No. 871

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by

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July 2000

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Date of Design: July 12 2000

【日本語の題名】　動学的パネルデータの指数回帰

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【作成日】　2000 年 7 月 12 日

【日本語の要旨】

この論文の中では、付加的固定効果を有する動学的パネルデータモデルを乗法的固定効果を有する指数回帰の型へと変換することによってこのモデルを一致推定する方法を示す。

Exponential regression of dynamic panel data models

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Abstract

This short paper shows that we can consistently estimate dynamic panel data models with additive fixed effects by transforming them into a form of exponential regressions with multiplicative fixed effects.

Keywords: Dynamic panel data; Exponential regression; Generalised method of moments;

JEL classification: C23

1. Introduction

A problem specific to the consistent estimation of dynamic panel data models in case of large N and fixed T is how to handle their additive fixed effects. Many estimators have been developed for solving this problem: Holtz-Eakin et al. (1988), Arellano and Bond (1991), Ahn and Schmidt (1995), Arellano and Bover (1995), and Blundell and Bond (1998). In this short paper, it is shown that the models are transformed into a form of exponential regressions with multiplicative fixed effects and

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then the consistent GMM estimators for the models are obtained. It will be recognised that one of these estimators uses the moment restrictions on the framework of the quasi-differencing transformation proposed by Wooldridge (1997) and Chamberlain (1992).

2. Model, transformation, and moment restrictions

For simplicity, we consider a AR(1) dynamic panel data model with additive fixed effects:

$$y_{it} = \alpha y_{i,t-1} + \eta_i + v_{it}. \tag{1}$$

In this model, y_{it} is the dependent variable, α is the parameter of interest, η_i is the fixed effect, and v_{it} is the disturbance with the mean zero. The subscripts i and t denote the individual and the time respectively, where $i = 1,\dots,N$ and $t = 2,\dots,T$ with N large and T fixed.

We impose an assumption on the disturbances:

$$v_{it} \sim iidN(0, \sigma_i^2), \qquad \text{for } t = 2,\dots,T. \tag{2}$$

This is an important assumption in this paper. The variances of the disturbances are different among individuals but equal over time. The σ_i^2 will be regarded as a latent fixed effect.[1]

We transform (1) into an exponential form as below:

$$\exp(y_{it} - \alpha y_{i,t-1}) = \exp(\eta_i)\exp(v_{it}), \qquad \text{for } t = 2,\dots,T. \tag{3}$$

[1] The unconditional moment restrictions related to the assumption of the intertemporal homoscedasticities in v_{it} proposed by Ahn and Schmidt (1995) are obtained from the elimination of this effect.

Taking a conditional expectation $E[\bullet \mid \eta_i, y_i^{t-1}]$ of both sides of (3), we obtain

$$E[\exp(y_{it} - \alpha y_{i,t-1}) \mid \eta_i, y_i^{t-1}] = \mu_i, \qquad \text{for } t = 2, \ldots, T, \tag{4}$$

where y_i^{t-1} is $(y_{i1}, \ldots, y_{i,t-1})$. The newly-defined fixed effect $\mu_i = \exp(\eta_i + \sigma_i^2/2)$ is obtained from the relationship $E[\exp(v_{it}) \mid \eta_i, y_i^{t-1}] = \exp(\sigma_i^2/2)$ stemming from $E[v_{it} \mid \eta_i, y_i^{t-1}] = 0$ based on (2). Taking the first-difference of (4) to eliminate μ_i and then applying the law of iterated expectations, we obtain the following conditional moment restrictions:

$$E[\exp(y_{it} - \alpha y_{i,t-1}) - \exp(y_{i,t-1} - \alpha y_{i,t-2}) \mid y_i^{t-2}] = 0, \qquad \text{for } t = 3, \ldots, T. \tag{5}$$

When multiplying both sides of (5) by $\exp(\alpha y_{i,t-2})$, we can obtain the conditional moment restrictions on the basis of the quasi-differencing transformation. The conditional moment restrictions (5) can provide the following $(T-2)(T-1)/2$ unconditional moment restrictions for estimating α consistently as below:

$$E[y_{is}\{\exp(y_{it} - \alpha y_{i,t-1}) - \exp(y_{i,t-1} - \alpha y_{i,t-2})\}] = 0, \text{ for } t = 3, \ldots, T; s = 1, \ldots, t-2. \tag{6}$$

Further, when we specify the initial condition making y_{it} mean-stationary:

$$y_{i1} = \eta_i/(1-\alpha) + w_{i1}, \qquad \text{where } w_{i1} \text{ is } \textit{iid} \text{ with the mean zero,} \tag{7}$$

we can use the following $(T-2)(T-1)/2$ unconditional moment restrictions for estimating α consistently:

$$E[\Delta y_{is} \exp(y_{it} - \alpha y_{i,t-1})] = 0, \qquad \text{for } t = 3, \ldots, T; s = 2, \ldots, t-1.^{2} \tag{8}$$

[2] Since the variables η_i, v_{it}, and Δy_{is} (for $s = 2, \ldots, t-1$) are independent each other and $E[\Delta y_{it}] = 0$ from (1), (2), and (7), the restrictions (8) are obtained from (3).

3. Monte Carlo experiments

In order to investigate the small sample performances of the GMM estimators based on the unconditional moment restrictions (6) and (8), Monte Carlo experiments are carried out. The data generating process is designed as below:

$$y_{i1} = \eta_i /(1-\alpha) + v_{i1} /(1-\alpha^2)^{1/2},$$

$$y_{it} = \alpha y_{i,t-1} + \eta_i + v_{it}, \qquad \text{for } t = 2,\ldots,T,$$

$$\eta_i \sim iidN(0, 0.25), \qquad v_{it} \sim iidN(0, 1), \qquad \text{for } t = 1,\ldots,T,$$

where $i = 1,\ldots,N$. In the experiments, $N = 100$ and $T = 7$. The number of replications (NR) is 500. The values of α vary from 0.0 to 0.9. The two GMM estimators of α are constructed using the unconditional moment restrictions (6) and (8). We will call the estimator based on (6) as the GMM (EXP-DIF) and the estimator based on (8) as the GMM (EXP-LEV). These two estimators are compared with the GMM (DIF) and GMM (SYS) estimators in Blundell and Bond (1998). Monte Carlo results are indicated in Table 1, where the results using each estimator are presented in each column. The GMM (EXP-DIF) and GMM (EXP-LEV) estimators are upward-biased. The higher the true value of α, the larger these biases. However, for high values of α these estimators perform better than the GMM (DIF).[3] A merit of these two estimators is that their standard deviations are small and the means of their estimated standard errors predict them well. These two estimators will probably perform well in some situations.

[3] The downward-biases of the GMM (DIF) and the standard deviations of its estimates are substantial for high values of α in small sample size. These are well-known facts.

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Table 1. Monte Carlo results for $N = 100$, $T = 7$, and $NR = 500$

α	GMM (EXP-DIF)	GMM (EXP-LEV)	GMM (DIF)	GMM (SYS)
0.0	**0.024**	**0.022**	**-0.010**	**-0.003**
	(0.080, 0.053)	(0.085,0.059)	(0.057, 0.056)	(0.057, 0.040)
	[15.00, 14]	[14.91, 14]	[14.88, 14]	[20.25, 19]
0.1	**0.131**	**0.126**	**0.088**	**0.096**
	(0.083, 0.056)	(0.088, 0.062)	(0.060, 0.059)	(0.059, 0.041)
	[15.06, 14]	[14.94, 14]	[14.89, 14]	[20.25, 19]
0.2	**0.241**	**0.230**	**0.184**	**0.195**
	(0.085, 0.059)	(0.090, 0.065)	(0.064, 0.062)	(0.061, 0.042)
	[15.14, 14]	[14.97, 14]	[14.90, 14]	[20.27, 19]
0.3	**0.352**	**0.335**	**0.280**	**0.294**
	(0.087, 0.062)	(0.092, 0.068)	(0.068, 0.066)	(0.063, 0.044)
	[15.23, 14]	[15.02, 14]	[14.92, 14]	[20.28, 19]
0.4	**0.466**	**0.442**	**0.375**	**0.393**
	(0.087, 0.066)	(0.092, 0.071)	(0.073, 0.071)	(0.065, 0.045)
	[15.36, 14]	[15.07, 14]	[14.95, 14]	[20.31, 19]
0.5	**0.582**	**0.551**	**0.467**	**0.491**
	(0.086, 0.071)	(0.092, 0.074)	(0.080, 0.078)	(0.068, 0.046)
	[15.47, 14]	[15.14, 14]	[14.98, 14]	[20.34, 19]
0.6	**0.699**	**0.662**	**0.554**	**0.589**
	(0.083, 0.074)	(0.088, 0.076)	(0.091, 0.088)	(0.071, 0.048)
	[15.54, 14]	[15.24, 14]	[15.03, 14]	[20.38, 19]
0.7	**0.812**	**0.775**	**0.632**	**0.685**
	(0.074, 0.075)	(0.079, 0.075)	(0.107, 0.104)	(0.075, 0.048)
	[15.51, 14]	[15.36, 14]	[15.10, 14]	[20.43, 19]
0.8	**0.910**	**0.883**	**0.681**	**0.781**
	(0.057, 0.066)	(0.062, 0.067)	(0.140, 0.134)	(0.079, 0.048)
	[15.37, 14]	[15.49, 14]	[15.22, 14]	[20.51, 19]
0.9	**0.977**	**0.969**	**0.610**	**0.884**
	(0.033, 0.041)	(0.035, 0.045)	(0.226, 0.205)	(0.079, 0.040)
	[15.19, 14]	[15.50, 14]	[15.23, 14]	[20.60, 19]

Note: (i) Each boldfaced figure is the Monte Carlo mean of estimates of α using each estimator. (ii) In each parenthesis below the Monte Carlo mean, values of the first and second elements are the Monte Carlo standard deviations of estimates and the Monte Carlo mean of the estimated standard errors. (iii) In each bracket below the parenthesis, a value of the first element is the *Sargan* test statistic for over-identifying restrictions. This statistic is chi-square-distributed with the degree of freedom being a value of the second element. (iv) Estimates of the GMM (DIF) estimator are calculated with the one-step estimator allowing for MA (1) disturbances. Estimates of other estimators are calculated with the two-step estimators when the weighting matrices of the one-step estimators are non-optimal.