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A Fuzzy Linear Regression Analysis for Fuzzy Input-Output Data Using the Least Square Method under Linear Constraints and Its Application to Fuzzy Rating Data

by

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Abstract – A fuzzy linear regression analysis using the least square method under linear constraints, where input data, output data, and the coefficients are represented by triangular fuzzy numbers, was proposed and then compared to the possibilistic linear regression analysis proposed by Sakawa and Yano(1992) using fuzzy rating data in a psychological study. The major findings of the comparison were as follows: (1) Under the proposed analysis, the width between the upper and lower values of the predicted model was nearer to the width of the dependent variable than that of the possibilistic linear regression analysis, (2) In addition, the representative value of the predicted value by the proposed analysis was also nearer to that of the dependent variable, compared with that of the possibilistic linear regression analysis. Key words: Fuzzy regression analysis, Possibilistic linear regression analysis, Least square method, Fuzzy rating.

1 Introduction

In order to measure the vagueness of human judgment, the fuzzy rating method has recently been proposed and developed [2]. In the fuzzy rating method, respondents select a representative rating point on a scale and indicate lower or upper rating points if they wish, depending on the relative vagueness of their judgment. For example, the fuzzy rating method would be useful for measuring perceived temperature indicating the representative value and the lower or upper values. This rating scale allows for asymmetries, and overcomes the problem, identified by Smithson [6], of researchers arbitrarily deciding the most representative value from a range of scores. By making certain simplifying assumptions (not uncommon within fuzzy set theory), the rating can be viewed as a triangular fuzzy number, hence making possible the use of fuzzy set theoretic operations [2]. An example of the fuzzy rating scale and of the representation of the rating data by the triangular fuzzy number was shown in Fig.1.

To analyze fuzzy rating data, it would be useful to apply fuzzy linear regression analysis, in which observed values and estimated values are assumed to have fuzziness. The original version of the possibilistic linear regression analysis proposed by Tanaka et al. [11] assumed that while output data and parameters are fuzzy numbers, input data are not fuzzy numbers. More recently, Sakawa and Yano [5] formulated and developed a possibilistic linear regression analysis, where both input data, output data, and parameters are fuzzy numbers. This method could be very effective for human-related sciences such as psychology, sociology, and ergonomics, because most of the input data and output data for such sciences, are considered to be fuzzy.

Unfortunately, some application studies using fuzzy rating data have indicated that the predicted variable for possibilistic linear analysis for fuzzy input-output data had too large a spread of fuzzy number for meaningful interpretation [7] [8]. Moreover, practically, it takes a lot of resources to compute the solution for the possibilitistic linear regression analysis by computer.

On the other hand, Diamond [1] proposed a fuzzy least square regression, in which both input data and output data were represented by the triangular fuzzy number. In his model of the analysis, the regression coefficients were assumed to be crisp numbers because a product by a multiplication of triangular fuzzy numbers is not a triangular fuzzy number. However, firstly, the assumption that the regression coefficients have fuzziness is rather natural for many human related sciences such as psychology. Secondly, a true product by a multiplication of triangular fuzzy number, which is a fuzzy number, can approximate a triangular fuzzy number [3] [4]. Thus, we adopted the assumption that both input-output data and regression coefficients had fuzziness which was represented by the triangular fuzzy number. However, this approach is more heuristic than Diamond's approach [1] because the present approach is considered to be the natural extension of the ordinary least square method for the crisp data rather than the extension of the fuzzy set theory.

2 Fuzzy Linear Regression Model for Fuzzy Input-Output Data

Fuzzy linear regression model (where both input and output data are fuzzy numbers) is represented as follows:

$$Y(X_i) = A \otimes X_i, i = 1, \dots, k \tag{1}$$

where

$$A = (A_0, \dots, A_n), X_i = (1, X_{i1}, \dots, X_{in})^T \tag{2}$$

the predicted value $Y(X_i)$, the parameter, a_j, and the observed input data, X_{ij} are fuzzy numbers, and '$\otimes$' is the product operator based on the extension principle [13].

In the following example, for simplicity, we assume that the fuzzy input data X_{ij}, fuzzy output data Y_i, and fuzzy parameter a_j are given as triangular fuzzy numbers defined by

$$\begin{aligned}
\mu_{X_{ij}}(x_{ij}) &= 0, \qquad x_{ij} \le x_{ij1}, \\
&\quad \frac{(x_{ij} - x_{ij1})}{(x_{ij2} - x_{ij1})}, \quad x_{ij1} \le x_{ij} \le x_{ij2}, \\
&\quad \frac{(x_{ij3} - x_{ij})}{(x_{ij3} - x_{ij2})}, \quad x_{ij2} \le x_{ij} \le x_{ij3}, \\
&\quad 0, \qquad x_{ij} \ge x_{ij3}
\end{aligned} \tag{3}$$

$$\begin{aligned}
\mu_{Y_i}(y_i) &= 0, \qquad y_i \le y_{i1}, \\
&\quad \frac{(y_i - y_{i1})}{(y_{i2} - y_{i1})}, \quad y_{i1} \le y_i \le y_{i2},
\end{aligned}$$

$$\begin{aligned} & \frac{(y_{i3}-y_i)}{(y_{i3}-y_{i2})}, \quad y_{i2} \le y_i \le y_{i3}, \\ & 0 \qquad\qquad\qquad y_i \ge y_{i3} \end{aligned} \tag{4}$$

$$\mu_{A_j}(a_j) = \begin{cases} 0, & a_j \le a_{j1}, \\ \frac{(a_j - a_{j1})}{(a_{j2}-a_{j1})}, & a_{j1} \le a_j \le a_{j2}, \\ \frac{(a_{j3}-a_j)}{(a_{j3}-a_{j2})}, & a_{j2} \le a_j \le a_{j3}, \\ 0 & a_j \ge a_{j3} \end{cases} \tag{5}$$

where x_{ij1}, y_{i1}, and a_{j1} are lower values, x_{ij2}, y_{j2} and a_{j2} are representative values, and x_{ij3}, y_{i3}, and a_{j3} are upper values on the set of real numbers. Clearly, the grades of memberships are zero for the lower and upper values for the fuzzy variables $(X_{ij}, Y_i, and\ a_j)$, and the grades of memberships are one for the representative values for the fuzzy variables $(X_{ij}, Y_i, and\ a_j)$.

Triangular fuzzy numbers such as X_{ij}, Y_i, and a_j are often symbolically represented by

$$\begin{aligned} X_{ij} &= (x_{ij1}, x_{ij2}, x_{ij3}), \\ Y_i &= (y_{i1}, y_{i2}, y_{i3}), \\ A_j &= (a_{j1}, a_{j2}, a_{j3}) \end{aligned} \tag{6}$$

For simplicity, we assume that X_{ij} and Y_i are positive triangular fuzzy numbers such that:

$$x_{ij1} > 0, y_{i1} > 0 \tag{7}$$

In the following section, firstly, the possibilistic linear regression analysis for both input and output data [5]. Then, to explore an alternative way to examine fuzzy input-output data, a fuzzy linear regression analysis using the least square method under linear constraints, in which the fuzzy input-output data and the regression coefficients were represented by the triangular fuzzy numbers, was proposed. The proposed analysis was compared to the possibilistic linear regression analysis proposed by Sakawa and Yano [5] using fuzzy rating data in psychological studies.

3 Possibilistic Linear Regression Analysis for Fuzzy Input-Output Data.

In the Sakawa and Yano' s [5] original formulation of the possibilistic linear regression analysis for fuzzy input-output data, it was assumed that input - output data and parameters were the symmetric L-L fuzzy numbers. In the following, the formulation of the possibilistic linear regression analysis for fuzzy-input data is slightly generalized by using the asymmetric triangular fuzzy numbers, because the fuzzy rating data can be often represented by asymmetric fuzzy numbers.

In the possibilistic linear regression model, the α-level set of $A \otimes X_i$ in (2) can be obtained as follows:

$$(A \otimes X_i)\,\alpha = [Z^L_{i\alpha}, Z^R_{i\alpha}] \tag{8}$$

where

$$Z_{i\alpha}^{L} = \sum_{j=0}^{n} \{\min \{(a_{j1} + \alpha (a_{j2} - a_{j1})) (x_{ij1} + \alpha (x_{ij2} - x_{ij1})) \ , \\ (a_{j1} + \alpha (a_{j2} - a_{j1})) (x_{ij3} + \alpha (x_{ij3} - x_{ij2}))\}\} \ , \quad (9)$$

$$Z_{i\alpha}^{R} = \sum_{j=0}^{n} \{\max \{(a_{j3} + \alpha (a_{j3} - a_{j2})) (X_{ij1} + \alpha (X_{ij2} - X_{ij1})) \ , \\ (a_{j3} + \alpha (a_{j3} - a_{j2})) (X_{ij3} + \alpha (X_{ij3} - X_{ij2}))\}\} \ . \quad (10)$$

It should be noted that $Z_{i\alpha}^{L}$ and $Z_{i\alpha}^{R}, i = 1, \dots, k$, involve the minimization and maximization operators. In order to deal with these operators in $Z_{i\alpha}^{L}$ and $Z_{i\alpha}^{R}, i = 1, \dots, k$, assume that the following relation hold for any fixed degree α:

$$\begin{aligned} a_{j1} + \alpha(a_{j2} - a_{j1}) &\geq 0, \ j \in J_1, \\ a_{j1} + \alpha(a_{j2} - a_{j1}) &\leq 0, \ a_{j3} + \alpha(a_{j3} - a_{j2}) \geq 0, \ j \in J_2, \\ a_{j3} + \alpha(a_{j3} - a_{j2}) &\leq 0, \ j \in J_3, \end{aligned}$$

where

$$J = \{0, \dots, n\} = J_1 \cup J_2 \cup J_3, J_i \cap J_j = \phi, i(\neq j) = 1, 2, 3. \quad (11)$$

On the basis of the above assumptions, the following set which depends on the index sets J_1, J_2, and J_3 is defined .

$$L(J_1, J_2, J_3) = \Big\{(\bar{a}_1, \bar{a}_2, \bar{a}_3) \in R^{3(n+1)} \mid a_{j1} + \alpha (a_{j2} - a_{j1}) \geq 0, a_{j1} \leq a_{j2} \leq a_{j3}, j \in J_1 \ , \\ a_{j1} + \alpha (a_{j2} - a_{j1}) \leq 0, a_{j3} + \alpha (a_{j3} - a_{j2}) \geq 0, a_{j1} \leq a_{j2} \leq a_{j3}, j \in J_2, \\ a_{j3} + \alpha (a_{j3} - a_{j2}) \leq 0, a_{j1} \leq a_{j2} \leq a_{j3}, j \in J_3\} \quad (12)$$

where

$$\begin{aligned} \bar{a}_1 &= (a_{01}, \dots, a_{j1}, \dots, a_{n1}) \\ \bar{a}_2 &= (a_{02}, \dots, a_{j2}, \dots, a_{n2}) \\ \bar{a}_3 &= (a_{03}, \dots, a_{j3}, \dots, a_{n3}) \end{aligned} \quad (13)$$

Then, $Z_{i\alpha}^{L}$ and $Z_{i\alpha}^{R}, i = 1, \dots, k$,can be expressed as :

$$Z_{i\alpha}^{L} = \sum_{j \in J_1} (a_{j1} + \alpha (a_{j2} - a_{j1})) (x_{ij1} + \alpha (x_{ij2} - x_{ij1})) \\ + \sum_{j \in J_2 \cup J_3} (a_{j1} + \alpha (a_{j2} - a_{j1})) (x_{ij3} + \alpha (x_{ij3} - x_{ij2})) \ , \quad (14)$$

$$Z_{i\alpha}^{R} = \sum_{j \in J_1 \cup J_2} (a_{j3} - \alpha (a_{j3} - a_{j2})) (x_{ij3} + \alpha (x_{ij3} - x_{ij2})) \\ + \sum_{j \in J_3} (a_{j3} - \alpha (a_{j3} - a_{j2})) (x_{ij1} + \alpha (x_{ij2} - x_{ij1})) \ , \quad (15)$$

The possibilistic linear model where both input and output data are fuzzy numbers can be constructed as follows:

$$P(\alpha) : \min J(\bar{a}_1, \bar{a}_2, \bar{a}_3) = \sum_{i=1}^{k} \{ (Z_{i1}^L - Z_{i0}^L) + (Z_{i0}^R - Z_{i1}^R) \}$$

subject to

$$\begin{aligned} Pos\,(Y_i = a \otimes X_i) \geq \alpha, i = 1, \ldots, k, \\ (\bar{a}_1, \bar{a}_2, \bar{a}_3) \in L\,(J_1, J_2, J_3) \end{aligned} \tag{16}$$

where the value $\alpha\,(0 \leq \alpha \leq 1)$ represents the degree of comformity between the fuzzy out put data, Y_i, and the fuzzy linear regression model, $a \otimes X_i$.

The inequality of problem $P(\alpha)$ can be transformed to the following inequality:

$$Pos\,(Y_i = a \otimes X_i) \geq \alpha, \;\; i = 1, \ldots, k$$

if and only if

$$\begin{aligned} y_{i3} - \alpha\,(y_{i3} - y_{i2}) &\geq Z_{i\alpha}^L, \\ y_{i1} + \alpha\,(y_{i2} - y_{i1}) &\leq Z_{i\alpha}^R \end{aligned} \tag{17}$$

Therefore, the problem $P(\alpha)$ can be written as follows:

$$P\,(\alpha : J_1, J_2, J_3) : \min J\,(\bar{a}_1, \bar{a}_2, \bar{a}_3) = \sum_{i=1}^{k} \{ (Z_{i1}^L - Z_{i0}^L) + (Z_{i0}^R - Z_{i1}^R) \}$$

subject to

$$\begin{aligned} y_{i3} - \alpha\,(y_{i3} - y_{i2}) &\geq Z_{i\alpha}^L, \;\; i = 1, \ldots, k, \\ y_{i1} + \alpha\,(y_{i2} - y_{i1}) &\leq Z_{i\alpha}^R, \;\; i = 1, \ldots, k \\ a_{j1} + \alpha\,(a_{j2} - a_{j1}) &\geq 0, \;\; j \in J_1, \\ a_{j1} + \alpha\,(a_{j2} - a_{j1}) &\leq 0, \;\; a_{j3} + -(a_{j3} - a_{j2}) \geq 0, \;\; j \in J_2, \\ a_{j3} + \alpha\,(a_{j3} - a_{j2}) &\leq 0, \;\; j \in J_3, \\ a_{j1} \leq a_{j2} &\leq a_{j3},.\; j0, 1, \ldots, n \end{aligned} \tag{18}$$

The above problem can be reduced to the linear programming problem with respect to (a_1, a_2, a_3). Therefore, the problem can be solved by the linear programming method.

4 A Fuzzy Linear Regression Analysis Using the Least Square Method Under Linear Constraints.

In the proposed analysis, a fuzzy linear regression analysis using the least square under linear constraints, it was assumed that both input-output data and regression coefficients had fuzziness which was represented by the triangular fuzzy number as shown in (3), (4), (5). This method is more heuristic than Diamond's [1] approach because the present approach is considered to be the natural extension of the ordinary least square method for the crisp data rather than the extension of the fuzzy set theory.

In order to estimate fuzzy parameters, we adopt the least square method under linear constraints ,$i.e. a_{j1} \leq a_{j2} \leq a_{j3}$ for all j. That is, the problem is to obtain fuzzy parameters $a_j = (a_{j1}, a_{j2}, a_{j3}), j = 1, \ldots, k$, that minimize $F(a_1, a_2, a_3)$ subject to the above constraints. The problem $FP(w_1, w_2, w_3)$ is defined as follows:

$$\begin{aligned} FP(w_1, w_2, w_3) \quad &: \quad \min F(\bar{a}_1, \bar{a}_2, \bar{a}_3) \\ &= \sum_{i=1}^{n} \left\{ w_1 \left(y_{i1} - \left(a_{01} + \sum_{j=1}^{k} a_{j1} x_{ij1} \right) \right)^2 + w_2 \left(y_{i2} - \left(a_{02} + \sum_{j=1}^{k} a_{j2} x_{ij2} \right) \right)^2 \right. \\ &+ \left. w_3 \left(y_{i3} - \left(a_{03} + \sum_{j=1}^{k} a_{j3} x_{ij3} \right) \right)^2 \right\} \end{aligned}$$

subject to

$$a_{j1} \leq a_{j2} \leq a_{j3}, \; j = 1, \ldots, k \tag{19}$$

where w_1, w_2, or w_3 is relative importance of lower value, representative value, and upper value, respectively. In ordinal cases, w_1, w_2, and w_3 should be equal. This minimizing formula as shown in (19) can be reduced to the quadratic programming problem with respect to $(\bar{a}_1, \bar{a}_2, \bar{a}_3)$. Therefore, it is easy to solve the problem by the quadratic programming method.

As described the above, the proposed analysis can be interpreted as the natural extension of the ordinary least square regression analysis. Assume that input and output data are crisp numbers.

The problem $FP(w_1, w_2, w_3)$ in (19) can be reduced to the following ordinary least square problem.

$$\min F(a_2) = \sum_{i=1}^{n} \left(y_{i2} - \left(a_{02} + \sum_{j=1}^{k} a_{j2} x_{ij2} \right) \right)^2 \tag{20}$$

5 Application of the Analysis to Psychological Study and Comparison Between the Proposed Analysis and the Posiibilistic Linear Regression Analysis

5.1 Study 1: On the effect of perceived temperature and humidity on an unpleasantness rating.

5.1.1 Overview

This study was conducted to examine the effect of perceived temperature and perceived humidity on unpleasantness using the fuzzy rating method to measure vagueness in judgment.

5.1.2 Method

Subjects: Eight adults (Age: 21 - 34 years old).

Procedure: The experiment was conducted in hot midsummer (2th August 1995). Subjects were instructed to rate representative values, lower values, and upper values from 0 to 100 for perceived humidity(0: not humid at all; 100: very humid), perceived temperature(0: not hot at all; 100: very hot), and unpleasantness(0: not unpleasant at all; 100: very unpleasant). For example, a rating of the perceived humidity is 70, 85, 95 for the lower value, the representative value, and the upper value, respectively. In the same way, for example, a rating of the perceived temperature is 30, 40, 55 , and a rating of the unpleasantness is 60, 80, 85. The fuzzy rating data were represented by the triangular fuzzy numbers as shown in Fig.1.

5.1.3 Result of the fuzzy linear regression analysis using the least square method under linear constraints

For evaluating the fitness of the identified fuzzy linear function, there are the following types of measures for the fitness [9] [10] [12].

$$Pos\,(Y_i = a \otimes X_i) = \sup_{x \in R} \min \{\mu_{Y_i}(x), \mu_{A \otimes X_i}(x)\}, \tag{21}$$

$$Nes\,(Y_i \supseteq a \otimes X_i) = \inf_{x \in R} \max \{\mu_{Y_i}(x), 1 - \mu_{A \otimes X_i}(x)\}, \tag{22}$$

$$Nes\,(Y_i \subseteq a \otimes X_i) = \inf_{x \in R} \max \{1 - \mu_{Y_i}(x), \mu_{A \otimes X_i}(x)\}, \tag{23}$$

where $Pos\,(Y_i = A \otimes X_i)$ is the possibility measure concerning the equality, $Y_i = A \otimes X_i, Nes\,(Y_i \supseteq A \otimes X_i)$ is the necessity measure concerning the relation, $Y_i \supseteq A \otimes X_i, Nes\,(Y_i \subseteq A \otimes X_i)$ the necessity measure concerning the relation, $Y_i \subseteq A \otimes X_i, \mu_{Y_i}(x)$ is a membership function of the observed dependent data, Y_i, and $\mu_{A \otimes X_i}(x)$ is a membership function of the predicted model $A \otimes X_i$. The values of the all measure ranges from 0 to 1.

In this study, the possibility measure concerning the predicted fuzzy variable and dependent fuzzy variable for each observation was used in order to compare with the results of the possibilistic linear regression analysis which maximize the possibility measure under the certain constraints. The possibility measure is considered to be useful for contrasting with the possibilistic linear regression analysis because this value of possibility is the same as the criterion for the possibilistic linear regression analysis.

The mean values of the possibility and the regression coefficients for the proposed analysis were shown in Table 1. The mean value of $Pos\,(Y_i = a \otimes X_i)$ was .65 for the inside room, and .74 for the outside. The estimated parameter was (.37, .46, .53) for the perceived temperature, and (.36, .43, .53) for perceived humidity in the room, respectively. Outside the room, the parameter was (.70 ,.78, .86.) for the perceived temperature, and (.51, .60, .62) for perceived humidity. The mean value of the predicted variable was (29.39, 37,73, 69.60) for the inside room, and (59.90, .73.54, .83.88) for the outside , respectively, whereas the mean value of the dependent variable was (30.63, 37.75, 46.38) for the inside room, and (63.87, .73.38, .80.62) for the outside. As shown in the result, the fitness defined by the possibility measure tended to be reasonably high.

5.1.4 Comparison to the possibilistic linear regression analysis.

We analyzed the data by the possibilistic linear regression analysis method, which is the most popular version of fuzzy linear regression analysis. For the analysis, we manipulated α level as a threshold for minimum possibility from 0.5 to 0.1 (That is, 0.5, 0.4, 0.3, 0.2, 0.1). As a result, when the α level was less than 0.2, meaningful parameters were obtained. when the α level was greater than 0.2, no meaningful parameters were obtained. Thus, we adopted the parameters in the case where α level equals 0.2.

The mean values of the possibility and the regression coefficients for the possibilistic linear regression analysis in comparison with the proposed analysis were shown in Table 1. The mean value of $Pos\,(Y_i = a \otimes X_i)$ was .57 for the inside room, and .52 for the outside. The estimated parameter was -(-.02, .08, .38) for the perceived temperature, and (.85, .85, .85) for perceived humidity in the room, respectively. Outside the room, the parameter was (.95 ,.96, .96.) for the perceived temperature, and (.00, .00, .00) for perceived humidity. The mean value of the predicted variable was (11.12, 28.30, 42.57) for the inside room, and (55.68, 64.06, 70.53) for the outside , respectively, whereas the mean value of the dependent variable was (30.63, 37.75, 46.38) for the inside room, and (63.87, .73.38, .80.62) for the outside.

The fitness of the model tended to be reasonably high. However, as shown in the results of Table 1, the width between the upper and lower values of the predicted model by possibilistic linear regression analysis was not nearer to the width of the dependent variable than that of the proposed fuzzy linear regression analysis. It was also found that the representative value of the predicted value for the proposed analysis was also nearer to that of the dependent variable compared with that of the possibilistic linear regression analysis.

For the both analyses, the spreads of the estimated fuzzy parameters were very small, and the paramerter were near to crisp numbers. The results indicate that the weight of attributes can approximate to crisp numbers.

5.2 Study 2 Consumer attitudes research.

5.2.1 Overview

This study was conducted to examine the effect of multi-attribute attitudes on behavioral intention using fuzzy rating method to measure vagueness in judgment and decision making.

5.2.2 Method

Subjects: One hundred and seventy-seven male and female adults who were living in Taiwan.

Procedure: Subjects were instructed to rate representative values, lower values, and upper values within 0 and 100 for attitude scales and behavioral intention of shopping behavior for fashion goods by indicating the points on the scales in the questionnaire. The scales were as follows: (1)traffic access to the store, (2)price of goods, (3)variety of goods in the store, (4) customer service, (5)quality of the goods, (6)atmosphere of the store, (7)environment of the store, (8)perceived value of buying goods in the store. The former seven items were independent variables, and the last item was a dependent variable. These scales were extracted from a pilot study for forty-two men and women. The rating data were represented by the triangular fuzzy numbers as same as in the study 1.

5.2.3 Result of the fuzzy linear regression analysis using the least square method under linear constraints

We computed a value of possibility for intersection between the predicted fuzzy variable and the fuzzy dependent variable as fitness of the model for each observation. That is, we defined the possibility measure $Pos\,(Y_i = A \otimes X_i)$ as the same manner in the study 1.

The mean values of the possibility and the regression coefficients for the proposed analysis were shown in Table 2. The mean value of $Pos\,(Y_i = A \otimes X_i)$ was .91. The estimated parameter was (1) (.24, .40, .44) for the traffic access, (2) (.27, .27, .30) for the price of goods, (3) (.44, .44, .48) for the variety of goods in the store, (4)(.28, .43, .43) for customer service, (5)(.02, .02, .02) for the quality of the goods, (6) (.09, .55, .55) for the atmosphere of the store, (7)(.20, .39, .39) for the environment of the store respectively. The mean value of the predicted variable was (23.02, 60.10, 88.04), whereas the mean value of the dependent variable was (53.79, 60.38, 66.76). As shown in the results of Table 2, the fitness defined by the possibility measure for the proposed analysis tended to be reasonably high.

5.2.4 Comparison to the possibilistic linear regression analysis

The data were analyzed by using the possibilistic linear regression analysis, which is the most popular version of fuzzy linear regression analysis. For the analysis, we manipulated α level as a threshold for minimum possibility from 0.5 to 0.1 (That is, 0.5, 0.4, 0.3, 0.2, 0.1). As a result, when the α level was less than 0.5, meaningful parameters were obtained. Thus, we adopted the parameters in the case where α level equals 0.5.

The mean values of the possibility and the regression coefficients for the possibilistic linear regression analysis in comparison with the proposed analysis were shown in Table 2. The mean value of $Pos\,(Y_i = a \otimes X_i)$ was .84. The estimated parameter was (1) (..08,.08,.08) for the traffic access, (2) -(-.08, .44, .44) for the price of goods, (3) (.06, .39, .39) for the variety of goods, (4)(.10, .10, .10) for the customer service (5) (.00, .00, .00) for the quality of the goods, (6) (.60, .60, .60) for the atmosphere of the store, and (7) (.00, .00, .00) for the environment of the store, respectively. The mean value of the predicted variable was (-42.00, 27.24, 132.35), whereas the mean value of the dependent variable was (53.79, 60.38, 66.76).

The fitness of the model tended to be reasonably high. Moreover, in the possibilistic linear regression model, we may have larger possibility grades by assigning larger values (e.g., 0.8) to α. Of course, this may have larger widths of the fuzzy parameters. As shown in the results of Table 2, the width between the upper and lower values of the predicted model by possibilistic linear regression analysis was not nearer to the width of the dependent variable than that of the proposed fuzzy linear regression analysis. It was also found that the representative value of the predicted value by the proposed analysis was also nearer to that of the dependent variable than that of the possibilistic linear regression analysis.

The results of the both analyses indicated that the spreads of the estimated fuzzy parameters were very small, and the parameters were near to crisp numbers. This property is the same as the finding in study 1, which suggests that the weights of attributes can approximate to crisp numbers.

6 Conclusion.

In order to measure the vagueness of judgment and decision making by using the fuzzy set theory, the fuzzy rating method [2] was introduced. The method of analysis for the fuzzy rating data is described through a comparison between the possibilistic linear regression analysis and the fuzzy regression analysis using the least square method. The fuzzy linear regression analysis using the least square method under linear constraints, where input data, output data, and the regression coefficients were represented by triangular fuzzy numbers the shapes of which are basically assymmetric, was proposed. The solutions of estimated parameters can be easily obtained by the ordinal quadratic programming method.

The psychological studies were undertaken in order to examine the effects of fuzzy independent variables on fuzzy dependent variable using fuzzy rating method. The results of these applications to psychological study indicated that the The fuzzy regression analysis using the least square was compared to the possibilistic linear regression analysis using fuzzy rating data in the psychological studies. The major finding of the comparison were as follows: (1) Under the fuzzy linear regression analysis using the least square method, the width between the upper and lower values of the predicted model was nearer to the width of the dependent variable than that of the possibilistic linear regression analysis, (2) The representative value of the predicted variable by the fuzzy linear regression analysis using the least square method was also nearer to that of the dependent variable compared with that of the possibilistic linear regression analysis, (3) For the both analyses, the estimated weights of the attributes (fuzzy parameters) were nearer to crisp numbers rather than fuzzy numbers with larger spreads. However, these findings might be restricted on the rating data of the present studies. Further research which examines the validity of the several methods for analyzing the fuzzy rating data will be needed.

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Table 1 Mean Values of the Regression Coefficients and the Possibilities for the Proposed Analysis and the Possibilistic Linear Regression Analysis in Study 1.

Model of Analysis	Inside the Room		Outside the Room	
	The Proposed Analysis	Possibilistic Linear Regression Analysis	The Proposed Analysis	Possibilistic Linear Regression Analysis
Regression Coefficients				
Perceived Temperature	(.37, .46, .53)	− (−.02, .08, .38)	(.70, .78, .86)	(.95, .95, .95)
Perceived Humidity	(.36, .43, .53)	(.85, .85, .85)	(.51, .60, .62)	(.00, .00, .00)
Possibility Measure	.65	.57	.74	.52

Table 2 Mean Values of the Regression Coefficients and the Possibilities for the Proposed Analysis and the Possibilistic Linear Regression Analysis in Study 2.

Model of Analysis		The Proposed Analysis	Possibilistic Linear Regression Analysis
	Traffic Access	(.24, .40, .44)	(.08, .08, .08)
	Price	(.27, .27, .30)	−(.08, .44, .44)
Regression	Variety	(.44, .44, .48)	(.06, .39, .39)
Coefficients	Service	−(.28, .43, .43)	(.10, .10, .10)
	Quality	−(.02, .02, .02)	(.00, .00, .00)
	Atmosphere	(.09, .55, .55)	(.60, .60, .60)
	Environment	−(.20, .39, .39)	(.00, .00, .00)
Possibility Measure		.91	.84

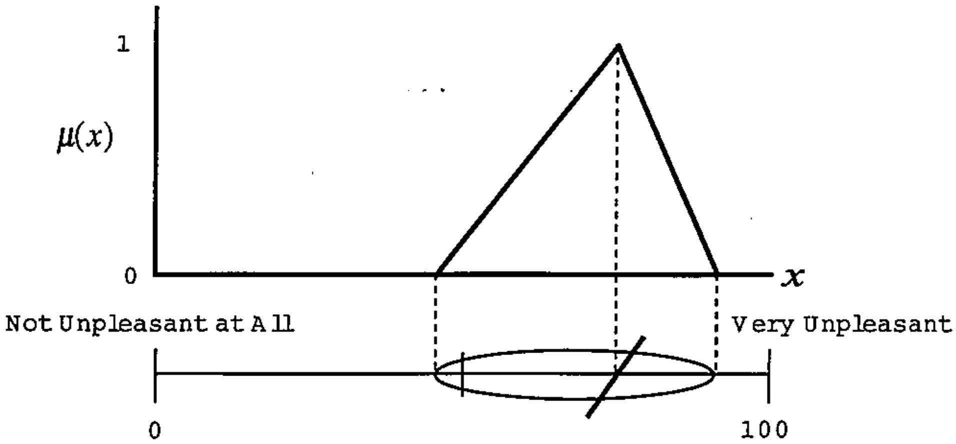

Figure 1: An Example of Fuzzy Rating Data and Its Representation by the Triangular Fuzzy Number