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Approximation of Nonlinear Term Structure Models

by

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Abstract

We propose a method to approximate the nonlinear term structure of interest rates. In the Cox, Ingersoll, and Ross (CIR) framework a nonlinear term structure model is obtained by assuming a process of the short-term rate to have a nonlinear drift which recent empirical studies suggest. Applying the proposed method to the process, we derive a closed-form model of the term structure. Our empirical analysis shows that the approximate model shows much better performance than the original CIR model.

1 Introduction

In valuing interest-rate-sensitive securities including the term structure, it is often assumed that a single state variable, which is usually the risk-free (short-term) rate, follows a time-homogeneous Markov diffusion with a linear drift. Part of these examples is an equilibrium term structure model of Vasicek [1977], Cox, Ingersoll and Ross (CIR)[1985], and Brennan and Schwartz [1979]. Chan et al. [1992], who examined the dynamic behavior of the short-rate by their relatively unrestricted diffusion model, assume a linear drift.

Why is a linear drift often assumed? In view of term structure models, linearity both in a local mean and variance of changes in the short-rate enables to derive a long yield which is linear in the underlying short-rate. (See, for example, Duffie and Kan [1996].) However, there are recent papers suggesting nonlinearity in the drift of a short-rate process. For example, Pfann et al. [1996] examined nonlinearity by piecewise linear functions containing threshold parameters. Aït-Sahalia [1996] tested existing models on a short-rate process by comparing a marginal distribution derived from particular specification of each model with that directly estimated from data by nonparametric procedures. Stanton [1997] estimated shapes of approximated drift functions also nonparametrically. Although each econometric procedure is different, one of the common key findings is that the short-rate behaves like a random walk in its middle range but mean-reverts sharply at extremely high or high-and-low levels. This implies that the degree of mean reversion depends on levels of the short-rate, hence that the drift is nonlinear. Conley et al. [1997] also found nonlinearity in the drift, although their result is somewhat different from above studies in that the sharp mean reversion occurs at lower levels of the short-rate and random walk appears even at extremely high levels. (This is due to their choice of the local variance which is highly sensitive to changes in the short-rate.)

If a linear drift is not good enough to capture the dynamics of the short-rate, prediction of yields by a linear term structure model is not accurate enough since the conditional distribution of the short-rate which is used to take expectation of a future payoff of a discount bond is not correctly specified. Instead, if we assume a nonlinear drift, cross-sectional relations between the short-rate and long yields become nonlinear. However, it is extremely difficult to derive a nonlinear term structure model since a partial differential equation valuing a discount bond has no closed-form solution except for very special cases. Such special cases are Longstaff [1989], and Beaglehole and Tenney [1992] who obtained the nonlinear term structure model in the CIR framework. In their model, the process of the short-rate is derived by making nonlinear transformation of a single state variable which is assumed to follow a Gaussian process. Therefore, their specification is still limited and not flexible enough to capture the actual behavior of the short-rate, which leads to the same problem as that of a linear term structure model. Hence the fundamental trade-off is yet unsolved between capturing the short-rate dynamics accurately and deriving functional relations between the short-rate and long yields.

In this paper, we consider approximation of nonlinear term structure models. By this approximation, we can choose any functional form of a drift to capture the dynamics of the short-rate flexibly and obtain an analytical expression for the term structure of interest rates. We apply a local linearization method (LLM), which is originally developed by Shoji and Ozaki [1997], to a nonlinear drift in order to approximate it by a linear function. Under the approximated process, we can solve the partial differential equation for the term structure and the solution becomes a nonlinear function of the short-rate. To derive a nonlinear term structure model, we use the CIR framework and compare performance between the original CIR and our nonlinear model. However, it is important to note that nonlinear models can be obtained from other single-factor term structure models such as Vasicek [1977], and multi-factor models such as Longstaff and Schwartz [1992], and Pearson and Sun [1994].

In empirical analysis, we choose a polynomial function as a nonlinear drift, which is simple and flexible enough to capture the actual behavior of the short-rate, including that suggested by the recent empirical studies. We divide a sample period into two; the first period is for estimation of parameters and the second period is for performance comparison of term structure models based on root mean square error of predicted yields. But before proceeding to this comparison, it is important to examine whether the actual behavior of the short-rate is captured better by the nonlinear drift. This is because if it little improves the descriptive ability, performance of a nonlinear term structure model may be worse than that of the linear model due to approximation error by the LLM. (See Shoji [1998] in which the maximum order of approximation error is proved.) Hence we test nonlinearity in the drift by using in-sample data.

The rest of this paper is organized as follows. Section 2 introduces the LLM and provides a nonlinear term structure model in the CIR framework. Section 3 presents empirical results of (non)linearity in the drift and performance comparison of term structure models. Section 4 concludes, relating to further course of this study.

2 A Nonlinear Term Structure Model

We model dynamics of a single state by the following nonlinear stochastic differential equation,

$$dx = g(x)dt + \upsilon\sqrt{x}dZ_t \,, \tag{1}$$

where g is a twice-continuously differentiable function, and $\{Z_t\}$ is a standard Brownian motion.

In the CIR model economy, it is assumed that preference of a representative consumer is logarithmic and that local means and variances of rate of return on production technologies are both proportional to the underlying state (CIR [1985], *Assumption 2*). These assumptions lead to the short-rate which is also proportional to the state as,

$$r_t = cx_t \,, \tag{2}$$

where c is assume to be positive constant (see CIR [1985], footnote 6). By applying Ito formula to equation (2), we have

$$dr = f(r)dt + \sigma\sqrt{r}dZ_t \,, \tag{3}$$

where $f(r) \equiv cg(r/c)$, and $\sigma \equiv \upsilon\sqrt{c}$.

Note that in equation (3), the local variance of changes in the short-rate is still proportional to its current level. Thus, the risk premium, which is a covariance of changes in the short-rate with changes in the production technologies, is also proportional to the short-rate as λr. (See also Campbell et al. [1997], Chapter 11, for the affine term structure.) Therefore, an equilibrium discount bond price at time t with maturity date T, $P(r,t,T)$, follows the fundamental valuation equation,

$$\tfrac{1}{2}\sigma^2 r\frac{\partial^2 P}{\partial r^2} + f(r)\frac{\partial P}{\partial r} + \frac{\partial P}{\partial t} - \lambda r\frac{\partial P}{\partial r} - rP = 0 \,, \tag{4}$$

with the boundary condition $P(r,T,T)=1$.

Obviously, it is difficult to solve equation (4) unless specifying f. In addition, only extremely limited functions allow us to obtain a closed-form solution. A linear drift is one of these very special cases as shown in the original CIR model, however, this causes loss of descriptive ability of a term structure model. On the other hand, if we employ numerical method for solving equation (4), the market risk parameter λ must be estimated before computation. But if we estimate λ, we have to solve equation (4) in advance. Or it is theoretically possible to solve equation (4) and estimate λ simultaneously by the following procedures. For an initial value of λ given, we solve the partial differential equation for a discount bond numerically, then compare the solution with actual data to obtain the next value of λ, and so on, until we obtain the optimal estimate of λ. But these procedures require for intensive computation in both solving the partial differential equation and searching for the optimal estimate recursively. Thus the numerical method is not practical. In contrast with this method, our method to approximate the nonlinear term structure has at least two advantages. First, we can choose any functional form of a drift to capture the dynamics of the short-rate flexibly. Second, we can explicitly derive a term structure model which enables us to estimate the market risk parameter simply. We derive a nonlinear term structure model by the following procedures.

First, by applying Ito formula to the drift function $f(r)$ in equation (3), we obtain

$$\begin{aligned} df &= \frac{df}{dr}(r)dr + \tfrac{1}{2}\frac{d^2f}{dr^2}(r)(dr)^2 \\ &= \frac{df}{dr}(r)dr + \tfrac{1}{2}\frac{d^2f}{dr^2}(r)\sigma^2 r dt\,, \end{aligned} \tag{5}$$

where the second equality is derived from quadratic variation in equation (3). Integrating both sides of equation (5) on time interval $[t, u]$ provides

$$\int_t^u df = \int_t^u \left(\frac{df}{dr}(r)dr + \tfrac{1}{2}\frac{d^2 f}{dr^2}(r)\sigma^2 r dt \right) .$$

By assuming that the first order derivative and the second order derivative multiplied by $\sigma^2 r$ are locally constant during this time interval, we have

$$f(r_u) - f(r_t) = \frac{df}{dr}(r_t)(r_u - r_t) + \tfrac{1}{2}\frac{d^2 f}{dr^2}(r_t)\sigma^2 r_t(u - t) .$$

Rearranging terms gives

$$f(r_u) = L_t r_u + M_t u + N_t \qquad (6)$$

where

$$\begin{aligned} L_t &\equiv \frac{df}{dr}(r_t) , \\ M_t &\equiv \tfrac{1}{2}\sigma^2 r_t \frac{d^2 f}{dr^2}(r_t) , \\ N_t &\equiv f(r_t) - L_t r_t - M_t t . \end{aligned}$$

Thus, we obtain the linear stochastic differential equation of the short-rate,

$$dr_u = (L_t r_u + M_t u + N_t)du + \sigma\sqrt{r_u}dZ_u , \qquad (7)$$

and the partial differential equation valuing a discount bond,

$$\tfrac{1}{2}\sigma^2 r_u \frac{\partial^2 P}{\partial r_u^2} + (L_t r_u + M_t u + N_t)\frac{\partial P}{\partial r_u} + \frac{\partial P}{\partial u} - \lambda r_u \frac{\partial P}{\partial r_u} - r_u P = 0 , \qquad (8)$$

under the boundary condition $P(r, T, T) = 1$. Equation (8) can be solved explicitly as

$$P(r, t, T) = A(t, T)\exp(-B(t, T)r) , \qquad (9)$$

where

$$A(t,T) \equiv \left[\frac{(2\gamma_t)^{M_t(T-t)+h_t}\exp(C(t,T))}{g(t,T)^{h_t}}\right]^{2/\sigma^2},$$

$$B(t,T) \equiv \frac{2(e^{\gamma_t(T-t)}-1)}{g(t,T)},$$

$$C(t,T) \equiv (\lambda - L_t + \gamma_t)(\tfrac{T-t}{2})((\tfrac{T-t}{2})M_t + h_t) - M_t \int_t^T \log(g(u,T))du,$$

$$g(t,T) \equiv (\lambda - L_t + \gamma_t)(e^{\gamma_t(T-t)}-1) + 2\gamma_t,$$

$$h_t \equiv f(r_t) - L_t r_t,$$

$$\gamma_t \equiv ((\lambda - L_t)^2 + 2\sigma^2)^{1/2}.$$

It is important to note that $A(t,T)$ and $B(t,T)$ contain the current information on the level of the short-rate. Therefore, a yield to maturity, $Y(r,t,T) = -(\log P(r,t,T))/(T-t)$, becomes a nonlinear function of the short-rate.

3 The Empirical Analysis

Our empirical analysis consists of two steps. The first step is to examine the nonlinearity in the drift of a short-rate process and to estimate structural parameters by using in-sample data. The second step is to compare performance between the linear and nonlinear term structure model by using out-of-sample data.

Our data set is monthly Eurodollar deposits, maturing in one-, three-, and six-month(s). The sample period covers from January 1971 to Jun 1999, providing 342 observations in total. The source for the data is the H-15 Federal Reserve Statistical Release (Selected Interest Rate Historical Data). We take the one-month rate as a proxy for the short-rate, and it explains the three- and six-month yields. The first sample period is from January 1971 to December 1993 (276 observations), and the second period is from January 1994 to Jun 1999 (66 observations). The historical data of the one-month Eurodollar deposit rate is shown in Figure 1.

3.1 *Examining nonlinearity in the drift of a short-rate process*

We specify the nonlinear drift as a polynomial function of third power. Hence the stochastic behavior of the short-rate is described by the following nonlinear stochastic differential equation,

$$dr = (\alpha_3 r^3 + \alpha_2 r^2 + \alpha_1 r + \alpha_0)dt + \sigma\sqrt{r}dZ_t\,. \tag{10}$$

This is one of the simplest models to cover the dynamics that the short-rate behaves like a random walk in its mean range and reverts sharply to this range at extremely high and low levels.

On the other hand, CIR [1985] assume

$$dr = (\alpha_1 r + \alpha_0)dt + \sigma\sqrt{r}dZ_t\,. \tag{11}$$

Clearly, our nonlinear drift model nests the linear model. Hence, for examining nonlinearity in the drift, we implement the F-test where the null hypothesis is $\alpha_2 = \alpha_3 = 0$.

Before implementing the F test, each continuous-time model of the short-rate is discretized by Euler method to obtain

$$\begin{aligned} \frac{r_{t+\Delta t} - r_t}{\sqrt{r_t}} &= \frac{f(r_t)\Delta t}{\sqrt{r_t}} + \epsilon_t\,, \\ \epsilon_t &\sim N(0,\, \sigma^2\Delta t)\,, \end{aligned} \tag{12}$$

where $f(r) = \alpha_3 r^3 + \alpha_2 r^2 + \alpha_1 r + \alpha_0$ for the nonlinear drift model and $f(r) = \alpha_1 r + \alpha_0$ for the linear drift model, and $\Delta t = 1/12$ for monthly data.

The test statistics, following F distribution asymptotically with degree of freedom 2 and 271, is 3.658 and its p-value is 0.027. Therefore, we can reject the CIR model against the nonlinear model at a conventional significance level.

3.2 *Parameter estimation*

By using the equation (12), we employ the maximum likelihood method to estimate parameters of each short-rate model. Panel A of Table 1 provides the result of estimation. All these estimates are consistent with the behavior of interest rates. That is, the short-rate does not explode for $\alpha_3 < 0$ in the nonlinear model and $\alpha_1 < 0$ in the linear model, nor reach to zero for $2\alpha_0/\sigma^2 \geq 1$ in both models, in finite time. (See Karlin and Taylor [1981], chapter 15, section 6, for boundary classifications.) Figure 2 shows the estimated shape of the drift. It is important to note that random walk is implied by the nonlinear model at middle range of the short-rate. But sharp mean reversion occurs at higher levels. The short-rate is also pulled to its mean range at lower levels around 0 %, although the historical minimum is beyond this range.

In the in-sample period, we also estimate the market risk parameter λ. The original CIR model for a discount bond is

$$P(r, t, T) = A(t, T) \exp(-B(t, T) r) \tag{13}$$

where

$$A(t, T) \equiv \left[\frac{2\gamma e^{(\lambda - \alpha_1 + \gamma)(T-t)/2}}{(\lambda - \alpha_1 + \gamma)(e^{\gamma(T-t)} - 1) + 2\gamma} \right]^{2\alpha_0/\sigma^2},$$

$$B(t,T) \equiv \frac{2(e^{\gamma(T-t)}-1)}{(\lambda-\alpha_1+\gamma)(e^{\gamma(T-t)}-1)+2\gamma},$$

$$\gamma \equiv ((\lambda-\alpha_1)^2+2\sigma^2)^{1/2}.$$

(Note that equation (13) follows the parameter notation of the short-rate model in equation (11).)

Yield to maturity of a discount bond is obtained by $-\log P(r,t,T)/(T-t)$, thus from equation (9) and (13) we obtain the following regression equation in each model,

$$Y(r,t,T) = \frac{-1}{T-t}(\log A(t,T;\lambda) - B(t,T;\lambda)r) + u_{t,T}, \tag{14}$$

where $u_{t,T}$ is assume to be independently and identically distributed. Hence, we implement the least square method to estimate λ. Panel B of Table 1 provides the result of estimation, which shows that the both estimates are close around −0.173.

3.3 *Comparison of the term structure models*

By measuring root mean square error (RMSE), we compare performance between the linear and nonlinear term structure model. The results are summarized in Table 2. The nonlinear term structure model shows better performance in describing both three- and six-month yields. This results are consistent with that of the F-test to the short-rate models. The RMSE of the nonlinear model on three- and six-month yield is 13.2 basis points (bp) and 27.6 (bp), respectively, which is less than that of the linear model by 13.0% and 10.3%, respectively. The components of RMSE show that the source of this better performance of the nonlinear model is substantially small bias of pricing errors. Bias of the nonlinear model on three-month yield is −1.6 (bp), which is nearly one-fifth in magnitude of that of the linear model, −7.8 (bp). On six-month yield, the bias of the former (–3.6 (bp)) is around one-fourth as large as that of the latter (–14.6 (bp)). This effect exceeds a negative effect of slightly higher standard deviation of the nonlinear model. Therefore, our approximation method is successful in improving the descriptive ability of a term structure model by reducing bias of pricing errors.

Comparison based on the time-series property also indicates better performance of the nonlinear model. Figure 3 plots absolute pricing errors of the linear model minus those of the nonlinear model (hence the positive (negative) region means better (poorer) performance of the nonlinear model). Cleary, in most of the times the nonlinear model has smaller absolute errors. However, from 1994 through the first quarter of 1995, the nonlinear model is less accurate in explaining both actual yields. Looking at Figure 1, in this period, interest rates increased sharply from historically low levels of 3% to above 6%. But the nonlinear drift implies random walk in this range (see Figure 2) so that it failed to capture this trend. In the subsequent period, interest rates behaved like a random walk in the meddle range and performance of the nonlinear model is almost always better. By this result, we confirm that a term structure model, in which a process of the short-rate is specified more flexibly to capture the actual behavior, predicts yields better in cross-sections.

4 Concluding Remarks

We derived a nonlinear term structure model of interest rates in which the short-rate is modeled by a nonlinear stochastic differential equation. By applying a local linearization method (LLM) to a nonlinear drift, a closed-form solution for the term structure is obtained. Our empirical results using the U.S. interest rate data showed that a nonlinear term structure model predicts yields better, particularly by reducing bias of pricing errors. In addition, we showed that the market risk parameter is easily estimated in our model. It is also emphasized that the LLM can be applicable to wide range of valuation models. Hence, greater flexibility for constructing models is obtained.

In this paper, we derived a nonlinear term structure model by introducing a nonlinear drift in a process of the short-rate. However, many empirical studies pointed out that higher sensitivity of a local variance to levels of the short-rate also plays a key role in capturing its dynamics (for example, Chan et al. [1992], Conley et al. [1997]). These empirical results suggest that modeling nonlinearity in the local variance may further improve the descriptive ability of a term structure model. Therefore, it is a further study to obtain a term structure model under an arbitrary process of the short-rate by applying the LLM to a nonlinear local variance as well as to a nonlinear drift.

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Table 1: Result of estimation

Panel A: Parameter estimates of short-rate models

	α_3	α_2	α_1	α_0	σ (%)
Nonlinear					
estimate	–136.4	33.70	–2.455	0.0549	0.0928
(t-value)	(–1.287)	(0.982)	(–0.717)	(0.524)	(42.82)
Linear					
estimate			–0.2018	0.0157	0.0940
(t-value)			(–1.362)	(1.030)	(46.90)

Panel B: Parameter estimates of the market risk

	λ
Nonlinear	
estimate	–0.1727
(t-value)	(–11.88)
Linear	
estimate	–0.1730
(t-value)	(–11.93)

Each continuous-time model of the short-rate is discretized by Euler method to obtain

$$\begin{aligned} \frac{r_{t+\Delta t} - r_t}{\sqrt{r_t}} &= \frac{f(r_t)\Delta t}{\sqrt{r_t}} + \epsilon_t \, , \\ \epsilon_t &\sim N(0, \, \sigma^2 \Delta t) \, , \end{aligned}$$

where $f(r) = \alpha_3 r^3 + \alpha_2 r^2 + \alpha_1 r + \alpha_0$ for the nonlinear drift model and $f(r) = \alpha_1 r + \alpha_0$ for the linear model. Parameters of the short rate models are estimated by the maximum likelihood method. The market risk parameter λ is estimated by minimizing sum of squared errors in the following regression equation,

$$Y(r, t, T) = -\frac{1}{T - t}(\log A(t, T) - B(t, T)r) + u_{t,T} \, ,$$

where $u_{t,T}$ is assume to be independently and identically distributed.

Table 2: Comparison of term structure models

		3-month yield		6-month yield	
		Linear	Nonlinear	Linear	Nonlinear
RMSE	(bp)	15.14	13.16	30.78	27.61
Mean	(bp)	−7.82	−1.57	−14.57	−3.61
Std.dev.	(bp)	12.96	13.07	27.11	27.37

Comparison between the linear and nonlinear term structure model is made in the out-of-sample period from January 1994 to Jun 1999 (66 observations). Root mean square error (RMSE) of each model is obtained by calculating the difference between the actual and theoretical (model-implying) yields of three- and six-month Eurodollar deposit.

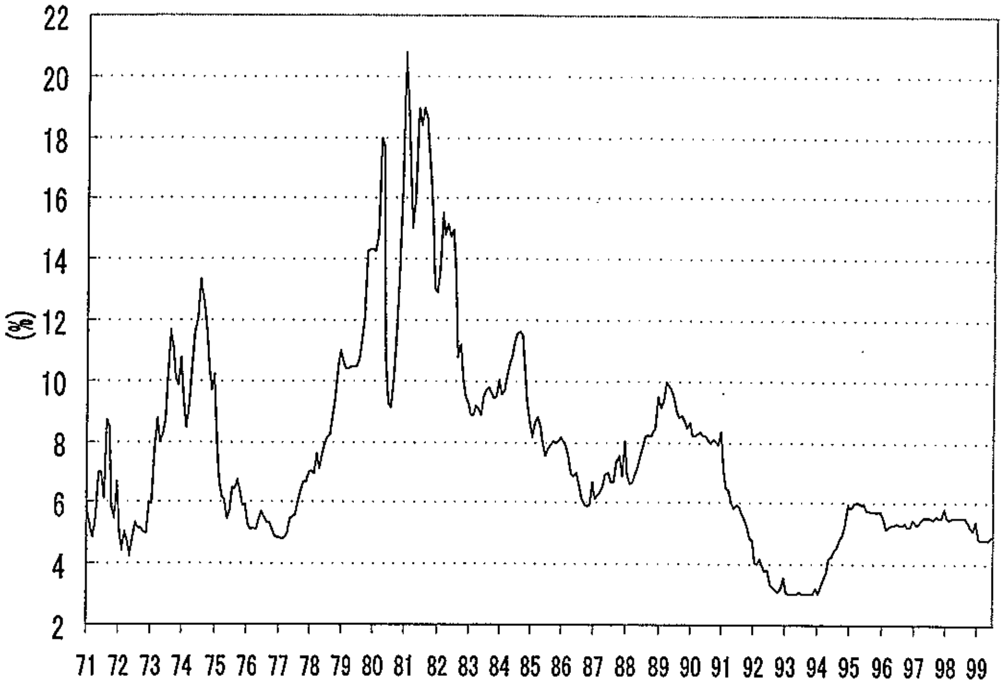

Figure 1: Historical Data of One-month Eurodollar Rate

The sample period is divided into two for comparing performance between a linear and nonlinear term structure model in the out-of-sample period. The first period is from January 1971 to December 1993 and the second period is from January 1994 to Jun 1999.

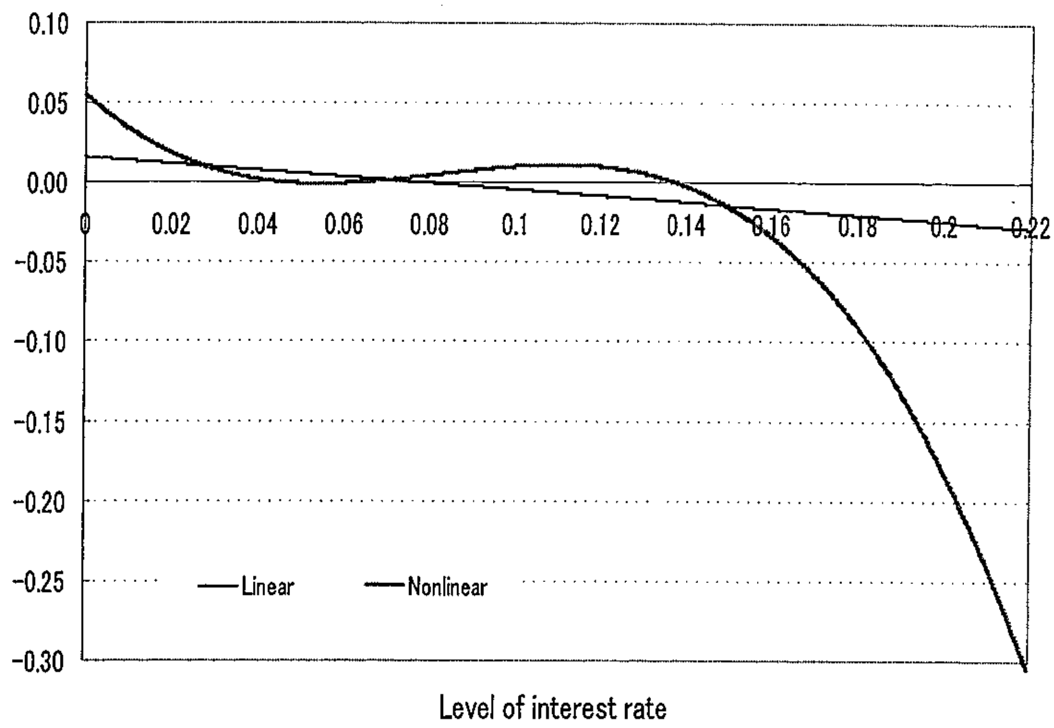

Figure 2: Estimated Shape of Drift

The linear and nonlinear diffusion models of the short-rate are described by

$$\begin{aligned} dr &= (\alpha_1 r + \alpha_0)dt + \sigma\sqrt{r}dZ\,, \\ dr &= (\alpha_3 r^3 + \alpha_2 r^2 + \alpha_1 r + \alpha_0)dt + \sigma\sqrt{r}dZ\,, \end{aligned}$$

respectively. Parameters are estimated by the maximum likelihood method, after discretizing the continuous-time models by Euler method. The sample is monthly frequency and the estimation period is from January 1971 to December 1993.

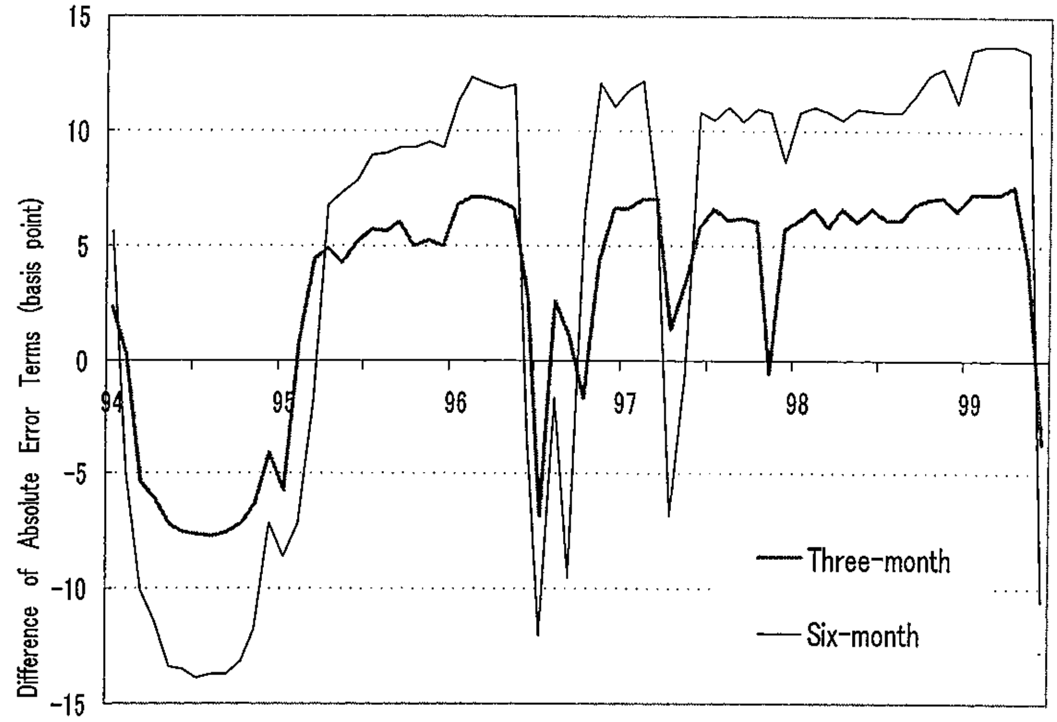

Figure 3: Performance Comparison between a Linear and Nonlinear Term Structure Model

Figure 3 shows the difference of absolute pricing errors between the linear and nonlinear term structure model. The positive (negative) region implies that absolute error of the nonlinear model is smaller (larger) than that of the linear model. Comparison is made in the out-of-sample period from January 1994 to Jun 1999.