No. 821

Dislike for Inflation Creates Endogenous Price Fluctuations: An Analysis under a Logarithmic Utility Function

by

Toshiaki Koide

May 1999

Dislike for Inflation Creates Endogenous Price Fluctuations: An Analysis under a Logarithmic Utility Function

Toshiaki Koide*

Doctoral Program in Policy and Planning Sciences
University of Tsukuba
Tsukuba, Ibaraki 305-8573, Japan

This version: May, 1999

Abstract. Inflation is always viewed as a bad thing by the public, but it is not easy to answer why. The reason for it may be not what is meaningful economically but what is subjective or psychological. This study investigates how such a "dislike-for-inflation psychology" of the agents affects the dynamics of an overlapping generations economy. It is shown that it can create endogenous price fluctuations under a logarithmic utility function which never generates periodic equilibria in standard overlapping generations models. A numerical example in which a chaotic dynamics emerges is also presented.

Keywords: Dislike for inflation; Endogenous price fluctuations; Flip bifurcation

JEL classification: E31

*E-mail. tkoide@shako.sk.tsukuba.ac.jp

1 Introduction

Inflation is always viewed as a bad thing. It affects not only economic decision making of individuals but also political matters such as governmental elections. In the last decade, some industrial countries have adopted a new monetary policy known as "inflation targeting."[1] Nowadays inflation control is an important task for the world's monetary authorities.

It is not easy to answer why inflation is disliked by the public. Although there is a substantial literature on the social cost of inflation, Romer (1996, p.429) points out that there is a wide gap between the view of the cost of inflation by the public and that by economists. The reason why people dislike inflation may be not what is meaningful economically but what is subjective or psychological. For example, Shafir, Diamond and Tversky (1997) provide survey evidence that inflation tends to make the public reluctant to buy goods even if it does not change their real income. They explain that this phenomenon is due to "money illusion," a bias in the evaluation of the real value of goods, induced by nominal price representation. Moreover, Katona, Strumpel and Zahn (1971, chapter 4) argue that inflation detracts from satisfaction at progress in one's personal situation and lowers the extent of optimism.[2] Romer (1996, p.433) mentions the possibility that the

[1] Bernanke and Mishkin (1997) provide a good assessment of inflation targeting policy regime.

[2] Katona and Strumpel (1978, chapter 3) report that the series of monthly consumer price indices in the mid 1970s is highly negatively correlated to the data of the index of con-

public's aversion to inflation comes from the fact that the periods of high inflation coincides with periods of low real economic growth. Such subjective resistance to inflation of the agents is designated as a "*dislike-for-inflation psychology*" in this note. This psychology may affect the balance between the income and the substitution effects and may lead to endogenous price fluctuations through the mechanism pointed out by Grandmont (1985) and Fukuda (1993). This study investigates the possibility that the agents' resistance to inflation affects the dynamics of an economy inhabited by overlapping generations. We find that such a "dislike-for-inflation psychology" of the agents can create endogenous price fluctuations under a logarithmic utility function which never generates periodic equilibria in standard overlapping generations models.

2 The model

We construct an overlapping generations model with money. Each individual lives for two periods except one generation who live only in the initial period. It is a pure exchange economy with no population growth. We assume that there is only one kind of commodity which is perishable, so it is required to use money to store wealth in this economy.

All agents are identical and have a kind of "dislike-for-inflation psychol-

sumer sentiment, which is constructed by the Survey Research Center, University of Michigan. For details on the index of consumer sentiment, see the Survey Research Center's web page located on `http://athena.sca.isr.umich.edu/PDFs/SurveyInfo/Icscalc.pdf`.

ogy." An individual born at time t maximizes his/her lifetime utility given by

$$U_t = \log c_{1t} + \beta \log c_{2,t+1}, \tag{1}$$

where c_{1t} and c_{2t} are consumptions of the young and the old agents at t, respectively, while $\beta \in (0,1)$ indicates the subjective discount factor. The budget constraints of the agents born at t are defined as

$$\hat{w}_1 = c_{1t} + s_t, \quad \hat{w}_2 + \frac{p_t}{p_{t+1}} s_t = c_{2,t+1},$$

where s_t is their saving. The vector $(\hat{w}_1, \hat{w}_2)$ will be called *the effective endowment vector*, which is defined as

$$(\hat{w}_1, \hat{w}_2) \equiv \left(w_1, \left(\frac{p_t}{p_{t+1}}\right)^{\alpha} w_2 \right), \quad \alpha > 0,$$

where $(w_1, w_2) \in \mathbf{R}^2_{++}$ is *the real endowment vector* which is time-invariant and satisfies the following condition:[3]

$$\frac{\beta w_1}{w_2} > 1. \tag{2}$$

That is, agents' "dislike-for-inflation psychology" is reflected in their second period endowment in this model. Parameter α, which is assumed to be constant over time, represents intensity of their "dislike-for-inflation psychology": the larger α gets, the more their effective endowment in the second period depreciates by inflation.[4]

[3]This condition ensures that this economy is Samuelson-type.

[4]The standard overlapping generations model with money corresponds to $\alpha = 0$ in this model.

The saving function for the young agents at time t is given by

$$s_t \equiv \arg\max_s \log(w_1 - s) + \beta \log\left\{ \left(\frac{p_t}{p_{t+1}}\right)^\alpha w_2 + \frac{p_t}{p_{t+1}} s \right\},$$

and the first-order condition will be

$$\frac{1}{w_1 - s_t} = \frac{\beta}{(p_t/p_{t+1})^{\alpha-1} w_2 + s_t}. \tag{3}$$

We assume that nominal money supply is constant and denote it as M. This money is distributed to the old members in the initial period as a gift. Defining m_t as the real money balance at t, i.e., $m_t \equiv M/p_t$, the clearing condition for the money market is $m_t = s_t$ for any t. Substituting these relations into (3) and solving it for m_{t+1}, the dynamics of this economy is characterized by the following first-order difference equation with respect to m_t indexed by parameter α:

$$m_{t+1} = \phi(m_t; \alpha) \equiv m_t \left\{ \frac{\beta w_1 - (1+\beta) m_t}{w_2} \right\}^{1/(\alpha-1)}. \tag{4}$$

In the next section, we analyze the properties of (4) excluding the case that $\alpha = 1$ since the dynamics cannot be defined for that case.[5]

3 Emergence of endogenous price fluctuations

First, we will see a proposition on the number and the value of the steady states in this economy.

[5]It is easily shown that $m_t = (\beta w_1 - w_2)/(1+\beta)$ for all t if $\alpha = 1$, i.e., the saving is independent of the rate of inflation and the dynamics of this economy disappears.

Proposition 1. *The number and the value of the steady states of* $m_{t+1} = \phi(m_t; \alpha)$ *are the same as those of the standard overlapping generations model with money and are independent of intensity of the agents' "dislike-for-inflation psychology."*

Proof. Substituting $m_t = m_{t+1} = \bar{m}$ into (4) we obtain

$$\bar{m} = \bar{m}\left\{\frac{\beta w_1 - (1+\beta)\bar{m}}{w_2}\right\}^{1/(\alpha-1)},$$

which has two solutions: $\bar{m} = 0$ (nonmonetary steady state) and $\bar{m} = m^* \equiv (\beta w_1 - w_2)/(1+\beta) > 0$ (monetary steady state). These are absolutely the same as those of the standard overlapping generations model with money and m^* is independent of α, so that the proposition is proved. □

This proposition implies that the phase curve $m_{t+1} = \phi(m_t; \alpha)$ intersects the 45° line at the origin and at (m^*, m^*).

Next consider the stability of the steady states. The following proposition states the slope of the phase curve at the stationary points.

Proposition 2. *The slope of* ϕ *is less than one at the nonmonetary steady state while greater than one at the monetary steady state if* $\alpha < 1$. *When* $\alpha > 1$, *by contrast, the above relations are completely overturned: the slope of* ϕ *is greater than unity at the nonmonetary steady state while less than unity at the monetary steady state.*

Proof. They are shown immediately by calculating the slope of ϕ at $m_t = 0$

and at $m_t = m^*$ as follows.

$$\frac{\partial}{\partial m_t}\phi(0;\alpha) = \left(\frac{\beta w_1}{w_2}\right)^{1/(\alpha-1)} \begin{cases} < 1 & (\text{if } \alpha < 1) \\ > 1 & (\text{if } \alpha > 1) \end{cases}$$

$$\frac{\partial}{\partial m_t}\phi(m^*;\alpha) = \frac{1}{1-\alpha}\left(\frac{\beta w_1}{w_2} - \alpha\right) \begin{cases} > 1 & (\text{if } \alpha < 1) \\ < 1 & (\text{if } \alpha > 1) \end{cases}$$

□

Propositions 1 and 2 imply that, if $\alpha < 1$, the dynamic property of this economy is the same as that of the standard overlapping generations model with money. But it is quite different when $\alpha > 1$: the nonmonetary steady state loses its stability while the monetary stationary point can be asymptotically stable. Therefore we concentrate our attention on the case that the parameter α exceeds unity, i.e., on the case that the effective endowment of the agents in the second period depreciates faster than their saving by inflation.

The phase curve ϕ intersects the horizontal axis not only at the origin but also at $m_t = \hat{m} \equiv \{\beta/(1+\beta)\}w_1$ when $\alpha > 1$. In addition, we can show that

$$\frac{\partial}{\partial m_t}\phi(m_t;\alpha)\{\gtreqless\}0 \quad \text{if} \quad m_t\{\lesseqgtr\}\tilde{m} \equiv \left(\frac{\alpha-1}{\alpha}\right)\left(\frac{\beta}{1+\beta}\right)w_1,$$

that is, ϕ is a single-peaked function in $m_t \in [0, \hat{m}]$. Hence their is a possibility that the mapping $m_{t+1} = \phi(m_t;\alpha)$ takes a form like the logistic system as shown in figure 1.[6]

Figure 1 about here

In the following, we investigate whether periodic solutions exist or not in this economy. We denote α^* as the value of α which satisfies $(\partial/\partial m_t)\phi(m^*;\alpha) = -1$, i.e., $\alpha^* \equiv \{1 + (\beta w_1/w_2)\}/2$. The existence of periodic equilibria is claimed in the next proposition.

Proposition 3. *A stable two-period cycle emerges for α in the left neighborhood of the bifurcation point α^* in this economy.*

Proof. We apply the flip (or period-doubling) bifurcation theorem to our model.[7] One can verify that, from (2), the nondegeneracy condition is met and that the Schwarzian derivative is negative at $(m_t, \alpha) = (m^*, \alpha^*)$ as follows:

$$\frac{\partial}{\partial \alpha}\phi(m^*;\alpha^*)\frac{\partial^2}{\partial m_t^2}\phi(m^*;\alpha^*) + 2\frac{\partial^2}{\partial m_t \partial \alpha}\phi(m^*;\alpha^*) = \frac{8w_2}{\beta w_1 - w_2} \neq 0,$$

$$\frac{\frac{\partial^3}{\partial m_t^3}\phi(m^*;\alpha^*)}{\frac{\partial}{\partial m_t}\phi(m^*;\alpha^*)} - \frac{3}{2}\left\{\frac{\frac{\partial^2}{\partial m_t^2}\phi(m^*;\alpha^*)}{\frac{\partial}{\partial m_t}\phi(m^*;\alpha^*)}\right\}^2 = -\frac{2\beta w_1(1+\beta)^2(\beta w_1 + w_2)}{w_2^2(w_2 - \beta w_1)^2} < 0.$$

[6]It becomes impossible to define the dynamics in $[0, \hat{m}]$ when $\phi(\tilde{m};\alpha) > \hat{m}$, i.e., when the maximum of $\phi(m_t;\alpha)$ exceeds the upper bound of its domain. But the dynamics is still defined locally in the neighborhood of m^* even for that case. For $\phi(\tilde{m};\alpha) \leq \hat{m}$ to hold, parameter α should satisfy $\beta w_1/w_2 \leq \alpha\{\alpha/(\alpha-1)\}^{\alpha-1}$.

[7]See, for example, Guckenheimer and Holmes (1983, Theorem 3.5.1) on the flip bifurcation theorem.

Then the flip bifurcation theorem guarantees that a stable two-period cycle bifurcates from m^* for α in one side of α^*. In addition, it is shown that

$$\frac{\partial}{\partial \alpha}\left\{\frac{\partial}{\partial m_t}\phi(m^*;\alpha)\right\} = \frac{\beta w_1 - w_2}{(\alpha-1)^2 w_2} > 0,$$

that is, the slope of ϕ at $m_t = m^*$ is increasing in α. Therefore the monetary fixed point loses its stability and a periodic orbit emerges when $\alpha < \alpha^*$. This completes the proof. □

Fluctuations of the real money balance is equivalent to fluctuations of the price level since the nominal money supply is constant in this economy. Hence proposition 3 implies the possibility that the "dislike-for-inflation psychology" of the agents is one of the factors that create endogenous price fluctuations.

A periodic equilibrium never emerges in a standard overlapping generations model with money ($\alpha = 0$ in our model) under the logarithmic utility function (1) in which the substitution effect always dominates the income effect. But, in our model, the agents' aversion to inflation reinforces the income effect through the process that inflation depreciates the second period endowment of the consumers. Then the balance between the substitution and the income effects is affected and so is the dynamics of the economy.

Example. Now consider a numerical example in which a complicated dynamics emerges. Suppose that $w_1 = 100$, $w_2 = 30$ and $\beta = 0.8$. In this case, $m^* \approx 27.778$ and $\alpha^* \approx 1.833$. The bifurcation diagram of this example is depicted in figure 2. The sequence of real money balance converges to m^* whenever $\alpha > \alpha^*$. But if α gets smaller and reaches to α^*, the steady state loses its stability and a stable cycle of period 2 bifurcates. At $\alpha \approx 1.65$, the two-period cycle becomes unstable and a stable cycle of period 4 emerges. The real balance exhibits very complicated fluctuations for α between 1.53 and 1.64.

Figure 2 about here

It is noteworthy that this economy exhibits a stable cycle of period 3 when $\alpha \approx 1.578$. Therefore, we can claim that, applying the famous theorem by Sarkovskii (1964), there exists a cycle of any period $k \geq 2$ in this economy.

4 Conclusion

This study investigated the dynamic property of an overlapping generations economy inhabited by agents who feel aversion to inflation. We found that this economy has periodic equilibria for a certain intensity of their dislike for inflation under a logarithmic utility function. As is well known, in standard overlapping generations model with money, periodic equilibria never exist under such a utility function. Therefore we conclude that a "dislike-for-inflation psychology" of the agents can be one source of endogenous price fluctuations.

Acknowledgement

The author wishes to thank Professors Yoshihiko Otani and Hiroaki Hayakawa for their helpful comments on an earlier draft of this paper. Any remaining errors and shortcomings are the author's.

References

[1] Bernanke, B. S. and F. S. Mishkin, 1997, Inflation targeting: A new framework for monetary policy? Journal of Economic Perspectives 11, 97-116.

[2] Fukuda, S., 1993, The emergence of equilibrium cycles in a monetary economy with a separable utility function, Journal of Monetary Economics 32, 321-334.

[3] Grandmont, J.-M., 1985, On endogenous competitive business cycles, Econometrica 53, 995-1045.

[4] Guckenheimer, J. and P. Holmes, 1983, Nonlinear oscillations, dynamical systems, and bifurcations of vector fields (Springer-Verlag, New York).

[5] Katona, G. and B. Strumpel, 1978, A new economic era (Elsevier, New York).

[6] Katona, G., B. Strumpel and E. Zahn, 1971, Aspirations and affluence (McGraw-Hill, New York).

[7] Romer, D., 1996, Advanced macroeconomics (McGraw-Hill, New York).

[8] Sarkovskii, A., 1964, Coexistence of cycles of a continuous map of a line into itself, Ukrainian Mathematical Journal 16, 61-71.

[9] Shafir, E., P. Diamond and A. Tversky, 1997, Money illusion, Quarterly Journal of Economics 112, 341-374.

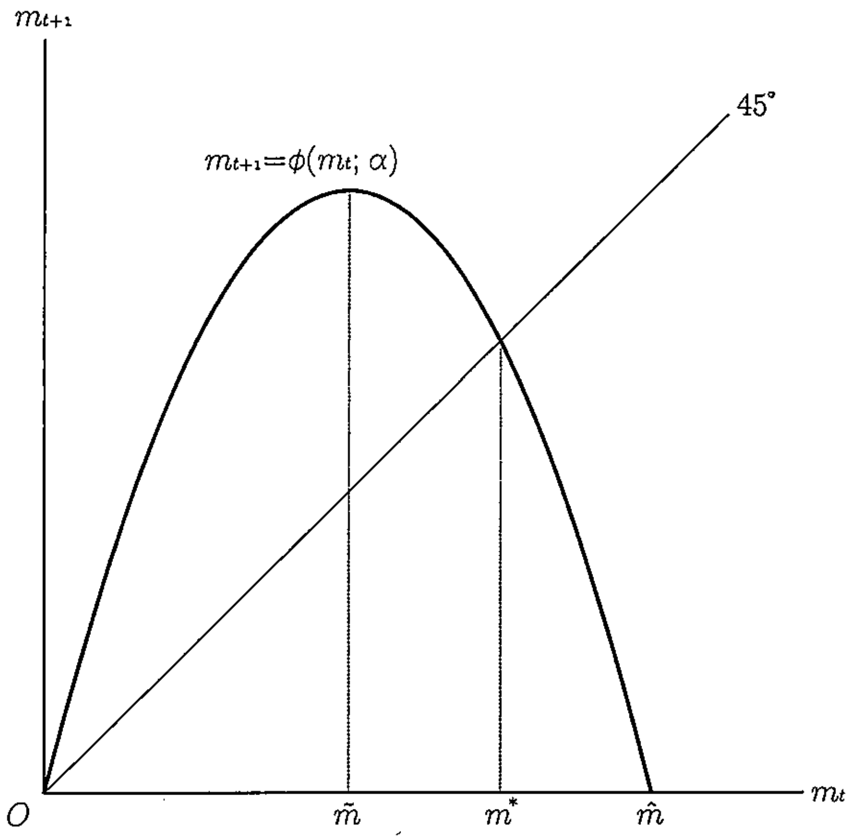

Figure 1: Phase diagram of $m_{t+1} = \phi(m_t; \alpha)$ for the case that $\alpha > 1$.

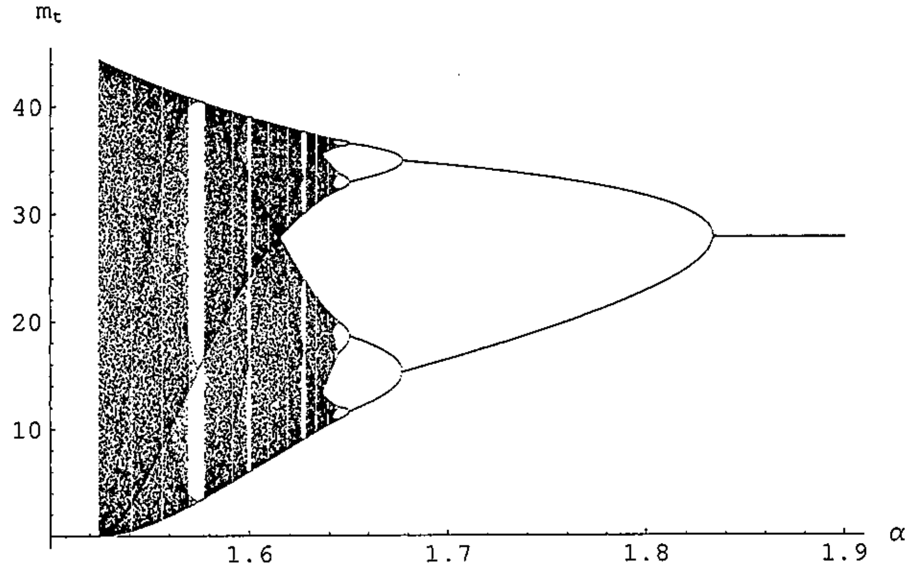

Figure 2: Bifurcation diagram for the example. The parameters are: $w_1 = 100$, $w_2 = 30$, $\beta = 0.8$.