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Estimation of Persistence in Log-Volatility Using Panel Data

by

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Abstract

This paper proposes a stochastic volatility model for panel data, and estimation

methods of its persistence parameter, in the case of large number of individuals and

small number of time periods, (presumably) for the first time. In this paper, two types of

the estimators for this model are presented, in accordance with frameworks of the

dynamic panel data model and the generalised method of moments. To examine and

compare the two types of the estimators, Monte Carlo experiments are carried out.

Furthermore, an empirical application to data of stock returns is implemented using

these estimators mentioned above.

Keywords: Stochastic volatility model, Dynamic panel data model, Generalised method

of moments

JEL code: C23

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【日本語の題名】

パネルデータを用いた対数ボラティリティの持続性の推定

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【日本語の概要】

この論文は、はじめて、個別主体数が大きく時点数が少ない場合におけるパネルデータのための確率的ボラティリティモデルとその持続性パラメーターの推定法を提唱する。この論文では、動学的パネルデータと一般化積率法の枠組にしたがって、このモデルに対する2つの型の推定量が提示される。2つの型の推定量を吟味比較するために、モンテカルロ実験が実行される。さらに、上記の推定量を用いて、株価収益率のデータへの応用が実施される。

キーワード: 確率的ボラティリティモデル、動学的パネルデータモデル、 一般化積率法

Journal of Economic Literature の分類コード: C23

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I. INTRODUCTION

Today, the *stochastic volatility* model (the *SV* model) proposed by Clark (1973) is attracting interests of many financial econometricians. This is one of approaches capturing the time-varying volatility in financial time series, such as *ARCH* by Engle (1982), *GARCH* by Bollerslev (1986), and *EGARCH* by Nelson (1991), etc.

In many literatures, a great deal of estimation methods of the SV model is developed, discussed, and applied to financial data in the framework of time series analyses (Melino and Turnbull 1990; Jacquier et al. 1994; Harvey et al. 1994; Andersen and Sørensen 1996, etc). What seems to be lacking is, however, that the SV model is talked about in the context of panel data analyses.

In this paper, the panel data version SV model is created (presumably) for the first time, and techniques of estimating the persistence parameter in this model are propounded, using panel data with large number of individuals over small (fixed) number of time series. The advantage of using this model and these techniques for this type of panel data model is that these allows us to observe short-run movements of the volatility in the whole stock market. These techniques incorporate a series of ideas on estimation methods of dynamic panel data models, recently developed and discussed in Holtz-Eakin et al (1988), Arellano and Bond (1991), Ahn and Schmidt (1995), Arellano and Bover (1995), and Blundell and Bond (1998). These techniques embrace two kinds of estimators; One is based on Holtz-Eakin et al. (1988) and Arellano and Bond (1991), and the other based on Arellano and Bover (1995). It is conjectured that the latter estimators are superior to the former since using more information. Therefore, to examine performances of two kinds of estimators and corroborate the superiority of the latter estimators, Monte Carlo experiments are carried out. Finally, an empirical

application of the techniques is put into operation, using the panel data composed of the daytime stock returns listed on the first and second sections of the *Tokyo Stock Exchange* in 1998.

The paper is organised as follows: the section 2 presents the stochastic volatility model for the panel data with large number of individuals and small (fixed) number of time series. Here, we also look into the estimation methods for the model. In section 3, we investigate the small sample performances of the estimators by carrying out the Monte Carlo experiments. In section 4, the application of the estimators to the data of the stock returns is implemented. Section 5 concludes.

II. MODEL

In this section, a stochastic volatility model for panel data is delineated and then estimation methods of its persistence parameter are proffered. We will call the model as the panel data stochastic volatility (PDSV) model.

In this model, subscripts i (i=1,...,N) and t (t=1,...,T) respectively imply the individual (stock issue, etc) and the time period with large N and small (fixed) T, and there is a series of observable variables $\{y_{ii}\}$ whose structure we assume as:

$$y_{it} = \sigma_{it} \varepsilon_{it}$$
, for $t = 1,...,T$, (Observable Variables) (1)

$$\varepsilon_{ii} \sim iidN(0,1),$$
 (2)

where the log-volatility ($\log \sigma_u^2$) of y_u is unobservable (latent) and has the following driving process of the *PDSV* model:

$$\log \sigma_{i1}^2 = \frac{g_i}{1 - \phi} + \nu_{i1}, \qquad \text{(Initial Condition)}$$
 (3)

$$\log \sigma_{ii}^2 = \phi \log \sigma_{i,i-1}^2 + g_i + \nu_{ii}, \qquad \text{for } t = 2, ..., T,$$
 (4)

where g_i is the individual specific fixed effect, and V_{ii} is the disturbance with zero mean which is not correlated serially, cross-sectionally, and with g_i . In this situation, we incorporate the notion that the (persistence) parameter ϕ ($|\phi|<1$) is common through all individuals and the individual effect g_i controls for the individual heterogeneity of the log-volatility, according to the prevalent panel data model. Due to the initial condition (3), $\log \sigma_{ii}^2$ is mean-stationary with mean $g_i/(1-\phi)$ at any time period.

In this paper, we are only interested in estimating ϕ . For this purpose, we make the reduced form of the SV model using (1), (2), (3), and (4), according to Harvey et

al. (1994):

$$(\log y_{i1}^2 + 1.27) = \frac{g_i}{1 - \phi} + \omega_{i1}, \qquad \text{(Initial Condition)}$$
 (5)

$$(\log y_{it}^2 + 1.27) = \phi(\log y_{i,t-1}^2 + 1.27) + g_i + \omega_{it},$$
 for $t = 2,...,T$, (6)

where $\omega_{i1} = v_{i1} + \xi_{i1}$ and $\omega_{it} = v_{it} + \xi_{it} - \phi \xi_{i,t-1}$ with $\xi_{it} \sim iid(0, \pi^2/2)$. See APPENDIX for deriving this reduced form. Originated from the initial condition (5), this is a form of the mean-stationary dynamic panel data model, in which the variable $\log y_{it}^2 + 1.27$ driving the process composed of (5) and (6) is observable. Allowing for the fixed effect g_i and the first order serial correlation of the disturbance ω_{it} for $t=2,\ldots,T$, two types of the moment restrictions prepared for estimating ϕ consistently are described in frameworks of the generalised method of moments (GMM) and the dynamic panel data analysis when asymptotics rely on the assumption $N\to\infty$ under T fixed.

Firstly, the type of the moment restrictions proposed in Holtz-Eakin et al. (1988) and Arellano and bond (1991) is presented:

$$E[(\Delta \log y_{it}^2 - \phi \Delta \log y_{i,t-1}^2)(\log y_{is}^2 + 1.27)] = 0,$$
for $t = 4, ..., T; s = 1, ..., t - 3,$

$$(7)$$

where Δ is the first-differencing operator. The moment restrictions (7) imply that for the purpose of estimating ϕ we use the third and higher lags of level dependent variables as instruments for first-differenced equations of (6). From this point on, these are called as the *standard moment restrictions*. These moment restrictions do not use the information on the initial condition (5). Therefore, the estimators based on these moment restrictions may not be efficient when the initial condition (5) is specified.

Next, looking at (5) and (6), the reduced form is written in a form of the dynamic panel data model, in which the mean-stationarity through time with respect to $(\log y_{ii}^2 + 1.27)$ is satisfied. In order to estimate ϕ under the efficient information on this mean-stationarity, this paper intensively adopts the idea on the mean-stationarity moment restrictions proposed in Arellano and Bover (1995) and discussed in Ahn and Schmidt (1995) and Blundell and Bond (1998). That is:

$$E[\{(\log y_{it}^2 + 1.27) - \phi(\log y_{i,t-1}^2 + 1.27)\} \Delta \log y_{is}^2] = 0,$$
for $t = 4, \dots, T$; $s = 2, \dots, t-2$.

The moment restrictions (8) imply that for the purpose of estimating ϕ we use the second and higher lags of first-differenced dependent variables as instruments for level equations of (6). From this point on, these are called as the *stationarity moment* restrictions. The estimators using (8) may be superior to using (7) in the sense that the information on the mean-stationarity is incorporated in the estimators using (8).

The two types of the *GMM* estimators can be constructed using the standard moment restrictions (7) and the stationarity moment restrictions (8), respectively. Hereafter, we call the *GMM* estimators using the former moment restrictions as the *STD-GMM* estimators, and using the latter the *STA-GMM* estimators.

The (consistent) STD-GMM estimators are described as below:

$$\hat{\phi}_{n}^{STD} = \frac{\left(\sum_{i=1}^{N} \Delta Y_{i,(-1)}^{\prime} Z_{i}\right) A_{n}^{STD} \left(\sum_{i=1}^{N} Z_{i}^{\prime} \Delta Y_{i}\right)}{\left(\sum_{i=1}^{N} \Delta Y_{i,(-1)}^{\prime} Z_{i}\right) A_{n}^{STD} \left(\sum_{i=1}^{N} Z_{i}^{\prime} \Delta Y_{i,(-1)}\right)},$$
(9)

where fixed are the $(T-3)\times 1$ vectors of $\Delta Y_i = [\Delta \log y_{i4}^2, \Delta \log y_{i5}^2, ..., \Delta \log y_{iT}^2]'$ and $\Delta Y_{i,(-1)} = [\Delta \log y_{i3}^2, \Delta \log y_{i4}^2, ..., \Delta \log y_{i,(T-1)}^2]'$, and the $(T-3)\times m$ block diagonal matrix:

$$Z_{i} = \begin{bmatrix} (\log y_{i1}^{2} + 1.27) & 0 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & (\log y_{i1}^{2} + 1.27) & (\log y_{i2}^{2} + 1.27) & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & (\log y_{i1}^{2} + 1.27) & \cdots & (\log y_{iT-3}^{2} + 1.27) \end{bmatrix},$$

with m = (T-3)(T-2)/2. The (non-optimal) 1-step, (efficient) 2-step, and iterated STD-GMM estimators ($\hat{\phi}_i^{STD}$, $\hat{\phi}_2^{STD}$, and $\hat{\phi}_i^{STD}$) are defined, choosing as A_n^{STD} ($m \times m$ weighting matrix) $A_1^{STD} = \left[\frac{1}{N}\sum_{i=1}^N Z_i'Z_i\right]^{-1}$, $A_2^{STD} = \left[\frac{1}{N}\sum_{i=1}^N Z_i'\Delta\hat{u}_{i(1)}\Delta\hat{u}_{i(1)}'Z_i\right]^{-1}$, and $A_i^{STD} = \left[\frac{1}{N}\sum_{i=1}^N Z_i'\Delta\hat{u}_{i(1)}\Delta\hat{u}_{i(1)}'Z_i\right]^{-1}$, respectively. In addition, the realised 1-step, 2-step, and iterated STD-GMM estimates are noted down as $\hat{\phi}_1^{STD^*}$, $\hat{\phi}_2^{STD^*}$, and $\hat{\phi}_i^{STD^*}$, respectively. In this case, $\Delta\hat{u}_{i(1)}$ is the vector of the (consistent) 1-step residuals so that $\Delta\hat{u}_{i(1)} = \Delta Y_i - \hat{\phi}_1^{STD^*}\Delta Y_{i,(-1)}$. Furthermore, since the iterated estimator is obtained by updating the weighting matrix until the estimate of ϕ converges starting from A_i^{STD} , $\Delta\hat{u}_{i(I)}$ is the vector of iterated residuals so that $\Delta\hat{u}_{i(I)} = \Delta Y_i - \hat{\phi}_i^{STD^*}\Delta Y_{i,(-1)}$.

The (consistent) STA-GMM estimators are described as below:

$$\hat{\phi}_{n}^{STA} = \frac{\left(\sum_{i=1}^{N} Y'_{i,(-1)} \Delta Z_{i}\right) A_{n}^{STA} \left(\sum_{i=1}^{N} \Delta Z'_{i} Y_{i}\right)}{\left(\sum_{i=1}^{N} Y'_{i,(-1)} \Delta Z_{i}\right) A_{n}^{STA} \left(\sum_{i=1}^{N} \Delta Z'_{i} Y_{i,(-1)}\right)},$$
(10)

where fixed are the $(T-3)\times 1$ vectors of $Y_i = [(\log y_{i4}^2 + 1.27), (\log y_{i5}^2 + 1.27), ..., (\log y_{iT}^2 + 1.27)]'$ and $Y_{i,(-1)} = [(\log y_{i3}^2 + 1.27), (\log y_{i4}^2 + 1.27), ..., (\log y_{i,(T-1)}^2 + 1.27)]'$, and the $(T-3)\times m$ block diagonal matrix:

$$\Delta Z_i = \begin{bmatrix} \Delta \log y_{i2}^2 & 0 & 0 & \cdots & 0 & \cdots & 0 \\ 0 & \Delta \log y_{i2}^2 & \Delta \log y_{i3}^2 & \cdots & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & \Delta \log y_{i2}^2 & \cdots & \Delta \log y_{i,T-2}^2 \end{bmatrix},$$

with m=(T-3)(T-2)/2. The (non-optimal) I-step, (efficient) 2-step, and iterated STA-GMM estimators $(\hat{\phi}_1^{STA}, \hat{\phi}_2^{STA}, \text{ and } \hat{\phi}_I^{STA})$ are defined, choosing as A_n^{STA} ($m \times m$ weighting matrix) $A_1^{STA} = \left[\frac{1}{N}\sum_{i=1}^N \Delta Z_i' \Delta Z_i\right]^{-1}$, $A_2^{STA} = \left[\frac{1}{N}\sum_{i=1}^N \Delta Z_i' \hat{u}_{i(1)} \hat{u}_{i(1)}' \Delta Z_i\right]^{-1}$, and $A_I^{STA} = \left[\frac{1}{N}\sum_{i=1}^N \Delta Z_i' \hat{u}_{i(I)} \hat{u}_{i(I)}' \Delta Z_i\right]^{-1}$ respectively. In addition, the realised I-step, 2-step, and iterated STA-GMM estimates are noted down as $\hat{\phi}_1^{STA^*}$, $\hat{\phi}_2^{STA^*}$, and $\hat{\phi}_I^{STA^*}$, respectively. In this case, $\hat{u}_{i(1)}$ is the vector of the (consistent) I-step residuals so that $\hat{u}_{i(1)} = Y_i - \hat{\phi}_1^{STA^*} Y_{i,(-1)}$, and $\hat{u}_{i(I)}$ is the vector of the iterated residuals so that $\hat{u}_{i(I)} = Y_i - \hat{\phi}_1^{STA^*} Y_{i,(-1)}$.

III. MONTE CARLO

With the intention of investigating the limited sample performances of the STD-GMM and STA-GMM estimators for the PDSV model in previous section and corroborating the superiority of the STA-GMM estimators, Monte Carlo experiments are carried out. ⁶

A design of the data generating process (DGP) for the PDSV model is described as below:

$$y_{ii} = \sigma_{ii} \varepsilon_{ii}$$
, for $t = 1,...,T$, (Observable Variables) $\varepsilon_{ii} \sim iidN(0,1)$,

$$\log \sigma_{i1}^2 = \frac{g_i}{1 - \phi} + \frac{w_{i1}}{\sqrt{1 - \phi^2}},$$
 (Initial Condition)

$$\log \sigma_{ii}^2 = \phi \log \sigma_{i,i-1}^2 + g_i + w_{ii}, \quad \text{for } t = 2,...,T, \quad \text{(Driving Process)}$$

$$g_i \sim iidN(0,1),$$

$$w_{ij} \sim iidN(0,1)$$
, for $t = 1,...,T$,

where i=1,...,N. The parameter values $\phi=0.0,\ 0.2,\ 0.5,\ 0.8,\ 0.9,\ 0.95,\ 0.98$ are used in the experiments. The combinations of the sample sizes are $N=100,\ 500,\ 1000$ and T=8. The number of replications is set to NR=500.

In the experiments, the estimations of ϕ are carried out by the 1-step, 2-step, and iterated STD-GMM estimators (9), and the 1-step, 2-step, and iterated STA-GMM estimators (10) in previous section.

The results are shown in the Table 1A, Table 1B, and Table 1C for N=100, N=500, and N=1000, respectively. We can recognise that the STD-GMM estimators for ϕ are substantially downward-biased, while the STA-GMM estimators for ϕ are fairly precise in all cases, judging from the Monte Carlo means (MCM) for the

estimators of ϕ and the root mean squared errors (RMSE) for the estimators of ϕ . The behaviours of the STA-GMM estimators are markedly good in precision. From these Monte Carlo results, it is corroborated that the STA-GMM estimators are superior to the STD-GMM estimators in finite sample size, and are probably practical estimators for empirical researches of the PDSV model, as long as the mean-stationarity of the log-volatility is satisfied.

After this, we shall concentrate on the behaviours of the STA-GMM estimators (the right-hand sides of Table 1A, Table 1B, and Table 1C), with such a reason. From the tables, we can see the Monte Carlo fact that the I-step and 2-step estimators perform better than the iterated estimator in all cases, comparing their MCM and RMSE. It is remarkable for the high values (0.9, 0.95, and 0.98) of ϕ . For the moderate values (0.2, 0.5, and 0.8) of ϕ , the larger the sample size (from N = 100, N = 500 to N = 1000), the better the performances of the estimators. Comparing the MCSD with the MCSE for the I-step, 2-step, and iterated estimators reveals that inferences on the estimates of ϕ are problematic when we use the estimates of the iterated standard errors, as these are nontrivially downward-biased. Note that the MCSD and the MCSE are the Monte Carlo standard deviation for the estimators of ϕ and the Monte Carlo mean of the estimated standard errors for the estimators of ϕ , respectively. The means of Sargan tests (SARGAN) do not reject the validity of the stationarity moment restrictions used for estimating ϕ at the significant level 5% throughout. This test statistic is asymptotically chi-square-distributed with degree of freedom DF.

IV. EMPIRICAL APPLICATION

The estimators for the *PDSV* model discussed in previous sections are applied to the data of the stock returns in this section, with the aim of investigating the estimated values of the persistence parameter in particular short-run time spans. In the application, the simple *PDSV* model composed of (1), (2), (3), and (4) is assumed. 8

Time series analyses on the SV model estimate the model by using the long-run time series with large sample size and a particular stock issue or option to get high (near to unity) values of the persistence parameter. In contrast, panel data analyses on the PDSV model in this paper estimate the model to grasp the short-run movement of the volatility in the whole stock market by using the short-run time series with small sample size and the stock issues with large sample size.

Data used is a panel data set of the daytime stock returns (DTSR) composed of the stock issues listed on the first and second sections of the Tokyo Stock Exchange (TSE) from February 18 to December 4 in 1998 except for holidays. The size of this panel data set is 1839 (number of stock issues) by 200 (number of time periods). The data source is Stock Price Chart CD-ROM Published In New Year 1999 (Kabuka Chart CD-ROM 1999 Nen Shinshun Gou, in Japanese) published by Toyo Keizai Inc. (Toyo Keizai Shinpou Sha, in Japanese).

The definition of the DTSR y_{it} of ith issue at period t is

$$y_{it} = \log CP_{it} - \log OP_{it},$$

where CP_{it} and OP_{it} are respectively the closing price and opening price at period t.

As a preparation for estimating the *PDSV* model, we select sub-panel data sets without serial correlations in the panel data set above, since assuming (1) and (2). The numbers of time periods in the sub-panel data sets are set up at eight through this

application, allowing for the number of time periods used in the Monte Carlo experiments in previous section.

Firstly we single out the 192 distinct sub-panel data sets with the sequential eight time periods and the 1839 stock issues from this panel data set, by establishing the starting time period of each sub-panel data set with each time period in the original panel data set. Secondly, we eliminate the stock issues inappropriate for the data analyses in the sub-panel data sets. These inappropriate issues are comprised of both issues whose transactions are not implemented at one day in the eight time periods and whose stock returns are zero at one day in the eight time periods.

We obtain the balanced 192 sub-panel data sets trimmed in above way at this stage, with distinct numbers of the stock issues. From these sub-panel data sets, we opt for the sub-panel data sets without the serial correlation of k th order in y_{it} , where $t=1,\ldots,T$ and $k=1,\ldots,T-1$ with T=8 in this case. In this process, the statistic testing the null hypothesis that y_{it} and $y_{i,t-k}$ are not correlated:

$$lm_{(-k)} = \frac{\overline{\mu}}{\sqrt{s^2/\tau}} \sim N(0,1),$$

is used, where $\overline{\mu} = (1/\tau) \sum_{i=1}^{N} y_{i*}' y_{i(-k)}$ and $s^2 = [1/(\tau - 1)] \sum_{i=1}^{N} [y_{i*}' y_{i(-k)} - \overline{\mu}]^2$ with $\tau = N(T-k)$, $y_{i(-k)} = [y_{i1}, y_{i2}, \dots, y_{i,T-k}]'$, and $y_{i*} = [y_{i,k+1}, y_{i,k+2}, \dots, y_{iT}]'$. 9 This statistic is based on the second order serial correlation test proposed in Arellano and Bond (1991). 10

Using the $lm_{(-k)}$ tests, we obtain seven sub-panel data sets without serial correlations. Hereafter, the application of the PDSV model is presented, using proper five sub-panel data sets picked out of these seven sub-panel data sets.¹¹ Itemisations of

these five sub-panel data sets are depicted in Table 2. The results of the $lm_{(-k)}$ tests in these five sub-panel data sets are indicated in Table 3. The cross-sectional sample size (number of stock issues) used for the estimation is roughly 500 in each sub-panel data set.

Using these five sub-panel data sets, estimations of the persistence parameter in the *PDSV* model are implemented.

Looking at Table 4, we can recognise that all the STD-GMM estimates of the persistence parameter ϕ are negative. These are similar to the result of the Monte Carlo experiments in similar sample size N=500 shown in previous section. These results would be unreliable. On the other hand, the STA-GMM estimates of ϕ are positive and significant, and the Sargan test statistics pronounce that the used moment restrictions are valid at the significant level 5%. We consider that these results would be reliable, taking into consideration the small sample performances of the STA-GMM estimators in the Monte Carlo experiments. Therefore, we set the discussion forward, focusing on the STA-GMM estimates of ϕ from now on.¹²

In Table 4, the 1-step, 2-step, and iterated STA-GMM estimates of ϕ at both spans from June 12 to June 23 (0.853, 0.984, and 0.987) and from June 16 to June 25 (0.806, 0.929, and 0.939) are considerably higher than those at the other spans. Especially seeing the 2-step and iterated estimates in all spans, we can find that in the above two spans the extremely high (close to unity) estimates of the persistence parameter ϕ are estimated significantly (see t-values in Table 4).

In the above two spans that double, both the Federal Bank of New York (Fed) and the Bank of Japan (BOJ) held the joint intervention into the foreign exchange market at June 17. Just before the intervention, the sharp fall of yen against US dollar

occurred and it was likely that the volatility of the yen-dollar exchange rate get higher, reflecting the exacerbation of the Japanese fundamentals. There was an apprehensive scenario that the fall would probably give rise to the global recession triggered by the possible devaluation of Chinese yuan. The Fed and BOJ embarked on the intervention with the aim of breaking such a situation. By nature, the intervention sends to the market participants the signal that the Japanese government shall ameliorate the Japanese future fundamentals to support the strong and stable yen by prompting the resolution of the bad loan in the banking system and the implementation of the permanent tax reduction. ¹³ It is pointed out, however, that the market participants do not believe the signal positively. In fact, Dominguez (1998a) obtains the result that intervention operations generally increase exchange rate volatility against the central banks' intentions of decreasing it, using the dollar-mark and dollar-yen daily exchange rate data. Accordingly it can be considered that in the period before and after the intervention the successive high volatility (volatility clustering) of the exchange rate is provoked in foreign exchange market.

Further, Dominguez (1998b) finds the empirical fact that many Japanese companies are exposed to the yen-dollar exchange rate risk, and states that it is because their imports and exports are invoiced in dollar and they do not fully hedge against the exchange risk. Taking account of this fact, it seems reasonable to consider that the strong volatility clustering in the exchange market propagates to the stock market in the *TSE*, by bringing out the uncertainty of the speculators.

By the aid of a series of the explanations above, it is no wonder that the much higher (close to unity) persistent parameters ϕ are estimated in the above two spans including the date of the joint intervention than in other comparatively tranquil spans.

Thus, it is shown that the values of the persistence parameter of the SV model probably differ in spans, when we allow for the movement in the whole stock market. Looking at the SV model from a different angle by using panel data specification instead of time series, we found a new fact that there are both types of the spans with the high persistence of the volatility and with the moderate persistence.

V. CONCLUSION

This paper has advanced the panel data version stochastic volatility model, and the techniques for estimating its persistence parameter, using panel data with large number of individuals over fixed number of time series. The procedure of this technique converts this stochastic volatility model into the dynamic panel data model, and then implements the 1-step, 2-step, and iterated GMM estimations using the pertinent moment restrictions. The two kinds of the GMM estimators were presented: those using the standard moment restrictions and those using the stationarity moment restrictions.

In order to examine the small sample performances of the estimators and corroborate the superiority of the estimators using the stationarity moment restrictions in precision, the limited Monte Carlo experiments were performed for the sample sizes 100, 500, and 1000. Results indicate that the estimators using the standard moment restrictions have considerable downward-biases while the estimators using the stationarity moment restrictions are highly precise.

Furthermore, the empirical application of the estimators to the daytime stock returns' data of the issues listed on the Tokyo Stock Exchange in 1998 has implemented. In the application, the five spans composed of eight periods without the serial correlations were singled out, and then the estimations were carried out using the five spans. The estimates using the standard moment restrictions are negative and considerably low, which coincide with the results of the Monte Carlo experiments. On the other hand, the estimates using the stationarity moment restrictions show different high values between zero and unity in the five spans. Those in mid June, in which term the joint intervention into yen-dollar exchange market was done, exhibit the values extremely close to unity. It can be reckoned that the persistence parameters in the

whole stock market vary in terms. It is envisaged that further empirical researches using the techniques in this paper will exhume unknown facts in many stock markets.

APPENDIX: Derivation of (5) and (6)

Squaring both sides of (1) and then taking its logarithm, we get

$$\log y_{it}^2 = \log \sigma_{it}^2 + \log \varepsilon_{it}^2. \tag{A1}$$

From (2), $\log \varepsilon_u^2 \sim iid(-1.27, \pi^2/2)$ holds, according to Abramowitz and Stegan (1970).

Therefore,

$$\log \sigma_{ii}^2 = \log y_{ii}^2 + 1.27 - \xi_{ii}. \tag{A2}$$

Introducing (A2) into (3) and (4), we obtain (5) and (6).

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ENDNOTES

- 1. Author hopes that readers recognise handling the SV model by using short-run panel data is a variety of ideas for seizing the structure of financial data from a different angle, without negating it unsparingly.
- 2. Stricter value of 1.27 is 1.27036284546.... The stricter value is used in Monte Carlo experiments and empirical application that are put into effect in later sections.
- 3. The GMM is advocated in Hansen (1982).
- 4. In the context of the AR(1) dynamic panel data model, Holtz-Eakin et al. (1988) and Arellano and Bond (1991) take up as a representative example the first-differenced GMM estimators of the AR(1) coefficient under the serially uncorrelated disturbance, which are constructed on the basis of the standard moment restrictions using the second and higher lags of level dependent variables as instruments for first-differenced equations. Blundell and Bond (1998), however, show in their Monte Carlo experiments that the first-differenced GMM estimators perform poorly in small sample when the AR(1) coefficient is high (near to unity).
- 5. In the context of the mean-stationary AR(1) dynamic panel data model, Arellano and Bover (1995) contrive the system GMM estimators using jointly the mean-stationarity moment restrictions and the standard moment restrictions. Blundell and Bond (1998) make sure in their Monte Carlo experiments that under the serially uncorrelated disturbance,

the system *GMM* estimators perform much better than the first-differenced *GMM* estimators in small sample, even when the AR(1) coefficient is high (near to unity). Under the serially uncorrelated disturbance, the system *GMM* estimators are constructed based on the standard moment restrictions using the second and higher lags of level dependent variables as instruments for first-differenced equations and the mean-stationary moment restrictions using the first lags of first-differenced dependent variables as instruments for level equations.

- 6. The econometrics software TSP 4.4 (Hall et al., 1997) is used for the experiments.
- 7. Improvement of the behaviour of the STD-GMM estimators requires very large cross-sectional size, e.g. N = 25000.
- 8. The econometrics software TSP 4.4 is used for the empirical application.
- 9. The DTSR y_{ii} may include the additional fixed effect f_i so that we may come up with the structure $y_{ii} = f_i + \sigma_{ii} \varepsilon_{ii}$. The statistic $lm_{(-k)}$ also tests absence of the fixed effect f_i .
- 10. The TSP procedure of carrying out the $lm_{(-1)}$ and $lm_{(-2)}$ tests is provided by TSP international. On the basis of this procedure, the procedure of carrying out $lm_{(-k)}$ test is designed.
- 11. The applications using the two remaining sub-panel data sets are not presented in this

paper, because the moment restrictions in these cases are not valid often when we use the STA-GMM estimators.

- 12. Some people may point out cross-sectional correlations of the disturbances in (5) and (6) when we deal with actual stock returns' data. If it is the case, both the STD-GMM and STA-GMM estimators are less efficient but still consistent.
- 13. The Japanese central bank, BOJ, is not independent of the government tolerably in this time. Therefore, it is not denied that the intention of the government is in line with that of the BOJ in many respects.

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Table 1A. Monte Carlo results for the estimators of ϕ in the PDSV model [1]

N=100, T=8, NR=500

		STD-GMM		STA-GM	M		
	True	1-step	2-step	iterated			iterated
MCM	0.0	-0.477	-0.452	-0.380			0.076
RMSE		0.519	0.531	0.578	0.311		0.618
MCSD		0.203	0.278	0.436			0.613
MCSE		0.249	0.204	0.205			0.201
SARGAN (D)	F=14)	14.66	14.33	13.92			13.44
MCM	0.2	-0.455	-0.427	-0.351	0.201	0.191 (0.142
RMSE		0.685	0.684	0.699	0.286).392 (0.616
MCSD		0.201	0.275	0.430	0.286).391 (0.614
MCSE		0.248	0.204	0.205	0.259		0.193
SARGAN (DI	T=14)	14.79	14.45	14.01	14.64	l 4.14	13.36
MCM	0.5	-0.414	-0.367	-0.273	0.401).388 (0.290
RMSE		0.937	0.914	0.892	0.285).385 (0.641
MCSD		0.205	0.287	0.445	0.267).369 (0.605
MCSE		0.246	0.202	0.203	0.237 0).185 (0.172
SARGAN (DI	7 = 14)	15.42	14.99	14.43	14.61 1	4.14	13.56
<i>MCM</i>	0.8	-0.409	-0.360	-0.255	0.777	.779 (0.660
RMSE		1.227	1.198	1.158	0.172 0	.247 (0.548
MCSD		0.208	0.298	0.478			0.530
MCSE		0.246	0.204	0.205			0.112
SARGAN (DI	<i>T=14)</i>	15.60	15.14	14.43	14.65 1	.4.13 1	13.52
MCM	0.9	0.424	-0.387	-0.302			0.827
RMSE		1.339	1.320	1.290			0.437
MCSD		0.205	0.292	0.467).431
MCSE		0.247	0.206	0.207			0.070
SARGAN (DF	<i>'=14)</i>	15.18	14.78	14.15	14.45 1	4.01 1	l3.55
MCM	0.95	0.430	-0.401	-0.323).941
RMSE		1.395	1.381	1.353).279
MCSD		0.203	0.289	0.458).279
MCSE		0.248	0.206	0.208		.032 0	0.037
SARGAN (DF	=14)	14.95	14.57	14.00	14.32 1	3.97 1	3.70
MCM	0.98	-0.431	-0.400	-0.321	0.995 0	.996 0	.971
RMSE		1.426	1.411	1.381			.210
MCSD		0.210	0.293	0.463		.019 0	.210
MCSE	.	0.250	0.206	0.206	· ·		.019
SARGAN (DF	<i>'=14)</i>	14.75	14.39	13.94	14.29 1	3.94 1	3.70

⁽i) The True implies the true value of ϕ in the Monte Carlo.

⁽ii) When the true value of ϕ is 0.98, the numbers of individuals N fluctuate ranging from 84 to 99 with their mean 91.30 in the experiments, due to our elimination of the data with numeric errors.

Table 1B. Monte Carlo Results for the estimators of ϕ in the PDSV model [2]

N=500, T=8, NR=500

14-500, 1-0, 1	111-000	STD-GMM			STA-G	STA-GMM		
	True	1-step	2-step	iterated	1-step	2-step	iterated	
MCM	0.0	-0.486	-0.467	-0.421	0.128	0.127	0.114	
RMSE		0.524	0.543	0.565	0.300	0.421	0.584	
MCSD		0.196	0.278	0.378	0.271	0.401	0.573	
MCSE		0.256	0.234	0.234	0.270	0.245	0.234	
SARGAN (DF	=14)	14.10	13.71	13.39	14.15	13.65	12.91	
<i>MCM</i>	0.2	-0.437	-0.403	-0.346	0.211	0.205	0.162	
RMSE		0.669	0.666	0.665	0.256	0.372	0.547	
MCSD		0.204	0.283	0.379	0.256	0.372	0.546	
MCSE		0.251	0.229	0.229	0.257	0.231	0.221	
SARGAN (DF	=14)	14.71	14.30	13.88	14.22	13.79	13.14	
MCM	0.5	-0.309	-0.187	-0.047	0.442	0.449	0.410	
RMSE		0.838	0.755	0.684	0.216	0.289	0.445	
MCSD		0.221	0.323	0.410	0.208	0.285	0.436	
MCSE		0.239	0.214	0.212	0.202	0.182	0.176	
SARGAN (DF	=14)	17.57	16.41	15.17	14.24	13.95	13.68	
MCM	0.8	-0.310	-0.155	0.089	0.793	0.805	0.749	
RMSE		1.134	1.019	0.863	0.134	0.200	0.387	
MCSD		0.231	0.355	0.489	0.134	0.199	0.384	
MCSE		0.245	0.220	0.215	0.123	0.108	0.106	
SARGAN (DF=14)		19.00	17.64	15.53	14.11	13.72	13.55	
MCM	0.9	-0.380	0.288	-0.130	0.926	0.933	0.881	
RMSE		1.299	1.238	1.149	0.085	0.133	0.337	
MCSD .		0.225	0.348	0.510	0.081	0.129	0.336	
MCSE		0.251	0.229	0.227	0.076	0.068	0.066	
SARGAN (DF	=14)	16.63	15.97	14.93	13.84	13.46	13.33	
MCM	0.95	-0.417	-0.364	-0.267	0.980	0.983	0.950	
RMSE		1.384	1.354	1.303	0.049	0.068	0.244	
MCSD		0.219	0.326	0.464	0.039	0.060	0.244	
MCSE		. 0.255	0.234	0.233	0.039	0.036	0.037	
SARGAN (DF=	=14)	15.09	14.67	14.16	13.55	13.32	13.23	
MCM	0.98	-0.431	-0.394	-0.325	0.996	0.997	0.975	
RMSE		1.427	1.407	1.374	0.024	0.029	0.200	
MCSD		0.214	0.305	0.429	0.018	0.023	0.200	
MCSE		0.259	0.238	0.236	0.020	0.019	0.020	
SARGAN (DF=	<i>-14)</i>	14.39	14.00	13.55	13.45	13.24	13.14	

⁽i) The True implies the true value of ϕ in the Monte Carlo.

⁽ii) When the true values of ϕ are 0.9 and 0.98, the numbers of individuals N fluctuate ranging from 498 to 500 with their mean 499.98 and from 436 to 470 with 455.08 in the experiments, respectively, due to our elimination of the data with numeric errors.

Table 1C. Monte Carlo Results for the estimators of ϕ in the PDSV model [3]

N=1000, T=8, NR=500

N=1000, T=8	, IVR=000	STD-C	71.67.6	·	CM4 C		
	True			#4 4T	STA-G		
16016		1-step	2-step	iterated	1-step		iterated
MCM DMCE	0.0	-0.493	-0.461	-0.414	0.128	0.122	0.112
RMSE		0.531	0.541	0.556	0.300	0.424	0.582
MCSD		0.197	0.282	0.371	0.271	0.406	0.571
MCSE	.	0.256	0.237	0.237	0.270	0.248	0.238
SARGAN (DI	4=1 <i>4)</i>	14.04	13.71	13.41	13.91	13.36	12.64
MCM	0.2	-0.408	-0.346	-0.276	0.211	0.204	0.183
RMSE		0.642	0.616	0.604	0.245	0.355	0.501
MCSD		0.205	0.284	0.372	0.245	0.355	0.501
MCSE		0.245	0.226	0.227	0.242	0.224	0.215
SARGAN (DI	7=14)	14.95	14.48	14.02	14.01	13.67	13.25
MCM	0.5	-0.201	-0.013	0.139	0.456	0.467	0.455
RMSE		0.734	0.590	0.486	0.178	0.227	0.309
MCSD		0.219	0.291	0.325	0.173	0.225	0.306
MCSE		0.225	0.196	0.193	0.169	0.158	0.155
SARGAN (DF	T=14)	19.21	16.81	15.17	14.08	13.93	13.88
<i>MCM</i>	0.8	-0.206	0.045	0.322	0.795	0.809	0.795
RMSE		1.033	0.834	0.630	0.108	0.144	0.227
MCSD		0.236	0.355	0.410	0.108	0.143	0.227
MCSE		0.237	0.206	0.197	0.099	0.090	0.090
SARGAN (DF	=14)	22.02	18.87	15.77	14.03	13.84	13.88
MCM	0.9	-0.331	-0.183	0.031	0.919	0.926	0.893
RMSE		1.252	1.141	1.009	0.075	0.108	0.260
MCSD		0.226	0.359	0.513	0.073	0.105	0.260
MCSE		0.251	0.230	0.225	0.066	0.060	0.060
SARGAN (DF	<i>=14)</i>	18.04	16.91	15.19	13.86	13.67	13.64
MCM	0.95	-0.402	-0.330	-0.206	0.975	0.980	0.948
RMSE		1.368	1.321	1.250	0.045	0.058	0.234
MCSD		0.212	0.325	0.477	0.037	0.049	0.234
MCSE		0.257	0.240	0.239	0.037	0.035	0.036
SARGAN (DF	=14)	15.25	14.76	14.02	13.61	13.42	13.35
MCM	0.98	-0.427	-0.384	-0.306	0.995	0.995	0.975
RMSE		1.424	1.401	1.356	0.024	0.031	0.183
MCSD		0.220	0.317	0.431	0.020	0.027	0.183
MCSE		0.261	0.244	0.242	0.020	0.019	0.021
SARGAN (DF	=14)	14.20	13.80	13.38	13.42	13.32	13.30

⁽i) The True implies the true value of ϕ in the Monte Carlo.

⁽ii) When the true values of ϕ are 0.9 and 0.98, the numbers of individuals N fluctuate ranging from 999 to 1000 with their mean 999.97 and from 888 to 937 with 911.99 in the experiments, respectively, due to our elimination of the data with numeric errors.

Table 2. Itemisations of the selected sub-panel data sets

T=8				· · · · · · · · · · · · · · · · · · ·	
	May 15 ~ May 26	June 12 ~ June 23	June 16 ~ June 25	July 28 ~ August 6	Nov 13 ~ Nov 25
N	442 [100]	420 <i>[100]</i>	434 [100]	408 [100]	492 [100]
FMC	35 <i>[7.92]</i>	38 <i>[9.05]</i>	36 <i>[8.29]</i>	32 [7.84]	36 <i>[7.32]</i>
F	23 <i>[5.20]</i>	22 <i>[5.24]</i>	24 <i>[5.53]</i>	24 <i>[5.88]</i>	28 <i>[5.69]</i>
TPP	12 <i>[2.71]</i>	14 <i>[3.33]</i>	19 <i>[4.38]</i>	17 <i>[4.17]</i>	18 <i>[3.66]</i>
COR	59 <i>[13.35]</i>	59 <i>[14.05]</i>	50 <i>[11.52]</i>	55 <i>[13.48]</i>	59 <i>[11.99]</i>
GC	10 <i>[2.26]</i>	12 [2.86]	11 [2.53]	7 [1.72]	15 <i>[3.05]</i>
INM	28 <i>[6.33]</i>	21 <i>[5.00]</i>	26 <i>[5.99]</i>	19 <i>[4.66]</i>	29 <i>[5.89]</i>
M	42 <i>[9.50]</i>	36 <i>[8.57]</i>	34 <i>[7.83]</i>	28 <i>[6.86]</i>	39 <i>[7.93]</i>
EM	48 <i>[10.86]</i>	50 <i>[11.90]</i>	49 <i>[11.29]</i>	60 [14.71]	70 <i>[14.23]</i>
TPO	49 <i>[11.09]</i>	49 <i>[11.67]</i>	58 <i>[13.36]</i>	44 [10.78]	48 <i>[9.76]</i>
C	36 <i>[8.14]</i>	42 [10.00]	44 [10.14]	40 [9.80]	54 <i>[10.98]</i>
BIR	54 <i>[12.22]</i>	41 [9.76]	40 <i>[9.22]</i>	47 <i>[11.52]</i>	47 <i>[9.55]</i>
TS	46 <i>[10.41]</i>	36 <i>[8.57]</i>	43 [9.91]	35 <i>[8.58]</i>	49 <i>[9.96]</i>

- (i) The year is 1998. The "Nov" implies November.
- (ii) The T and N are the number of time periods and the number of the selected stock issues, respectively.
- (iii) Stock issues are classified into the following industry groups with their codes in brackets. (The small numbers of issues are, however, classified into inappropriate groups):
- FMC: Fishery, Mining and Construction (1300-1999). F: Foods (2000-2999).
- TPP: Textiles and Pulp & Paper (3000-3999). COR: Chemicals, Oil & Coal Products, and Rubber Products (4000-5199). GC: Glass & Ceramics (5200-5399). INM: Iron & Steel, Non-ferrous Metals, and Metal Products (5400-5999).
- M: Machinery (6000-6499). EM: Electrical Machinery (6500-6999).
- TPO: Transport Equipment, Precision Instruments, and Other Products (7000-7999).
- C: Commerce (8000-8299). BIR: Banking & Insurance and Real Estate (8300-8999).
- TS: Land Transport, Maritime Transport, Air Transport, Warehousing, Communication, Electricity & Gas, and Services (9000-9999).
- (iv) The numbers of stock issues used for the estimations per the industry groups and their percentages in the selected spans are depicted in columns. The percentages are in brackets.

Table 3. Results of the serial correlation tests in the sub-panel data sets

T=8

	May 15	June12	June 16	July 28	Nov 13
N	~ May 26 442	~ June 23 420	~ June 25 434	~ August 6 408	~ Nov 25 492
lm ₍₋₁₎	-0.475	-0.089	1.173	-0.011	0.894
$lm_{(-2)}$	-0.294	0.424	-0.302	-0.701	-0.697
lm ₍₋₃₎	-1.164	0.963	-0.221	1.771	-0.573
lm ₍₋₄₎	-0.872	-0.683	-0.726	0.186	-0.255
lm ₍₋₅₎	0.810	-0.267	-0.160	0.572	1.106
lm ₍₋₆₎	0.346	0.798	1.080	0.678	0.060
lm ₍₋₇₎	-1.299	1.155	0.649	-0.767	1.953

- (i) The year is 1998. The "Nov" implies November.
- (ii) The T and N are the number of time periods and the number of stock issues used for estimations, respectively.
- (ii) The $lm_{(-k)}$ is the test of serial correlation of k th order, where $k=1,\ldots,T-1$.

Table 4. Estimates of the persistence parameter ϕ in the PDSV model

	STD- G	MМ		STA-GI	MM	
	1-step	2-step	iterated	1-step	2-step	iterated
May 15	-0.514	-0.610	-0.855	0.590	0.617	0.624
~ May 26	(-4.13)	(-5.41)	(-8.48)	(5.67)	(6.21)	(6.33)
[<i>T=8, N=442, df=14</i>]	[42.37]	[41.40]	[36.39]	[10.63]	[10.42]	[10.38]
June 12	-0.614	-0.637	-0.647	0.853	0.984	0.987
~ June 23	(-4.82)	(-5.41)	(-5.51)	(7.17)	(9.39)	(9.45)
[<i>T=8, N=420, df=14</i>]	[32.76]	[32.53]	[32.45]	[17.61]	[14.12]	[14.07]
June 16	-0.578	-0.588	-0.588	0.806	0.929	0.939
~ June 25	(-3.36)	(-3.56)	(-3.56)	(7.13)	(11.63)	(11.90)
[<i>T=8, N=434, df=14</i>]	[17.67]	[17.63]	[17.63]	[19.68]	[16.12]	[15.98]
July 28	-0.214	-0.177	-0.174	0.629	0.739	0.800
~August 6	(-0.72)	(-0.66)	(-0.65)	(3.41)	(5.13)	(6.15)
[T=8, N=408, df=14]	[17.35]	[17.15]	[17.13]	[9.68]	[8.89]	[8.55]
November 13	-0.374	-0.343	-0.241	0.589	0.569	0.552
~ November 25	(-1.67)	(-1.59)	(-1.11)	(4.21)	(4.12)	(3.95)
[T=8, N=492, df=14]	[23.65]	[23.63]	[23.28]	[14.79]	[14.79]	[15.12]

- (i) The year is 1998.
- (ii) The T and N are the number of time periods and the number of stock issues used for estimations, respectively.
- (iii) The t-values of estimates are reported in parentheses below estimates.
- (iv) The Sargan test statistics of over-identifying restrictions are reported in brackets below t-values. These are asymptotically chi-square-distributed with degree of freedom df.