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The Behavior of Agricultural Households under constrained Off-Farm Wage Employment: An Alternative Decomposition of Their Comparative Statics Analysis

by

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Abstract

In case the off-farm wage employment is constrained, the "internal wage" plays an important role of equilibrating the demand for labor with its supply within the household. Once this distinct role is recognized, responses of quantities demanded and supplied respectively are decomposed into the direct effects of changes in exogenous variables and the "internal wage effects" of changes in the internal wage caused by those in the same exogenous variables. This method of decomposition proves to be more readily amenable to the standard microeconomic theory to enhance the tractability of agricultural household models under constrained off-farm wage employment.

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1. Introduction

Several causes are responsible for making agricultural household models nonseparable [1] (e.g., Singh, Squire, and Strauss [15]) in the sense that the household's organization of production and its choice of consumption are to be jointly determined. Among such causes constrained off-farm wage employment seems to be most commonly observed in Japanese agriculture. Actually, Arayama [1], Kang and Maruyama [7], and Sonoda and Maruyama [17] verified the relevance of this constraint in their empirical studies of Japanese rice farmers. These evidences seem to be closely related to the practice of off-farm employers who offer a "higher than equilibrium wage" to use the resulting unemployment or excess supply of labor as a worker discipline device (Shapiro and Stiglitz [13]; Bulow and Summers [4]).

In case off-farm wage employment is constrained, agricultural households seek additional employment at the wage lower than the market wage and are obliged to put their endowed time in excess of their off-farm employment into their own farm activity. Then, it is not the market wage but the discounted "internal wage" [2] that is relevant in determining both their production organization and their consumption choice (Sonoda and Maruyama [17]). At this wage the residual profit imputable to their production activity is maximized but no incentive remains for them to seek additional employment. Hence, this wage can be regarded as an equilibrium wage within the households and furthermore, the determination of it combines their organization of production and their choice of consumption inseparably to render the associated agricultural household model nonseparable.

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[1] Sasaki and Maruyama [12] refer to this property as "indecomposable", while Jorgenson and Lau [6] as "not block-recursive". Here, a term of popular use is delegated though it ordinarily refers to the property of functions but not of simultaneous equations (inequalities). The term "indecomposable" may be preferred since it refers to the property of simultaneous equations (inequalities).

[2] This wage is closely related to the virtual price of labor due to Neary and Roberts [11] and to the implicit price of labor or endowed time due to Maruyama [10].

So far two types of approaches have been taken to the detailed comparative statics analysis of the nonseparable agricultural household model under absent or constrained off-farm employment. One directly works with the optimality conditions associated with the household's welfare maximization, which may be referred to as a primal approach (e.g., Sasaki and Maruyama [12]; Maruyama [9]). The other first defines the internal wage (or the virtual price of labor) implicitly in terms of the expenditure and profit functions, then makes the comparative statics analysis of quantity variables by use of "pseudo" demand and supply functions, which include in their arguments the endogenous internal wage in place of the exogenous market wage. This second approach may be referred to as a dual approach (e.g., Strauss [18]; Besley [3]).

The primal approach tries to decompose all terms of the comparative statics analysis into the income and substitution effects. This gives rise to the "commodity-factor cross substitution effect" (Sasaki and Maruyama [12]) and the income effect in the analysis of production organization and the commodity-factor cross substitution effect in the analysis of consumption choice. These cross effects are not readily amenable to the standard theory of microeconomic analysis in which the production organization is analyzed separately from the consumption choice, which seems to have reduced the tractability of nonseparable agricultural household model in empirical applications.[3]

On the other hand, the dual approach decomposes responses of the quantities demanded or supplied into the direct effect of changes in exogenous variables with the internal wage fixed at its equilibrium level and the indirect effect of changes in the internal wage caused by those in the same exogenous variables. Thus, all terms in the comparative statics analysis turn out to be amenable to the standard theory of microeconomic analysis because changes in the internal wage are treated as changes in a factor price in the analysis of production organization and as those in a commodity

[3] In separable agricultural household models, by contrast, the analysis of production organization is independent of that of consumption choice, and furthermore the latter is affected by the former only through changes in full income, i.e., "profit effects" (Strauss [18]). Hence, the separable models are quite amenable to the standard theory of microeconomic analysis and have been frequently used for empirical studies (e.g., Lau, Lin, and Yotopoulos [8]).

price in the analysis of consumption choice. Nonetheless, few studies have followed the dual approach and estimated structural parameters of the nonseparable agricultural household model to take full advantage of the results in its comparative statics analysis. This is partly because the internal wage is not observable but is to be estimated ordinarily by use of production functions as in Jacoby [5] and Skoufias [16]. However, popular and recent use of flexible production functions (e.g., of the traslog and the generalized Leontief forms) makes it difficult to derive the properties of the corresponding profit functions required for its full comparative statics analysis.

One way to ease these difficulties may be to follow the primal approach but to decompose responses of the quantities demanded or supplied in a way similar to the dual approach. This is actually done by Sonoda and Maruyama [17] who studied the behavior of agricultural households under constrained off-farm wage employment. The purpose of this article is to propose an alternative decomposition in the line of their comparative statics analysis. Special note is paid to the distinct role played by the internal wage which equates the demand for to the supply of labor within the household. A clear recognition of its distinct role helps establish the formal equivalence between the optimality conditions of agricultural household under constrained off-farm wage employment and those in a competitive labor market within and without the household. The formal equivalence in turn paves the way to an alternative decomposition of comparative statics analysis relative to the agricultural households under this circumstance.

The following section introduces the model of agricultural household under constrained off-farm wage employment and discusses its equilibrium. The third section proposes an alternative decomposition of its comparative statics analysis and compares the results with those in terms of the conventional decomposition. The fourth section applies the proposed method of decomposition to analyze the supply response of farm commodity with special reference to the possibility of its downward sloping supply curve. The final section concludes the article.

2. The Model

It is assumed that a sufficient consensus is observed among different members of the household so that their welfare function exists. The welfare function takes on the following form:

$$W = U(C_1, C_2, Z; G), \tag{1}$$

where C_1, C_2, and Z denote the amounts consumed of a home produced farm commodity, purchased commodities, and leisure, respectively. G represents the vector of shift factors of the function U(·). The welfare function U(·) is assumed to be well-behaved in the usual sense.

The household is subject to a number of constraints, the first of which is the budget constraint.[4]

$$pC_1 + p'C_2 + wZ \leq M, \tag{2}$$

where p, p', and w denote the prices of a farm commodity, purchased commodities, and the market wage, respectively. M denotes the full income (Becker [2]) to be defined in the following way:

$$M \equiv wTe + \pi, \quad \pi \equiv pX - wL_1 - qF, \tag{3}$$

where X, L_1, Te, q, and F denote the amount of a farm commodity produced, hours of farm labor, endowed time of this household, the price of current inputs, and their quantity, respectively.

The household is assumed to produce an amount X of a farm commodity and consume C_1 (< X) of it within the household. The amount produced, X, is bounded by the production possibility.

$$X \leq f(L_1, F; K), \tag{4}$$

where K denotes the vector of fixed factors. The production function f(·) is assumed to be well-behaved in the usual sense.

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[4] For simplicity, tax, savings, and unearned incomes are omitted in this study.

Finally, it is assumed that the household is self-employing and perceives off-farm employment open to it to be constrained as in Kang and Maruyama [7] and Sonoda and Maruyama [17].[5]

$$L_2 = Te - Z - L_1 \leq L = \text{constant}. \quad (5)$$

This constraint seems to be closely related to the practice of off-farm employers who offer a "higher than equilibrium wage" (e.g., Shapiro and Stiglitz [13]; Bulow and Summers [4]) to use the resulting excess supply of labor as a worker discipline device.

The household is assumed to be a price taker in all markets and to maximize its welfare, W, subject to the constraints (2)-(5). The Kuhn-Tucker-Lagrange conditions of optimality associated with this problem imply the following relations,

$$w^* \equiv w - \mu/\lambda \leq w, \qquad w^* = w \quad \text{for } \mu = 0,$$

$$pf_1(L_1,F;K) - w^* \leq 0, \quad (6.1)$$

$$pf_2(L_1,F;K) - q \leq 0, \quad (6.2)$$

$$U_1(C_1,C_2,Z;G) - \lambda p \leq 0, \quad (6.3)$$

$$U_2(C_1,C_2,Z;G) - \lambda p' \leq 0, \quad (6.4)$$

$$U_3(C_1,C_2,Z;G) - \lambda w^* \leq 0, \quad (6.5)$$

$$-pC_1 - p'C_2 - w^*Z + Y \geq 0, \quad (6.6)$$

$$Z + L_1 + L - Te \geq 0, \quad (6.7)$$

where $\lambda \geq 0$ and $\mu \geq 0$ denote the Lagrange multipliers associated with the constraints (2) and (5), respectively, and f_i and U_j denote respectively the first derivatives of the functions $f(\cdot)$ and $U(\cdot)$. The complementary slackness conditions are suppressed for simplicity. Y and π^* denote the full income and the residual profit imputable to the farm production activity, respectively, evaluated at the wage w^*.

$$Y \equiv w^*Te + (w - w^*)L + \pi^*, \quad \pi^* \equiv pX - w^*L_1 - qF.$$

The inequalities in (6.1) and (6.2) that are directly associated with the determination of production organization share the Lagrange multipliers λ and μ or the newly defined variable w^* with the inequalities in (6.3)-(6.7) that are directly associated with the

[5] In the relations in (5), it is assumed that dependents consume all their endowed time for leisure.

determination of consumption choice. Hence, the system of inequalities in (6.1)-(6.7) is nonseparable in the sense that production organization and consumption choice are to be jointly determined. Nonseparability of this system has a significant impact on its comparative statics, so that the income effect and others inherent in the comparative statics of consumption choice creep into those of production organization and render both the supply of commodities and demand for factors less elastic. In extreme cases, these effects give rise to downward-sloping supply and upward-sloping demand curves.

For interior solutions, (6.1), (6.3), and (6.5) imply that

$$pf_1(\cdot) = pU_3(\cdot)/U_1(\cdot) = w^* = w - \mu/\lambda < w. \tag{7}$$

The supply or reservation wage, $pU_3(\cdot)/U_1(\cdot)$, falls short of the market wage, w, due to severe constraint on off-farm employment and the household still seeks additional employment at the wage lower than the market wage. The term μ/λ represents the amount of discount it is prepared to allow. Since no additional off-farm employment is available, the household is obliged to put the remainder of its endowed time into its own farm production activity. Hence the marginal revenue product of farm labor or the demand wage of this household, $pf_1(\cdot)$, in turn falls short of the market wage by the amount μ/λ.

Thus, in case off-farm employment is constrained, it is not the market wage w but the discounted wage w^* that is relevant in determining both the household's organization of production and its choice of consumption. The relations in (7) together with (6.7) in equality imply that at the discounted wage w^* the sum of demands for on- and off-farm labor, $L_1 + L_2$, is equilibrated with its supply, $Te - Z$, or equivalently its demand for on-farm labor, L_1, with its supply in excess of off-farm employment, $Te - Z - L_2$, within the household. Hence, the wage w^* can be regarded as a kind of "equilibrium wage within the household" or simply "internal wage" in the internal market for labor.

In case the off-farm employment constraint is not binding hence μ vanishes, the system of inequalities in (6.1)-(6.7) turns out to be separable so that the determination of production organization is independent of that of consumption choice. For the convenience of subsequent analyses, the optimality conditions for this case are presented.

$$pf_1(L_1,F;K) - w \leq 0, \tag{8.1}$$

$$pf_2(L_1,F;K) - q \leq 0, \tag{8.2}$$

$$U_1(C_1,C_2,Z;G) - \lambda p \leq 0, \tag{8.3}$$

$$U_2(C_1,C_2,Z;G) - \lambda p' \leq 0, \tag{8.4}$$

$$U_3(C_1,C_2,Z;G) - \lambda w \leq 0, \tag{8.5}$$

$$-pC_1 - p'C_2 - wZ + M \geq 0, \tag{8.6}$$

$$Z + L_1 + L - Te \geq 0. \tag{8.7}$$

For interior solutions, (8.1), (8.3), and (8.5) imply that the supply or reservation wage, $pU_3(\cdot)/U_1(\cdot)$, is equated to the market wage w so that the household now has no incentive to seek additional employment at this wage. The marginal revenue product of labor, $pf_1(\cdot)$, is also equated to the market wage w so that the residual profit imputable to the farm production activity is maximized at this wage. Hence, the constraint (8.7) on off-farm employment turns out to be redundant, though it is kept to show a clear correspondence to the system (6.1)-(6.7) above.

It is clear from the preceding analysis that the optimality conditions in (6.1)-(6.7) for the nonseparable model and those in (8.1)-(8.7) for the separable model share a formal equivalence. In the former, the residual profit imputable to production activity is maximized and the household has no incentive to seek additional employment at the internal wage w^*. Thus, when the household is in equilibrium, the constraint (6.7) on off-farm employment ceases to be binding and can be suppressed without loss of analytical rigor as the corresponding constraint (8.7) can be in the separable model. This will prove to be extremely instrumental in highlighting the important role played by the internal wage w^* in the subsequent comparative statics analysis. However, a fundamental difference between these two models still remains. The internal wage w^* is an endogenous variable to be determined in the nonseparable model, while it is identically equal to the exogenous market wage in the separable model.

3. An Alternative Decomposition of the Comparative Statics Analysis under Constrained Off-Farm Wage Employment

How does the household respond to changes in the prices of current inputs, purchased commodities, and a home-produced farm commodity under constrained off-farm employment? Its response can be examined by the comparative statics analysis of the optimality conditions in (6.1)-(6.7) for interior solutions. The result of this analysis is shown compactly in a matrix expression.

$$\begin{bmatrix} pf_{11} & pf_{12} & 0 & 0 & 0 & 0 & -1 \\ pf_{21} & pf_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & U_{11} & U_{12} & U_{13} & -p & 0 \\ 0 & 0 & U_{21} & U_{22} & U_{23} & -p' & 0 \\ 0 & 0 & U_{31} & U_{32} & U_{33} & -w^* & -\lambda \\ 0 & 0 & -p & -p' & -w^* & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} dL_1 \\ dF \\ dC_1 \\ dC_2 \\ dZ \\ d\lambda \\ dw^* \end{bmatrix} = \begin{bmatrix} 0 & 0 & -f_1 \\ 0 & 1 & -f_2 \\ 0 & 0 & \lambda \\ \lambda & 0 & 0 \\ 0 & 0 & 0 \\ C_2 & F & C_1 - X \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} dp' \\ dq \\ dp \end{bmatrix}, \qquad (9)$$

or equivalently,

$$\begin{bmatrix} pf_{11} & pf_{12} & 0 & 0 & 0 & 0 \\ pf_{21} & pf_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & U_{11} & U_{12} & U_{13} & -p \\ 0 & 0 & U_{21} & U_{22} & U_{23} & -p' \\ 0 & 0 & U_{31} & U_{32} & U_{33} & -w^* \\ 0 & 0 & -p & -p' & -w^* & 0 \end{bmatrix} \begin{bmatrix} dL_1 \\ dF \\ dC_1 \\ dC_2 \\ dZ \\ d\lambda \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ -\lambda \\ 0 \end{bmatrix} dw^* = \begin{bmatrix} 0 & 0 & -f_1 \\ 0 & 1 & -f_2 \\ 0 & 0 & \lambda \\ \lambda & 0 & 0 \\ 0 & 0 & 0 \\ C_2 & F & C_1 - X \end{bmatrix} \begin{bmatrix} dp' \\ dq \\ dp \end{bmatrix}, \qquad (10.1)$$

$$dL_1 + dZ = 0. \qquad (10.2)$$

Equation (10.2) can be suppressed without loss of analytical rigor when the household is in equilibrium, although it must be taken into consideration in case responses of the internal wage itself are to be examined. Then, (10.1) can be written in a separated form.

$$\begin{bmatrix} pf_{11} & pf_{12} \\ pf_{21} & pf_{22} \end{bmatrix} \begin{bmatrix} dL_1 \\ dF \end{bmatrix} = \left\{ \begin{bmatrix} 0 & 0 & -f_1 \\ 0 & 1 & -f_2 \end{bmatrix} + \begin{bmatrix} \partial w^*/\partial p' & \partial w^*/\partial q & \partial w^*/\partial p \\ 0 & 0 & 0 \end{bmatrix} \right\} \begin{bmatrix} dp' \\ dq \\ dp \end{bmatrix}, \qquad (11.1)$$

$$\begin{bmatrix} U_{11} & U_{12} & U_{13} & -p \\ U_{21} & U_{22} & U_{23} & -p' \\ U_{31} & U_{32} & U_{33} & -w^* \\ -p & -p' & -w^* & 0 \end{bmatrix} \begin{bmatrix} dC_1 \\ dC_2 \\ dZ \\ d\lambda \end{bmatrix} = \left\{ \begin{bmatrix} 0 & 0 & \lambda \\ \lambda & 0 & 0 \\ 0 & 0 & 0 \\ C_2 & F & C_1 - X \end{bmatrix} + \lambda \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \partial w^*/\partial p' & \partial w^*/\partial q & \partial w^*/\partial p \\ 0 & 0 & 0 \end{bmatrix} \right\} \begin{bmatrix} dp' \\ dq \\ dp \end{bmatrix}. \quad (11.2)$$

For the convenience of comparison, the result of a similar analysis for the separable competitive case is presented.

$$\begin{bmatrix} pf_{11} & pf_{12} \\ pf_{21} & pf_{22} \end{bmatrix} \begin{bmatrix} dL_1 \\ dF \end{bmatrix} = \begin{bmatrix} 0 & 0 & -f_1 \\ 0 & 1 & -f_2 \end{bmatrix} \begin{bmatrix} dp' \\ dq \\ dp \end{bmatrix}, \quad w^* = w, \qquad (12.1)$$

$$\begin{bmatrix} U_{11} & U_{12} & U_{13} & -p \\ U_{21} & U_{22} & U_{23} & -p' \\ U_{31} & U_{32} & U_{33} & -w \\ -p & -p' & -w & 0 \end{bmatrix} \begin{bmatrix} dC_1 \\ dC_2 \\ dZ \\ d\lambda \end{bmatrix} = \begin{bmatrix} 0 & 0 & \lambda \\ \lambda & 0 & 0 \\ 0 & 0 & 0 \\ C_2 & F & C_1 - X \end{bmatrix} \begin{bmatrix} dp' \\ dq \\ dp \end{bmatrix}, \quad w^* = w. \qquad (12.2)$$

Equations (11.1) and (11.2) suggest that responses of quantity variables, L_1, F, C_1, C_2, and Z, can be decomposed into two parts, one representing a direct effect of changes in exogenous variables, p', q, and p, and the other an indirect effect of changes in the internal wage w^* caused by those in the same exogenous variables. The latter effect is not present in the separable competitive case (12.1) and (12.2), where w^* is identically equal to the market wage w and is not directly affected by changes in exogenous variables other than w. This indirect effect may be referred to as an "internal wage effect" by its construction, which is unique to the nonseparable case and plays an important role in coordinating changes in the production organization and those in the consumption choice. Responses of the internal wage w^* itself will be somewhat closely examined in the subsequent analysis.

Now, these results of the proposed decomposition will be compared with the corresponding ones of the conventional decomposition (e.g., Sasaki and Maruyama [12]) to characterize the former. Conventionally, the decomposition analysis has been directly applied to equation (9) without distinguishing the internal wage w^* from other endogenous variables, so that its role is submerged obscurely in the overall analysis of comparative statics. Whereas it is distinguished and given a distinct treatment in the proposed method as shown in (11.1) and (11.2), so that its role in the comparative statics analysis is highlighted. To see further details of the alternative method of decomposition, responses of farm labor, L_1, to changes in selected exogenous variables are examined. Similar responses of leisure consumption, Z, are examined for reference in Appendix below. To facilitate comparison, equations (9), (11.1), and (12.1) respectively are solved for dL_1 and the results are presented in this order.

a) Responses of farm labor L_1 to changes in the price p' of purchased commodities

$$\frac{\partial L_1}{\partial p'} = \frac{1}{|A|}(\lambda A_{41} + C_2 A_{61}), \tag{13.1}$$

$$\frac{\partial L_1}{\partial p'} = \frac{1}{|B|} p f_{22} \frac{\partial w^*}{\partial p'}, \tag{13.2}$$

$$\frac{\partial L_1}{\partial p'} = 0 \text{ and } w^* = w, \tag{13.3}$$

where A and B denote the matrices of coefficients on the left hand side (LHS for short) of (9) and (11.1), respectively, and A_{ij} denotes the cofactors associated with the elements a_{ij} in $|A|$. The first term on the right hand side (RHS for short) of (13.1) represents the commodity-factor cross substitution effect (Sasaki and Maruyama [12]) which describes the indirect substitution between farm labor and purchased commodities mediated through the substitution between leisure and purchased commodities. The second term on the RHS of the same equation represents the income effect on farm labor demand mediated through the income effect on leisure consumption. Thus, these two effects are taken care of by a single internal wage effect in the proposed method of decomposition in (13.2): Changes in the price p' of purchased commodities have no effect on the demand for farm labor in the separable competitive case, where its determination is independent of consumption choice.

b) Responses of farm labor L_1 to changes in the price q of current inputs

$$\frac{\partial L_1}{\partial q} = \frac{1}{|A|}(A_{21} + F A_{61}), \tag{14.1}$$

$$\frac{\partial L_1}{\partial q} = \frac{1}{|B|}(-pf_{12} + pf_{22}\frac{\partial w^*}{\partial q}), \tag{14.2}$$

$$\frac{\partial L_1}{\partial q} = \frac{1}{|B|}(-pf_{12}) \text{ and } w^* = w. \tag{14.3}$$

The first terms on the RHS's of these equations represent the factor substitution effects. The second term on the RHS of (14.1) represents the income effect, while that of (14.2) the internal wage effect. Here, only the income effect corresponds to the internal wage effect since the commodity-factor cross substitution effect is not relevant in this case.

c) Responses of farm labor L_1 to changes in the price p of farm commodity

$$\frac{\partial L_1}{\partial p} = \frac{1}{|A|}\{-f_1 A_{11} - f_2 A_{21} + \lambda A_{31} + (C_1 - X)A_{61}\}, \tag{15.1}$$

$$\frac{\partial L_1}{\partial p} = \frac{1}{|B|}(-pf_1 f_{22} + pf_2 f_{12} + pf_{22}\frac{\partial w^*}{\partial p}), \tag{15.2}$$

$$\frac{\partial L_1}{\partial p} = \frac{1}{|B|}(-pf_1 f_{22} + pf_2 f_{12}) \text{ and } w^* = w. \tag{15.3}$$

The first two terms on the RHS's of these equations represent the direct effects of changes in the price of farm commodity. The third and fourth terms on the RHS of (15.1) represent the commodity-factor cross substitution and the income effects, respectively, while the third term on the RHS of (15.2) the internal wage effect. Here, the pair of the commodity-factor cross substitution and the income effects corresponds to the internal wage effect.

Thus, for all these responses of farm labor the pairs of the commodity-factor cross substitution and the income effects in the conventional decomposition correspond to the internal wage effects in the proposed one wherever all these effects are relevant. Furthermore, it is shown in Appendix that the similar pairs in the conventional decomposition correspond to the internal wage and the income effects in the proposed decomposition of responses of leisure consumption wherever all these effects are relevant. Hence, it should be clear from this examination that the proposed decomposition can do without both the commodity-factor cross substitution and the income effects in the analysis of production organization and furthermore that it can do without the commodity-factor cross substitution effect in the analysis of consumption choice.[6]

Now, we turn to the examination of responses of the internal wage itself. The discussion above suggests that the response of any quantity variable, Q, to changes in an exogenous variable, v, can be decomposed in the following way:[7]

$$dQ = \left.\frac{\partial Q}{\partial v}\right|_{dw^*=0} dv + \frac{\partial Q}{\partial w^*}\frac{\partial w^*}{\partial v} dv .$$

To obtain the response of the internal wage, $\partial w^*/\partial v$, substitute this relation for $Q = L_1$ and for $Q = Z$ into equation (10.2) which has been suppressed so far, and solve for $\partial w^*/\partial v$, then,

$$\frac{\partial w^*}{\partial v} = -\left(\left.\frac{\partial L_1}{\partial v}\right|_{dw^*=0} + \left.\frac{\partial Z}{\partial v}\right|_{dw^*=0}\right)\Bigg/\left(\frac{\partial L_1}{\partial w^*} + \frac{\partial Z}{\partial w^*}\right). \tag{16}$$

The first and second terms in the numerator on the RHS respectively represent the response of farm labor, L_1, and leisure consumption, Z, to changes in the exogenous variable v with the internal wage w^* being fixed at its equilibrium level. Whereas the first and second terms in the denominator respectively represent their response to changes in the internal wage itself. Since both the terms in the denominator, $\partial L_1/\partial w^* = pf_{22}/|B|$ and $\partial Z/\partial w^* = \lambda C_{33}/|C|$, prove to be negative, the internal wage rises, falls, or is left invariant according as the sum of the response of farm labor and leisure consumption increases, decreases, or remains constant with the internal wage being fixed.

For selected exogenous variables, the signs of responses of the internal wage in (16) are determined under certain conditions. If the production function satisfies the condition $f_{12} > 0$, the following relations are verified from (13.2), (14.2), and (15.2) with

[6] It should be noted that this does not imply that the sum of the commodity-factor cross substitution effect and the income effect is equal to the corresponding internal wage effect. This point will be examined in Figure 1 to be introduced below.
[7] It is to be noted that $pf_{22}/|B| = \partial L_1/\partial w^*$ in equations (13)-(15) above and that $\lambda C_{33}/|C| = \partial Z/\partial w^*$ in equations (A1)-(A3) in Appendix below.

changes in the internal wage being equal to nil.

$$\left.\frac{\partial L_1}{\partial p'}\right|_{dw^*=0} = 0,\ \left.\frac{\partial L_1}{\partial q}\right|_{dw^*=0} < 0,\ \text{and}\ \left.\frac{\partial L_1}{\partial p}\right|_{dw^*=0} > 0. \tag{17}$$

Furthermore, suppose that leisure is a normal good, that leisure is a substitute for farm commodity and a gross substitute for purchased commodities, and that the household is a net seller of farm commodity. Then, the following relations are verified from (A1.2), (A2.2), and (A3.2) in Appendix below with changes in the internal wage being equal to nil.

$$\left.\frac{\partial Z}{\partial p'}\right|_{dw^*=0} > 0,\ \left.\frac{\partial Z}{\partial q}\right|_{dw^*=0} < 0,\ \text{and}\ \left.\frac{\partial Z}{\partial p}\right|_{dw^*=0} > 0. \tag{18}$$

The relations in (17) and (18) are combined with equation (16) to imply that

$$\frac{\partial w^*}{\partial p'} > 0,\ \frac{\partial w^*}{\partial q} < 0,\ \text{and}\ \frac{\partial w^*}{\partial p} > 0. \tag{19}$$

Thus, a rise in the prices of purchased commodities p', current inputs q, and a farm commodity p causes the internal wage w^* to rise, fall, and rise, respectively.

Finally, we will give an intuitive exposition of the properties of the internal wage w^*. Figure 1 illustrates how the demand for labor, $L_D = L_1 + L_2$, and its supply, $L_S = Te - Z$, are adjusted in the internal market for labor within the household. The demand function in period i, $L_{D,i}$ (i = 1, 2), consists of the horizontal sum of the demand wage (or marginal revenue product of farm labor) function and the market wage function representing the off-farm employment L at the constant market wage w. The supply function in period i, $L_{S,i}$ (i = 1, 2), consists of the supply wage (or reservation wage) function. The slopes of the demand and supply functions are evaluated to be $\partial L_D/\partial w^* = pf_{22}/|B| < 0$ and $\partial L_S/\partial w^* = -\lambda C_{33}/|C| > 0$, respectively, and the equilibrium of the internal market is determined at the point of intersection E_i of the two functions. w_i^*, $L_{1,i}$, $L_{2,i}$ and Z_i denote the internal wage and hours of farm work, off-farm work, and leisure in period i, where hours of farm work in period i, $L_{1,i}$, is the sum of $L_{1,i}^{(1)}$ and $L_{1,i}^{(2)}$ in this figure. In case the relations in (17) and (18) hold, a rise in the price p of farm commodity causes the demand and supply functions of labor to shift to the right and to the left, respectively. Hence, the internal wage rises from w_1^* to w_2^* as shown in this figure as well as in the relation (19) above.

Furthermore, the response of the demand for farm labor, L_1, is decomposed by the two alternative methods, where C-F cross substitution effect in the figure refers to the commodity-factor cross substitution effect. In the conventional decomposition (arrows with dotted lines), the direct effect of a rise in the price p (the first two terms on the RHS of (15.1)) is represented by the rightward shift of the labor demand function along the labor supply function in period 1, $L_{S,1}$. Furthermore, the commodity-factor cross substitution and income effects (the last two terms on the RHS of (15.1)), reflecting the leftward shift of the labor supply function,[8] reduce the demand for farm labor down to the level corresponding to the point E_2. On the other hand, in the proposed decomposition (arrows with solid lines), the direct effect of a rise in the price p (the first two terms on the RHS of (15.2)) is represented by the rightward shift of the labor demand function with the internal wage being fixed at w_1^*. Unlike the direct effect in the conventional decomposition, this direct effect is similar to the corresponding one in the separable competitive model, where the determination of labor demand is independent of that of labor supply. Since the demand for labor exceeds its supply at w_1^*, the internal wage should rise to w_2^* to restore the equilibrium between the demand for and the supply of labor within the household. The rise of the internal wage, in turn, has an internal wage effect (the last term on the RHS of (15.2)) to reduce the demand for farm labor down to the level corresponding to the point E_2.

[8] This leftward shift of the labor supply function is caused by the sum of the commodity substitution and the income effects on leisure consumption represented by the last two terms in equation (A3.1) in Appendix. Note that the sum of these two effects on leisure consumption corresponds to the sum of the similar effects on farm labor demand in equation (15.1), i.e.,

$$\{\lambda A_{31} + (C_1 - X)A_{61}\}/|A| = -\{\lambda A_{35} + (C_1 - X)A_{65}\}/|A|.$$

4. Application of the Proposed Decomposition to the Supply Response of Farm Commodity

Now, the supply response of farm commodity is examined by means of the proposed method of decomposition to illustrate its applicability in the analysis of agricultural households' behavior under constrained off-farm employment.

Differentiating the supply function of a farm commodity $X = f(L_1, F; K, T)$ with respect to its price p in equilibrium,

$$\frac{\partial X}{\partial p} = f_1 \frac{\partial L_1}{\partial p} + f_2 \frac{\partial F}{\partial p}. \tag{20}$$

The responses of farm labor L_1 and current inputs F in (20) respectively are evaluated by use of (11.1).

$$\frac{\partial L_1}{\partial p} = \left.\frac{\partial L_1}{\partial p}\right|_{dw^*=0} + \frac{pf_{22}}{|B|}\frac{\partial w^*}{\partial p}, \ |B| > 0, \tag{21}$$

$$\frac{\partial F}{\partial p} = \left.\frac{\partial F}{\partial p}\right|_{dw^*=0} - \frac{pf_{21}}{|B|}\frac{\partial w^*}{\partial p}, \ |B| > 0, \tag{22}$$

where

$$\left.\frac{\partial L_1}{\partial p}\right|_{dw^*=0} = \frac{p(f_2 f_{12} - f_1 f_{22})}{|B|}, \quad \left.\frac{\partial F}{\partial p}\right|_{dw^*=0} = \frac{p(f_1 f_{21} - f_2 f_{11})}{|B|}.$$

Substituting (21) and (22) into (20), the supply response of farm commodity is evaluated by the proposed method of decomposition.

$$\frac{\partial X}{\partial p} = \left.\frac{\partial X}{\partial p}\right|_{dw^*=0} + \frac{\partial X}{\partial w^*}\frac{\partial w^*}{\partial p}, \tag{23}$$

where the response of the internal wage, $\partial w^*/\partial p$, is evaluated by use of equation (16) and

$$\left.\frac{\partial X}{\partial p}\right|_{dw^*=0} = \frac{p(2f_1 f_2 f_{12} - f_1^2 f_{22} - f_2^2 f_{11})}{|B|} > 0 \ \text{ for } f_{12} > 0, \tag{24}$$

$$\frac{\partial X}{\partial w^*} = \frac{p(f_1 f_{22} - f_2 f_{21})}{|B|} < 0 \ \text{ for } f_{12} > 0. \tag{25}$$

The first term on the RHS of (23) represents the direct effect of a change in the price of farm commodity and is positive for $f_{12} > 0$. On the other hand, the second term represents the internal wage effect unique to the behavior of agricultural households under constrained off-farm employment and is negative in case the relation $\partial w^*/\partial p > 0$ in (19) holds. Therefore, this internal wage effect should cause the supply of farm commodity to be reduced. The relative importance of the two effects can be evaluated only by empirical analyses to estimate parameters of the production and welfare functions, which have actually been done by Sonoda and Maruyama [17] for a Cobb-Douglas production function and a Stone-Geary type welfare function.

The decomposition of the supply response in (23) is illustrated in Figure 2. $X(w_i^*)$ (i = 1, 2) represents the supply function of farm commodity with the internal wage equal to w_i^*, while p_i and X_i represent its price and quantity supplied in period i, respectively, and E_i is the point of equilibrium corresponding to E_i in Figure 1. In case the price of farm commodity rises from p_1 to p_2, the household expands its supply from X_1 to X_1' as long as the internal wage remains at w_1^*, reflecting the direct effect given by (24). However, the internal wage should rise from w_1^* to w_2^* as shown in (19) as well as in Figure 1, which shifts the supply function of farm commodity from $X(w_1^*)$ to $X(w_2^*)$ due to the effect shown in (25). Therefore, the household will reduce its supply of farm commodity from X_1' to X_2, reflecting the negative internal wage effect shown in (23). In case the internal wage effect exceeds the direct effect as shown in Figure 2, the observed supply function will have a downward slope.

5. Conclusion

A number of authors have provided empirical evidences to suggest the constrained off-farm wage employment for Japanese rice farmers. In case off-farm wage employment is constrained, it is not the market wage but the internal wage that is relevant in determining both their production organization and their consumption choice. At this wage, the residual profit imputable to their production activity is maximized, and furthermore no incentive remains for them to seek additional employment. Hence, this wage can be regarded as an equilibrium wage within their households and the determination of this wage renders their organization of production and their choice of consumption nonseparable.

Conventionally, the comparative statics analysis of this nonseparable model has directly been applied to the optimality conditions of agricultural households without paying due attention to the difference in roles played by various endogenous variables. This gives rise to the income and the commodity-factor cross substitution effects in the analysis of production organization and the commodity-factor cross substitution effects in the analysis of consumption choice. These cross effects are not readily amenable to the standard theory of microeconomic analysis in which the production organization and the consumption choice are separately studied. Thus, their presence seems to have reduced the tractability of nonseparable agricultural household models in empirical researches.

The internal wage plays a distinct role in implementing the equilibrium of the demand for labor with its supply within the household, while other endogenous variables represent the quantities of commodities or production factors. This distinct role of the internal wage deserves a separate treatment in the comparative statics analysis, which enables us to replace the cross effects by more tractable internal wage effects. Thus the comparative statics analysis of agricultural household models under constrained off-farm employment turns out to be more readily amenable to the standard theory of microeconomic analysis to enhance their tractability in empirical applications.

Finally, it should be clear from our analysis that a similar method of decomposition can be applied in analyzing the behavior of agricultural households subject to the constraints on their output of home produced commodities. Such an analysis is addressed by Sicular [14] in connection with their sales quotas and by Becker [2] and others in connection with their home production with the corresponding "sales quotas" of them regarded as being equal to nil. In these cases the internal prices, $p^* = p + v/\lambda$, take the place of the internal wage, w^*, in this article, where v denote the Lagrange multipliers associated with the constraints on the home-produced commodities.

Appendix. Responses of Leisure Consumption to Changes in Selected Exogenous Variables

a) Responses of leisure consumption Z to changes in the price p' of purchased commodities

$$\frac{\partial Z}{\partial p'} = \frac{1}{|A|}(\lambda A_{45} + C_2 A_{65}), \tag{A1.1}$$

$$\frac{\partial Z}{\partial p'} = \frac{1}{|C|}(\lambda C_{23} + \lambda C_{33}\frac{\partial w^*}{\partial p'} + C_2 C_{43}), \tag{A1.2}$$

$$\frac{\partial Z}{\partial p'} = \frac{1}{|C|}(\lambda C_{23} + C_2 C_{43}) \text{ and } w^* = w, \tag{A1.3}$$

where C denotes the matrix of coefficients on the LHS of equation (11.2) and C_{ij} the cofactors associated with the elements c_{ij} in $|C|$. The first terms on the RHS's of these equations represent the commodity substitution effects, while the final terms the income effects. The second term on the RHS of (A1.2) represents the internal wage effect. The commodity-factor cross substitution effect does not arise in (A1.1) since both leisure consumption and the price of purchased commodities are directly related to the consumption choice.

b) Responses of leisure consumption Z to changes in the price q of current inputs

$$\frac{\partial Z}{\partial q} = \frac{1}{|A|}(A_{25} + FA_{65}), \tag{A2.1}$$

$$\frac{\partial Z}{\partial q} = \frac{1}{|C|}(\lambda C_{33}\frac{\partial w^*}{\partial q} + FC_{43}), \tag{A2.2}$$

$$\frac{\partial Z}{\partial q} = \frac{1}{|C|}(FC_{43}) \text{ and } w^* = w. \tag{A2.3}$$

The final terms on the RHS's of these equations represent the income effects. The first term on the RHS of (A2.1) represents the commodity-factor cross substitution effect, while that of (A2.2) the internal wage effect. Thus, the pair of the commodity-factor cross substitution and the income effects in the conventional decomposition corresponds to the pair of the internal wage and the income effects in the proposed one.

c) Responses of leisure consumption Z to changes in the price p of farm commodity

$$\frac{\partial Z}{\partial p} = \frac{1}{|A|}\{-f_1 A_{15} - f_2 A_{25} + \lambda A_{35} + (C_1 - X)A_{65}\}, \tag{A3.1}$$

$$\frac{\partial Z}{\partial p} = \frac{1}{|C|}\{\lambda C_{13} + \lambda C_{33} \frac{\partial w^*}{\partial p} + (C_1 - X)C_{43}\}, \tag{A3.2}$$

$$\frac{\partial Z}{\partial p} = \frac{1}{|C|}\{\lambda C_{13} + (C_1 - X)C_{43}\} \text{ and } w^* = w. \tag{A3.3}$$

The final terms on the RHS's of these equations represent the income effects, while the third term on the RHS of (A3.1) and the first term on the RHS's of (A3.2) and (A3.3) the commodity substitution effects. The first two terms on the RHS of (A3.1) represent the commodity-factor cross substitution effects, while the second term on the RHS of (A3.2) the internal wage effect.

Thus in all these equations relative to the response of leisure consumption the pair of the commodity-factor cross substitution and the income effects in the conventional decomposition corresponds to the pair of the internal wage and the income effects in the proposed one wherever all these effects are relevant.

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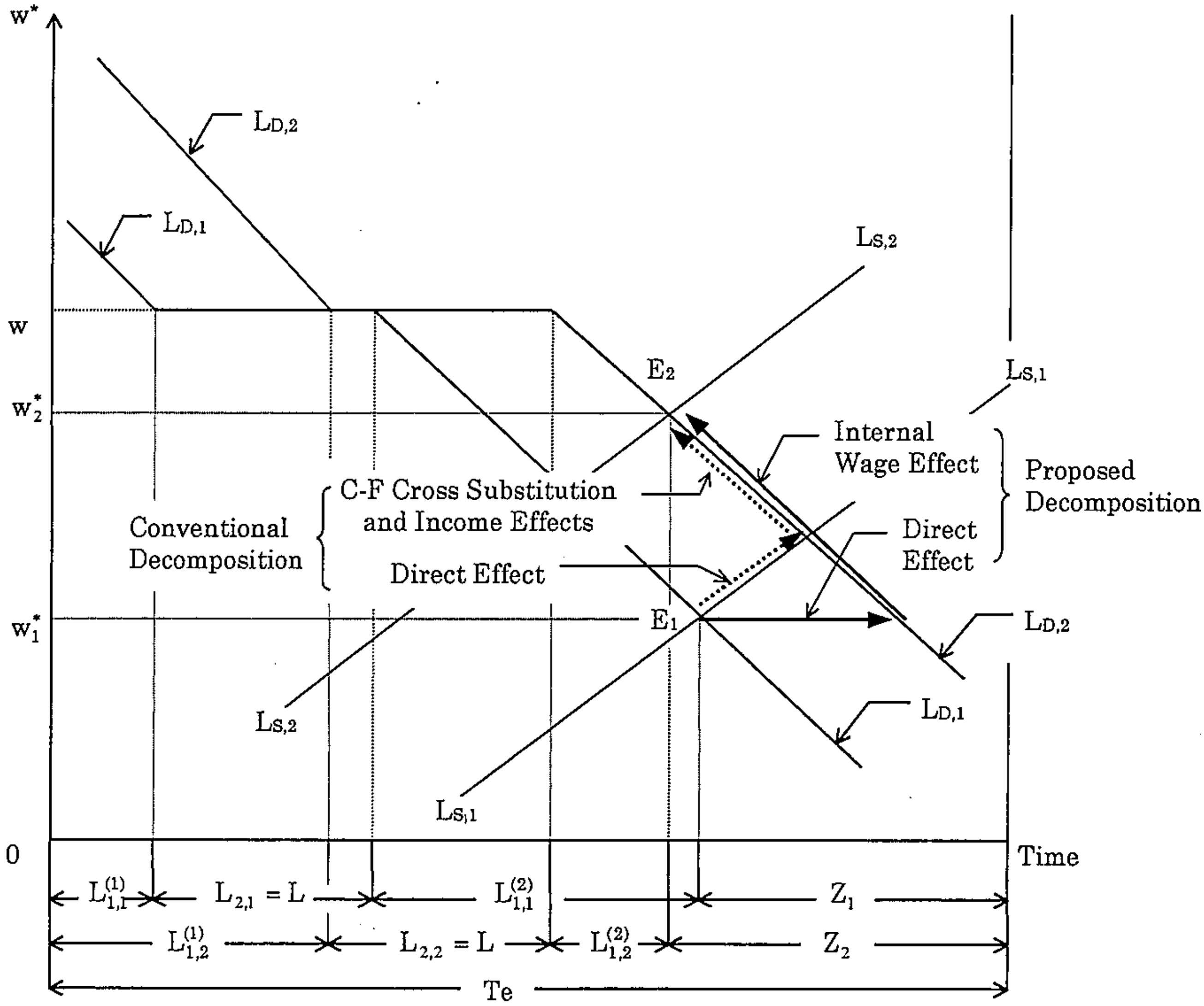

Figure 1. The internal market for labor within the household

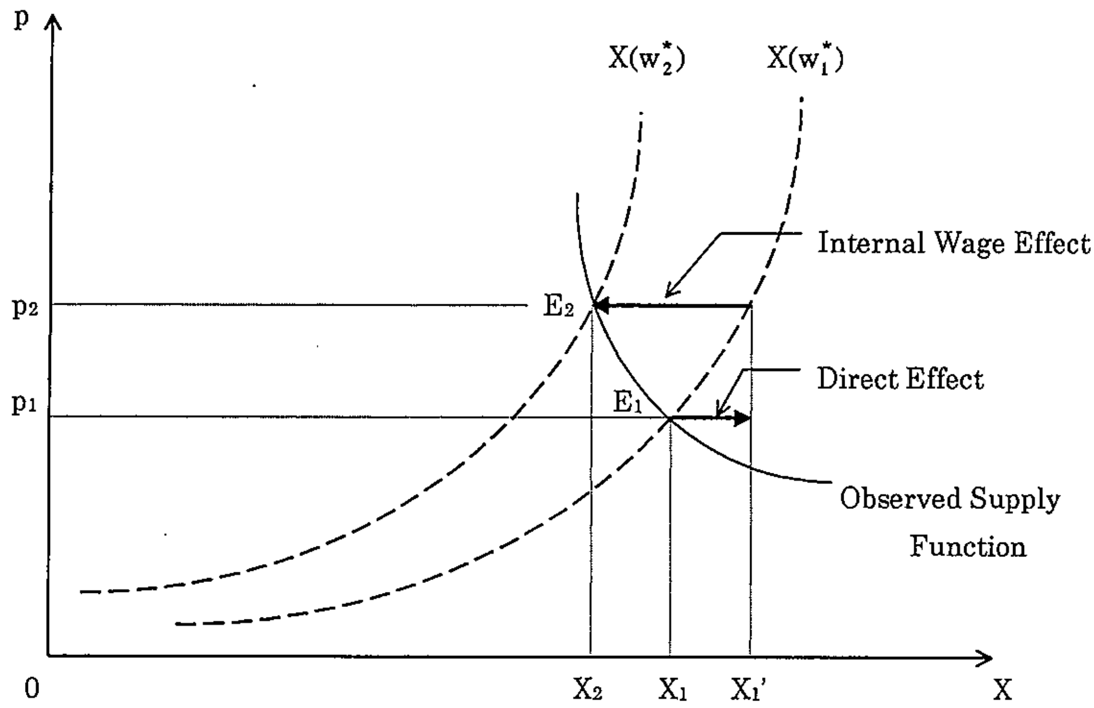

Figure 2. Response of the supply of farm commodity to a change in its price