Theorem 3. The assumption of the unique optimum of $(T_k(x))$, $\forall k \in \{1, \ldots, N\}$ leads to that (12) holds if and only if (13) holds.

Corollary 1. (Divisional solution at the optimum iteration)

$$\bar{w}_k^{t(\tau)} = \bar{w}_k(x^\tau) \qquad \text{and} \qquad \bar{y}_k^{t(\tau)} = \bar{y}_k(x^\tau), \ \forall k \in \{1, \ldots, N\}$$

Proof. Immediate from Lemma 4, Theorem 2 and 3. Q.E.D.

Lemma 7. (Dual solutions)

$$\bar{u}_k^{t(\tau)} = \bar{u}_k(x^\tau) = u_k^* \qquad \forall k \in \{1, \ldots, N\}$$

where $(u_k^*)_{k=1}^N$ denotes the dual optimum associated with the constraint (5) in (G).

Proof. $\bar{y}_k^{t(\tau)}$, $\bar{y}_k(x^\tau)$ and $y_k{*}$ are the primal optima to $(T_k(x^{t(\tau)-1}))$, $(T_k(x^\tau))$ and $(T_k(x^*))$ respectively, $\forall k \in \{1, \ldots, N\}$, and $\bar{w}_k^{t(\tau)}$, $\bar{w}_k(x^\tau)$ and $w_k{*}$ are their associated objective values respectively, $\forall k \in \{1, \ldots, N\}$. By Theorem 3 and Corollary 1,

$$\bar{y}_k^{t(\tau)} = \bar{y}_k(x^\tau) = y_k{*} \qquad \forall k \in \{1, \ldots, N\}$$

Also by Theorem 2 and Corollary 2,

$$\bar{w}_k^{t(\tau)} = \bar{w}_k(x^\tau) = w_k{*} \qquad \forall k \in \{1, \ldots, N\}$$

These equations implies Lemma. Q.E.D.

Theorem 4. (Another criterion of optimality)

The optimality criterion (8) holds at iteration t if and only if

(13) $$\hat{w}_k^t = \bar{w}_k^t \qquad \forall k \in \{1, \ldots, N\}$$

Proof. By Theorem 1 and Lemma 7. Q.E.D.

Remark on Theorem 4. Replacing (10) with (13) means the "monotonicity" of the objective value $\bar{w}_k^t$ of $(T_k(x^\tau))$ in t in a certain sense. When $\bar{w}_k^t$ is not monotonous in t, the upper bound $v^{\tau(t)}$ is not generally equal to v^t. Theorem 4 assures $v^{\tau(t)} = v^t$ when t is the optimum iteration and allows us to consider as if the divisional objective values decrease monotonously in t.

Our model, though its general frame is provided by the Benders partitioning procedure, has a special structure in two points: (i) the continuous variable part (the y-part) which represents the divisional tactics planning has the staircase structure and (ii) the discrete variable part (the x-part) which represents the corporate strategy and the corporate resource allocation has the zero-coefficient in the objective function. These two features are fully exploited in deriving Theorem 1-4 which provide the desired properties to increase the degree of divisional autonomy. More specifically these two features allow each division to judge the optimality with no intervention of the other divisions. Technically this is done by splitting the optimality criterion (8) to the divisional components (10) or (13) and by making the divisional objective value "monotonously decreasing."

Now the decision procedure stated at the beginnig of this section may be restated in the algorithmic terms, starting with $t = 1$.

Step 0. Each division k proposes the divisional efficiency target $\bar{u}_k^{\,t}$ in U_k to the central authority.

Step 1. The central authority proposes the corporate strategy $\hat{x}^t$ and the associated profit target $\hat{x}^t$ by solving $(\hat{S}(u^t))$.

Step 2. Each division k revises $\bar{u}_k^{\,t}$ to $\bar{u}_k^{\,t+1}$ and obtains its associated profit $\bar{w}_k^{\,t+1}$ by solving $(T_k(\hat{x}^t))$. It examines whether the equation (13) holds or not. If it holds, $\forall k \in \{1, \ldots, N\}$, then the procedure terminates and the optimum is now attained. If else, then go to Step 1 and repeat the procedure for iteration $t = t + 1$.

Remark on the efficiency concept: The word "efficiency" is used above in Step 0. As to the problem to interpret the dual solution to the integer problem as the efiiciency, see [1].

4. EVALUATION OF SYSTEMS ACCEPTABILITY

The primal purposes of decentralization, at least in Japanese management which is based on the bottomup system, are autonomy to energize the members and agreement among members. On the other hand our particular system analyzed herein has its own particular purpose to put a corporate strategy for immediate effect. A third criterion is the resource efficiency which is measured in terms of attainability of the global optimum.

Property 1. (Autonomy)

Each division is fully autonomous in deciding its own divisional tactics under the given target of its profit and the given corporate strategy.

Demonstration. The divisional decision is independent of the central authority in deciding its own tactics y_k by solving $(T_k(x^t))$ for given tactics x^t and in judging its own optimality by examining (10) or (13) for given target $\hat{w}_k{}^t$. Futhermore each division is independent of the other divisions in every decision making (Theorem 1 and 4).

Property 2. (Agreement-based decision)

The process of giving the target to each division is agreeable to each division. Moreover the global optimality is judged by a unanimous agreement between the central authority and all the divisions.

Demonstration. As the target values given to the divisions decrease uniformly to every division, a "tradeoff" between divisions which allows the increase to a division and commands the decrease to another can not happen (Lemma 1 and Remark on Theorem 1). This is quite agreeable to the divisions. Moreover the global optimality is judged when (10) or (13) holds for every division (Theorem 1 or 4).

Property 3. (Accomplishment of corporate strategy)

Corporate strategy goes through well with divisional tactics in an autonomous manner.

Demonstration. Corporate strategy is harmonized well with divisonal tactics in that the both of them reach their optima at the same time with the agreement of the both sides (Property 2).

Property 4. (Resource efficiency)

The system consumes the resources in the optimally efficient way.

Demonstration. By Lemma 3 and 4, the global optimum is attained. This means the resources are consumed in the optimally efficient way.

5. CONCLUSION

The combining system of corporate strategy and divisional tactics in a single-period planning is modeled and analyzed. This system reflects the need of strengthening the central control to get through the difficult time. Mathematically it is represented in a mixed-integer problem where the integer variables denote corporate strategy in yes-no form. This decentralized system is shown to be effective in attaining its goals and it is evaluated to be effective on the four criteria: (i) autonomy, (ii) agreeability among subsystems, (iii) accomplishment of corporate strategy, and (iv) resource efficiency.

6. APPENDIX: INTERRELATION OF STRATEGIES

Yes-no selection

$$0 \leqq v_1 \leqq 1 \quad \text{with} \quad v_1\text{: integer}, \quad v_1 = 1\text{: yes}, \quad v_1 = 0\text{: no}$$

Choice of some (p) among severals $(q$, with $p < q)$

$$\sum_{i=1}^{q} v_i = p \quad \text{and} \quad 0 \leqq v_i \leqq 1, \ \forall i \ \text{with}$$

$$v_i \text{ : intger}, \ \forall i \in \{1, \ldots, q\}$$

Choice of at least (at most) p among q, $p < q$

$$\sum_{i=1}^{n} v_i \leqq (\geqq) \ p \quad \text{and similar to above.}$$

Incompatibility

$$\sum_{i=1}^{n} v_i \leqq 1 \quad \text{and} \quad 0 \leqq v_i \leqq 1, \quad \text{with} \quad v_i : \text{ integer},$$

$$\forall i \in \{1, \ldots, n\}$$

Association or implication $(v_1 \rightarrow v_2)$

$$v_1 \leqq v_2 \quad \text{and} \quad 0 \leqq v_i \leqq 1 \quad \text{with} \quad v_i : \text{integer}, \quad \forall i \in \{1, 2\}$$

7. REFERENCES

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strategy (integer)	resource allocation	divisional tactics	
x_0	$x_1, \ldots\ldots, x_N$	$y_1, \ldots\ldots, y_N$	
0	$0, \ldots\ldots, 0$	$c_1, \ldots\ldots, c_N$	$\to \max$
A_0^1	$0, \ldots\ldots, 0$	$0, \ldots\ldots, 0$	$\leqq g_0^1$
A_0^2	$I, \ldots\ldots, I$	$0, \ldots\ldots, 0$	$\leqq g_0^2$
A_1^1	$0, \ldots\ldots, 0$	$B_1^1, 0, \ldots, 0$	$\leqq g_1^1$
0	$-I, 0, \ldots, 0$	$B_1^2, 0, \ldots, 0$	$\leqq 0$
$\vdots$	$\ddots$	$\ddots$	
A_N^1	$0, \ldots\ldots, 0$	$0, \ldots, 0, B_N^1$	$\leqq g_N^1$
0	$0, \ldots, 0, -I$	$0, \ldots, 0 \; B_N^2$	$\leqq 0$

Fig. 1 Structure of Matrix