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A Neo-Classical Approach to the Classic Export-Base Model:
A Supply-Constrained I-O System.

by

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"I racked my brain to construct new falsehoods" Eco (1990, p.370)

1. Introduction

It is common to think of the classic export-base model of regional growth as a demand-side model. Of course this is incorrect. The classic export-base situation is where the development of the region is founded upon the exploitation of a natural resource that is sold in a competitive external market. The development of the region is clearly supply constrained – constrained by the extent of the natural resource base. In this paper I use a variant of the trade-focussed 1-2-3 general equilibrium model to analyse the classic export base model from a neo-classical perspective (Devarajan *et al*, 1997). The key elements of the variant of the model presented here are that the export industries face parametric prices, that the natural resources enter into only the production of the export sectors and that other resources are freely available through in-migration. The model has some similarities to the forecasting model of Minford *et al*, (1994).

I have three main objectives. The first is to show conditions under which the neo-classical model will exhibit the fixed coefficients normally associated with standard export-base approach. It proves to be the case that where the stimulus to the export sector comes from an expansion in the natural resource base, the neo-classical model operates in a manner identical to the classic export-base model. Second to show that where the stimulus to the export sector comes from other sources, specifically an increase in the export price or labour augmenting technical progress, the model can differ radically from the classic export-base case. Third, that where regional growth does occur as a result of an expansion in the regional natural resource base, that the model can be extended to generate a full-blown supply-constrained Input-Output (I-O) system.

2. The Basic Model

The basic model is a variant of the 1-2-3 trade-focussed general equilibrium model in which there is *1* region, *2* sectors and *3* commodities (Devarajan *et al*, 1997). In the two sectors, one sector produces a traded good that is sold exclusively outwith the region and the other produces a non-traded good for regional consumption. The three commodities are the traded good, the non-traded good, and a regional import. The imported commodity goes solely to regional consumption. The traded good is produced under conditions of perfect competition by labour and a natural resource and is sold at a parametric price. The non-traded good is produced under perfect competition using labour alone. In the model outlined in this section, for pedagogic reasons, there is a fixed level of efficiency (or technology) in both the traded and non-traded sectors. The real consumption wage is determined exogenously through flow migration (Treyz *et al*, 1993).

2.1 The Traded Sector (Sector T)

Output, Q_T, is produced using a linear homogeneous production function requiring a natural resource, R, and labour, L_T. Thus

(2.1) $$Q_T = Q_T(R, L_T)$$

The traded sector is competitive and supplies all its output to exports at the parametric price P_T. The real consumption wage is determined exogenously by flow-equilibrium migration. This is explained in more detail in Section 2.2 but it implies that the nominal wage rate, W_L, is fixed by the regional consumer price index, CPI. The natural resource earns a rent, W_R. Linear homogeneity and the first-order conditions for cost minimisation imply that the labour-natural resource ratio in the traded sector is solely a function of the relative factor prices. This can be expressed as:

(2.2) $$L_T = Rf(W_R/W_L)$$

where $\partial L_T/\partial R$, $\partial L_T/\partial W_R > 0$ and $\partial L_T/\partial W_L < 0$. Note the neo-classical nature of the employment function. Employment is dependent on the level of natural-resource availability and the relative factor prices. The assumption of perfect competition in the traded sector means that there are zero profits. Price therefore equals unit cost, which is a linear homogeneous function of the input prices. This can be represented as:

(2.3) $$P_T = P_T(W_L, W_R)$$

All labour income is retained locally but only a proportion, β_R, of the natural resource is owned within the region. Therefore income generated in the traded sector contributing to regional GNP, Y_T, is:

(2.4) $$Y_T = L_T W_L + \beta_R R W_R$$

This local GNP generated in the traded sector is the source of the autonomous expenditure injection into the non-traded sector.

2.2 The Non-Traded Sector (Sector N)

In the non-traded sector, output is produced by labour alone. The assumption of constant returns means that employment is linearly related to output in this sector, so that:

(2.5) $$L_N = \gamma Q_N$$

where γ is the constant labour-output ratio in the non-traded sector. The assumption of perfect competition means that price equals unit cost, generating:

(2.6) $$P_N = \gamma W_L$$

The contribution of the non-traded sector to regional GNP, Y_N, can be expressed as::

(2.7) $$Y_N = P_N Q_N$$

Expenditure on the non-traded good is then:

(2.8) $$\phi(Y_T + Y_N) = Q_N P_N$$

where ϕ is the proportion of GNP spent on the non-traded good, as against the imported commodity (there are no savings).

As with the price of the traded good, adopting the small-region assumption renders the price of the imported commodity parametric. Further, the price of the imported good is assigned the role of numeraire. This has two main advantages. First, it simplifies those expressions that have the price of imports as an argument. Second, it means that the exogenous price of exports, P_T, represents the terms of trade, the ratio of the region's export price to its import price. ϕ is taken to be a function of the prices of the non-traded and imported good, so that

(2.9) $$\phi = \phi(P_N)$$

Finally, the real consumption wage is constant, fixed by flow-equilibrium migration (Treyz, *et al*, 1993), and equals the initial nominal wage, W^0_L, times the regional consumer price index, CPI, producing:

(2.10) $$W_L = W^0_L \, CPI$$

and:

(2.11) $$CPI = \phi P_N$$

all initial prices being scaled to unity.

2.3 Characteristics of the model

This is a supply-constrained neo-classical export-base model. It is clearly neo-classical in that all firms profit maximise; both sectors are in equilibrium; and where prices are exogenous to the region, this exogeneity is motivated by the region's operating in perfectly competitive factor and product markets (Gee, 1991). The model complements the neo-classical export-base models presented in McGregor *et al*, (2000) and McVittie and Swales (1999). In those models the scarce resource (labour) is used in both the traded and non-traded sectors, whereas in this model the scarce input, natural resources, is used only in the export sector. As such the present model is closer in conception to the classic export-base case, where regional development occurs as a result of exploiting the export potential of a regional-specific natural resource (Innis, 1920). The slightly different assumptions produce a number of key differences to the results. In particular, they allow a supply-constrained I-O model and a coherent motivation for the classic export-base (fixed coefficients) model.

In the present model, where a stimulus is applied to the traded sector - whether in the form of an increase in the resource base, export price or efficiency - output in that sector will rise. This is not always the case in the models of McGregor *et al* (2000) and McVittie and Swales (1999) where a stimulus to the traded sector can produce a reduction in traded output as scarce resources are switched to the non-traded sector. This switching reflects increased consumption demand for the non-traded good, fuelled by the increase in real income generated by the improvement in the regional terms of trade. In the present model such switching does not occur, as resources for the non-traded sector are assumed to be freely available through in-migration.

Second, where the stimulus to the traded sector takes the form of an increase in the natural resource base or natural-resource-augmenting technical progress, output and employment increase by the same proportionate amount in both the traded and non-traded sectors. The neo-classical model replicates the fixed coefficients of the traditional export-base account. Essentially, under these conditions the stimulus generates an expansion in output with no change in prices. The factor price ratio remains constant and therefore so does the labour-natural resource ratio in the traded sector. However, where the stimulus is given by an increase in the export price or by labour-augmenting technical progress, the results are less clear cut. Specifically, we no longer expect the economy to expand with fixed coefficients, so that the behaviour of the economy depends upon the nature of the stimulus to the export base.

In Section 3 we outline the log derivative form of the model. This model is then used to rigorously derive the results for an expansion in the traded sector generated by: an increase in the resource base (Section 4); an improvement in the regional terms of trade (Section 5); and labour-augmenting technical progress in the traded sector (Section 6). Section 7 investigates the values of the export-base output and employment multipliers where the model does not replicate the classic export-base results. The elasticity of substitution in the production of the traded good is identified as a key parameter value. In Section 8, the classic export-base case associated with an increase in resource availability is extended to incorporate multiple traded and non-traded goods and also intermediate inputs. This section therefore demonstrates that the neo-classical export-base model extends straightforwardly to a supply-constrained Input-Output system. Section 9 is a short conclusion.

3. The Log Derivative Model

In this section the model is presented in log derivative form. The lower case variables are the proportionate changes in the upper case variables whose level forms were discussed in the previous section. Therefore, for example, $w_R = dW_R/W_R$.

3.1 The Traded Sector

In the traded sector, from the assumption of cost minimisation within perfect competition:

(3.1) $$q_T = \alpha(l_T + \lambda_L) + (1 - \alpha)(r + \lambda_R)$$

where α is the share of labour in the total output of the sector and λ is factor augmenting technical change, with the subscript identifying the appropriate factor. Using the definition of the elasticity of substitution in production, σ, together with the implications of cost minimisation in perfectly competitive factor markets:

(3.2) $$l_T + \lambda_L = r + \lambda_R + \sigma(w_R - \lambda_R - w_L + \lambda_L)$$

In this expression the inputs and prices of factors are implicitly measured in efficiency units. That is, the increase in labour inputs, in efficiency units, is $l_T + \lambda_L$ and the change in the price of labour, in efficiency units, is $w_L - \lambda_L$. Exactly the same adjustment in made for the natural-resource input and its rental rate. Using Shephard's lemma,

(3.3) $$p_T = \alpha(w_L - \lambda_L) + (1-\alpha)(w_R - \lambda_R)$$

The proportionate change in regional income derived from the traded sector is the weighted sum of changes in labour and resource income in this sector:

(3.4) $$y_T = \varphi(l_T + w_L) + (1-\varphi)(r + w_R)$$

where $\varphi = \alpha/(\alpha+(1-\alpha)\beta_R)$ and recall that β_R is the proportion of resources owned within the region. This implies that φ, the share of labour income in regional income from the traded sector, is greater than α, the share of labour income in the production of the traded good, as long as $\beta_R < 1$.

The external price change, p_T, resource growth, r, and the factor augmenting technical progress, λ_L and λ_R, are taken to be exogenous. Further, we will see that the change in the nominal wage, w_L, is determined in the non–traded sector. There are therefore four equations to determine the four additional endogenous variables associated with the traded sector, the change in employment, l_T, output, q_T, natural-resource rental, w_R, and regional income, y_T.

3.2 The Non-Traded sector

In the non-traded sector, from the supply side, the proportionate change in output is simply the proportionate change in the labour input:

(3.5) $$q_N = l_N$$

and the proportionate change in non-traded price is simply the proportionate change in the wage:

(3.6) $$p_N = w_L$$

The proportionate change in regional income derived from the non-traded sector is given by:

(3.7) $$y_N = p_N + q_N$$

The proportionate change in output in the non-traded sector is determined by the change in demand for the traded good, so that:

(3.8) $$\phi_N p_N + \tau y_T + (1-\tau) y_N = q_N + p_N$$

where τ is the share of initial regional income generated in the traded sector (Y_T/(Y_T+ Y_N) and ϕ_N is the elasticity of ϕ with respect to the price of the non-traded good. Through flow equilibrium migration, the real wage is held constant, so that:

(3.9) $$w_L = cpi$$

Further, changes in the consumer price index depend only on changes in the price of the non-traded good, generating.

(3.10) $$cpi = \phi p_N$$

The proportionate change in income from the traded sector is determined in the traded sector. There are therefore six equations, (3.5) to (3.10), to determine the six remaining endogenous variables. These are the proportionate changes in: the consumer price index, cpi, the non-traded labour input, l_N, price and output of the non-traded good, p_N and q_N, the nominal wage, w_L, and regional income from the non-traded sector, y_N.

3.3 The Marginal Export-Base Multiplier

In the analysis that follows, a central concern is how the outcomes compare to the traditional export-base multiplier. For variable Z, where in our case Z = L, Q or Y, the generic marginal export base multiplier, $M^M{}_{Z}$, is given by:

(3.11) $$M^M{}_{Z,} = dZ/dZ_T = (dZ_T + dZ_N)/dZ = 1 + dZ_N/dZ_T$$

where dZ, dZ_N and dZ_T are the changes in the total, non--traded and traded-sector values of variable Z. In the log derivative model we obtain results for the corresponding proportionate values z, z_N, and z_T. However, the value of the multiplier can be expressed as a function of these proportionate values and the average multiplier ($M^A{}_{Z}$). Specifically:

(3.12) $$M^M{}_{Z,} = 1 + (z_N/z_T)(Z_N/Z_T) = 1 + (M^A{}_{Z} - 1)(z_N/z_T)$$

where $M^A{}_{Z}$ is the initial average multiplier, which equals $(Z_T + Z_N)/Z_T > 1$.

There are two key points about the expression represented as equation (3.12). First, where the proportionate change in the variable is constant across sectors, so that $z_N = z_T$, then the marginal export base multiplier equals the initial average base multiplier for that variable: $M^M{}_{Z,} = M^A{}_{Z}$. Second, the value of the marginal export base multiplier for a particular variable is increasing in the ratio of the non-traded to traded rate of growth of that variable. That is to say:

(3.13) $$\partial M^M{}_{Z,}/\partial(z_N/z_T) = M^A{}_{Z} - 1 > 0$$

4. Expansion in the Resource Base and Resource-Augmenting Technical Progress

In this paper I focus exclusively on shocks to the traded sector, beginning with an expansion in the resource base through either new discovery or resource augmenting technical progress. However, it will prove useful initially to consider the non-traded sector. Equations (3.6), (3.9) and (3.10) simultaneously determine the changes in the consumer price index, non-traded output price and the nominal wage and give:

(4.1) $$\phi p_N = cpi = w_L = p_N$$

Given $\phi < 1$, (4.1) is consistent only with

(4.2) $$cpi = w_L = p_N = 0$$

Equation (4.2) simply identifies the fact that in this model, the wage, the price of the non-traded good and the consumer price index are all fixed to import prices. Specifically, they are independent of the level of activity within the region. With no change in these three price variables, the equations for the non-traded sector can be substantially simplified. Equation (4.2) replaces equations (3.6), (3.9) and (3.10) and equations (3.7) and (3.8) become:

(4.3) $$y_N = q_N$$

and

(4.4) $$\tau y_T + (1-\tau)y_N = q_N$$

Combining equations (4.3) and (4.4) produces a very important result. This is that:

(4.5) $$y_T = y_N$$

This implies that with this model, from equation (3.12), the marginal and average values of the export-base income multiplier are the same. The proportionate increase in traded income retained in the region is equal to the proportionate change in non-traded income. This is irrespective of the source of the initial increase in traded income.

However, consider whether the same can be said for changes in output or employment. Begin with the most straightforward case. This is the classic example where the expansion in the export sector comes through an increase in the natural resource base, so that of the four exogenous variables associated with the traded sector, $r > 0$ and λ_L, λ_R and $p_T = 0$. Focus first on the change in the natural-resource rental, w_R, which can be determined from equation (3.3). With no technical change, the change in the price of the traded good is simply the weighted sum of wage and resource rentals. The change in the traded price is zero, as is the change in the wage from equation (4.2) so that there is no change in the resource rental and therefore no change in relative factor prices.

Substituting this result into equations (3.1), (3.2) (3.4), (3.5), (4.3) and (4.5) produces:

(4.6) $$r = l_T = q_T = y_T = l_N = q_N = y_N$$

With relative factor prices fixed and a linear homogeneous production function in the traded sector, there is no change in the cost-minimising choice of technique and therefor no change in the factor intensity in the traded sector. Output, labour and natural-resource inputs all expand by the same proportionate amount (equations 3.1 and 3.2). This generates an equal proportionate increase in income from the traded sector (equation 3.4). With no change in prices in the non-traded sector, the expansion in demand generates an equal proportionate expansion in non-traded employment, output and income (equations 3.5, 4.3 and 4.5). Essentially, as a neo-classical system, the regional economy expands to fully employ the increased supply of natural resources. With fixed prices and linear homogeneity, this expansion occurs in a linear manner.

An almost identical story can be told where the increase in the output of the traded sector is generated by natural-resource-augmenting technical progress. In this case $\lambda_R > 0$ and r, λ_L, and $p_T = 0$. Again, begin with equation (3.3). Here, the fact that p_T, w_L, and λ_L are all zero generates the result that $w_R = \lambda_R$. The natural-resource rental increases by the rate of resource-augmenting technical progress, which means that measured in efficiency units, the resource rental remains unchanged. Substituting this result into equation 3.2 gives $l_T = \lambda_R$. Essentially the traded sector is again expanding with constant factor prices, where the factors (and natural resources specifically) are measured in efficiency units. Using exactly the same set of equations and arguments that were used for analysing the expansion of the natural-resource base, produces the result that:

(4.7) $$\lambda_R = l_T = q_T = y_T = l_N = q_N = y_N$$

The regional economy expands linearly at the rate of resource-augmenting technical progress.

With the expansion in the natural-resource base (a supply-side expansion in the key input for the region's export sector), the neo-classical model operates in exactly the way predicted by the classic export-base model. The linear expansion of the model means that for the marginal values for the income, output and employment multipliers will equal their respective average values. But note the model is not demand driven, but supply driven. Further, it will be shown in Section 8, that the model can be extended to encompass any number of traded and non-traded sectors. In that case, an expansion in the resource base with parametric export and import prices and with a fixed real wage and cost of capital produces a neo-classical model that replicates Input-Output results. That is to say, this neo-classical model has generated a supply-driven I-O system.

5. An Improvement in the Terms of Trade

In this neo-classical model, an increase in the natural resource base produces a linear expansion in the regional economy. In that case, the neo-classical and classic export-base models are observationally equivalent. However, this is not generally true for other stimuli to the regional export sector. Consider an improvement in the regional terms of trade, that is an increase in the price of the region's exports relative to its imports. Given that we take the import price to be the numeraire, this is represented by an increase in p_T, so that in this case $p_T > 0$ and r, λ_R, and $\lambda_L = 0$.

Again, begin by looking at equation 3.3. Given that w_L, λ_R, and λ_L all equal zero, the increase in the price of the traded good generates a corresponding rise in the resource rental rate, so that:

(5.1) $$w_R = p_T/(1-\alpha)$$

This increase in the resource price is central to the neo-classical account of the subsequent expansion in the traded sector. The available, unchanged, level of natural resources will still be fully employed, but the increase in the resource rental, relative to the wage, will mean that production will become less natural-resource intensive, and employment in the traded sector will rise, along with output. Traded-sector employment is determined by substituting the value for w_R into equation (3.2) (all the other variables are zero) to give:

(5.2) $$l_T = \sigma p_T/(1-\alpha)$$

and the subsequent increase in traded output is derived from equation (3.1) as:

(5.3) $$q_T = (\alpha\sigma/(1-\alpha))\ p_T$$

An alternative way of thinking about this process is that the increase in the price of the traded good temporarily produces positive profits in the traded sector. This generates an increase in employment in the sector as, labour is the only variable input. The rise in employment increases the marginal product of the natural resource, and therefore its rental. This process continues until resource rentals increase to such an extent that unit costs in the export sector have risen by the full amount of the initial increase in the price, thereby eliminating any positive profits.

Substituting the values for l_T and w_R from equations (5.1) and (5.2) into equation (3.4) gives the proportionate increase in regional income derived from the traded sector. As we know from equations (3.5), (4.3) and (4.5), this also equals the growth in non-traded employment, output and income, so that:

(5.4) $$y_T = l_N = q_N = y_N = p_T(\varphi\sigma + (1-\varphi))/(1-\alpha)$$

Compare the results in equations (5.2), (5.3) and (5.4), with the results derived in Section 4. It is clear that in the neo-classical model developed here, the values of the employment and output export-base multipliers will not be independent of the nature of the stimulus to the export sector. Specifically, where the stimulus to the export sector takes the form of an improvement in the terms of trade, in general marginal and average employment and output multipliers are not the same.

A detailed account of the multiplier values for the different stimuli to the export sector is presented in Section 7. However, a short discussed is given here to illustrate the differential behaviour of the model under different shocks to the basic sector. From equation (3.12), for average and marginal multipliers to be equal, the proportionate change in both the traded and non-traded values must be the same. For employment the position is as follows. From equations (5.2) and (5.4), $l_T = l_N$ iff

(5..5) $$p_T(\varphi\sigma + (1-\varphi))/(1-\alpha) = \sigma p_T/(1-\alpha)$$

which holds only where σ is unity. For the output multiplier, the situation is more straightforward. From equations (5.3) and (5.4), given that $\varphi > \alpha$, $q_N > q_T$. As a result of the improvement in the region's terms of trade, the proportionate increase in regional income from the traded sector is always greater than the proportionate increase in the output of that sector. The marginal output multiplier is therefore always greater than the average output multiplier under an expansion in the traded sector generated by an increase in export prices.

6. Labour–Augmenting Technical Change in the Traded Sector

Finally, output in the traded sector might be stimulated by a labour-augmenting efficiency improvement in that sector. This is the most complicated case, corresponding to the set of values for the exogenous variables: $\lambda_L > 0$ and r, λ_R, and $p_T = 0$. Again, beginning at equation (3.3), the "incidence" of the improvement in labour efficiency is an increase in the rental of the fixed factor, natural resources:

(6.1) $$w_R = (\alpha/(1-\alpha))\,\lambda_L$$

Substituting equation (6.1) into equation (3.2) produces

(6.2) $$l_T = \lambda_L\,(\alpha+\sigma-1)/(1-\alpha)$$

This equation reveals the possibility that employment change in the traded sector could be negative with labour-augmenting technical change. In particular, if $\alpha+\sigma < 1$, $l_T < 0$. For employment to rise with labour-augmenting technical progress, the wage-elasticity of demand for labour must be greater than unity. Where this is not the case, employment falls. An example which is straightforward to envisage would be where the traded sector has Leontief technology, so that $\sigma = 0$. Here, there are fixed coefficients and the supply of the other factor, natural resources, is unchanging. Under these circumstances, an improvement in labour efficiency of λ_L would simply produce an identical proportionate reduction in employment.

Substituting equation (6.2) into (3.1) gives the output change in the traded sector:

(6.3) $$q_T = \lambda_L\,\alpha\sigma/(1-\alpha)$$

This change in traded output is non-negative. However, the same cannot be said for the change in regional income derived from the traded sector. Recall from equations (3.5), (4.3) and (4.5), that the rate of growth in traded income also equals the rate of growth of income, output and employment in the non-traded sector. Substituting equations (6.1) and (6.2) into equation (3.4) and using equations (3.5), (4.3) and (4.5) therefore produces:

(6.4) $$y_T = l_N = q_N = y_N = \lambda_L\,(\alpha+\varphi(\sigma-1))/(1-\alpha)$$

Recall that we assume that $\varphi > \alpha$, that is that the share of income going to labour in production in the traded sector, α, is less than the share of wage income in retained regional income derived from the traded sector, φ. This means that at low values of σ, ($<1-(\alpha/\varphi)$), y_T will be negative. The reason is that with low values of $\sigma < 1$, an increase in labour efficiency will increase natural resource income as a share of total payments to factors of production in the traded sector. But we assume that a certain proportion of the natural resources is owned outwith the region, unlike wage income which is all retained in the region. Therefore if the increase in total traded output is small, which is also more likely to be the case where σ is low, its impact on retained regional income can be dominated by the adverse distributional effect, so that regional retained income and expenditure falls. These possibilities are not as far fetched as they may sound. A combination of increased productivity, increased output, but falling employment and local income might well have characterised many UK coal mining communities since the mid 1980s.

The possibility of negative employment change in the traded sector and negative employment and output change in the non-traded sector, means that both the marginal employment and output multipliers associated with labour-augmenting technical change can take negative as well as positive values. This is clearly a major divergence from the classic export-base case and is investigated in more detail in the next section.

7. Marginal Export-Base Multiplier Values

A key aim of this paper is to show that the classic export-base model can be rigorously grounded in neo-classical theory. That is to say, that we can derive a neo-classical model that has the exploitation of a natural resource as the source of the region's export base and which generates fixed coefficient multipliers. Section 4 demonstrates that growth in the export base predicated on increased supplies of the natural resource produces a linear expansion in the regional economy. In that case, for employment, output and income the value of the marginal export-base multiplier equals the value of the corresponding average export-base multiplier. The region expands with fixed coefficients.

However, a second aim is to demonstrate that where regional exports are stimulated through other economic mechanisms, this form of linear expansion in activity is unlikely to occur. Section 5 focuses on an improvement in regional terms of trade and Section 6. on labour-augmenting technical change in the export sector. In this section I investigate in more depth the determination of the appropriate multiplier values in these two cases. Note that in these circumstances the key variable in fixing the value for the multipliers is the elasticity of substitution in the production of the export good. Multiplier values are compared across different export-base stimuli and values of this parameter.

7.1 An Improvement in the Regional Terms of Trade

We use the generic expression for the marginal export-base multiplier (equation 3.12) introduced in Section 3.3. Where expansion occurs through an improvement in the regional terms of trade, the marginal output multiplier is given by substituting equations (5.3) and (5.4) into equation (3.12). This gives:

(7.1) $$[M^M{}_Q]^{PT} = 1 + (M^A{}_Q - 1)((\varphi(\sigma - 1) + 1)/\alpha\sigma)$$

with the additional superscript, PT, indicating the source of the stimulus to the export base. In order to graph the relationship between the multiplier values and σ, it will prove useful to determine the sign of their first and second derivatives with respect to σ, and the values of the multiplier where σ approaches the extreme values of 0 and ∞. Also with some of the multipliers there are other key points which require identification, such as the value where the marginal multiplier is equal to the average multiplier.

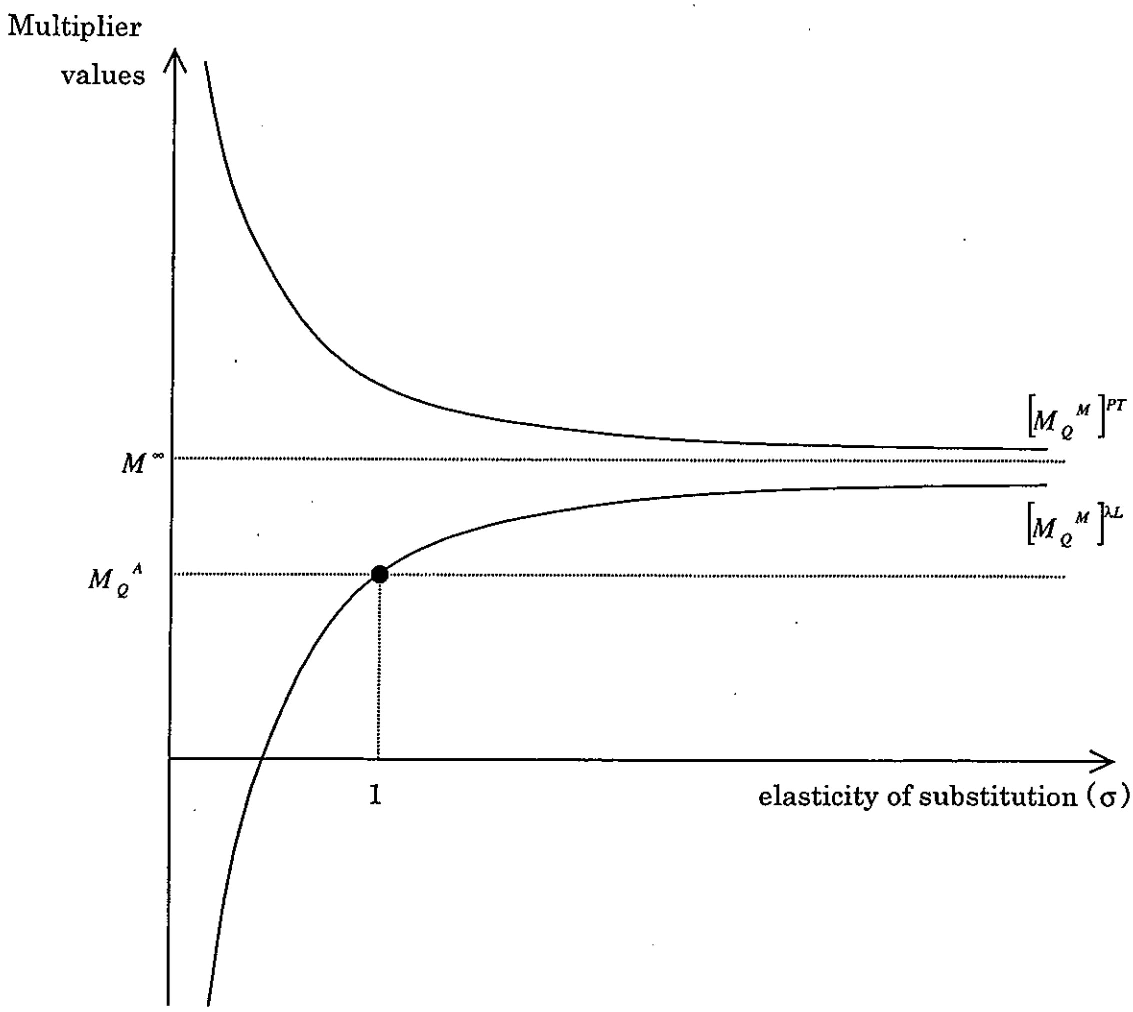
For the output multiplier associated with an improvement in the terms of trade:

$$\partial[M^M{}_Q]^{PT} / \partial\sigma = -(M^A{}_Q - 1)(\alpha(1-\varphi)/(\alpha\sigma)^2) < 0$$

$$[\partial[M^M{}_Q]^{PT}]^2 / \partial^2\sigma = 2\alpha(M^A{}_Q - 1)(\alpha(1-\varphi)/(\alpha\sigma)^3) > 0$$

and as $\sigma \rightarrow 0$, $[M^M{}_Q]^{PT} \rightarrow \infty$, and as $\sigma \rightarrow \infty$, $[M^M{}_Q]^{PT} \rightarrow (1 + (M^A{}_Q - 1)\varphi/\alpha) > M^A{}_Q$. The variation the value of the marginal output multiplier as σ varies is shown in Figure 1. Where σ takes the value zero, so that technology is Leontief, there can be no increase in traded output with the natural-resource base fixed. However, the improvement in the terms of trade increases regional income and subsequently

Figure1. Variation in marginal output multiplier values as σ varies



Notes: $M^{\infty} = 1 + (M_Q^{A} - 1)\varphi / \alpha$

expenditure on non-traded goods, so that there is an increase in non-traded output. Consequently as the elasticity of substitution tends to zero, the marginal output multiplier tends to infinity. As σ increases, allowing a greater expansion in the traded output, the value of marginal multiplier falls, but always remains above the value of the corresponding average multiplier. The increase in income associated with improvements in the terms of trade, which occurs independently of any expansion in traded output, maintains the marginal output multiplier above the average multiplier. Distributional issues also come into play, as at values of $\sigma > 1$, the distribution of income in production moves in favour of labour which increases the impact on retained income and subsequent expenditure on the non-traded commodity.

The employment multiplier is found by substituting equations (5.2) and (5.4) into equation (3.13). This produces the expression:

(7.2) $$[M^M{}_L]^{PT} = 1 + (M^A{}_L - 1)((\varphi(\sigma - 1)+1)/\sigma)$$

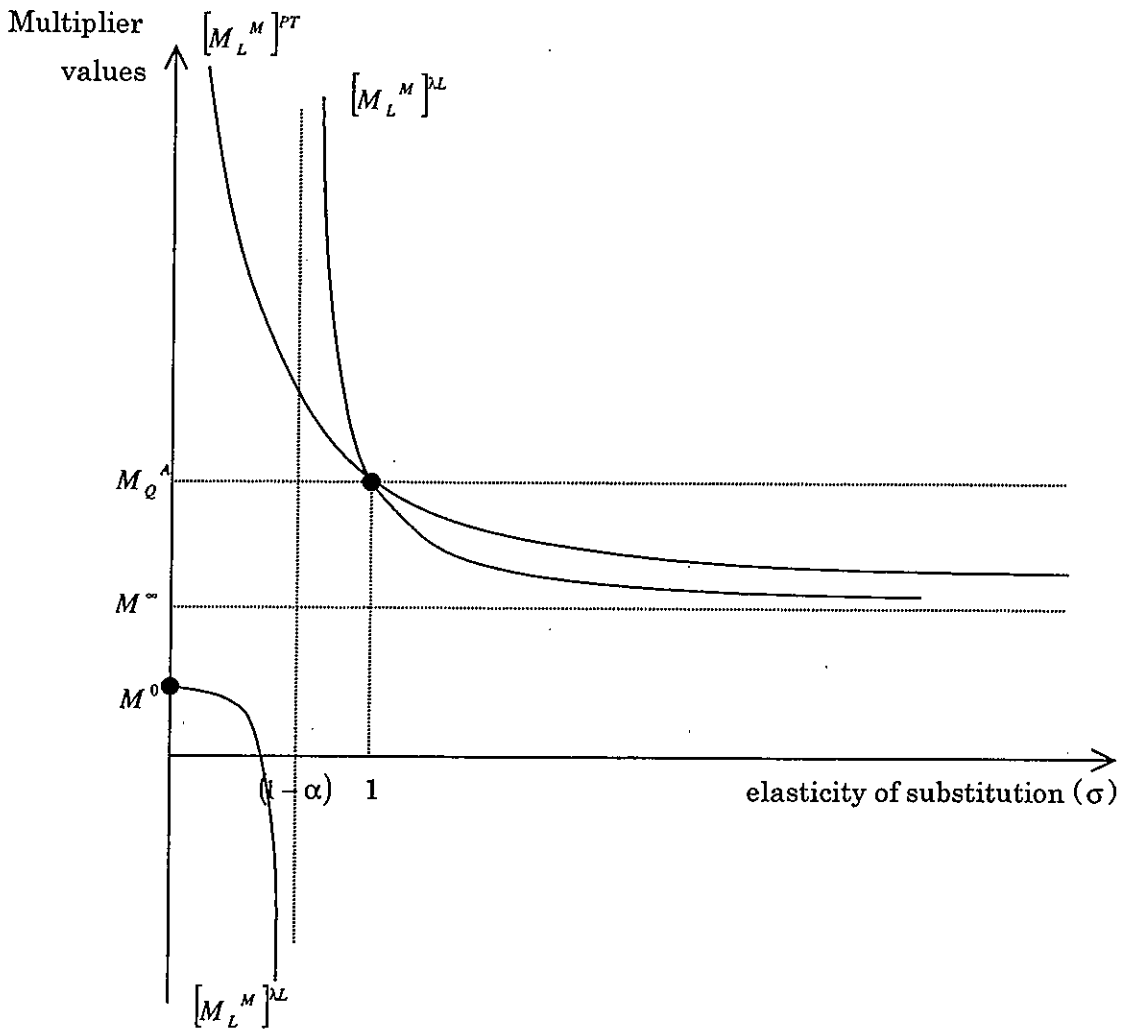
This has the following characteristics:

$$\partial[M^M{}_L]^{PT}/\partial\sigma = -(M^A{}_L - 1)((1-\varphi)/\sigma^2) < 0, \text{ and}$$

$$[\partial[M^M{}_L]^{PT}]^2/\partial^2\sigma = 2(M^A{}_L - 1)((1-\varphi)/\sigma^3) > 0.$$

As $\sigma \to 0$, $[M^M{}_L]^{PT} \to \infty$, as $\sigma \to \infty$, $[M^M{}_L]^{PT} \to 1 + (M^A{}_L - 1)\varphi < M^A{}_L$, and where $\sigma = 1$, $M^M{}_L = M^A{}_L$. The variation in the marginal employment multiplier as σ varies is shown in Figure 2. In this case, the employment multiplier changes in a qualitatively similar way to the output multiplier. With Leontief technology, the employment multiplier is infinitely large. Employment in the traded sector remains unchanged, whilst employment in the non-traded sector rises. This is in response to the increased expenditure associated with the increased regional income from the improvement in the terms of trade. However, as it becomes easier to substitute factor inputs in the traded sector, the employment multiplier falls. One important difference, as against the output multiplier, is that where $\sigma = 1$, the Cobb-Douglas case, the marginal and average employment multiplier are the same. This implies that for values of $\sigma > 1$, $M^A{}_L > M^M{}_L$. A key point here is that because there is a fixed input, natural resources, the input of the other factor, labour, must rise more rapidly than output, so that for any value of σ, the marginal employment multiplier will be lower than the corresponding marginal output multiplier.

Figure2. Variation in marginal employment multiplier values as σ varies



Notes: $M^0 = 1 + (M_L^A - 1)(\varphi - \alpha)/(1-\alpha)$
$M^\infty = 1 + (M_L^A - 1)\varphi$

7.2 Labour-Augmenting Technical Progress

For labour-augmenting technical progress, substituting equation (6.3) and (6.4) into equation (3.12) produces:

(7.3) $$[M^M_Q]^{\lambda L} = 1 + (M^A_Q - 1)((\alpha+\varphi(\sigma - 1))/\alpha\sigma)$$

where

$$\partial[M^M_Q]^{\lambda L}/\partial\sigma = (M^A_Q - 1)(\alpha(\varphi-\alpha)/(\alpha\sigma)^2) > 0$$

$$[\partial[M^M_Q]^{\lambda L}]^2 /\partial^2\sigma = -2\alpha(M^A_Q - 1)(\alpha(\varphi-\alpha)/(\alpha\sigma)^3) < 0$$

and as $\sigma \rightarrow 0$, $[M^M_Q]^{\lambda L} \rightarrow -\infty$, and as $\sigma \rightarrow \infty$, $[M^M_Q]^{\lambda L} \rightarrow (1 + (M^A_Q - 1)\varphi/\alpha) > M^A_Q$, and where $\sigma = 1$, $[M^M_Q]^{\lambda L} = M^A_Q$. The relationship between the marginal output multiplier and the elasticity of substitution in production in the traded sector is given in Figure 1. Note that in this case, where σ falls to zero the value of the marginal output multiplier is $-\infty$. There is no change in the output, product price or the wage in the traded sector but employment falls. There is a redistribution of the income generated in that sector towards the resource owners. But because a proportion of resources are owned outwith the region, regional income, expenditure and non-traded activity fall. As the elasticity of substitution increases the output multiplier rises. Where σ is unity, marginal and average multiplier values are the same, so that at higher values of σ, the marginal output multiplier is greater than the original average value.

It should be noted that for labour-augmenting technical change in the traded sector, deviations in the marginal from the average output multiplier values are generated solely by distributional factors. Imagine that all resources were owned within the region, so that the share of income going to labour in the traded sector, α, equals the proportion of wages in regional income generated in the traded sector, φ (i.e. assume that $\beta_R = 1$). Substituting $\alpha=\varphi$ into equation (7.3) produces the result that $[M^M_Q]^{\lambda L} = M^A_Q$. In this case, not only is the marginal multiplier independent of the value of σ, but it also equals the average multiplier.

The reason the marginal value varies with σ at present can be explained as follows. Where the relative price of a factor falls, the change in its share of the total product depends on the elasticity of substitution. Specifically, where this is greater than unity, the share rises. Therefore, where the price of labour falls in efficiency units, as a result of labour augmenting technical progress, the share of the traded income going to labour will rise if $\sigma > 1$. Because all of labour income is retained in the region, under these circumstances regional income from the traded sector, and subsequent expenditure on the non-traded good, will rise more rapidly than traded output. The marginal value will exceed the average for the output multiplier. Where $\sigma < 1$, the reverse holds. Where the elasticity of substitution tends to infinity, the factors become perfect substitutes and the rise in income goes exclusively to labour. The value of the multiplier in this case tends to the value where traded output is generated by an increase in labour and the whole increase in income goes to labour.

The determination of the marginal employment multiplier for the labour-augmenting technical progress case proves to be the most complex. Substituting equations (5.2) and (6.4) into (3.12) gives the expression:

(7.4) $$[M^M{}_L]^{\lambda L} = 1 + (M^A{}_L - 1)((\alpha+\varphi(\sigma - 1))/(\alpha+\sigma-1)$$

with

$$\partial[M^M{}_L]^{\lambda L} / \partial\sigma = -(M^A{}_L - 1)\alpha(1-\varphi)/(\alpha+\sigma-1)^2) < 0, \ \sigma \neq 1-\alpha,$$

$$[\partial[M^M{}_L]^{\lambda L}]^2 / \partial^2\sigma = 2\alpha(M^A{}_L - 1)(\alpha(1-\varphi)/(\alpha+\sigma-1)^3), \ \sigma \neq 1-\alpha$$

and : where $\sigma = 0$, $[M^M{}_L]^{\lambda L} = 1 + (M^A{}_L - 1)(\varphi-\alpha)/(1-\alpha) < M^A{}_L$,

where $\sigma \rightarrow \infty$, $[M^M{}_L]^{\lambda L} \rightarrow 1 + (M^A{}_L - 1)\varphi < M^A{}_L$,

where $\sigma = 1$, $[M^M{}_L]^{\lambda L} = M^A{}_L$, and where $\sigma = 1-\alpha$, $[M^M{}_L]^{\lambda L}$ is undefined.

The relationship between the value of the marginal employment multiplier and σ is shown in Figure 2. There is a discontinuity in the curve where the elasticity of substitution equals $1-\alpha$. This is because at this value, employment change in the traded sector falls to zero. Consider first the portion of the curve to the left-hand side of this discontinuity.

Where the elasticity of substitution is zero, that is where there is Leontief technology in the traded sector, the introduction of labour-augmenting technical progress leads to employment falling in both the traded and non-traded sectors producing a positive multiplier. Employment falls in the traded sector because with

Leontief technology and resources fixed, output is fixed so that the increased labour efficiency simply generates a reduction in the number of jobs. Employment falls in the non-traded sector as a result of the redistribution of income towards resource owners and the subsequent reduction in regional expenditure. The proportionate reduction is greater in the traded sector so that the marginal multiplier is less than the average at this point.

As σ increases, the employment change increases in both sectors. Initially this implies simply a lower fall in employment in both sectors. Where $\sigma = 1-\alpha/\varphi$, employment change in the non-traded sector becomes positive whilst employment change in the traded sector is still negative. At a value of σ between $1-\alpha/\varphi$ and $1-\alpha$ the total employment change becomes positive whilst employment change in the traded sector is still negative so that the marginal employment multiplier now becomes negative. Further, as the negative employment change in the traded sector becomes smaller and smaller, the absolute size of the multiplier increases, approaching $-\infty$ as σ approaches $1-\alpha$ from below

In the portion of the curve on the right hand side of the discontinuity, both traded and total employment changes are positive. However, employment change in the traded sector is initially very small, so that the marginal employment multiplier value approaches ∞ as σ approaches $1-\alpha$ from above. As the value of σ increases, the marginal employment multiplier falls continuously, equals the average employment multiplier where $\sigma = 1$ and is below the average value from that point on. Note that the second derivative changes sign at the discontinuity in the function, so that for $\sigma < 1-\alpha$ the function is concave but where $\sigma > 1-\alpha$ it is convex.

7.3 Marginal Multipliers for Different Export-base Stimuli

The marginal output multiplier values for the terms of trade and technical progress stimuli to the traded sector can be directly compared in Figure 1 (recall that for an expansion in the resource base, the marginal multiplier equals the average multiplier). The results are very different. It is true that both approach the same value where $\sigma \rightarrow \infty$. However, the marginal multiplier for the terms of trade approaches this value from above, the labour-augmenting efficiency change, from below. The key point is that with the terms of trade stimulus, variations in the multiplier as σ varies depend on more than just distributional issues. If we set $\alpha=\varphi$ into equation (7.1), the marginal output multiplier still varies with σ (though it now approaches the average value as $\sigma \rightarrow \infty$). As argued before, a key factor here is that the improvement in the terms of trade gives an income boost to the region, experienced as an increase in resource rentals. This income expands expenditure on the non-traded commodity in relationship to the increase in output in the traded sector. However, the larger the value of σ, the bigger the increase in traded output, so the smaller proportionate impact of the income gain from the improvement in the terms of trade.

A similar comparison can be made using Figure 2 for the marginal employment multipliers. (Remember that for an increase in natural resources, the marginal multiplier always equals the average multiplier.) At low levels of σ the multiplier values are radically different. But where $\sigma > (1-\alpha)$ the way that the multiplier values change as σ varies is similar, both take the average multiplier value where σ is unity and both tend to the same value, $(1+(M^A{}_L-1)\varphi < M^A{}_L$ as $\sigma \rightarrow \infty$.

8. Supply-Constrained Input-Output Model

The analysis undertaken up to now has been limited in terms of the number of sectors, commodities and non-produced inputs. There are only two sectors (traded and non-traded), and three commodities (traded, non-traded and imported) and two non-produced inputs, (homogeneous labour and natural resources). Further, we have no intermediate inputs. The simplicity of the model is motivated partly for pedagogic reasons and partly to replicate the very simple export-base approach. However, in this section the results presented in Section 4 for the variant of the model where the stimulus to the export sector comes through an expansion in the resource base are generalised to produce a full supply-based Input-Output system. That is to say, a neo-classical supply-constrained model will in this case replicate I-O results.

It is perhaps prudent at the outset to make it clear that there is no correspondence between the supply-constrained I-O system identified here and the supply-driven Ghoshian I-O approach (Ghosh, 1958). Here, I am arguing that the familiar Leontief input coefficients will remain unchanged, that the neo-classical model replicates a standard Leontief I-O model. The system is driven solely through changes in exports but these are determined by changes in the region's resource base. For a detailed discussion of the Ghoshian analysis, see Oosterhaven (1988) and Dietzenbacher (1997).

Imagine a neo-classical economy in which there are n purely export sectors, based around n natural resources, and where each of these traded commodities faces a parametric price in external markets. There are m non-traded goods, k imported commodities and capital and labour. Finance is freely available at a fixed extra-regional interest rate, ρ, labour is available at a given real wage through migration, and production also involves non-traded commodities and imports as intermediate inputs. Production takes place everywhere under conditions of perfect competition with constant returns to scale. It will be shown that in response to an expansion in the resource base, such a neo-classical system expands as an I-O system would in response to an increase in export demand, with no changes in prices and therefore with fixed coefficients. However, note that there will be no change in demand here, rather a change in supply, and that this is a neo-classical system replicating I-O results.

It is most straightforward to begin with prices in the non-traded sector. We here assume perfect competition which implies zero profits, so that in each sector commodity price, $P^N{}_i$, equals minimum unit cost. Unit costs are a function of the vector of prices for all non-traded commodities, $\mathbf{P}^N$, the vector of prices of imported commodities, $\mathbf{P}^K$, the interest rate, ρ, and the nominal wage, W_N. This takes into account the use of non-traded commodities and imports as intermediate inputs. For non-traded commodity i this can be expressed as:

(8.1) $$P^N{}_i = P^N{}_i(\mathbf{P}^N, \mathbf{P}^K, \rho, W_N)$$

where $P^N{}_i$ is the unit cost function. All the prices for imported commodities are exogenous, as is the interest rate. The nominal wage is then given as a function of non-traded and import prices

(8.2) $$W_N = W_N(\mathbf{P}^N, \mathbf{P}^K)$$

We therefore have m+k+2 equations to determine the prices of the m non-traded goods, the k imported commodities, the interest rate and the nominal wage. Note that the nominal wage and the non-traded prices are set independently of prices in the traded sector. Also once the real wage, interest rate and import prices are determined, in this neo-classical model, the non-traded commodity prices are also determined independently of the level of activity in these sectors. This argument essentially follows that of McGregor *et al* (1996): where non-produced resources are freely available at the existing rental rates, prices in a neo-classical model are not affected by changes in the level of demand.

In the traded sectors the situation is very different. Here we do have region-specific resources whose prices are set endogenously. Again, we have n equations that set the commodity price, P^T_i, equal to the minimum unit cost, although we now also have n resources whose rentals, W^i_R, need to be determined. This is represented as:

(8.3) $$P^T_i = P^T_i(\mathbf{P}^N, \mathbf{P}^K, \rho, W_L, \mathbf{W}_R)$$

where $\mathbf{W}_R$ is the vector of natural-resource rentals. Given that the m traded commodity prices are set parametrically, we have 2m additional equations for the m traded commodity prices and the m resource rentals. Note that the number of export sectors must equal the number of resources, though we do not require a resource to be used exclusively in one export sector. This is a familiar result from trade theory, related to the Stolper-Samuelson factor-price equalisation theorem (Silberberg, 1978). In a standard neo-classical model, if commodity prices are set parametrically in external markets, these prices determine the resource rentals.

Mechanisms have therefore been identified in both the traded and non-traded sectors whereby the n+m+k commodity prices, the n natural resource rentals, the nominal wage and the interest rate are all set independently of demand. But if these prices are fixed, so are the production and consumption coefficients. If we use the conventional notation:

(8.4) $$a_{i,j} = a_{i,j}(\mathbf{P}^N, \mathbf{P}^K, \rho, W_L, \mathbf{W}_R)$$

(8.5) $$c_i = c(\mathbf{P}^N, \mathbf{P}^K)$$

(8.6) $$e_j = e_j(\mathbf{P}^N, \mathbf{P}^K, \rho, W_L, \mathbf{W}_R)$$

(8.7) $$v_{i,j} = v_{i,j}(\mathbf{P}^N, \mathbf{P}^K, \rho, W_L, \mathbf{W}_R)$$

where $a_{i,j}$ is the input of commodity i in the production of one unit of commodity j, c_i is consumption of commodity i per unit of consumer expenditure, e_j is the labour input per unit of output in sector j, and $v_{i,j}$ is the input of resource i in the production of commodity j. Of course, there is no intermediate or consumption demand for traded goods and there is no use of natural resources as inputs in the non-traded sector.

What is implied here is that the neo-classical system will operate in a way that is observationally equivalent to a consumption-endogenous I-O system, in the sense that it has fixed prices and fixed production and consumption coefficients. However, note that this is only where the shock to the export sector takes the form of a relaxation of the natural resource constraint (through an expansion of natural resources or the introduction of resource-augmenting technical progress in export sectors.). A demand disturbance, in the form of an increase in the export price or a supply-side shock produced by labour-augmenting technical progress, will fail to generate I-O results. This is because these shocks leads to a change in relative factor rentals which has an effect on the choice of technique and distribution of income as analysed in Section 5.

The effect of relaxing the resource constraint is determined through the mechanism underlying the Rybczynski theorem (Rybczynski, 1955). First construct an nxn matrix, $\mathbf{\Omega}$, whose i,jth element is $v_{i,j}$. Full employment of the natural resources requires:

(8.8) $$\mathbf{\Omega}\,\mathbf{Q} = \mathbf{R}$$

where $\mathbf{Q}$ is an nx1 vector of the gross outputs of the traded sector and $\mathbf{R}$ is an nx1 vector of the resource constraints. Pre-multiplying both sides of equation (8.8) by the inverse of the resource coefficients matrix gives:

(8.9) $$\mathbf{Q} = \mathbf{\Omega}^{-1}\,\mathbf{R}$$

The natural resource endowment, together with the matrix of resource coefficients, determines the outputs in the traded sectors. Given that these are purely export sectors, the natural resource endowments essentially determines the region's exports. The outputs of the non-traded sector are then subsequently generated through I-O linkages.

If there is a unit increase in the availability of resource i, the impact on the region's exports can be determined by reading down the ith column in the inverse of the resource coefficients matrix. The most straightforward case is where each industry exploits only one resource, with industry i using resource i. In this case, the resource-input matrix and its inverse are diagonal. The only non-zero element in the ith column is $(v_{i,j})^{-1}$. Here an increase in the ith resource of ΔR_i produces an increase in exports and output in the ith sector of $\Delta R_i/v_{i,j}$ with the subsequent change in regional output determined by the standard type-II multiplier. But note that this result is being derived for a supply-constrained neo-classical model, rather than the traditional demand driven I-O model. But this is appropriate because the classic export base model is a model of resource exploitation. Where some individual resources are used in more than one industry, the inverse matrix will be more complex. An expansion in one resource might well be associated with reductions in exports and output in some traded sectors, together with an expansion in others. However, the model still operates in a manner equivalent to a traditional I-O system responding to export-demand shocks.

9. Conclusions

The classic export-base model is a model of a supply-constrained regional economy where the region's exports are dependent upon the exploitation of a natural resource. In this paper I have attempted to model this from a neo-classical perspective, maintaining the key classic export-base assumption that the natural resource is the only region-specific non-produced input. Finance, labour and imports are freely available at the existing national market prices. The central model used in the paper is a very simple variant of the trade-based 1-2-3 general equilibrium model. I show that where the expansion in the export sector is driven by an expansion in the effective natural-resource base, this neo-classical model exhibits classic export-base properties. The regional economy expands in a linear fashion, with constant marginal (= average) output and employment multipliers. Moreover, the model can be straightforwardly disaggregated in this case to cover any number of export and non-traded sectors, and as such becomes a supply-constrained Input-Output system. However, where the stimulus to the export sector comes through an improvement in the terms of trade or labour-augmenting technical progress in the traded sector, the model fails to replicate classic export-base results. Average and marginal income multipliers are always equal. However, the more important marginal output and employment multipliers differ from their average values and can even be negative. In the determination of these marginal multipliers, the elasticity of substitution in production is a key variable.

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