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A Mental Model of Creative Process in Naturalistic Decision Making : Its Qualitative and Mathematical Representations

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Abstract:

Decision makers often create mental representation of the decision problems. In decision making, even if the completely same alternatives are formed in the same objective situation, the decision made may be different, depending on the way of creating mental representation. In this chapter, we use the term "mental ruler" as a metaphorical model of the mental representation of the created decision problem and use it to describe and to explain the decision phenomena which are very context dependent and often violate some axioms of utility theories such as the expected utility theory or the nonlinear utility theory. The concept of mental ruler is similar to the decision frame which was proposed by Tversky and Kahneman (1981). The concept of mental ruler, however, has a more concrete image. This chapter introduces a new model of contingent decision making in which utility theories (including nonlinear utility theories such as the rank-dependent utility theory) are often violated. In this model, called "Mental Ruler" theory (Takemura, 1998), it is assumed that people construct a mental ruler to evaluate options for judgment and decision. A mental ruler is assumed to have two endpoints (reference points) like an ordinal physical ruler. It is assumed that a mental ruler is constructed on the support for a subjectively framed situation which is dependent on the focused situation. Contrary to the most of the utility theories and the prospect theory, the evaluation function is an S-shaped function, which is concave below and convex above a certain point between the endpoints of the support for the mental ruler.

1. Introduction

People judge or make decisions under various situations in complex real world settings(Klein, Orasanu, Calderwood, & Zsambok, 1993). A consumer, for example, decides on a purchase almost impulsively in one situation and the same person may be very deliberate in another situation. Is it possible to explain this kind of decision making phenomenon from a unified perspective?

Utility theory is a representative system of the theories which explain decision making phenomena. It uses mathematical methods and is introduced most frequently in consumer activity researches in economics.

Although utility theory explains most of decision making phenomena, it does not completely explain the contingent decision making. This paper firstly explains briefly why utility theory cannot completely explain the contingent decision making, secondly introduces the decision frame model (Tversky & Kahneman, 1981) and the psychological purse model (Kojima, 1959: Kojima, 1994) from a critical point of view, and then proposes the "mental ruler" model.

The mental ruler model assumes that people subjectively construct circumstance in order to easily create a one-dimensional mental ruler, and make a decision using the mental ruler created on the subjective circumstance as a support. I do not insist at all that the decisions are always one-dimensional. Rather, I insist that despite of multi-dimensional nature of the object, there is a possibility that one-dimensional decisions are made with considerable frequency in naturalistic situations, while of course multi-dimensional judgments and decisions are made in some cases as assumed in various decision theories. Unlike recent utility theory or prospect theory, the mental ruler model treats utility or value and subjective probability not as different functions but basically as the same evaluation function. Furthermore, unlike the other theories, this model forecasts instability of the judgment beyond the endpoints of the mental ruler, and also forecasts some other natures.

This chapter first explains the phenomenon, describing metaphorically the qualitative nature of the mental ruler model. Secondly, as a starting point for more precise formulation of the mental ruler model, explanation of the model in the set theory is done, partially with mathematical explanation of the evaluation function. Thirdly, experimental outcomes pertaining to contingent decision making which have been reported so far are taken up to be interpreted using the mental ruler model. Finally come conclusion and perspective.

2. Contingent Decision Making and the Problems in Its Modeling

2. 1. Contingent Decision Making

Contingent decision making is regarded as the most typical decision making phenomenon. "Contingency (situation-dependency)" can be observed very frequently and widely as seen at least the following phenomena (Takemura, 1995). Contingency mentioned in (1) to (7) below are not repulsing one another, but may occur concurrently. Here, contingency is defined widely, in order to enable more general discussion.

(1) Contingency on time: the phenomenon that a different mental process is observed, or a different judgment or decision is made, depending on the time when the judgment or the decision is made. A chronological change of the mental process in a judgment or a decision, usually within a few-day time period in shorter term. In longer term, there is the developmental change.

(2) Contingency on place: the phenomenon that a different mental process is observed, or a different judgment or decision is made, depending on the place where the judgment or the decision is made.

(3) Contingency on personal relations: the phenomenon that a different mental process is observed, or a different judgment or decision is made, depending on the personal relations in the situation where the judgment or the decision is made. There are two cases of the personal relations here: a case of different person and a case of different status of the person.

(4) Contingency on procedure: the phenomenon that a different mental process is observed, or a different judgment or decision is made, depending on the procedure of the judgment or the decision. This phenomenon includes response mode effect which violates procedure invariance (Tversky, Slovic, & Kahneman, 1990) and brings a different outcome depending on the decision making procedure, such as matching task or choice task.

(5) Contingency on description: the phenomenon that a different mental process is observed, or a different judgment or decision is made, depending on the descriptive form of the necessary information for the judgment or the decision. This phenomenon includes framing effect which violates description invariance (Tversky & Kahneman, 1986).

(6) Contingency on other external environment: the phenomenon that a different mental process is observed, or a different judgment or decision is made, depending on other external environment when the judgment or the decision is made.

(7) Contingency on internal condition: the phenomenon that a different mental process is observed, or a different judgment or decision is made, depending on the internal condition of the person when the judgment or the decision is made. For instance, emotional condition is included.

2.2. Why is It Difficult to Explain Contingent Decision Making by Utility Theory?

In order to solve this question, let us consider a problem of "contingency on description" as an example. For instance, suppose an answering phone whose standard price is $ 198.00 sells for $148.50 at an electric goods store. The effect of the POP advertisement (point of purchase advertisement) may be different whether it says "$49.50 off the standard price" or "25% off the standard price", although the both description convey the same discount information. In fact, Kojima (1986) reported that if an article is considered as a top-level brand by consumers, it sells more when the discount percentage is indicated than the discount amount is indicated, while on the contrary, if an article is considered as a second- or lower-level brand, it sells more when the discount amount is indicated than the discount percentage is indicated.

This kind of phenomenon indicates that the mathematically same decision problem may elicit different decisions psychologically, which cannot be explained by utility

theory as it tacitly assumes the mathematical descriptive uniqueness. Most of utility theory and mathematical models disregard the difference in descriptive forms in order to generalize the explanation (Takemura, 1994, 1996).

Recently there are new models which try to explain contingency on decision making under the framework of utility theory (Fishburn, 1988; Tversky, Sattath, & Slovic, 1988; Tversky & Kahneman, 1992; Tversky & Simonson, 1993; Takemura, 1994). These models can only explain part of the contingent decision making (for example, preference reversal of choice and matching, framing effect under risk, context effect by alternative positioning), but there is no theory that can explain the contingency of decision making from a unified perspective at the moment.

2. 3. Existing Qualitative Models Explaining Contingency of Decision Making

Let us next consider how to deal with contingent decision making which is so difficult to be systematized mathematically. One possible approach is to capture the complicated decision making phenomena by describing contingency of the decision making qualitatively or metaphorically. This kind of approach is sometimes effective in marketing, etc., since it helps to conceptualize an on-going problem. Decision frame model and psychological purse model are the two representative methods of the approach.

(1) Decision Frame Model

Tversky and Kahneman (1981) proposed the concept as a psychological framework to recognize decision making problems. They indicated that the decision making process comprises the editing stage to recognize the problem and the evaluating stage to evaluate the alternatives according to the recognition of the problem, and that a different decision can be made for the same problem depending on how the decision frame is constructed in the former stage. Although they gave little explanation on the nature or the function of the decision frame, they explained contingency by using an intuitively easily understandable word, "frame", and proved by experiment that consumers make a different choice on the purchase if they are given a different frame.

(2) Psychological purse Model

More than 20 years before the research done by Tversky and Kahneman (1981), Kojima (1959) already indicated that consumers are affected greatly by the contingent recognition of a problem when they decide on purchase or regarding the satisfaction after the purchase. He elucidated the contingent recognition of a problem by a constructive concept called "psychological purse". According to this model, consumers act as if they have different plural purses, and pay from different psychological purses according to the article, the kind of services, or the situation when they purchase it. Even if they pay the same amount for the same article, if the purse they pay from is different they feel different degree of satisfaction or mental pain pertaining to the expense (Kojima, 159; Kojima, 1994). Kojima, Akamatsu, Hama (1983) conducted factor analytical research based on questionnaire and clarified from what kinds of psychological purses various merchandise are purchased.

2. 4 Problems of the Qualitative Models

What they only took up for their qualitative decision frame model are positive

frame corresponding to value function in gain area in the prospect theory (Kahneman & Tversky, 1979; Tversky & Kahneman, 1992) and negative frame corresponding to value function in loss area. Nevertheless, more and more kinds of frames are assumed to exist in real decision making situations. Although it is easily understood that they can be roughly classified into positive and negative frames, there must be some kind of phenomenon which cannot be classified into neither frame. For instance, the judgment or the decision on "which is more beautiful", "which is bigger", or "which is more generous", cannot be easily classified, and it is unknown whether it can be interpreted into the prospect theory corresponding to decision frame. We cannot explain the judgment on "how generous the person is", as the research done by Hsee (1998) in 5.2., only by positive and negative frames.

Whereas, psychological purse model proved what kinds of psychological purses exist by factor analytical research and so forth, which should be highly evaluated. If we evaluate it very strictly, however, this model is only on the stage of situation classification. Moreover, psychological purse model is naturally a model of consumers' purchasing behavior and might be applicable to other activities than purchasing behavior, such as corporate activities or accounting activities, but it cannot explain people's other everyday decision problems.

Furthermore, both the decision frame model and the psychological purse model have not completely indicated how the structural concept functions and how the decision is elicited. That is, while the both models deal with internal structure of decision making problem and indicate that the internal structure has great effect on decision making, they do not explain sufficiently on how decision maker psychologically constructs the situation, what kind of nature or function the decision frame or the psychological purse has, and how the judgment or the decision is elicited from them.

3. Qualitative Description of "Mental Ruler"

3. 1. Basic Hypothesis of the Model and Basic Property of Mental Ruler

Here I present "mental ruler" model in order to solve the aforementioned problems, as well as developing the basic ideas of decision frame model (Tversky & Kahneman, 1981) and psychological purse model (Kojima, 1959; Kojima, 1994).

The basic hypothesis of this model postulates that people make decisions as if they have a ruler. In everyday life it is often said metaphorically that every person uses a ruler with different value to make decisions. Although this kind of metaphor is valid only in our daily conversations, if we consider this metaphor thoroughly and scientifically, it is more useful than we might think to explain contingent decision making. Objects of the mental ruler can basically be divided into gain and loss areas just as the decision frame model, but phenomena which cannot be classified into gain and loss areas can be included, such as the judgment on personal impression like generosity or calmness, or the judgment on probability.

Let us first consider the basic meaning of "ruler". Ruler is to measure "length". The reason why people use a ruler is, of course, that only looking at an object is not good in judgment on length as it causes unevenness or distortion. Or psychologically, people cannot judge with confidence without a ruler. We use a ruler as a standard for judgment. A physical ruler enables us to judge on length with relief. If we do not have a physical ruler, what do we do? I assume people construct a ruler

internally in their mind in a sense in such a situation. This can be regarded as a creative process in recognition of decision making problem.

Then consider the characteristics of the mental ruler further, and capture and discuss the characteristics of decision making metaphorically.

(1) Basic Property 1: The ruler has graduation.

I assume that people make a decision, based on the graduation of the mental ruler, which can be fine or rough, just as units of millimeter or centimeter on the graduation of a physical ruler. For example, let us consider the judgment on price. With fine graduation, consumers must be sensible of a difference by even 1 cent. On the contrary, with rough graduation, they can be insensible of a difference by several $100. Such difference of sensibility about prices can be described using the fineness or roughness of the graduation of the mental ruler. As I will state later, we can imagine the roughness of the graduation of the ruler may change in the same person depending on the situation.

(2) Basic Property 2: The length of the ruler is bounded (boundedness)

This property seems quite basic, but the metaphor denotes a great deal. For example, in judgment on price, we cannot easily judge if the price of an object exceeds the length of the mental ruler greatly in both directions, i.e. when the price is extremely too high or extremely too low. Consumers may joint several rulers when the ruler is too short, but the elicited judgment probably varies widely.

(3) Basic Property 3: The ruler is one-dimensional.

A physical ruler measures a one-dimensional property called length. I presume that even though people make judgment founded on multi-dimensional information, it is quite possible that they finalize the judgment one-dimensionally. In Japan, many people think that the education based on the standardized value of test score is not good, while at the same time they tend to be concerned about the standardized value of test score so much. People like to check various kinds of ranking, such as a "best seller" ranking at a shop. These tendencies seem to indicate a human nature, one-dimensional judgment.

3. 2. Basic Function of Mental Ruler

Based on the above-mentioned basic property of the mental ruler, I hereby present some theoretical prediction about its basic functions.

(1) Basic Function 1: People construct an appropriate mental ruler, depending on the situation.

People construct a mental ruler with appropriate graduation and of appropriate size, depending on the situation. People do this so naturally that they usually do not perceive it themselves. This phenomenon, however, can often be perceived if we compare purchasing situations. For example, in Japan, if a person think of purchasing a new car, he/she constructs a mental ruler with graduation of 10,000 yen (approx. 120 yen/$) unit when he/she needs to negotiate with a car dealer on the price or optional equipment, as a brand-new car usually costs more than 1 million yen. In such a situation, price area of several 100 yen is treated as an error, and is seldom taken into consideration. The same consumer, however, goes to a supermarket after the car dealer and can be satisfied with the price of a package of 10 eggs which is lower than usual by 20 yen, or be disappointed with a higher price than usual by

30 yen and he/she may not buy the eggs. The person concerns about a price of 10 yen unit to make a judgment or a decision in this situation. Like this way, we can presume that people focus on the on-going situation and construct the situation subjectively, and construct a mental ruler upon the situation.

(2) Basic Function 2: Reference point or endpoints of the ruler are applied differently, depending on the situation.

For example, in judging on price, a reference point changes according to the object group compared. Either a price is lower than another shop or than the price before, makes the reference point of the ruler move to a different position, and the judgment on the price or the decision on the purchase may be changed. The endpoints of the ruler is also assumed to change according to the situation such as the comparing group of objects.

(3) Basic Function 3: Graduation of the ruler becomes especially finer around the reference point and the endpoints (non-linearity of the ruler) .

This is a property which does not apply to a physical ruler. For instance, a consumer who is trying to buy an article for the budget of $100 becomes more sensible of the difference between $95 and $100, than the one between $50 and $55. If the comparing prices exceed the endpoints, it becomes extremely difficult to evaluate. For instance, if the budget is $100, the consumer becomes insensible of the difference between $150 and $155, and the evaluation becomes unstable.

(4) Basic Function 4: More knowledge or more involvement creates finer graduation of the ruler.

If a consumer has a lot of knowledge on an article or if a consumer is involved in an article very much, the graduation of the ruler becomes finer, then the consumer becomes sensible of a little difference, which leads to that he/she classifies similar articles very precisely. Therefore, it happens that the consumer tends to buy the article at a higher price if and even if its quality is only a little better than others.

(5) Basic Function 5: Even if information is given multi-dimensionally, a one-dimensional judgment is elicited by the mental ruler.

This does not only denote that people only simplify the problem, avoiding the data processing load. Consumers may construct another ruler to cope with the situation as a kind of creative process in the recognition of decision making problem. For instance, by reading fashion magazines or through repeated shopping experience, consumers construct a ruler such as "good taste" based on the complicated information about clothes to make a purchase decision. The mental ruler in this case is also basically one-dimensional.

(6) Basic Function 6: It is difficult to compare different mental rulers.

I presume it is very difficult for consumers to compare and evaluate the various mental rulers themselves which they constructed mentally, depending on the situation. Such contradictory judgments or decisions between situations as the example of car purchase and egg purchase, cannot be perceived by consumers themselves.

It is because, since people usually focus on the situation and construct the situation subjectively and construct a mental ruler on the situation, it becomes difficult to construct more than two rulers on one situation from the cognitive load point of view. People sometimes use a different ruler for the same value from the economic rationality point of view, or use the same ruler for the situations they should use different rulers.

3．3．Compatibility of Stimulus-Response Structures as Construction Principle of Mental Ruler

Lastly, I discuss on the construction principle of the mental ruler.

I presume the compatibility of stimulus-response structures plays an important role in constructing the mental ruler. The compatibility of stimulus-response structures denotes the compatibility between structural characteristics of input mode and response mode(Selart, 1997). If they match or correspond well, the efficiency of the information process in a judgment or a decision increases. I assume that a consumer constructs a mental ruler as an input mode corresponding to a given response mode. For instance, the purchase choice situation "to buy or not to buy" has the two-valued response mode and the consumer constructs a two-valued mental ruler, "good or bad". On the other hand, if a consumer is asked to evaluate an article by ranking or by points, the consumer constructs a mental ruler of multi-values.

If the stimulus-response structure does not correspond well, a consumer has difficulty in the judgment. For instance, if a mental ruler has already been constructed, a consumer cannot judge precisely if the ruler only has rough graduation, and vice versa.

From this compatibility of stimulus-response structure point of view, too, the reason why the mental ruler is one-dimensional might be explained. Since the structure of the environment requires a one-dimensional response mode of the judgment or the decision, the mental ruler becomes one-dimensional. In addition, I can also assume we often use linguistic terms of dual values such as "good or not good" to evaluate merchandise and so forth, which is because decisions are constructed by dual-value response modes such as "to buy or not to buy".

4．An Explanation of the Mental Ruler by Set Theory and Its Mathematical Description

For simplification, I here present an explanation by set theory for the mental ruler and its partial mathematical description. Below, I elucidate the qualitative and metaphorical description I indicated before, by adding the structure. Therefore it is not the perfectly retrieved qualitative and metaphorical description. Nevertheless, in order to create a psychometric model or to conduct various quantitative experiments, we will need the formulation to some extent. That's why I try the following.

4．1．Definition of the Situation

Let X' denote the whole situation to be discussed. X' is generally regarded as finite set. Let S' (S'$\subset$ X') , which is a subset of X', denote the focused situation. For instance, suppose X' denotes the purchasing situation in a supermarket, and S can be a situation whether to buy a coke or not, or whether to

buy a set of five notebooks or not, etc. The problem here is the focused situation which is cut out cognitively by the decision maker. In fact, although it is more natural to suppose that a situation S' denotes a subset of X' on the Cartesian product ($S' \subset X' \times X' \times \cdots \times X'$) since situations are often a set of relationship in a situation, suppose a sate S a subset of X' in order to make it simple. An important thing here is that S' is subject to how the decision maker pays attention, that is, S' will have different element if the same person focuses on another side of the same situation, according to one's mood. Nevertheless, the hypothesis here is that S' is a commonly subjective situation that can be recognized by other people, too. S' is a set of events which exist over an individual's subject and can be denoted extensively. For example, whether an article of $10 sells for $2 off or for 20% off means the same situation, as long as the meaning of the event is stated denotatively.

4. 2. Definition of Subjective Situation

Let us discuss the subjective situation. Let the limited set X denote the whole subjective situation, and S the subjective situation focused by a decision maker. X', the set of whole situation corresponds to X, and S', the focused situation corresponds to S. One element in a situation, however, can have more than two elements in a subjective situation, as the compatibility of X' and X, and S' and S are a many-to-one mapping (univalent correspondence) from the subjective situations to the objective situations. For instance, although the both descriptions, "$2 off" and "20% off" for an article of $10 means the same choice as long as they denotes a event extensionally, they can be different elements in a subjective situation. Furthermore, even if in the same situation S', plural subjective situations are considered to exist, such as the subjective situation S_1, S_2, . . . , S_n which is subject to the way of the mental structure. Thus, the mapping f to the situation S' is considered to be subject to the way of the mental structure on the decision making problem and can exist like f_1, f_2, . . . , f_n. The set of these function F (f_1, f_2, . . . , $f_n \in$ F) is considered to be constrained according to the cognitive ability of human being (e. g., Holyoak & Thagard, 1995). Lastly, the mapping from the subjective situation to the objective situation, f, is not onto mapping generally. Therefore elements of the subjective situation do not necessarily cover the elements of a situation but can be omitted partially, which can be considered because of the cognitive constraints of the decision maker's attention, memory capacity or searching ability. I presume the mapping has the direction to promote the stimulus-response compatibility stated earlier, or to create the dominance structure (Montgomery, 1983, 1993) in decision making problem.

4. 3. Structure of Mental Ruler

I hereby define the mental ruler. The mental ruler approximately differs with positive and negative areas, just like the value function in the prospect theory. The greater number the better in one case and the worse in another. Nevertheless, as stated earlier, the objects of the mental ruler model also includes a rather neutral one like probability judgment, not only the gain and loss areas. For simplification, however, I only discuss in the positive area. In addition, I first discuss the mental ruler model in a case where the evaluation object as a element of the subjective situation S can be objectively described using an additive measurement such as price,

length or size. Then I describe the mental ruler as a set function from the subsets of the subjective situation S to one-dimensional real number space R.

First consider a case where a element x of the subjective situation S can be objectively described as an additive function to price, length, proportion, probability, and so forth, $m(x) \in R$. Consider a function m from S to one-dimensional real number space R, $m : S \rightarrow R$. For instance, let $m(x)$ denote the discount rate m for an article x. Moreover, consider the mental ruler using the function v from one-dimensional real number space R which is mapped by m, to one-dimensional real number space R which describes the evaluation value, $v : R \rightarrow R$. Here, v has the following property.

$$m(x) = 0 \Rightarrow v(m(x)) = 0 \quad \ldots\ldots (1)$$

$$x^* = \underset{x \in S}{\operatorname{argmax}}\, m(x) \Rightarrow v(m(x^*)) = k, \text{ where } k \text{ is a constant} \quad \ldots (2)$$

$$m(x) \geqq m(y) \Rightarrow v(m(x)) \geqq v(m(y)) \ldots\ldots (3)$$

Formulas (1) and (2) denote the boundedness of the mental ruler. That is, for example, the evaluation for the relative income of \$0 is 0, where the evaluation for the evaluation object which has the most value in the subjective situation is a real number k. x^* denotes x which maximizes $m(x)$. For instance, when the upper limit of a relative income is considered \$10,000, the alternative which gives the \$10,000 is x^*. Or, considering the evaluation for a price using the mental ruler, if the upper limit of the budget is \$100, the article equivalent to the \$100 is equivalent to x^*. Formula (3) describes the monotonicity of the mental ruler. It leads to that the evaluation using the mental ruler does not exceed k in the subjective situation S. Also, if the mental ruler is unique with regard to the positively proportional transformation(the similarity transformation), by an adequate scale transformation,

$$v(m(x^*)) = 1, \text{ where } x^* = \underset{x \in S}{\operatorname{argmax}}\, m(x) \ldots\ldots\ldots (4)$$

As stated earlier, x^* denotes x which maximizes $m(x)$. (x^* always denotes the same in the following discussion.) Also, for simplification, presume the evaluation function of the mental ruler always holds the formula (4) in the discussion below.

4. 4. Subadditivity of the Mental Ruler and Its Mathematical Description

Although the mental ruler has the monotonicity of the formula (3), it does not have the additivity such as the following:

$$v(m(x) + m(y)) = v(m(x)) + v(m(x)) \quad \ldots (5)$$

The mental ruler is considered to hold the following two kinds of subadditivity (Tversky & Fox, 1995; Tversky & Wakker, 1995).

(i) Lower Subadditivity $v(m(x)) \geqq v(m(x)+m(y)) - v(m(y))$, where $m(x)+m(y) \geqq 1-\varepsilon$, $\varepsilon \geqq 0$ . . . . (6)

The formula (6) describes that the evaluation function becomes concave downward when $m(x)$ is relatively low. This property is the same as that of the weighting function for relatively lower probability in the prospect theory, and also the same as that of the diminishing the marginal utility in the utility theory.

(ii) Upper Subadditivity $v(m(x^*)) - v(m(x^*)-m(x)) \geqq v((m(x)+m(y)) - v(m(y))$, where $m(x) \geqq \varepsilon'$, $\varepsilon' \geqq 0$, ... (7)

This property denotes the event that the evaluation function of the mental ruler becomes convex downward when $m(x)$ is relatively high. This is the same as the property of the certainty effect indicating the weighting of probability 1 is much more than the probability less than 1, as explained by the prospect theory.
The mental ruler model, however, forecasts that this property hold not only with the weighting probability but also with the values of the outcomes. This forecast is completely contrary to the property of the diminishing marginal utility in the utility theory or in the prospect theory. In the utility theory or the prospect theory, a function which is concave downward is always assumed, whereas the mental ruler model assumes on the contrary that there exists a function which is convex downward around the upper bound. For example, such phenomenon that when negotiating on a discount for the price, the sensibility rises around the target. Let $m(x^*)$ denote the targeted gain for the negotiation, and the function becomes convex downward around the targeted price, while it becomes concave downward around the zero gain, where $m(x) = 0$. The function which holds the property of the formulas (6) and (7), is an S-shaped function. An S-shaped mathematical description which holds this kind of property is such as:

$$v(m(x)) = \exp(-(-\ln(m(x)/m(x^*)))^{\gamma}) \quad \ldots\ldots (8),$$

This is originally the function Prelec (1995) used as a weighting function for probability (See Figure 1). Here, $m(x)/m(x^*)$ takes interval $[0,1]$, and price interval is also $[0,1]$, then the fixed point becomes $1/e \fallingdotseq 0.36$ regardless of the value of γ (Wu & Gonzalez, 1996).

Another such function is:

$$v(m(x)) = (m(x)/m(x^*))^{\gamma} / ((m(x)/m(x^*))^{\gamma} + (1-m(x)/m(x^*))^{\gamma})^{\lambda} \quad \ldots\ldots (9)$$

Here, if $\lambda = 1$, it is the same as probability weighting function of Karmarker (1978) (See Figure 2) ; if $\lambda = 1/\gamma$, it is the same as probability weighting function of Tversky and Kahneman (1992) (See Figure 3).

Wu and Gonzalez(1996) conducted a psychological experiment to investigate the weighting function to probability, applied many kinds of functions, and proved that

the function by Tversky and Kahneman(1992) relatively showed high applicability as well as that the function of Plerec (1995) also applies well. They also proved that although the weighting function is concave downward around the probability 0.40, it becomes convex downward if it exceeds around 0.40. Although the weighting function they obtained is only against the probability, it can be assumed that the same kind of expertise can be obtained for the value such as money, from the mental ruler point of view. Also, it is already proved that the evaluation function of the number of surviving lives is S-shaped, as shown in the evaluation experiment later.

Moreover, the following function has been proposed neither in the research on the existing probability weighting function nor in the research on the prospect theory, but is now being proposed in Takemura(1998), and is assumed to be an evaluation function which satisfies upper subadditivity and lower subadditivity (See Figure 4).

$$v(m(x)) = w_1 (m(x)/m(x^*))^{\alpha} + w_2 (1 - (1 - m(x)/m(x^*))^{\beta}),$$
$$\text{where } w_1 \geqq 0,\ w_2 \geqq 0,\ w_1 + w_2 = 1 \quad \ldots\ldots (10)$$

In this evaluation function, let w_1 denote the strength in relative focus against the upper bound (strength of attention), w_2 the strength in relative focus against the upper bound, α and β index of power function against the upper bound and the upper bound respectively. This evaluation function is assumed to be applicable to not only the weighting function to probability, but also to psychophysical and perceptual judgment such as length or size, or economic values such as money.

4. 5. Threshold as Graduation of the Mental Ruler

I stated in the Section 3 that the mental ruler has graduation. The description as measure which satisfies the monotonicity and subadditivity above, however, is not sufficient in describing the concept of the graduation appropriately. Therefore, I assume that the threshold exists when a judgment or a decision is made using the mental ruler, and it becomes higher or lower according to the subjective situation. For instance, if a shopper is paying attention to the $100 unit, compared to when one is paying attention to the 1 cent unit, the threshold is assumed to be high. Or, the threshold is assumed to differ with the situations when a Japanese is shopping in yen in Japan, in dollars in the United States, or in yuan in China, even though these units can be proportionally exchanged. The threshold is also assumed to be low around the endpoints of the ruler, and high around the middle of the ruler.

Preference becomes indifferent within the threshold. The indifference can be denoted as binary relationship I on S when a strict preference relation R on set S (i.e., a relationship that can indicates which is preferred) is assumed. Against $x, y \in S$, when

$$x\,I\,y \Leftrightarrow \text{not } [x\,R\,y] \ \&\ \text{not } [x\,R\,y], \quad \ldots\ldots (11)$$

x and y are indifferent.

Thus, in a judgment or in a decision making, there must be a relationship when which is preferred cannot be mentioned. Therefore, when preference relationship R in a set S is considered, a real-valued function where an indifference occurs regarding a certain difference and which has the threshold can be assumed as follows. For all $x, y \in S$,

$$x R y \Leftrightarrow v(x) > v(y) + \delta(x, y) \quad \ldots\ldots (12)$$

where v is assumed to be an evaluation function of the mental ruler, and δ a positive value function of the threshold, which is subject to the objects x, y. Also, both v and δ are assumed to be dependent on the subjective situation S. For simplification, suppose the threshold is constant in the situation S, then the formula (12) becomes as follows. For all $x, y \in S$,

$$x R y \Leftrightarrow v(x) > v(y) + \delta \quad \ldots\ldots (13)$$

where δ is a positive constant.

The necessary and sufficient condition for the formula (13) is, according to the theorem by Scott and Suppes (1958), that the preference structure (S, R) is the following semiorder. For all w, x, y, z,

(i) not[xRx]
(ii) wRx and yRz $\Rightarrow$ [wRz or yRz]
(iii) wRx and xRy $\Rightarrow$ [wRz or zRy] .....(14)

The condition (14) is the necessary and sufficient condition for the formula (13). Therefore, assume the threshold is constant in the subjective situation, then preference structure is to be semiorder, and vice versa.

Then, how does this threshold correspond to the graduation of the mental ruler as stated in the Section 3? Here the big δ means the rough graduation of the ruler, and the small δ means the fine graduation. δ is therefore a function of the subjective situation S, which becomes small when the knowledge rate or the involvement rate of a decision maker is high, and vice versa.

4. 6. Restructure of the Subjective Situation and the Mental Ruler

I have explained that the mental ruler takes the value within the limited interval [0, k] or [0, 1] as a function of additive measure m (x) of the element x in the subjective situation S. In naturalistic decision-making situations, however, the subjective situation is changing at every moment. The actual problem in complex real world settings is that the subjective situation becomes larger. There are many situations where one has to extend the subjective situation in the past. For instance, such situation when a person who seldom goes shopping has to go to buy daily goods for the week for one's family because of a sudden illness of one's spouse, when one is going to purchase a car or a real estate for one's first time, or when an inexperienced teacher has to evaluate candidate students for admission. How people construct the mental ruler in these cases?

In the utility theory, the non-linear utility theory, or the prospect theory

in the past, the evaluation function was denoted on the universal set, therefore it was assumed that the evaluation function is already made up. In the mental ruler model, however, it is assumed that people cannot use the mental ruler appropriately in a new situation. For instance, let T (T $\supset$ S) denote a larger set which includes a set S. In constructing the mental ruler in this situation, the judgment is considered to be difficult to be made. The evaluation function of the mental ruler may have an error term (or a disturbance term) in such area, or the value of the disturbance term in the past is assumed to become much greater. This kind of phenomenon is considered to hold even in such cases as the evaluation function is described by the formulas (8), (9), and (10). For example, the formula (10) becomes the formula (15) to (17), according to the area of the value of m(x).

(i) If $m(x \in T) \in [0, m(x^*)]$,

$$v(m(x)) = w_1 (m(x)/m(x^*))^{\alpha} + w_2 (1 - (1 - m(x)/m(x^*))^{\beta}) + e(m(x)),$$

where $w_1 \geqq 0$, $w_2 \geqq 0$, $w_1 + w_2 = 1$, $e(m(x)) \in R$, $\varepsilon \geqq e(m(x)) \geqq 0$, ε is a positive constant. . . . (15)

(ii) If $m(x \in T) \notin [0, m(x^*)]$, and also $m(x) \geqq 0$,
$v(m(x)) = 1 + e(m(x))$, where $e(m(x)) \in R$,
$\delta \geqq e(m(x)) > \varepsilon$, ε is a positive constant. . . . (16)

(iii) If $m(x \in T) \notin [0, m(x^*)]$, and also $m(x) < 0$,
$v(m(x)) = -e(-m(x))$, where $e(m(x)) \in R$, $\zeta < e(m(x)) \leqq \eta$, ζ and η are positive constants. . . . .(17)

Although a concrete representation of the function e (m (x)) which indicates the disturbance term is unknown, its disperse changes according to the value of m (x) and it is a probabilistic variable whose average value is the monotone increasing function of m (x). The disperse of e (m(x)) is thought to become rapidly greater outside the interval [0, m (x*)].

Moreover, threshold δ is considered to become rapidly greater outside the interval [0, m (x*)] in case of preference declaration. This means that the graduation of the ruler becomes rougher.

Let us consider next how the mental ruler would be when a set of the subjective situations such as T, which does not include S, is constructed. If a set of the subjective situations and the existing set S are both independent and have no common property, it is almost impossible to construct a stable mental ruler. If they have a common property or a similar property, the existing subjective situation can be corresponded through an inference involved by analogy (e. g., Holyoak & Thagard, 1995), and the mental ruler is constructed on the set of the situations. In this case, the attribute not corresponding to the one in the existing subjective situation is disregarded. Also when the compatibility by analogy is not enough, the disturbance term of the evaluation function becomes greater, and the threshold also becomes high.

4. 7. The Mental Ruler as a Set Function

I here discuss the mental ruler as a set function, as expanding the idea in the past considering the mental ruler using the evaluation function ($v : R \rightarrow R$) against the additive function $m(x) \in R$ of the element x in the subjective situation S. Suppose that the evaluation function V of the mental ruler is a set function from the set of all subsets in the subjective situation S, 2^S, to a one-dimensional real number space R, $(V : 2^S \rightarrow R)$. V has the following properties.

$$V(\phi) = 0 \quad \ldots\ldots (18)$$

$$V(S) = K, \text{ where K is a constant} \quad \ldots\ldots (19)$$

$$S \supseteq A,\ S \supseteq B,\ A \supseteq B \Rightarrow V(A) \geqq V(B) \quad \ldots\ldots (20)$$

The formulas (18) and (19) indicate the boundedness of the mental ruler as a set function. Formula (20) indicates the monotony of the mental ruler. For example, let S denote a set of patients in a certain area who need medical care ($\{x_1, x_2, , , , x_n \in S\}$), let A ($\{x_1, x_2, , , , x_m \in A\}$, where $m < n$) and B ($\{x_1, x_2, , , , x_l \in B\}$ where $l < m$) denote a set of patients in S whose lives will be saved by a certain remedy. The desirability here has the monotonicity of V which is indicated in the formula (20). $V(\phi) = 0$ means nobody is saved, and $V(S) = K$ means all the patients are saved.

If the mental ruler is unique with regard to the similarity transformation, an appropriate scale transformation elicits the formula (21).

$$V(S) = 1 \quad \ldots\ldots (21)$$

The mental ruler as a set function here is mathematically the same as the fuzzy measure in fuzzy theory, the capacity in the integral theory, or the non-additive probability in non-linear utility theory, but is different from them in its interpretation. The fuzzy measure, the capacity, and the non-additive probability are generally interpreted as a weighting function to the subjective index for uncertainty or risk, while the mental ruler can be interpreted not only as a subjective index for uncertainty or risk, but also as the evaluation function of judgment or decision.

Note that the subjective situation is constructed optionally. Even if the situation S' is the same, there can be these events that the subjective situation S_1 is included in S_2 and S_1 is smaller the S_2. In these events, according to the formulas (18), (19), (20) or (21), the value of the mental ruler for the subset A are described as the formula (22). V_1 against S_1 becomes equal to or more than V_2 against S_2.

$$A \subseteq S_1 \subseteq S_2 \Rightarrow V_1(A) \geqq V_2(A) \quad \ldots\ldots (22)$$

This indicates that taking the broader comparison area in decision making reduces the relative value of the object and lowers the evaluation. For example, let (A) denote a set of articles for which a \$5 to \$6 discount is possible, (S_2) a set of articles which can be purchased at less than \$500, and (S_1) a set of articles which can be purchased at less than \$50. When one evaluates (A), the value of the mental ruler for (S_1) is assumed to be higher than the value for (S_2).

While the mental ruler has the monotonicity indicated by the formula (20), generally it does not have the additivity as in the formula (23).

$$A \cap B = \phi \Rightarrow V(A \cup B) = V(A) + V(B) \quad \ldots \quad (23)$$

The mental ruler V is, as well as the mental ruler v, assumed to have the following two kinds of subadditivities (Tversky & Fox, 1995; Tversky & Wakker, 1995) .

(i) Lower Subadditivity $V(A) \geqq V(A \cup B) - V(B)$, where A and B are independent and $V(A \cup B)$ takes lower value than around the upper bound value $(K-\varepsilon, \varepsilon \geqq 0)$ . . . . . (24)

This property indicates that the impact of a set A against the nullset is bigger than the impact of a set A against a set B. For example, let S denote a set of patients in a certain area who need medical care ($\{x_1, x_2, , , , x_n \in S\}$), let A ($\{x_1, x_2, , , , x_j \in A\}$, where $j < n$) and B ($\{x_1, x_k, , , , , x_p \in B\}$ where $j < k < p < n$) denote a set of patients in S whose lives will be saved by a certain remedy. The evaluation of the desirability here is assumed to be in the following relationship by the lower subadditivity of V indicated in the formula (24): $V(A) \geqq V(A \cup B) - V(B)$. More concretely, for example, the lower subadditivity is satisfied by the event when there are 100 patients ($\{x_1, \ldots x_{100}\}$) and the impact that the number of lives to be saved becomes one ($\{x_1\}$) from none (ϕ) is bigger than the impact that the number becomes 51 ($\{x_1, \ldots x_{51}\}$) from 50 ($\{x_2, \ldots x_{51}\}$) .

(ii) Upper Subadditivity $V(S)-V(S-A) \geqq V(A \cup B) - V(B)$, where A and B are independent and V (B) takes higher value than around the upper bound value (ε' , $\varepsilon' \geqq 0$) . . . . . (25)

This property indicates that the impact when a set A is taken away from the set of the subjective situation, S, is bigger than the impact when a set A is taken away from its subset $A \cup B$. The uncertainty effect explained by the prospect theory, that the impact of certain event is much greater than the impact of uncertain event, has the same property. The mental ruler model, however, presumes that the same property holds not only on the probability weight but on the values of the outcomes. This presumption means the complete contrary to the property of the diminishing the marginal utility n in the utility theory or the prospect theory, just like the case of property v of the mental ruler. For instance, let S denote a set of patients in a certain area who need medical care ($\{x_1, x_2, , , , x_n \in S\}$), let A ($\{x_1, x_2, , , , x_j \in A\}$, where $j < n$) and B ($\{x_k, , , , , x_p \in B\}$ where $j < k < p < n$) denote a set of patients in S whose lives will be saved by a certain remedy. The evaluation of the desirability here is assumed to be in the following relationship by the upper subadditivity of V indicated in the formula (25): $V(S) - V(S-A) \geqq V(A \cup B) - V(B)$. More concretely, for example, the upper subadditivity is satisfied by the event when there are 100 patients ($\{x_1, \ldots x_{100}\}$) and the impact that the number of lives to be saved becomes 100 ($\{x_1, \ldots x_{100}\}$) from 99 ($\{x_2, \ldots x_{100}\}$) is bigger than the impact that the number

becomes 51 ($\{x_1, \ldots x_{51}\}$) from 50 ($\{x_2, \ldots x_{51}\}$) .

5. Explanation of the Experimental Findings

Below, I qualitatively explain some of the experimental outcomes in the past, using the mental ruler model just proposed.

5.1. An Interpretation of the Experimental Results by Tversky and Kahneman (1981)

I try to discuss the outcome of the experiment conducted by Tversky and Kahneman (1981) on purchase decision making, using the mental ruler. They conducted the following experiment in order to confirm that the decision frame will change the decision (Problem 9.10) .

They separated 181 college students into two groups, and two versions of question were presented to the different groups.

> Version 1: Imagine that you are about to purchase a jacket for $125, and a calculator for $15. The calculator salesman informs you that the calculator you wish to buy is on sale for $10 at the other branch of the store, located 20 minutes drive away. Would you make the trip to the other store?
>
> Version 2: Imagine that you are about to purchase a jacket for $15, and a calculator for $125. The calculator salesman informs you that the calculator you wish to buy is on sale for $120 at the other branch of the store, located 20 minutes drive away. Would you make the trip to the other store?

The outcome is, 68 percent of the respondents were willing to make an extra trip to save $5 on a $15 calculator; only 29 percent were willing to exert the same effort when the price of the calculator was $125. This is obviously contradictory to the utility theory, as both versions presented the same choice between a total amount of $140 for a jacket and a calculator in a near-by shop and $135 for the same articles in a distant shop. As long as the total amount is concerned, both versions should be indifferent. This experimental outcome is contradictory to the utility theory under the hypothesis that the choice rate is a monotone increasing function of the utility.

Tversky and Kahneman (1981) interpreted this outcome using their decision frame concept that the subjects used different decision frames for each item, not for the total amount. Their interpretation seems basically right, but unfortunately they did not explicate why the different frames for the two articles had happened.

I use the mental ruler model to interpret the process of obtaining this outcome. First, the situation of this problem can be illustrated as follows:

Version 1: $S'_1 =$ { ($125 jacket, $15 calculator, no extra trip) , ($125 jacket, $10 calculator, an extra 20 min. trip)}

Version 2: $S'_2 =$ { ($15 jacket, $125 calculator, no extra trip) , ($15 jacket, $120 calculator, an extra 20 min. trip)}

Next, in the subjective situations, common information are canceled and deleted, and, especially, the unmentioned information "no extra trip" is not taken into consideration:

Version 1: $S_1 = \{(\$15 \text{ calculator}), (\$10 \text{ calculator, an extra 20 min. trip})\}$

Version 2: $S'_2 = \{(\$125 \text{ calculator}), (\$120 \text{ calculator, an extra 20 min. trip})\}$

A comparison can be made here as only the calculator is mentioned, for different prices, in both versions. Subjects are assumed to construct an evaluation function of the mental ruler, v, which is denoted in the formulas (8), (9), and (10), and consider on the extra trip, in order to evaluate the discount. Here, the evaluation function of the discount amount of price, F, can be described by v_1 and v_2, the functions of the mental rulers which have different $m(x^*)$:

Version 1: $F(\$5 \text{ discounted from } \$15) = v_1(\$5)$
Version 2: $F(\$5 \text{ discounted from } \$125) = v_2(\$5)$

Nevertheless, if the evaluation function F is applied to the evaluation function v in the formulas (8), (9), and (10), $m(x^*)$ becomes \$15 for v_1 in Version 1 and \$125 for v_2 in Version 2, while $m(x)$ is \$5 for both v_1 and v_2.

According to the property of the evaluation function v in the positive area, which is denoted in the formulas (8), (9), and (10), $v_1(\$5) > v_2(\$5)$ is elicited for both evaluation formulas. Therefore, $F(\$5 \text{ discounted from } \$15) > F(\$5 \text{ discounted from } \$125)$.

5. 2. An Interpretation of the Experiment by Hsee (1998)

Hsee (1998) conducted an experiment to confirm the "less is better effect" (Study 1).

Subjects are 83 college students separated into two groups, and each group received one of the two versions of the questionnaire below.

Version 1: Imagine that you are about to study abroad and have received a good-bye gift from a friend. It is a wool coat, from a nearby department store. The store carries a variety of wool coats. The worst costs \$50 and the best costs \$500. The one your friend bought you costs \$55.

Version 2: Imagine that you are about to study abroad and have received a good-bye gift from a friend. It is a wool scarf, from a nearby department store. The store carries a variety of wool scarves. The worst costs \$5 and the best costs \$50. The one your friend bought you costs \$45.

In both conditions participants were asked how generous they thought the friend was. Answers were given on a 0-6 point scale where 0 indicated 'not generous at all', and 6 indicated 'extremely generous'. The result was, although the \$55 coat was certainly more expensive than the \$45 scarf, those receiving the scarf considered

their gift giver to be significantly more generous than those receiving the coat (The mean rating values equal 5.63 and 5.00, respectively).

An interpretation of this experiment is shown in figures as follows:

Version 1: {$55 wool coat, worst $50, best $500}
Version 2: {$45 wool coat, worst $5, best $50}

Subjects are assumed to construct a mental ruler using the given information. In Version 1, the ruler is assumed to be constructed between $50 and $500; in Version 2, between $5 and $50.

Suppose that the subjects take the highest price as a comparative object. The evaluation function F can be illustrated by the following functions of the mental ruler, v_1, v_2, which have different $m(x^*)$:

Version 1: $F(\$55 \text{ wool coat}) = v_1(\$55)$
Version 2: $F(\$45 \text{ wool scarf}) = v_2(\$45)$

Nevertheless, if the evaluation function F is applied to the evaluation function v in the formulas (8), (9), and (10), $m(x)$ becomes $55 for v_1 in Version 1 and $45 for v_2 in Version 2, while $m(x^*)$ is $500 for v_1 and $50 for v_2. Also, if a psychological reference point is applied to the lowest price and the highest price, $m(x)$ becomes $5 for v_1 in Version 1 ($55 - $50), and $40 for v_2 in Version 2 ($45 - $5), while $m(x^*)$ is $450 for v_1 ($500 - $50), and $45 for v_2 ($50 - $5).

Regardless of the reference point, the following is elicited according to the property of the evaluation function v in the positive area which is denoted by the formulas (8), (9), and (10) for both the evaluation formulas:

$$v_1(\$55) < v_2(\$45)$$

therefore, $F(\$55 \text{ wool coat}) < F(\$45 \text{ wool scarf})$

Hsee (1998) conducted another experiment which asks how much the subjects are willing to pay for a serving of ice cream, presenting two versions: 8 oz of ice cream in a 10 oz cup, and 7 oz of ice cream in a 5 oz cup (Study 2). The experiment indicated less-is-better effect in separate evaluation(the between-subject design). The average price for 8 oz ice cream is $1.66, while $2.26 for 7 oz ice cream. This result implies that if the subjective situation is set very widely, the decision will be unstable, as the subjective situation is a support on which people construct the mental ruler to make a decision. In the mental ruler model, $m(x)$ is 8 oz and $m(x^*)$ is 10 oz in case of a 10 oz cup, while $m(x)$ is 7 oz and $m(x^*)$ is 5 oz in case of a 5 oz cup. By the formulas (8), (9), (10), and (16), it is elicited that the 7 oz of ice cream in a 5 oz cup is evaluated preferable to the 8 oz of ice cream in a 10 oz cup.

Hsee (1998) found that a clear preference reversal occurs in joint evaluations. That is, in within-subject evaluation, the subjects presented higher price for 8 oz ice cream than 7 oz ice cream. This can be interpreted as follows, using the mental

ruler model: in the joint evaluation, the focus is set on 'whether 7 oz or 8 oz' to construct the mental ruler in the subjective situation, as both versions are presented at the same time. In each version, the evaluation function of the mental ruler has m(x) of 7 oz and 8 oz respectively, and m (x*) of 8 oz, under which a decision is made.

As Hsee (1998) indicated, this kind of experimental result is difficult to be explained by decision making theories in the past such as the ones describing contingency. For example, decision frame model and the prospect theory are not sufficient to expound the phenomenon, as the both versions are regarded as gain frames. To apply decision frame model and the prospect theory to the evaluation such as generosity shown in Study 1 earlier is also difficult. Moreover, psychological purse model cannot explain this kind of phenomenon, either. These models or theories can only presume after the fact that there might have existed a different purse or a different frame. In addition, as Hsee (1998) indicated, the prominence hypothesis by Tversky et al. (1988) that the conspicuous attribute got weight is not sufficient, either.

Hsee (1998) presented evaluability hypothesis that the mutually evaluable attributes such as cup size and amount of ice cream are combined and, as a result, they produce an effect on the decision making. It presumes that the cup size does not have a great effect on the decision making in joint evaluation as the comparison of the amount of ice cream is easier than that of the cup size. This explanation is not contradictory to the explanation by the mental ruler model stated earlier. The evaluability hypothesis by Hsee (1998), however, did not explain how the subjective situation is constructed and evaluation is made on it, and what kind of judgment or decision emerges; it only states what kind of attributes tend to mutually relate to produce an effect on the decision making, which is meaningful for understanding the structure of the subjective situation in the mental ruler model.

5. 3. An Interpretation of the Evaluation Experiment on the Value of Saved Lives

Takemura(1998) conducted an experiment where 17 each of male and female college students answered to the following questionnaire, which is slightly transformed from the questionnaire of the Asian disease problem by Tversky & Kahneman (1981):

> Question: Imagine that a certain local area in Japan is preparing for the outbreak of an unusual disease, which is expected to kill 100 people. It is considered important that as many people as possible recover from the disease and do not die. Answer the status of preference for the following cases according to your subjective value, on a 0-100 point ruler where 0 indicates 'nobody recovers and all people die', and 100 indicates 'all people recovers and nobody dies'.

The subjects evaluated the preference on each case where the number of saved lives is 1, 2, 3, 5, 10, 20, 30, 40, 50, 60, 70, 80, 90, 95, 97, 98, 99.

Both positive and negative sides of the situation in a sentence was presented as follows, in order to suppress the elements of the framing effect.

(1) One person is saved; 99 persons die. (points) ,
(2) Two persons are saved; 98 persons die (points) ,

The average evaluation values in each case are shown in Figure 5. The result implies, as indicated in Figure 5, the S-shaped evaluation function which can be presumed by the mental ruler model. Especially, the property of the evaluation function obtained from this result is implied to be similar to the property of the evaluation function denoted by the formula (10).

5. 4. An Interpretation of the Perceptual Judgment Experiment by Miwa & Takemura (1998)

Study 1 by Miwa & Takemura (1998) was an experiment investigating the perceptual judgment on line length, circle area size, and rectangular area size using the magnitude estimation method. The subjects were 7 college and graduate students, who were presented 15 kinds of comparative stimulus in a random order, and asked to judge four times per stimulus. The response time to each judgment was measured. There was no significant difference among the response time for line length, and circle or rectangular area sizes, although there was a tendency that with only a little difference, area sizes were judged more quickly than length. This result indicates that there is no significant difference between the response time for a line which is a physically one-dimensional object, and for a circle or a rectangular which is physically two-dimensional.

Study 2 was an experiment investigating the perceptual judgment on the area size, the length and the width of a rectangular in each different session, using the magnitude estimation method. The subjects were 8 college and graduate students, who were presented 12 kinds of comparative stimulus in a random order, and asked to judge 10 times per stimulus. The dispersion of the responded value for each area, length, or depth was measured. As a result, the dispersion of area judgments (the difference of the upper bound and the upper bound of the assumed value) was much smaller than the product of the dispersions of length and width. This result implies that the subjects did not consciously evaluate the length and the width of a rectangular and then duplicate them in order to judge on the area size of the rectangular, but they perceptually judge on area.

These results imply that the subjects did not judge on sides or a diameter of the subject serially first and then integrated the information into an area, but did construct a one-dimensional mental ruler directly according to the dimension of values such as length or size. These results, however, do not eliminate a possibility that the subjects unconsciously figured out the data in length for each dimension of the subject by the parallel distributed processing and integrated the data afterwards to judge the area. They do not eliminate a possibility, either, that the judgment on length or area size is a response after processing physical property of the object which is of a lower dimension than the length. Although fully admitting these possibilities, I still consider the explanation using the mental ruler is effective, as a macro and metaphorical interpretation which is not contradictory to these possibilities.

6. Conclusion and Future Perspectives

In this chapter, I briefly indicated that the mathematical system such as the utility theory has difficulty in completely explaining the contingent decision making at the present stage, introduced the qualitative decision frame model (Tversky & Kahneman, 1981) or the psychological purse model (Kojima, 1959; Kojima, 1994) on contingent decision making, and finally introduced the "mental ruler" model to explain the contingent decision making qualitatively. I considered on the basic hypotheses, the basic function of the mental ruler, and the basic structural theories. The basic idea of the model is that people judge or make decisions using a mental ruler constructed on a subjective situation which was structured as a support so that one-dimensional mental ruler can be easily made on it. The main characteristic of the model is that contrary to the recent utility theory or prospect theory, it treats utility or value and subjective probability as basically the same evaluation function. I also mentioned the instability of the judgment in the area beyond the length of the mental ruler, not like the previous theories. I illustrated the mathematical model for its more rigorous formulation in the future.

I have focused on the one-dimensionality of the evaluation in judgment and decision making in this chapter. The discussion, however, has restrictions. There are cases where people evaluate multi-dimensional attributes and evaluate information multi-dimensionally. For example, a case is often observed where people make a judgment or a decision, consciously considering multi-dimensional information as assumed in multi-attribute attitude theory or multi-attribute decision making theory. It would be necessary in the future to clarify in which situation the one-dimensional evaluation as assumed in the mental ruler model is easily done, and in which situation the evaluation considering multi-dimensional information is done more often.

In addition, the model presented in this chapter is essentially qualitative and there are still vague parts to be precisely formulated in the future. Some forecasts or interpretation of the model can be also elicited by experiential tests. There are various possible researches, such as the ones on perceptual judgment forecast by evaluation function suggested in this chapter, evaluation function forecast in the area of social judgment or decision, re-interpretation of possibility weight function, risk evaluation forecast, or consumer activity forecast. In the future, we need to work on the experiential tests on these forecasts to examine the model more exquisitely.

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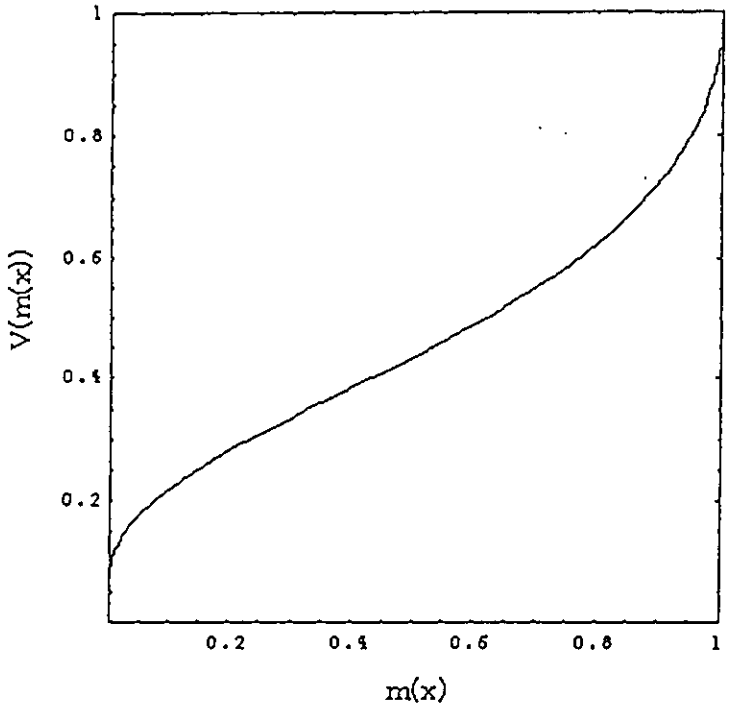

Figure1. Prelec's(1995) type of the evaluation function (γ=0.5)

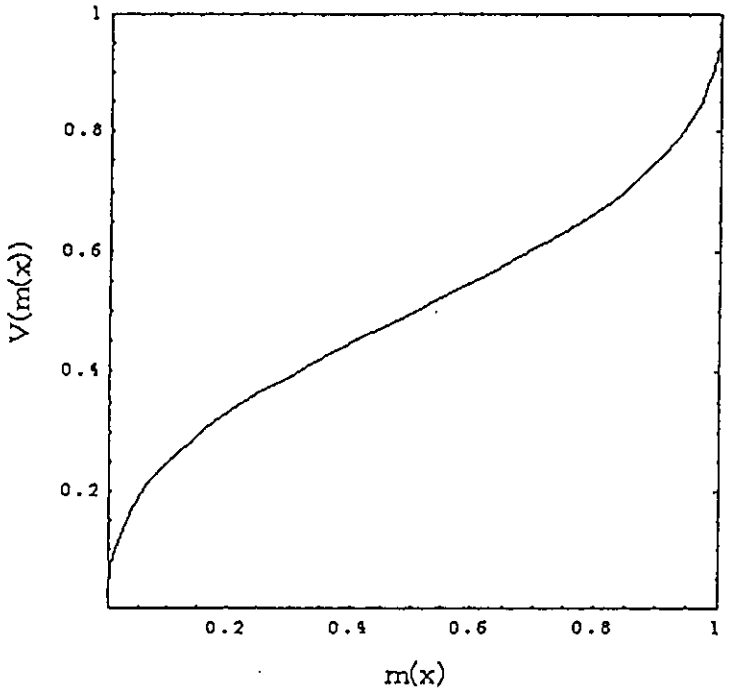

Figure2. Karmarker's(1979) type of the evaluation function (λ=1, γ=0.5)

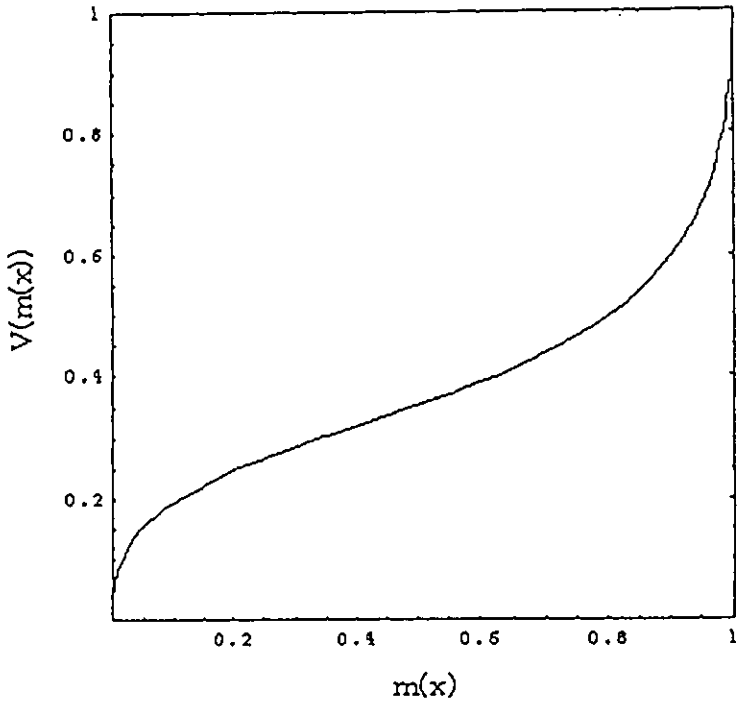

Figure3. Tversky & Kahneman's(1992) type of the evaluation function
($\lambda = 1/\gamma$, $\gamma = 0.5$)

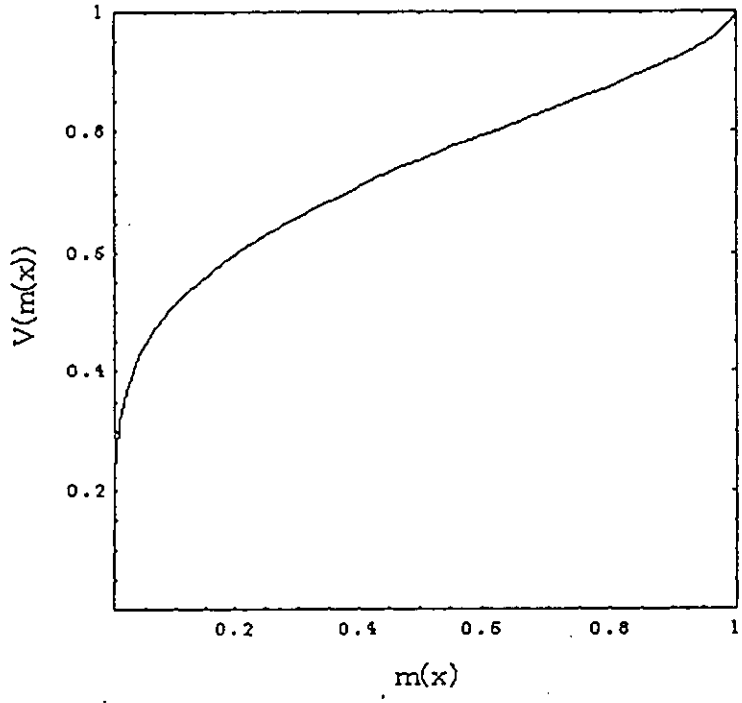

Figure4. Takemura's(1998) type of the evaluation function
($\alpha = 0.2$, $\beta = 0.5$, $w_1 = 0.8$, $w_2 = 0.2$)

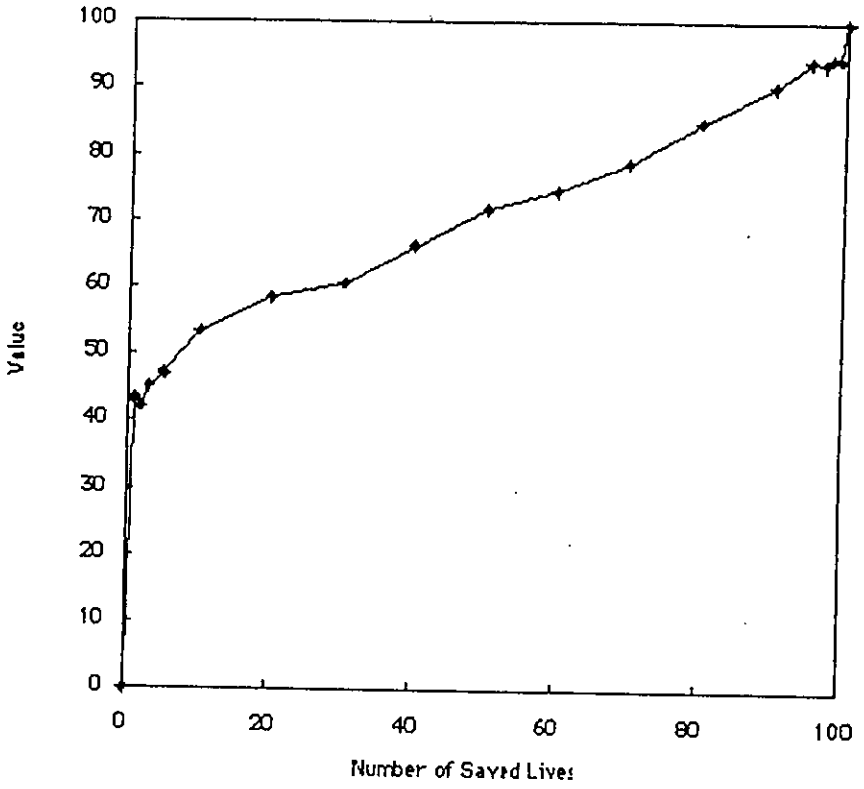

Figure5. Mean rating value for the number of saved lives