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City Size Distribution and Income Distribution in Space*

by
Koichi Mera

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Either approach is insufficient. The conventional city size approach neglects the spatially differentiated impact of a city on its surrounding[1/] and the agropolitan approach neglects the benefit of agglomeration economies within a city.

The spatially differentiated impact of a city on its surrounding has been documented by Social Physicists and is measured by Population Potential. According to Stewart (1947), an impact or "potential" of a city at a given location is defined to be proportional to its population and inversely proportional to the distance from the city. The population potential at a given location is the sum of all potentials created by human inhabitation outside of the location itself. On this basis, he as well as Zipf (1947) demonstrated that population potential is linearly correlated with a number of variables representing intensity of activities in rural areas. Warntz (1959) showed that the supply and demand prices of selected agricultural products are well associated with supplyand demand potentials, respectively, which are similarly defined. Then, he (1965) showed that per capita income variations within each of major regions in the U. S. can be explained well by population potential. This type of findings is consistent with the finding of Schults (1953), who found on the basis of economic analysis that agriculture was operated more efficiently near large urban centers.

In this paper, therefore, the question of city size is approached by taking into consideration agglomeration economies within a city and the service benefit of a city outside the city itself. In evaluating the system, two criteria will be used, as in Mera (1967), productive efficiency and urban-rural equity. The tradeoff relationship and the optimal city size for maximum productive efficiency will be derived for simple hypothetical cases. Then, the relevance to actual cases will be discussed.

2. Basic Assumptions

(a) City-Size-Production Relationship

The finding of all of the authors referred to above can be summarized that per capita product increases with city size but with a diminishing rate. Among functions which satisfy the above conditions, the following power function will be assumed:

$$y_U = a\,U_i^b, \qquad a > 0, \quad 0 < b < 1 \tag{1}$$

where

y_U is per capita product of city U_i

U_i is the population size of city U_i, and

a and b are constants.

This function has constant elasticity which is represented by b.

Empirical estimates of the elasticity coefficient so far made can be summarized as shown in Table 1. Some are estimated as the degree of homogeneity in the production functions, Mera (1973b) and Sveikaurkas (1975), and others are estimated as differential neutral technical level as related to city size. Some are concerned only with the manufacturing sector as a whole or individual two or three digid sectors, and some are concerned with the wage level in general. Some estimates are corrected for labor force composition, capital-labor ratio, and geographical price differentials and some are not.

Table 1. Summary of Estimated City Size Elasticity

Author	Country Examined	Sector	b
Mera (1973b)	Japan	Secondary	0.04
do	Japan	Tertiary	0.10
Sveikaurkas (1975)	U. S.	Two-digid Mfg	0.02 - 0.12
Segal (1976)	U. S.	Manufacturing	0.08
Yezer and Goldfarb (1978)	U. S.	Wage	0.04 - 0.05

On the whole, it can be said that if the manufacturing sector alone is taken, the elasticity coefficient would be in the order of 0.04 to 0.10 and if the entire economic activities in a city is taken, then the elasticity coefficient world be greater.

(b) Urban Service Benefit Function

In the literature of social physicists, the potential of a given city is assumed to decline in proportion to the inverse of the distance from the city. With this assumption, reasonable linear correlation has been observed between the population potential and intensity variables of human activities in space, Stewart (1947), Zipf (1947), Warntz (1959 and 1965). However, in the literature of urban economics, negative exponential functions have been widely used for representing spatially differentiated activity levels within an urban area such as population density and land value, Mills (1972), Muth (1969) and Clark (1967).

We shall employ a negative exponential or exponential decay function of distance from the city for representing per capita product in a rural area:

$$y_R = y_U \, e^{-ct}, \qquad c > 0 \tag{2}$$

where y_R is per capita product in rural area at distance t from the nearest city U, and c is a constant.

There are two specific points which should be discussed in connection with this assumption. One is the urban-rural continuity. In equation (2), per capita product declines continuously from a city to its hinterland. In the real world, it may well drop discontinuously from a city to rural areas. This is an empirical question. But, in most countries today, cities are not clearly bounded by land use or any other activity variable but gradually change into rural areas as a word such as suburbia or sprawl may imply. Therefore, continuity is assumed here.

The second is the impact of all other cities except the nearest city on a given rural area. The traditional population potential is based on the assumption that all population outside of the area in question will have some impact on any location within a nation. In a more elaborate study, such a general viewpoint would be necessary, but in this exploratory analysis, a simplifying assumption would be sufficiently worthwhile. Population potential is considered only from the city nearest to the rural area in question.

A more general question has to be answered. Although population potential based on inverse proportionality to distance has been widely accepted as having substantial explanatory power, the exponential decay function has not been tested much with empirical data with a notable exception of the urban population density distribution. Therefore, its empirical relevance can be questioned. But, there is a strong reason for preferring this function to the inversely proportional function. The inversely proportional fuction of distance cannot be defined in an infinitesimally close neighborhood of a city, i.e., potential is infinitely large. This cannot happen in the real world. So, the exponential decay function would better explain spatial distribution of activity levels. More pragmatically, if the inverse proportionallity function has a good correlation with certain spatially distributed variables, the exponential decay function should also have a good correlation because the correlation between $1/t$ and e^{-ct} is high.

(c) Urban Population, City Size and the Number of Cities

For any specific country, the total urban population is assumed fixed because the percentage of urban population or the total urban population is one of the most basic indicators of nation's development. Thus, the city size is considered to be a policy variable.

For the purpose of simplification and also for efficiency in achieving the performance criteria described below, all cities are assumed to be in equal size, i.e.,

$$U_i = \frac{U}{n}, \qquad i = 1, 2, \ldots, n \tag{3}$$

where

U_i is the population size of i-th city,

U is the total urban population of the nation, and

n is the number of cities in the nation.

(d) Spatial Distribution of Cities and the Nation

Two types of nation will be assumed, a linear and a circular. The linear nation has a total length of L and the cities will be distributed so that the hinterland of each city is identical. In other words, for each city its hinterland on one direction will be $L/2n$, as shown in Figure 1.

The circular nation has a total area of A and the radius of $\sqrt{A/\pi}$. When the number of cities is greater than one, the territory is is assumed to be torn off into equal amounts, each forming a circular region around each city. This assumption makes the average distance of

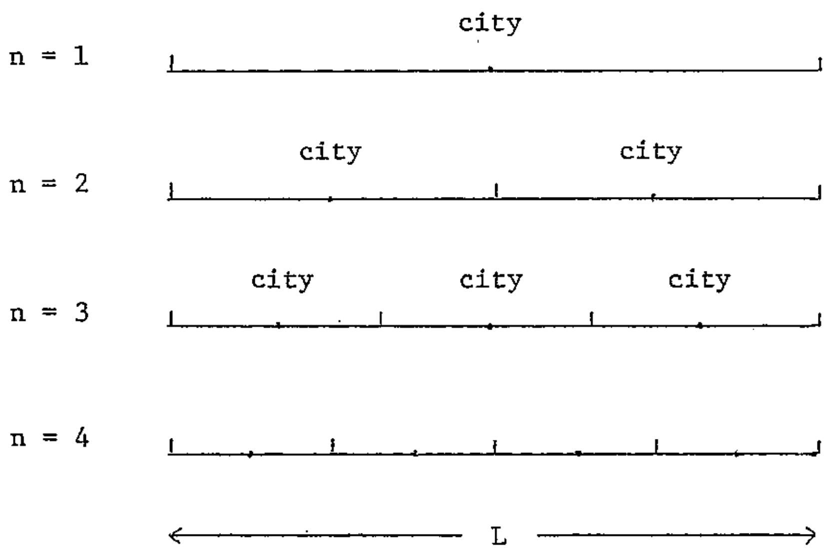

Figure 1. Spatial Distribution of Cities in a Linear Nation

the hinterland to the nearest city shorter than if the pattern of the territory is not changed, particularly when the number of cities is not large. But, the general characteristics obtainable with this assumption would not be so different from those obtainable otherwise. This simplifying assumption can be justified on the basis of operational ease. Then, when the area of the circular hinterland of each city is A/n, the radius of the circular hinterland of each city is $\sqrt{A/\pi n}$.

In addition, it is assumed that cities are located at points and the rural population is distributed uniformly over the entire territory.

(e) Evaluating Criteria

By changing the number of cities, per capita product of the urban population changes due to changes in agglomeration economies. At the same time, per capita product of the rural population changes due to changes in average accessibility to the nearest city and also changes in urban per capita product which defines the upper limit for rural per capita product.

Two criteria well be used for evaluation alternative situations. One is productive efficiency which is measured by national product as the sum of urban and rural products. It is possible to identify the number of cities that maximizes national product.

The second is the urban-rural equity. As the urban-rural imbalance is a significant issue for regional planning and policies, the distribution of income levels in two regions will be defined to be an important evaluating criterion. For this purpose, the ratio of urban per capita product to rural per capita product will be used,[2/] i.e.,

$$Ra = \frac{y_U}{\bar{y}_R} \tag{4}$$

where

Ra is the urban-rural equity index, and

$\bar{y}_R$ is the average rural per capita product.

The optimal value for Ra is of course unity. There may be a tradeoff relationships between two evaluating criteria. Whenever there is, no explicit weighting of the two indices will be made, but some preferable range of feasible combinations of the two indices will be pointed out.

3. Basic Equations

(a) The Case of the Linear Nation

On the basis of the assumptions described above, it is possible to derive equations which describe key indicators of the performances of the system when the number of cities, n, is given. The total national product is the sum of the urban product and the rural product:

$$Y = Y_U + Y_R \tag{5}$$

where

Y is the total national product,

Y_U is the urban product, and

Y_R is the rural product.

The urban product is per capita urban product times population:

$$Y_U = y_U \, U = a\left(\frac{U}{n}\right)^b U \tag{6}$$

and the rural product is the integral of rural product on one side of the hinterland of a city times 2n:

$$Y_R = 2n \int_0^{\frac{L}{2n}} y_t \; s \; dt \tag{7}$$

where s is the density of population and from the assumption of uniform distribution of rural population,

$$s = \frac{R}{L} \,. \tag{8}$$

By substituting (1), (2), (3) and (8) into (7), we obtain

$$Y_R = a\ R\ \frac{2n}{cL}\left(\frac{U}{n}\right)^b\ (1 - e^{-\frac{cL}{2n}}) \tag{9}$$

Then, the total national product is shown as

$$Y = a\left(\frac{U}{n}\right)^b\ \{U + R\ \frac{2n}{cL}\ (1 - e^{-\frac{cL}{2n}})\}\ . \tag{10}$$

From (9), average rural per capita product is

$$\bar{y}_R = a\ \frac{2n}{cL}\left(\frac{U}{n}\right)^b\ (1 - e^{-\frac{cL}{2n}})\ . \tag{11}$$

and, substituting (1), (3) and (11) into (4), the urban-rural equity index represented by

$$Ra = \frac{\frac{cL}{2n}}{1 - e^{-\frac{cL}{2n}}} \tag{12}$$

(b) The Case of the Circular Nation

For the circular nation, (5) and (6) hold without modification. The rural product is the integral of rural product within the circular hinterland of city times n:

$$Y_R = n\int_0^r yt\ s\ 2\pi\ t\ dt \tag{13}$$

where $s = \frac{R}{A}$ and (14)

$$r = \sqrt{\frac{A}{\pi n}} \tag{15}$$

Substituting (1), (2), (3) and (14) into (13), we obtain

$$Y_R = \frac{2aR}{(cr)^2}\left(\frac{U}{n}\right)^b \{1 - e^{-cr}(cr + 1)\} . \tag{16}$$

Then, the total national product is

$$Y = a\left(\frac{U}{n}\right)^b [U + \frac{2R}{(cr)^2}\{1 - e^{-cr}(cr + 1)\}] \tag{17}$$

The urban-rural equity index is, then:

$$Ra = \frac{(cr)^2}{2\{1 - e^{-cr}(cr + 1)\}} \tag{18}$$

(c) Major characteristics of the Equations

As far as the urban product is concerned, it is easily seen from (6) that it decreases as n increases regardless the shape of the nation. The rural product is a function of the urban per capita income as well as the rate of decay of urban influence as the distance increases from a city as shown in (9) and (16). The former part decreases as n increases, as stated above, but the latter part increases as n increases.[3/]

The change of total national product as related to a change in n can be known by differentiating Y with respect to n as if n

is a continuous variable. From (10), we have

$$\frac{dY}{dn} = a \frac{U^b}{n^{b+1}} [R\{h(1 - b)(1 - e^{-h}) - e^{-h}\} - bU] \tag{19}$$

where $h = \frac{cL}{2n}$, and

from (17), we have

$$\frac{dY}{dn} = a \frac{U^b}{n^{b+1}} \left(R[\frac{2}{(cr)^2} (1 - b)\{1 - e^{-cr}(cr + 1)\} - e^{-cr}] - bU\right). \tag{20}$$

In either case, the sign of $\frac{dY}{dn}$ depends on the value of parameters. We shall be further examining conditions for national product maximization in the next section through numerical cases.

It should be noted, however, that the sign of $\frac{dY}{dn}$ depends on the sign of the large parenthesis in (19) as well as in (20). Since the large parenthesis in either case is a homogeneous function of degree one with respect to U and R, the national product maximizing value of n is independent of the absolute value of U or R. Only the relative size of U to R is relevant. Thus, the only important characteristics of population is the percentage of urban population in the total population.

The behavior of Ra can be known by rewriting (12) and (18), respectively, as follows:

$$Ra = \frac{he^h}{e^h - 1} \tag{21}$$

where $h = \frac{cL}{2n}$

and

$$Ra = \frac{h^2 e^h}{2\{e^h - (1+h)\}} \tag{22}$$

where $h = cr$,

and, differentiating them with respect to h to obtain, respectively:

$$\frac{dRa}{dh} = \frac{e^h}{(e^h - 1)^2} \{e^h - (1+h)\}$$

and

$$\frac{dRa}{dh} = \frac{h\, e^h}{2\{e^h - (1+h)\}^2} \{2e^h - 2 - 2h - h^2\}.$$

By expanding the exponential term in the above equations, we have, respectively,

$$\frac{dRa}{dh} = \frac{e^h}{(e^h - 1)^2} \{\frac{h^2}{2!} + \frac{h^3}{3!} + \ldots\} > 0$$

and

$$\frac{dRa}{dh} = \frac{h\, e^h}{2\{e^h - (1+h)\}^2} \; 2 \times \{\frac{h^3}{3!} + \frac{h^4}{4!} + \ldots\} > 0$$

Therefore, the urban-rural equity index increases as h increases, i.e., it diminishes as n increases.

In addition, when n approaches infinity or h approaches zero, it can be shown in either case that[4/]

$$\lim_{h \to 0} Ra = 1$$

Therefore, it can be said that as the number of cities increases urban-rural imbalance diminishes and when it approaches infinity, imbalance is totally wiped out. This is presumably the case of "agropolitan development".

4. Numerical Assumptions

We shall now assume specific values for the parameters to examine the performances of the system when n changes. The following assumptions will be made.

(a) The Percentage of Urban Population, u

Two alternative values will be assumed for the percentage of urban population which is designated by u; 20 and 40 percent. The first value represents low income developing countries and the second middle income developing countries.

(b) Productivity Elasticity, b, with Respect to City Size

Some of the recent findings are shown in Table 1. Four alternative values which cover the range of values reported there will be assumed; 0.20, 0.10, 0.07 and 0.05. It is considered from them the two middle values are more representative than the other two.

(c) The Rate of Decay of Urban Service Benefit

In the case of the linear nation, the exponent in the exponential decay function always appears in the form of $-\frac{cL}{2n}$. This implies that the largest disparity takes place when n = 1 and between the city at the center and the edge on either side. The per capita product at the edge is, then, $e^{-\frac{cL}{2}}$ times the highest level.

The magnitude of disparities observable in developing countries is commonly several-fold as documented in Mera (1975), p.4. For the purpose of analysis here, two alternative values of $e^{-\frac{cL}{2}}$ are assumed here; $\frac{1}{2}$ and $\frac{1}{4}$. In terms of cL, these assumptions imply 1.3862 and 2.7730, respectively.

In the case of the circular nation, the exponent shows up in the form of $-c\sqrt{A/\pi n}$, and the largest disparity takes place when $n = 1$ between the center and an edge. The proportion in this case is $e^{-c\sqrt{A/\pi}}$. Again, two alternative values of $e^{-c\sqrt{A/\pi}}$ are assumed; $\frac{1}{2}$ and $\frac{1}{4}$. In terms of $c\sqrt{A/\pi}$, these imply 0.6931 and 1.3865, respectively.

5. The Results of the Numerical Cases

The results of the numerical cases assumed are summarized in Table 2 and 3 for the linear and circular nations, respectively. For either case, various combinations of b and $\frac{cL}{2}$ or cr are examined for a fixed level of urbanization. Then, for each combination, the number of cities that maximizes national product, n*, and the associated value of Ra* are shown. Furthermore, for each combination, the number of n that reduces the urban-rural disparity (which is defined to be Ra - 1) into a half is identified. At that point, the national product is always less than the maximum attainable, Y*. The ratio of the national product which is associated with a half-reduced imbalance to the maximum national product is shown to indicate the degree of tradeoff involved.

From the results for the linear nation shown in Table 2, it can be concluded:

Table 2. Numerical Results: Linear Nation

u = 20

$Exp(-\frac{cL}{2})$	b	National Product Maximization		Imbalance Reduced to a Half		
		n*	Ra*	n	Y/Y*	Ra
$\frac{1}{2}$	0.20	1	1.386	2	0.981	1.193
$\frac{1}{2}$	0.10	3	1.120	6	0.975	1.060
$\frac{1}{2}$	0.07	4	1.089	8	0.985	1.045
$\frac{1}{2}$	0.05	5	1.071	10	0.992	1.036
$\frac{1}{4}$	0.20	2	1.386	4	0.981	1.193
$\frac{1}{4}$	0.10	5	1.145	10	0.983	1.073
$\frac{1}{4}$	0.07	8	1.089	16	0.985	1.045
$\frac{1}{4}$	0.05	11	1.064	22	0.990	1.032

u = 40

$Exp(-\frac{cL}{2})$	b	National Product Maximization		Imbalance Reduct to a Half		
		n*	Ra*	n	Y/Y*	Ra
$\frac{1}{2}$	0.20	1	1.386	2	0.948	1.193
$\frac{1}{2}$	0.10	2	1.183	4	0.978	1.092
$\frac{1}{2}$	0.07	3	1.120	6	0.984	1.060
$\frac{1}{2}$	0.05	4	1.089	8	0.990	1.045
$\frac{1}{4}$	0.20	2	1.386	4	0.948	1.193
$\frac{1}{4}$	0.10	4	1.183	8	0.978	1.092
$\frac{1}{4}$	0.07	5	1.145	10	0.990	1.073
$\frac{1}{4}$	0.05	8	1.089	16	0.990	1.045

Note: n* is the national product maximizing number of cities, Y* is the national product and Ra* is the urban-rural equity index both when n = n*.

Table 3. Numerical Results: Circular Nation

u = 20

$Exp(-c\sqrt{\frac{A}{\pi}})$	b	National Product Maximization n*	Ra*	Imbalance Reduced to a Half n	Y/Y*	Ra
$\frac{1}{2}$	0.20	1	1.566	4	0.892	1.283
$\frac{1}{2}$	0.10	3	1.300	11	0.966	1.150
$\frac{1}{2}$	0.07	6	1.205	22	0.978	1.103
$\frac{1}{2}$	0.05	13	1.136	49	0.982	1.068
$\frac{1}{4}$	0.20	2	1.870	7	0.953	1.435
$\frac{1}{4}$	0.10	11	1.315	39	0.971	1.158
$\frac{1}{4}$	0.07	25	1.200	93	0.976	1.100
$\frac{1}{4}$	0.05	50	1.138	189	0.983	1.069

u = 40

$Exp(-c\sqrt{\frac{A}{\pi}})$	b	National Product Maximization n*	Ra*	Imbalance Reduced to a Half n	Y/Y*	Ra
$\frac{1}{2}$	0.20	1	1.566	4	0.849	1.283
$\frac{1}{2}$	0.10	1	1.566	4	0.976	1.283
$\frac{1}{2}$	0.07	3	1.300	11	0.978	1.115
$\frac{1}{2}$	0.05	6	1.205	22	0.985	1.103
$\frac{1}{4}$	0.20	1	2.382	3	0.934	1.691
$\frac{1}{4}$	0.10	5	1.495	17	0.973	1.248
$\frac{1}{4}$	0.07	12	1.300	43	0.978	1.150
$\frac{1}{4}$	0.05	25	1.200	93	0.984	1.100

Note: n* is the national product maximizing number of cities, and Y* is the national product and Ra* is the urban-rural equity index when n = n*.

(1) the greater is the productivity elasticity, b, the smaller n^* is.

(2) the greater is the rate of decay over space, cL, the greater n^* is.

(3) Ra is a function of n and cL and not of b, and the greater is n, the smaller Ra is.

(4) by comparing the cases of the percentage of urban population of 20 and 40, it can be known that the greater is u, the smaller or equal n^* is.

(5) the number of n that reduces urban-rural imbalance into a into a half is always twice greater than n^*.

(6) the ratio of national product to the maximum when urban-rural imbalance is halved from Y^* is generally greater than 95 percent and the greater is n^*, generally the greater the ratio is.

From the results for the circluar nation shown in Table 3, it can be said that (1), (2), (3) and (4) above also hold except that cL should be replaced by cr, whenever it appears. (5) and (6) above should be replaced by:

(5) the number of n that reduces urban-rural imbalance into a half is nearly four times greater than n^*.

(6) the ratio of national product to the maximum when urban-rural imbalance is halved from Y^* is generally greater than 85

percent and the greater is n^*, generally the greater the ratio is.

By comparing the cases of the linear and circular nations with each other, the following observations can be made: for the same combination of b and $\frac{cL}{2}$ or $c\sqrt{\frac{A}{\pi}}$, the circular nation is characterized by that

(1) n^* is greater when b is small.

(2) Ra is greater.

(3) a much greater increase in the number of cities is needed for reducing urban-rural imbalance into a half.

(4) the ratio of Y to Y^* which reduces urban-rural imbalance into a half from the level at Y^* is less.

6. Conclusions

Real nations would be much closer to the circular nation than the linear, and due to irregularity in the terrain, the above four characteristics may be more pronounced in real nations than the circular nation. Then, what would be the number of cities that maximizes national product in real nations? If the elasticity, b, is as great as 0.2, there is a good chance that the national product maximizing number of cities, n^*, is unity. Although it appears that this elasticity is relatively great in developing countries, the chance that b is so great does not appear to be good. There is, instead, a good chance that several cities would be needed for maximizing national product. This conclusion is a signinficant departure from what one tends to interprete the results of the city-size-productivity analyses.

It should be noted, however, that the national product maximizing number of cities crutially depends on the rate of decay of the urban influence as represented by $\frac{cL}{2}$ or $c\sqrt{A/\pi}$. This implies that not only c but the dimension of the territory is also a determining factor. Therefore, assuming c is common for all nations, the national product maximizing number of cities is greater in large nations than is small ones. The specific value of n^* crutially depends on c, however, which has not been exposed very much to empirical estimation. For this reason, quotation of any specific figure for n^* is not warranted at this time.

As for as efficiency-equity tradeoff is concerned, a move toward greater urban-rural equity away from maximum efficiency would probably involve development of a substantial number of cities. To reduce the existing urban-rural imbalance to a half may require the development of four times as many cities as they exist. However, the loss in productive efficiency would not be greater than four percentage points except when b is as great as 0.2, Notes

1/ Those who estimated city production functions should not be blamed for this deficiency because they did not intend to derive policy implications on the basis of the production functions estimated. Alonso (1971) was evidentally aware of the impact of the cities on the surroundings as he presented an city-income equation based on the potential concept as described below. Mera (1973a) took this aspect of urban influence by measuring national economic growth instead of urban economic growth when the implications of the growth of large cities were examined.

2/ In this system, no transfer mechanism between the urban population and rural population is assumed. If there is an adequate transfer system, only national product should be used as the evaluating criterion. When the transfer machanism is inoperative or inadequate, there is a question of tradeoff between productive efficiency and interregional equity as in Mera (1967).

3/ This can be seen by defining

$$z_1 = \frac{1}{h}(1 - e^{-h})$$

where $h = \frac{cL}{2n}$, and

$$z_2 = \frac{1}{h^2}\{1 - e^{-h}(h+1)\}$$

where $h = cr$,

corresponding, respectively, (9) and (16). In either case, h diminishes as n increases. By transforming them into

$$z_1 = \frac{e^h - 1}{b\,e^h}$$

and

$$z_2 = \frac{e^h - (h+1)}{h^2\,e^h}$$

and differentiating them with respect to h, we obtain

$$\frac{dz_1}{dh} = \frac{(1+h) - e^h}{h^2\,e^h}$$

and

$$\frac{dz_2}{dh} = \frac{(2+2h+h^2) - 2e^h}{h\,e^h}$$

By expanding the exponential term in the respective equation, we obtain

$$\frac{dz_1}{dh} = \frac{-\frac{h^2}{2!} - \frac{h^3}{3!} - \cdots}{h^2\,e^h} < 0$$

and

$$\frac{dz_2}{dh} = \frac{-2(\frac{h}{2!} + \frac{h}{3!} + \frac{h^4}{4!} + \ldots)}{h\,e^h} < 0$$

Therefore, z_1 and z_2 diminishes as h increases, i.e., z_1 and z_2 increases as n increases.

4/ By expanding the exponential term into an infinite series, (12) can be transformed into

$$Ra = \frac{1}{1 - \frac{h}{2!} + \frac{h^2}{3!} - \ldots}$$

and (18) into

$$Ra = \frac{1}{1 - \frac{2}{3} h + \frac{1}{4} h^2 - \ldots} .$$

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