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Cross-Border Shopping and Commodity Tax Harmonization

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Abstract

Corporate tax policy such as tax competition, tax harmonization and tax coordination has been a serious concern for the EU. In this paper, we formulate multi-country commodity tax competition models where each countrylying on a linear market chooses the tax rate that maximizes its tax revenue within a common band with an eye on strategic consideration. Then we explore the effect on the tax rates, the tax revenues and the number of the cross-border shoppers of narrowing the common band. Also, we explore the relationship between the tax revenues and the position and size of the countries.

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1. Introduction

In the EU today, different countries apply different commodity taxes without having border controls. This tax differentials with free movement of customers induces cross-border shopping within the EU. Since cross-border shopping has important effect on each government revenues, corporate tax policy such as tax competition, tax harmonization and tax coordination has been a serious concern for the EU: see Sinn(1990), Christiansen(1994), Radaelli(1997), Robson(1998). In the field of corporate tax policy, three main approaches can be considered: the first is an absolute equalisation at a common tax rate: the second is an approximation with a common band: the last is a complete tax competition. In general, the two former approaches are called *tax harmonization*. The tax equalization approach not only restricts the revenue losses associated with cross-border shopping, but also is necessary to move forward with economic and later political integration. On the other hand, the tax competition approach is based on the principle that the private (countries') benefit and social (EU's) benefit coincide: see Hitiris(1988). Also, it enables each country to control its tax rate for short-term economic policy purposes. Moreover, Edwards and Keen(1996) stated that tax competition mitigates political distortions. Thus, there is a trade-off between the tax equalization and the tax competition. It is evident that the tax approximation with a very wide common band reduces to a tax competition, and the tax approximation with a very narrow commonn band reduces to a tax equalization. Therefore, the tax approximation approach encompassing both the tax equalization and the tax competition approaches as extreme cases deserves more in-depth analysis. Central to this tax approximation approach is the problem of how to select the common band. Indeed, the EU has adopted the approximation approach with a minimum standard rate. From January 1993, a standard VAT rate of at least 15 per cent was applied.

The first scholor to give much attention to the tax approximation by taking advantage of geographical market rather than treating a spaceless market was Kanbur and Keen(1993). They formulated an analytical model of origin-based commodity tax competition between two governments lying on a linear market, in which each country sets its tax rate with a view to maximizing its tax revenue taking account of cross-border shopping. They examined how the tax approximation with minimum standard rate affect the equilibrium tax rates and revenues of two competing governments. However, the study of multi-country tax approximation has been strangely neglected by researchers.

In this paper, we formulate multi-country commodity tax competition models over a linear market where each country chooses its tax rate within a common band with the object of maximizing tax revenues. Then we analyze the effect of narrowing the common band. In particular, this paper focuses on the following questions. How does the common band affect the tax rates, demands, the revenues and the number of tax-induced cross-border shoppers at a Nash equilibrium ?; How do the relative sizes and positions of the countries affect their equilibrium tax rates and revenues in tax approximation ?; Under what conditions do Nash equilibria exist ?

Our model and Kanbur and Keen(1993)'s model can be regarded as the variations of Hotelling's spatial competition model: see Anderson et al.(1992), Gabszewicz and Thisse(1992). The current research extends the model in Kanbur and Keen(1993) in at least two directions. First, the number of countries is not restricted to be two. This enables us to explore the relationship between tax approximation and the position of the countries. Second, we deal with the role of maximum standard rates. This enables us to compare all corporate tax policies within only one framework. Third, while in Kanbur and Keen(1993) two countries differ by the density of customers, but have equal sizes, we assume that although customers are uniformly distributed over the whole market, the size of countries are different similarly as Ohsawa(1998). Ohsawa(1998) analyzed the outcomes of the tax competition among more than two countries to analyze how the position of countries affects tax rates and revenues at a Nash equilibrium. He showed that the size and positions of countries play an important role in the strategic tax design. This research generalizes Ohsawa(1998) in the introduction of these two standard rates, so it can be regarded as its continuation.

This paper is organized as follows. In the next section, our general tax competition model is formulated. Some properties on Nash equilibrium are also derived. In Section 3, we characterize the general effect of the tax approximation with a common band on tax rates, cross-border shoppers, demands and revenues in equilibrium. Also, by specializing a two-country model and a multi-country model having the same size, we explore the effect of the tax approximation, tax equalization, tax competition in more detail. Section 4 provides concluding comments. For convenience, all proofs are collected in Appendix.

2. Model

The model presented here is the same as that used in Ohsawa(1998). The assumptions used in both models are presented below for clarity. Since the notation is obviously burdensome, we try to match that used by Ohsawa(1998) as much as possible. Suppose that there exists a line segment along which $N(\geq 2)$ countries divide into N contiguous and nonoverlapping line segments. They are numbered in ascending order from one extremity of the whole line market. Let the size for ith country be denoted by $L_i(> 0)$, as shown in Figure 1. Denote the index set I be $I = \{1, 2, \cdots, N\}$. We make the following assumptions over the liner market:

1. Customers and firms are uniformly distributed in the line segment, so that the size of a country corresponds to its areal extension.
2. Transport costs are proportional to distance, and equal to $\gamma(> 0)$ per unit distance.
3. There is a single homogeneous commodity that is produced by all firms at constant marginal production cost.
4. Marginal production cost is zero, because our results do not depend on them.
5. Each government levies a source-based commodity tax, denoted by p_i, on the firms located within the corresponding country, such that $\underline{p} \leq p_i \leq \overline{p}$, where $\underline{p}$ and $\overline{p}$ are maximum and minimum standard rates, respectively.
6. Firms are non-cooperative, so that all firms in ith government would price at its tax-inclusive marginal cost, i.e., p_i.
7. Each customer purchases one unit of the commodity from the firm quoting the lowest *full price*, i.e., the mill price plus the transport cost between the firm and the customer, irrespective of its full price.

We define the *revenue* of ith government $\pi_i(p_1, \ldots, p_N)$ as the sum of the taxes from all firms within it. In basic Hotelling's spatial competition model literature, we call $p^*{}_i$'s in *equilibrium* if and only if $\underline{p} \leq p_i^* \leq \overline{p}$ and $\pi_i(p^*{}_1, \ldots, p_i, \ldots, p^*{}_N) \leq \pi_i(p^*{}_1, \ldots, p^*{}_i, \ldots, p^*{}_N)$ $(\forall i \in I)$: see Anderson et al.(1992), Gabszewicz and Thisse(1992). Thus we consider a Nash equilibrium of a non-cooperative N-person game whose players are governments, and where strategies are tax rates and payoffs are revenues. We also define the *demand* of ith government $D_i(p_1, \ldots, p_N)$ as the sum of the demands from all firms within it. Therefore, we have $\pi_i(p_1, \ldots, p_N) = p_i D_i(p_1, \ldots, p_N)$ We call this model *tax approximation* (constrained tax competition) model. As special cases, tax approximation with $\underline{p} = 0$ and $\overline{p} = \infty$ reduces to standard (unconstrained) tax competition, which has already been analyzed by Ohsawa(1998). The tax approximation with $\underline{p} = \overline{p}$ correspond to tax equalization. Thus, our formulation ranges between the one extreme of a complete tax competition and the other extreme of absolute tax equalization. In addition, the tax approximation with $\underline{p} > 0$ and $\overline{p} = \infty$ is called *minimum standard rate policy*. Also, tax approximation with $\underline{p} = 0$ and $\overline{p} < \infty$ is called *maximum standard rate policy*. To avoid misunderstanding, if $\underline{p_1} \geq \underline{p_2}$ and $\overline{p_1} \leq \overline{p_2}$ with strict inequality for at least one inequality, then tax approximation with $\underline{p} = \underline{p_1}$ and $\overline{p} = \overline{p_1}$ is called to have a more restrictive common band than the one with $\underline{p} = \underline{p_2}$ and $\overline{p} = \overline{p_2}$. To make our notation simpler, we often denote $\pi_i({p^*}_1, \ldots, {p^*}_N)$ by ${\pi_i}^*$ and $D_i({p^*}_1, \ldots, {p^*}_N)$ by ${D^*}_i$.

As shown by Ohsawa(1998), at a Nash equilibrium all adjoining countries have to define their market boundary. Therefore, the demand of ith government, denoted by $D_i(p_1, \ldots, p_N)$, can be expressed as

$$D_i(p_1, \ldots, p_N) = \begin{cases} L_1 + \frac{1}{\gamma}(p_2 - p_1), & i = 1; \\ L_i + \frac{1}{\gamma}(p_{i-1} + p_{i+1} - 2p_i), & 2 \leq i \leq N-1; \\ L_N + \frac{1}{\gamma}(p_{N-1} - p_N), & i = N. \end{cases} \tag{1}$$

This is illustrated in Figure 1, where the horizontal axis measures distance, the vertical axis measures full prices, two standard rates. It should be noted that peripheral governments face competition from one side only; interior governments face competition from both sides. Since $\pi_i(p_1^*, \ldots, p_i, \ldots, p_N^*) = D_i(p_1^*, \ldots, p_i, \ldots, p_N^*)p_i$ is quadratic with respect to p_i, p_i^*'s have to satisfy

$$\begin{aligned} {p^*}_1 &= \text{med}\{\underline{p}, \frac{\gamma L_1 + {p^*}_2}{2}, \overline{p}\}; \\ {p^*}_i &= \text{med}\{\underline{p}, \frac{\gamma L_i + {p^*}_{i-1} + {p^*}_{i+1}}{4}, \overline{p}\}, \qquad 2 \leq i \leq N-1; \\ {p^*}_N &= \text{med}\{\underline{p}, \frac{\gamma L_N + {p^*}_{N-1}}{2}, \overline{p}\}. \end{aligned} \tag{2}$$

where $\text{med}(x, y, z)$ indicates the median of x, y and z. To simplify notations, $D_i(p_1^*, \ldots, p_N^*)$ and $\pi_i(p_1^*, \ldots, p_N^*)$ are denoted by D_i^* and π_i^*, respectively. To derive closed forms for p_i^*'s seems to be very complicated. However, it follows from the system (2) that p_i^*'s are continuous and piece-wise linear with respect to $\underline{p}$ and $\overline{p}$, so do D_i^*'s, and π_i^*'s are continuous and piece-wise quadratic with respect to them. Let $\underline{I}$ and $\overline{I}$ denote $\{i \in I | p_i^* = \underline{p}\}$ and $\{i \in I | p_i^* = \overline{p}\}$, respectively. Since $\pi_i^* = p_i^* D_i^*$, we have the following standard optimal conditions:

$$\gamma D_i^* \begin{cases} \geq 2p_i^* & \text{for } i \in \overline{I} \\ = 2p_i^* & \text{for } i \in I \setminus (\underline{I} \cup \overline{I}), \\ \leq 2p_i^* & \text{for } i \in \underline{I} \end{cases} \qquad 2 \leq i \leq N-1 \tag{3}$$

$$\gamma D_i^* \begin{cases} \geq p_i^* & \text{for } i \in \overline{I} \\ = p_i^* & \text{for } i \in I \setminus (\underline{I} \cup \overline{I}), \\ \leq p_i^* & \text{for } i \in \underline{I} \end{cases} \qquad i = 1 \text{ or } i = N \tag{4}$$

The following uniqueness property, which can be proved by showing that the system (2) has a unique solution based on contractive-mapping theorem, facilitates our analysis on tax rates, demands and revenues in equilibrium.

Property 1 *If a Nash equilibrium exists, it is unique.*

To ascertain that the solution to the system (2) is in equilibrium, it still remains to exclude the possibility that some governments can increase their revenues by undercutting their competitors. The existence of a equilibrium in the standard Hotelling's model was examined by d'Aspremont et al.(1979). Some situations where there is no Nash equilibrium can be found in Ohsawa(1998). Even if a government undercut a peripheral government, it could not change its revenue discontinuously. Therefore, it suffices to show that any interior government cannot be undercut by its competitors. This also means that when the number of governments is two, there exists a Nash equilibrium.

3. Tax Harmonization

3.1. General Case

For notational purpose, define $L'_{min} = \min\{L_1, L_2/2, \cdots, L_{N-1}/2, L_N\}$ and $L'_{max} = \max\{L_1, L_2/2, \cdots, L_{N-1}/2, L_N\}$. Then we have the following proposition:

Proposition 1 *If $\gamma L'_{max} \leq \underline{p}$, then $p_i^* = \cdots = p_N^* = \underline{p}$ are in equilibrium. If $\overline{p} \leq \gamma L'_{min}$, then $p_i^* = \cdots = p_N^* = \overline{p}$ are in equilibrium.*

Neither very high minimum nor very low maximum standard rate policies is an absolute equalisation with respect to tax rates. Nevertheless, this Proposition shows that these policies establish a uniform structure across countries. An underlying mechanism is that very high minimum standard rates prevent any government, which would like to cut its tax rate to meet the optimality conditions (3) and (4), from doing so. A similar argument can apply to the case of very low maximum standard rates. In both case, $D_i^* = L_i$, and in the former case $\pi_i^* = \underline{p}L_i$, in the latter case $\pi_i^* = \overline{p}L_i$.

Throughout the remainder of this paper, we deal with only the situation where $\underline{p} < \gamma L'_{max}$ and $\gamma L'_{min} < \overline{p}$. In order to analyze their general behaviour of p_i^*, D_i^* and π_i^*, some local information such as their partial derivatives are useful. We are interested in the impacts of either raising the minimum standard rate $\underline{p}$ or lowering the maximum standard rate $\overline{p}$. Hence, we focus on the left-side partial derivatives with respect to $\underline{p}$ and the right-side one with respect to $\overline{p}$. Thus throughout this paper, the partial derivative with respect to $\underline{p}$ (resp. $\overline{p}$) should be understood to mean that the left-side (resp. right-side) partial derivative.

Property 2 *If a Nash equilibrium exists, then*

$$\gamma L'_{min} < p_i^* < \gamma L'_{max}, \qquad \text{for } \forall i \in I. \tag{5}$$

These inequalities indicate the lower and upper bounds on p_i^*'s, collectively. This Property means that the situation where $\underline{p} < \gamma L'_{max}$ and $\gamma L'_{min} < \overline{p}$ reduces to the one where $\gamma L'_{min} < \underline{p} \leq \overline{p} < \gamma L'_{max}$.

Property 3 *If a Nash equilibrium exists, then*

$$\frac{\partial p_i^*}{\partial \underline{p}} \begin{cases} = 0, & \text{for } i \in \overline{I}; \\ \in [0, \frac{1}{2}], & \text{for } i \in I \setminus (\underline{I} \cup \overline{I}); \\ = 1, & \text{for } i \in \underline{I}, \end{cases} \qquad \frac{\partial p_i^*}{\partial \overline{p}} \begin{cases} = 1, & \text{for } i \in \overline{I}; \\ \in [0, \frac{1}{2}], & \text{for } i \in I \setminus (\underline{I} \cup \overline{I}); \\ = 0, & \text{for } i \in \underline{I}. \end{cases}$$

From this Property four points become clear. First, raising minimum (resp. lowering maximum) standard rate increases (resp. decreases) any equilibrium tax rate. Second, when the standard rate $\underline{p}$ (resp. $\overline{p}$) changes, tax rates of the governments not belonging to $\underline{I}$ (resp. $\overline{I}$) change less rapidly than $\underline{p}$ (resp. $\overline{p}$). Hence, we recognize that tax approximation with a more restrictive common band results in reduction in the range of p_i^*'s, i.e., $\max\{p_1^*, \cdots, p_N^*\} - \min\{p_1^*, \cdots, p_N^*\}$. Third, tax approximation with a more restrictive common band reduces $|p_i^* - \underline{p}|$'s. So, there are less situation such that governments are undercut by ith government with $p_i^* \geq \underline{p}$. This means that to restrict the common band tends to induce a Nash equilibrium to exist. Finally, $\underline{I}$ and $\overline{I}$ are non-decreasing with $\underline{p}$ and $\overline{p}$, respectively.

Let CBS denote the number of tax-induced cross-border shoppers in the whole market, i.e., $CBS = \gamma^{-1} \sum_{i=1}^{N-1} |p_{i+1}^* - p_i^*|$.

Property 4 *If a Nash equilibrium exists, then*

$$\gamma \frac{\partial CBS}{\partial \underline{p}} \begin{cases} = 0, & \text{for } \underline{I} = \phi; \\ < -0, & \text{for } \underline{I} \neq \phi, \end{cases} \qquad \gamma \frac{\partial CBS}{\partial \overline{p}} \begin{cases} = 0, & \text{for } \overline{I} = \phi; \\ > 0, & \text{for } \overline{I} \neq \phi. \end{cases}$$

Thus we arrive at the important conclusion that any tax approximation with a more restrictive common band generates smaller cross-border shoppers over the whole market. Thus we see that the total number of cross-border shoppers in any tax approximation is below that in unconstrained Nash equilibrium. Thus, this result with the second point derived from Property 3 may justify that EU has adopted minimum standard rate policy for VAT rate in order to reduce the difference of tax rates. However, it should be noted that when $N \geq 3$ even if $\underline{p}$ increases or $\overline{p}$ decreases, the number of the cross-border shoppers of *some* borders may increase: see Example 1 in Appendix.

Property 5 *If a Nash equilibrium exists, then*

$$\gamma\frac{\partial D_i^*}{\partial \underline{p}}\begin{cases}\in [0,1], & \text{for } i \notin \underline{I};\\ \in [-1,0], & \text{for } i \in \underline{I}.\end{cases} \qquad \gamma\frac{\partial D_i^*}{\partial \overline{p}}\begin{cases}\in [-1,0], & \text{for } i \in \overline{I};\\ \in [0,1], & \text{for } i \notin \overline{I}.\end{cases}$$

There are at least one implications from this Property. Raising $\underline{p}$ (resp. lowering $\overline{p}$) increases (resp. decreases) the demands of governments not belonging to $\underline{I}$ (resp. $\overline{I}$). Obviously, the increment in D_i^* caused by changing $\underline{p}$ or $\overline{p}$ coincides with the one in the balance between inward and outward cross-border shoppers of ith government. Therefore, although both raising $\underline{p}$ and lowering $\overline{p}$) reduce cross-border shoppers, the former increases the balances of the governments not belonging to $\underline{I}$ and the latter decreases that not belonging to $\overline{I}$. Thus, the impacts on these balances are different.

Property 6 *If a Nash equilibrium exists, then*

$$\frac{\partial \pi_i^*}{\partial \underline{p}} \geq 0, \text{ for } i \notin \underline{I}, \qquad \frac{\partial \pi_i^*}{\partial \overline{p}} \geq 0, \text{ for } i \in I.$$

This Property has at least two implications. First, we recognize that lowering $\overline{p}$ harms all the governments. This leads to the important conclusion that the revenue of each government in tax equalization is below that in any minimum standard rate policy with the same rate as a minimum standard. Thus this result generalizes a two-country model developed by Kanbur and Keen(1993) to any number of governments. Second, we see that raising $\underline{p}$ improves at least the governments not belonging to $\underline{I}$. Combining this with Property 3 yields the fact that the tax revenue in any minimum standard rate policy is above that in unconstrained Nash equilibrium, for the governments whose tax rate in unconstrained Nash equilibrium is greater than the minimum standard. However, as Kanbur and Keen(1993) pointed out, raising $\underline{p}$ may harm some governments. This can be also seen in Example 2 in Appendix. Thus, we see that the impacts of $\underline{p}$ on π_i^*'s are not always symmetric with the ones of $\overline{p}$.

The following Proposition gives the sufficient condition for ensuring the existence of a Nash equilibrium. Let $\tilde{L}_{min}$ denote the size of the smallest interior country, i.e., $\tilde{L}_{min} = \min\{L_2, \cdots, L_{N-1}\}$.

Proposition 2 *If* $\min\{\overline{p}, \gamma L'_{max}\} - \max\{\underline{p}, \gamma L'_{min}\} \leq \gamma \tilde{L}_{min}$, *a unique Nash equilibrium exists.*

This condition has straightforward interpretations. For a fixed $\overline{p}$, the higher $\underline{p}$ prevents governments from doing undercutting. For a fixed $\underline{p}$, lowering $\overline{p}$ decreases p_i's, as we have seen, so it becomes less attractive to do undercutting. As special cases, in a two-country model, $\tilde{L}_{min}$ can be regarded as ∞, so a unique Nash equilibrium exists, irrespective of $\underline{p}$ and $\overline{p}$. In the case of $L_1 = L_2 = \cdots = L_N (\equiv L)$, $L'_{max} = \tilde{L}_{min} = L$ and $L'_{min} = L/2$. Since this meets that sufficient condition, it is ensured that a Nash equilibrium exists, irrespective of $\underline{p}$ and $\overline{p}$.

3.2. Two-Country Model

To understand how the size of countries affect the magnitudes of tax rates and that of per capita revenues in equilibrium, we restrict our attention to a duopolistic case. Remember that in this case there exists a unique Nash equilibrium.

Property 7 *In a duopolistic case, when* $L_1 < L_2$,

$$p_1^* < p_2^*, \tag{6}$$

$$\frac{\pi_1^*}{L_1} > \frac{\pi_2^*}{L_2}. \tag{7}$$

Thus the inequality (6) states that the small country sets a lower equilibrium tax rate than the big one, irrespective of $\underline{p}$ and $\overline{p}$. The inequality (7) means that per capita revenue is larger in the small country, irrespective of $\underline{p}$ and $\overline{p}$. Property 7 which generalizes results in unconstrained Nash equilibrium developed by Ohsawa(1998), is consistent with the findings in Kanbur and Keen(1993). This may explain the fact that the government of Luxembourg sets its VAT rate to the minimum standard rate.

3.3 Multi-country Model with Identical Size

To examine how the position of countries affect the ranking of tax rates and revenues in equilibrium, we focus attention to the case where more than two governments coexist with the restriction that the size of all countries are the same, i.e., $L_1 = L_2 = \cdots = L_N (\equiv L)$. In this case, as we have seen in Proposition 2 , there exists a unique Nash equilibrium.

Define the median M by $M = (N+1)/2$ if N is odd, $M = N/2$ otherwise. Of course, p_i^*'s, D_i^*'s and π_i^*'s are symmetric with respect to the median M, i.e., $p_i^* = p_{N+1-i}^*$, $D_i^* = D_{N+1-i}^*$ and $\pi_i^* = \pi_{N+1-i}^*$ for $1 \leq \forall i \leq M$. Here it should be noted from (5) that we consider only the case where $\gamma L/2 < \underline{p} \leq \overline{p} < \gamma L$. The following three properties also characterize them more precisely:

Property 8

$$
\begin{aligned}
p_{i+1}^* - p_i^* &> p_i^* - p_{i-1}^*, \qquad \textit{for } i \notin \underline{I} \cup \overline{I}. \\
p_1^* > p_2^* &\geq \cdots \geq p_M^*.
\end{aligned}
$$

The inequality $p_i^* \geq p_{i+1}^*$ strictly holds if and only if $p_i^* > \underline{p}$.

A close look at this Property reveals at least two points. First, Ohsawa(1998) showed that in unconstrained Nash equilibrium, U-shaped rate structure is established. An important property of Property 8 is that U-shaped rate structure is established irrespective of $\underline{p}$ and $\overline{p}$. This U-shaped structure may answer the question of why the Scandinavian government set higher VAT rate. As Ohsawa(1989) pointed out, the intuitive explanation of the U-shaped structure is simple. Peripheral countries enjoy a local monopoly, so they set the highest tax rate. The farther the country lies from the market boundary, the smaller its advantage due to competing with one peripheral country, and the larger its disadvantage due to competing with more interior countries. Second, from Property 8 , it is interesting to note that the cardinality of $\overline{I}$ is at most two, and that of $\underline{I}$ may exceed two. This means that while some tax approximation can eliminate cross-border shoppers of inner borders, any tax approximation such that $\underline{p} < \gamma L$ and $\gamma L/2 \leq \overline{p}$ cannot eliminate that of outermost borders.

This U-shaped tax rate structure also enables us to derive the following two Properties. In fact, this U-shaped tax rate structure implies p_i^*'s have unimodal. Therefore, if $\underline{I} \neq \phi$, then $CBS = 2(p_1^* - p_M^*)$.

Property 9

$$
-2 \leq \gamma \frac{\partial CBS}{\partial \underline{p}} \leq -1, \ \textit{for } \underline{I} \neq \phi, \qquad 1 \leq \gamma \frac{\partial CBS}{\partial \overline{p}} \leq 2, \ \textit{for } \overline{I} \neq \phi.
$$

Moreover, CBS is convex with respect to both $\underline{p}$ and $\overline{p}$.

The second claim of this Property means that CBS is convex with respect to raising $\underline{p}$, and CBS is concave with respect to lowering $\overline{p}$. Thus it can be concluded from this that in order to reduce cross-border shoppers, raising $\underline{p}$ is more effective than lowering $\overline{p}$ if we vary each standard rate by the same size.

It is straightforward to obtain the following M-shaped demand structure from the U-shaped rate structure.

Property 10

$$D_1^* < D_2^* \geq \cdots \geq D_M^*.$$

The inequality $D_i^* \geq D_{i+1}^*$ strictly holds if and only if $p_i^* > \underline{p}$.

The U-shaped rate structure together with the M-shaped demand structure yields $\pi_{i-1}^* < \pi_i^*$ for $3 \leq i \leq M$. However, the relationship between π_1^* and π_2^* is unclear.

Property 11

$$\pi_1^* < \pi_2^* \geq \cdots \geq \pi_M^*.$$

The inequality $\pi_i^* \geq \pi_{i+1}^*$ strictly holds if and only if $p_i^* > \underline{p}$.

Ohsawa(1998) revealed that in unconstrained Nash equilibrium both D_i^*'s and π_i^*'s have M-shaped structures. A noteworthy characteristics of Properties 10 and 11 is that $\underline{p}$ and $\overline{p}$ do not affect these structures. Thus, it can be concluded that although any tax approximation is introduced, the best location is either the second and $N-1$th positions.

As we have seen in Property 6, the sign of the revenue derivative $\frac{\partial \pi_i^*}{\partial \underline{p}}$ for $i \in \underline{I}$ depends on the spatial configuration. The following Property ensures that in the case of identical country size these signs are non-negative.

Property 12

$$\frac{\partial \pi_i^*}{\partial \underline{p}} \begin{cases} = 0, & p_{i-1}^* = p_i^* = p_{i+1}^*; \\ > 0, & \textit{otherwise,} \end{cases} \qquad \frac{\partial \pi_i^*}{\partial \overline{p}} \begin{cases} = 0, & p_{i-1}^* = p_i^* = p_{i+1}^*; \\ > 0, & \textit{otherwise.} \end{cases}$$

This Property means that raising not only $\overline{p}$ but also $\underline{p}$ improves all the countries.

In the tax equalization, the tax revenue increases with the common tax rate. The following Property compares the revenues in tax equalization with that in unconstrained Nash equilibrium.

Property 13 *$p_1^\circ, \cdots, p_N^\circ$ are the tax rates in unconstrained Nash equilibrium. For $1 < \forall i < M$ and $i = M$ with even N, there exists τ_0 between p_{i-1}° and p_i° such that tax equalization with $\tau(\geq \tau_0)$ improves the ith government's revenue and tax equalization with $\tau(< \tau_0)$ harms it. For the first government, and the Mth government with odd N, there exists τ_0 between p_1° and p_2° (resp. between p_{M-2}° and p_{M-1}°) such that tax equalization with $\tau(\geq \tau_0)$ improves first (resp. M-th) government, and tax equalization with $\tau(< \tau_0)$ harms it.*

This generalizes the result of a two-country model by Kanbur and Keen(1993).

To what extent does each country's tax revenue suffer from tax approximation ? In order to see these, we shall take up the following three simple cooperate tax policies where $N = 10$, $\gamma = 1$ and $L = 1$:

1. (minimum standard rate system) $\overline{p}$ is fixed at 1.0, $\underline{p}$ ranges from 0.5 to 1.0 at 0.05 intervals.

2. (maximum standard rate system) For fixed $\underline{p} = 0.5$, $\overline{p}$ ranges from 1.0 to 0.5 at 0.05 intervals.

3. (tax approximation) $\underline{p}$ ranges from 0.5 to 0.6 at 0.01 intervals and $\overline{p}$ ranges from 1.0 to 0.6 at 0.04 intervals.

For the first case, the corresponding tax rates in equilibrium are illustrated in Figure 2, and their equilibrium revenues are given in Figure 3. For the second case, the corresponding tax rates and revenues in equilibrium are displayed in Figures 4 and 5, respectively. For the third case, the corresponding tax rates and revenues in equilibrium are presented in Figures 6 and 7, respectively. They were solved by the procedure which we have given in Appendix. The results in these Figures are consistent with the theoretical results such as Properties 3, 6, 8, 11 and 12.

In addition, the number of all te cross-shoppers $\sum_{i=1}^{N-1} |p_{i+1}^* - p_i^*| \, (= 2|p_1^* - p_M^*|)$ is plotted in Figure 8. These plots are function with respect to the increment of the sum of $\underline{p}$ and $\overline{p}$, denoted by Δ. Thus, the plots for the first policy are function with respect to only the increment of $\underline{p}$, and the ones for the second policy are function with respect to only the increment of $\overline{p}$. From Figure 8, we can confirm the characteristics derived in Property 9.

6. Conclusions

Within the framework of tax competition such that a common band on tax rates is imposed, tax rates, demands, revenues and the number of cross-border shoppers in equilibrium of some strategic tax design were characterized. We generalized some results shown by Kanbur and Keen(1993) and Ohsawa(1998). We derived at least the following four conclusions, which may be useful to discuss the possible future of EU corporate tax policy.

1. It was shown that raising minimum standard rate increases any equilibrium tax rate, and lowering maximum standard rate decrease any equilibrium tax rate. It was also verified that tax approximation with a more restrictive common band reduces the range of tax rates, and generates less cross-border shoppers in equilibrium.

2. It was proved that both very high minimum and very low maximum standard rate policies establish a uniform structure across countries.

3. It was demonstrated that the tax revenues in any tax equalization is below that in minimum standard rate policy imposing the same rate as a minimum standard.

4. It was shown that raising the minimum standard rate is asymmetric with lowering the maximum standard rate in several respects. First, whereas lowering maximum rate always harms all governments, raising minimum rate may harm some countries and improve others simultaneously. Second, in the case of identical country size, while the former may eliminate cross-border shoppers of some border, the latter cannot eliminate cross-border shoppers of any border. Third, in the case of identical country size, the number of tax-induced cross-border shoppers in the whole market is convex with respect to raising $\underline{p}$, and it is concave with respect to lowering $\overline{p}$.

5. In the two-country model, the smaller government sets a lower tax rate and obtain more per capita revenue than the bigger one. It has been proved also that in the case of identical country size, any tax approximation establishes U-shaped rate structure, M-shaped demand and revenue structures. It has been concluded that the size and position of countries play a central role in tax approximation.

Appendix

Proof of Property 1

We express the system (2) in terms of a mapping of the form $\mathbf{p} = G(\mathbf{p})$. Any point satisfying $\mathbf{p} = G(\mathbf{p})$ is called a fixed point of G. We shall verify that the system (2) has a unique solution by showing that G has a unique fixed point. $\underline{\mathbf{p}}$ and $\overline{\mathbf{p}}$ stand for the vectors having all their entries equal to $\underline{p}$ and $\overline{p}$, respectively. For fixed $\underline{\mathrm{p}}$ and $\overline{\mathrm{p}}$, we define the closed set D_0 by $D_0 = \{\mathrm{q} \in \mathrm{R}^N | \underline{\mathrm{p}} \leq \mathrm{q} \leq \overline{\mathrm{p}}\}$. Now for $\forall \mathrm{p}, \forall \mathrm{q} \in D_0$ and $\forall i \in I$, let e_i denote ith element of the vector $G(\mathrm{p}) - G(\mathrm{q})$. Then one easily verifies that

$$e_i^2 \leq \begin{cases} \frac{1}{4}(p_2 - q_2)^2, \\ \frac{1}{16}(p_{i-1} - q_{i-1} + p_{i+1} - q_{i+1})^2, & 2 \leq i \leq N-1; \\ \frac{1}{4}(p_{N-1} - q_{N-1})^2. \end{cases}$$

Therefore, for all $\mathrm{p}, \mathrm{q} \in D_0$, $\|G(\mathrm{p}) - G(\mathrm{q})\|^2 = \sum_{i=1}^N e_i^2 < \frac{1}{2}\sum_{i=1}^N (p_i - q_i)^2 = \frac{1}{2}\|\mathrm{p} - \mathrm{q}\|^2$. This states that for all $\mathrm{p}, \mathrm{q} \in D_0$, $\frac{\|G(\mathrm{p}) - G(\mathrm{q})\|}{\|\mathrm{p} - \mathrm{q}\|} < \frac{1}{\sqrt{2}} < 1$. Thus we recognize that the mapping G is contractive on the closed set D_0. In addition, it is evident from the system (2) that $G(D_0) \subset D_0$. It follows from the contractive-mapping theorem that G has a unique fixed point in D_0: see Ortega and Rheinboldt(1970). Thus, it can be concluded that the system (2) has a unique solution.□

Proof of Proposition 1

Let us prove the first claim. Suppose in the contrary that $p_j^* > \underline{p}$ for some j. Define $k = \mathrm{argmax}_{i \in I}(p_i^* - \underline{p})$, so $\underline{p} < p_k^*$ and $D_k^* < L_k$. If $2 \leq k \leq N-1$, we have $L_k \leq 2L'_{max}$. Combining these inequalities with $\gamma L'_{max} \leq \underline{p}$ yields $\gamma D_k^* < \gamma L_k \leq 2\gamma L'_{max} \leq 2\underline{p} < 2p_k^*$. This contradicts the conditions (3) and (4). Otherwise, similar arguments can be applied.

Next let us prove the second claim. Suppose in the contrary that $p_j^* < \overline{p}$ for some j. Define $k = \mathrm{argmin}_{i \in I}(p_i^* - \overline{p})$, so $\overline{p} > p_k^*$ and $D_k^* > L_k$. If $2 \leq k \leq N-1$, we have $L_k \geq 2L'_{min}$. Combining these inequalities with $\gamma L'_{min} \geq \overline{p}$ yields $\gamma D_k^* > \gamma L_k \geq 2\gamma L'_{min} \geq 2\overline{p} > 2p_k^*$. This contradicts the conditions (3) and (4). Otherwise, similar arguments can be applied. □

Proof of Property 2

First let us prove the upper bounds (5). Suppose in the contrary that $p_j^* \geq \gamma L'_{max}$ for some j. Define $k = \mathrm{argmax}_{i \in I}(p_i^* - \gamma L'_{max})$, so $\gamma L'_{max} \leq p_k^*$ and $D_k^* < L_k$. If $2 \leq k \leq N-1$, we have $L_k \leq 2L'_{max}$. Combining these inequalities yields $\gamma D_k^* < \gamma L_k \leq \gamma L'_{max} \leq 2p_k^*$. This contradicts the conditions (3) and (4). Otherwise, similar arguments can be applied.

Second let us prove the lower bounds (5). Suppose in the contrary that $p_j^* \leq \gamma L'_{min}$ for some j. Define $k = \mathrm{argmin}_{i \in I}(p_i^* - \gamma L'_{min})$, so $\gamma L'_{min} \geq p_k^*$ and $D_k^* > L_k$. If $2 \leq k \leq N-1$, we have $L_k \geq 2L'_{min}$. Combining these inequalities yields $\gamma D_k^* > \gamma L_k \geq \gamma L'_{min} \geq 2p_k^*$. This contradicts the conditions (3) and (4). Otherwise, similar arguments can be applied. □

Proof of Property 3

For $i \in \underline{I} \cup \overline{I}$, the assertions are obvious. First we show that $\frac{\partial p_i^*}{\partial \underline{p}} \leq 1/2$ for $i \in I \setminus (\underline{I} \cup \overline{I})$. Suppose in the contrary that $\frac{\partial p_j^*}{\partial \underline{p}} > 1/2$ for some $j \in I \setminus (\underline{I} \cup \overline{I})$. Define $k = \text{argmax}_{i \in I \in I \setminus (\underline{I} \cup \overline{I})} \frac{\partial p_i^*}{\partial \underline{p}}$. If $2 \leq k \leq N-1$, we have $\frac{\partial p_k^*}{\partial \underline{p}} = \left(\frac{\partial p_{k-1}^*}{\partial \underline{p}} + \frac{\partial p_{k+1}^*}{\partial \underline{p}} \right) /4$. This contradicts either $\frac{\partial p_i^*}{\partial \underline{p}} \leq 1$ for $i \in \underline{I} \cup \overline{I}$ or the definition of k. Otherwise, similar arguments can be applied. Second we show that $\frac{\partial p_i^*}{\partial \underline{p}} \geq 0$ for $i \in I \setminus (\underline{I} \cup \overline{I})$. Suppose in the contrary that $\frac{\partial p_j^*}{\partial \underline{p}} < 0$ for some $j \in I \setminus (\underline{I} \cup \overline{I})$. Define $k = \text{argmin}_{i \in I \in I \setminus (\underline{I} \cup \overline{I})} \frac{\partial p_i^*}{\partial \underline{p}}$. If $2 \leq k \leq N-1$, we have $\frac{\partial p_k^*}{\partial \underline{p}} = \left(\frac{\partial p_{k-1}^*}{\partial \underline{p}} + \frac{\partial p_{k+1}^*}{\partial \underline{p}} \right) /4$. This contradicts either $\frac{\partial p_i^*}{\partial \underline{p}} \geq 0$ for $i \in \underline{I} \cup \overline{I}$ or the definition of k. Otherwise, similar arguments can be applied.

Similar argument leads to $0 \leq \frac{\partial p_i^*}{\partial \overline{p}} \leq 1/2$ for $i \in I \setminus (\underline{I} \cup \overline{I})$. □

Proof of Property 4

Let us show the first claim. If $\underline{I} = \phi$, the claim is obvious. Otherwise, suppose that $\underline{I} = \{u_1, u_2, \cdots, u_s\}$ with $u_j < u_{j+1}$. Define $I_1 = \{1, \cdots, \lfloor (u_1 + u_2 + 1)/2 \rfloor\}$, $I_2 = \{\lfloor (u_1 + u_2 + 1)/2 \rfloor, \cdots, \lfloor (u_2 + u_3 + 1)/2 \rfloor\}$, $I_s = \{\lfloor (u_{s-1} + u_s + 1)/2 \rfloor, \cdots, N\}$, where $\lfloor w \rfloor$ denotes the lower integer part of w. For $1 \leq \forall i \leq s$, we define $CBS_i = \sum_{j,j+1 \in I_i} |p_j^* - p_{j+1}^*|$. Therefore, $CBS = \sum_{i=1}^{s} CBS_i$. Therefore, it suffices to show that for $1 \leq \forall i \leq s$, $\frac{\partial CBS_i}{\partial \overline{p}} < 0$.

For each I_i, let $v_1, \cdots, v_t \in I_i$ be the indices such that $v_k \in I_i$, $v_k < v_{k+1}$ and v_kth government experiences only inward cross-border shoppers. Let $v_0 = \min_{j \in I_i} j$ and $v_{t+1} = \max_{j \in I_i} j$. For $0 \leq \forall k \leq t$, define the index $r(k)$ by $r(k) = \text{argmin}\{|u_i - j| \,|\, j = \text{argmax}_{v_k \leq w \leq v_{k+1}} p_w^*\}$. Therefore, we have $\sum_{v_k \leq j, j+1 \leq v_{k+1}} |p_{j+1}^* - p_j^*| = 2p_{r(k)}^* - p_{v_k}^* - p_{v_{k+1}}^*$. Note that $\frac{\partial p_j^*}{\partial \underline{p}}$ is non-increasing in $|j - u_i|$ for $\forall j \in I_i$. Therefore, although $\overline{p}$ increases by a sufficiently small amount, we have again $\sum_{v_k \leq j, j+1 \leq v_{k+1}} |p_{j+1}^* - p_j^*| = 2p_{r(k)}^* - p_{v_k}^* - p_{v_{k+1}}^*$. On the other hand, it is evident from the system (2) that $\frac{\partial p_{r(k)}^*}{\partial \underline{p}} \leq \frac{1}{4} \left(\frac{\partial p_{v_k}^*}{\partial \underline{p}} + \frac{\partial p_{v_{k+1}}^*}{\partial \underline{p}} \right)$. Therefore, we have $\frac{\partial}{\partial \underline{p}} (2p_{r(k)}^* - p_{v_k}^* - p_{v_{k+1}}^*) \leq -\frac{1}{2} \left(\frac{\partial p_{v_k}^*}{\partial \underline{p}} + \frac{\partial p_{v_{k+1}}^*}{\partial \underline{p}} \right)$. Thus, we have $\gamma^{-1} \frac{\partial CBS_i}{\partial \underline{p}} = \sum_{j,j+1 \in I_i} \frac{\partial |p_{j+1}^* - p_j^*|}{\partial \underline{p}} = \sum_{k=0}^{t} \frac{\partial}{\partial \underline{p}} (2p_{r(k)}^* - p_{v_k}^* - p_{v_{k+1}}^*) \leq -\frac{1}{2} \sum_{k=0}^{t+1} \frac{\partial p_{v_k}^*}{\partial \underline{p}}$. It should be noted that there exists v_k such that $v_k = u_i$, i.e., $\frac{\partial p_{v_k}^*}{\partial \underline{p}} = 1$. Hence, $\gamma^{-1} \frac{\partial CBS}{\partial \underline{p}} \leq -1/2 < 0$, as required. Similar argument leads to the second claim. □

Proof of Property 5

Combining the optimal conditions (3) and (4) with Property 3 yields this assertion. □

Proof of Property 6

Combining Properties 3 with 5 yields $\frac{\partial \pi_i^*}{\partial \underline{p}} \geq 0$ for $i \notin \underline{I}$ and $\frac{\partial \pi_i^*}{\partial \overline{p}} \geq 0$ for $i \notin \overline{I}$. However, the sign of $\frac{\partial \pi_i^*}{\partial \overline{p}} \geq 0$ for $i \in \overline{I}$ is unclear. Suppose that $p_i^* = \overline{p}$ with $2 \leq i \leq N-1$. Since $\gamma \pi_i^* = \overline{p}(\gamma L_i + p_{i-1}^* + p_{i+1}^* - 2\overline{p})$, we have $\gamma \frac{\partial \pi_i^*}{\partial \overline{p}} = \overline{p}\frac{\partial p_{i-1}^*}{\partial \overline{p}} + \overline{p}\frac{\partial p_{i+1}^*}{\partial \overline{p}} + (p_{i-1}^* + p_{i+1}^* + \gamma L_i - 4\overline{p})$. The substitution of $\frac{\partial p_{i-1}^*}{\partial \overline{p}} \geq 0$, $\frac{\partial p_{i+1}^*}{\partial \overline{p}} \geq 0$ and $p_{i-1}^* + p_{i+1}^* + \gamma L_i > 4\overline{p}$ into this partial derivative yields $\frac{\partial \pi_i^*}{\partial \overline{p}} \geq 0$. The same holds for $p_1^* = \underline{p}$ and $p_N^* = \underline{p}$. □

Proof of Proposition 2

Based on the upper and lower bounds (5), we have $p_i^* \leq \min\{\overline{p}, \gamma L'_{max}\}$ and $\max\{\underline{p}, \gamma L'_{min}\} \leq p_i^*$ for $\forall i \in I$. Hence, for $\forall i, \forall j \in I$, $p_i^* \leq \min\{\overline{p}, \gamma L'_{max}\} \leq \max\{\underline{p}, \gamma L'_{min}\} + \gamma \tilde{L}_{min} \leq p_j^* + \gamma \tilde{L}_{min}$. Thus, we have $p_i^* \leq p_{i-1}^* + \gamma L_i$ and $p_i^* \leq p_{i+1}^* + \gamma L_i (2 \leq \forall i \leq N-1)$. These two inequalities state that any interior government cannot be undercut by its two neighbours. □

Proof of Property 7

The inequality (6) is immediate from the system (2). Let us verify the inequality (7). The inequality (6) yields $D_2^* < L_2$. This leads to $\pi_2^* = p_2^* D_2^* < p_2^* L_2$. On the other hand, since p_1^* is a maximizer of $\pi_1(p_1, p_2^*)$, we have $\pi_1^* > \pi_1(p_2^*, p_2^*) = p_2^* L_1$. Combining these two inequalities results in $\pi_1^*/L_1 > p_2^* > \pi_2^*/L_2$, as required. □

Proof of Property 8

Substituting $L'_{min} = L$ into the lower bounds (5) yields $\gamma L < 2p_i^*$. This together with the system (2) gives $(p_{i+1}^* - p_i^*) - (p_i^* - p_{i-1}^*) = 2p_i^* - \gamma L > 0$, as required.

It follows from $\gamma L/2 < p_{i-1}^*$ and $\gamma L/2 < p_{i+1}^*$ that $(\gamma L + p_{i-1}^* + p_{i+1}^*)/4 < (p_{i-1}^* + p_{i+1}^*)/2 \leq \overline{p}$. Thus we see that

$$p_i^* < \overline{p}, \qquad (2 \leq \forall i \leq N-1). \tag{8}$$

Thus, if $p_1^* = \overline{p}$, then $p_1^* > p_2^*$, otherwise, it follows from the system (2) that $p_1^* > p_2^*$. So, $1 \notin \underline{I}$. If $\underline{I} \neq \phi$, assume that $k \notin \underline{I}$ and $k+1 \in \underline{I}$. Let us show that $p_i^* > p_{i+1}^*$, $(1 \leq i \leq k)$. Since $p_k^* - p_{k+1}^* > 0$, the first claim implies that $p_{k-1}^* - p_k^* > 0$. This argument together with (8) also establishes that $p_j^* > p_{j+1}^*$ for $1 \leq \forall j < k$. Otherwise for odd N $p_M^* = p_{M+1}^*$, and for even N $p_{M-1}^* = p_{M+1}^*$ and $p_{M-1}^* > p_M^*$ from the first claim. A similar argument establishes that $p_j^* > p_{j+1}^*$ for $1 \le \forall j < M-1$. □

Proof of Property 9

Let us prove the first assertion. In the case of $\underline{I} \neq \phi$, $\frac{\partial CBS}{\partial \underline{p}} = 2\frac{\partial p_1^*}{\partial \underline{p}} - 2$. Combining this with Property 3 yields $-2 \le \frac{\partial CBS}{\partial \underline{p}} \le -1$. In the case of $\overline{I} \neq \phi$, $\frac{\partial CBS}{\partial \overline{p}} = 2 - 2\frac{\partial p_M^*}{\partial \overline{p}}$. This together with Property 3 yields $1 \le \frac{\partial CBS}{\partial \overline{p}} \le 2$.

Let us prove the second assertion. It follows from the system (2) that $\frac{\partial p_1^*}{\partial \underline{p}}|_{p_1^*=u} \le \frac{\partial p_1^*}{\partial \underline{p}}|_{p_1^*=v}$ for $u < v$, so $\frac{\partial p_1^*}{\partial \underline{p}}$ is non-decreasing in $\underline{p}$. On the other hand, $\frac{\partial p_M^*}{\partial \underline{p}} = 1$ if $M \in \underline{I}$, $\frac{\partial p_M^*}{\partial \underline{p}} = 0$ otherwise, so $\frac{\partial p_M^*}{\partial \overline{p}}$ is non-increasing in $\overline{p}$. These imply that $\frac{\partial CBS}{\partial \underline{p}}$ (resp. $\frac{\partial CBS}{\partial \overline{p}}$) is non-decreasing in $\underline{p}$ (resp. $\overline{p}$). Thus, we see that CBS is convex with respect to both $\underline{p}$ and $\overline{p}$. □

Proof of Property 10

The U-shaped tax rate structure guarantees that for $2 \le i \le M$, $D_i^* \ge D_{i+1}^*$. Since $D_1^* < L$ and $D_2^* > L$, we have $D_1^* < D_2^*$. □

Proof of Property 11

Let us show that $\pi_1^* < \pi_2^*$. We consider four cases separately: *Case 1*; $\overline{p} > p_1^* > p_2^* > \underline{p}$; *Case 2*; $\overline{p} = p_1^* > p_2^* > \underline{p}$; *Case 3*; $\overline{p} > p_1^* > p_2^* = \underline{p}$; *Case 4*; $\overline{p} = p_1^* > p_2^* = \underline{p}$.

For the first case, it is easy to check from $p_1^* = (L+p_2^*)/2$ that $\gamma(\pi_2^* - \pi_1^*) = 2(p_2^*)^2 - p_1^* > 0 \Leftrightarrow p_1^* > ((4+\sqrt{2})/7)\gamma L(\approx 0.774\gamma L)$, irrespective of $\underline{p}$ and $\overline{p}$. We know from Ohsawa(1998) that $p_1^\circ = (3+\sqrt{3})/6\gamma L(\approx 0.789\gamma L)$. These together with $\frac{\partial p_1^*}{\partial \underline{p}} \ge 0$ yields $\pi_1^* < \pi_2^*$.

For the second case, $\gamma(\pi_2^* - \pi_1^*) = 2(p_2^*)^2 - \overline{p}(\gamma L + p_2^* - \overline{p})$. Hence, when $\underline{I} = \phi$, by simple recursive calculation of (2), we have $\frac{\partial p_2^*}{\partial \overline{p}} > 1/3$. This together with $p_2^* < \overline{p}$ gives $\gamma\frac{\partial(\pi_2^* - \pi_1^*)}{\partial \overline{p}} = \frac{\partial p_2^*}{\partial \overline{p}}(4p_2^* - \overline{p}) + (-p_2^* + 2\overline{p} - \gamma L) > p_2^*/3 + 5\overline{p}/3 - \gamma L > 2p_2^* - \gamma L > 0$. On the other hand, if $\overline{p} = p_2^*$, then $p_1^* = p_2^* = \cdots = p_N^*$ from the U-shaped rate structure, so we have $\pi_2^* - \pi_1^* = 0$. Thus we see that for $\underline{p}$ with $\underline{I} = \phi$, $\pi_2^* > \pi_1^*$. On the other hand, since $\frac{\partial p_2^*}{\partial \underline{p}} \ge 0$, $\gamma\frac{\partial(\pi_2^* - \pi_1^*)}{\partial \underline{p}} = \frac{\partial p_2^*}{\partial \underline{p}}(4p_2^* - \underline{p}) \ge 0$. Therefore, for $\underline{p}$ with $\underline{I} \neq \phi$, we have $\pi_2^* > \pi_1^*$.

For the third case, it follows from $p_1^* = (\gamma L + \underline{p})/2$ that $\gamma(\pi_2^* - \pi_1^*) = \underline{p}(\gamma L + p_1^* - \underline{p}) - (p_1^*)^2 = -\frac{3}{4}(\underline{p} - \frac{2}{3}\gamma L)^2 + (\gamma L)^2/12 > 0$ for $\gamma L/2 < \underline{p} < \gamma L$.

For the last case, unless $\overline{p}$ is imposed, $L + \frac{p_1^* - \overline{p}}{\gamma}$ is optimal for the first government. However, its revenue is below π_2^* based on the result for the third case. Thus, we have

$\pi_1^* < \pi_2^*$. □

Proof of Property 12

The claims for ith government with $p_{i-1}^* = p_i^* = p_{i+1}^*$ is straightforward to see. From the proofs of Properties 3 and 5, it suffices to prove $\frac{\partial \pi_i^*}{\partial \underline{p}} > 0$ with $p_{i-1}^* > p_i^* = \underline{p}$. Two cases may arise: *Case 1*: either N is even or N is odd but $i \neq M$; *Case 2*: N is odd and $i = M$. It should be noted that in the first (resp. second) case, cross-border shoppers are induced by tax rate differentials in only one side (resp. in both sides).

For the first case, $\gamma \pi_i^* = \underline{p}(\gamma L + p_{i-1}^* - \underline{p})$. Hence, substituting $\frac{\partial p_{i-1}^*}{\partial \underline{p}} > 0$ and $\gamma L > \underline{p}$ into $\gamma \frac{\partial \pi_i^*}{\partial \underline{p}}$ yields $\gamma \frac{\partial \pi_i^*}{\partial \underline{p}} = \underline{p} \frac{\partial p_{i-1}^*}{\partial \underline{p}} + (\gamma L - \underline{p}) + (p_{i-1}^* - \underline{p}) > 0$.

For the second case, taking advantage of the fact that $p_{M-1}^* = p_{M+1}^*$, we get $\gamma \pi_M^* = \underline{p}(\gamma L + 2p_{M-1}^* - 2\underline{p})$. When $N = 3$, i.e., $M = 2$, differentiating π_2^* with respect to $\underline{p}$ yields $\gamma \frac{\partial \pi_2^*}{\partial \underline{p}} = \underline{p}(2\frac{\partial p_1^*}{\partial \underline{p}} - 1) + (\gamma L - \underline{p}) + 2(p_1^* - \underline{p})$. Combining $\frac{\partial p_1^*}{\partial \underline{p}} \geq 1/2$ and $\gamma L > \underline{p}$ with this gives $\frac{\partial \pi_2^*}{\partial \underline{p}} > 0$. For $N = 5$, it is easy to check that $p_2^* > p_3^* < p_4^* \Leftrightarrow \underline{p} < 3\gamma L/5$. Moreover, p_{M-1}^*, p_M^* and p_{M+1}^* are decreasing function with respect to N. Hence, when $N \geq 5$, if $p_{M-1}^* > p_M^* < p_{M+1}^*$, then $\underline{p} < 3\gamma L/5$. The insertion of $\frac{\partial p_{M-1}^*}{\partial \underline{p}} \geq 1/4$ and $\underline{p} < 3\gamma L/5$ in $\frac{\partial \pi_M^*}{\partial \underline{p}}$ gives $\gamma \frac{\partial \pi_M^*}{\partial \underline{p}} = \underline{p}(2\frac{\partial p_{M-1}^*}{\partial \underline{p}} - \frac{1}{2}) + \frac{3}{2}(\frac{2}{3}\gamma L - \underline{p}) + 2(p_{M-1}^* - \underline{p}) > 0$, as required. □

Proof of Property 13

We consider four cases separately: *Case 1*: $i = 1$; *Case 2*: $i = 2$; *Case 3*: $2 < i < M$ or $i = M$ with even N; *Case 4*: $i = M$ with odd N. In the first case, an argument which is same with the proof of Proposition in Kanbur and Keen(1993) can be applied. When $\tau = p_2^\circ$, the revenue of the first government is below that in unconstrained Nash equilibrium because p_2° cannot be its best reply against p_2°. When $\tau = p_1^\circ$, its revenue is above that in unconstrained Nash equilibrium because it cannot experience any outward cross-border shopping.

In the second case, when $\tau = p_2^\circ$, the revenue of the second government is less than that in unconstrained Nash equilibrium because the U-shaped structure implies that the number of inward cross-border shoppers exceeds that of outward ones in unconstrained Nash equilibrium. When $\tau_0 = p_1^\circ$, its revenue is $p_1^\circ L$. It follows from the system (2) that $p_1^\circ = (L + p_2^\circ)/2$. On the other hand, its revenue in unconstrained Nash equilibrium is $2(p_2^\circ)^2$. Since N is greater than four, we know that $p_2^\circ < (1 + \sqrt{7})L/8 (< 0.640L)$ by Ohsawa(1998). Thus we have $p_1^\circ L - 2(p_2^\circ)^2 = (-1/2)(4(p_2^\circ)^2 - Lp_2^\circ - L^2) > 0$.

In the third case, when $\tau = p_i^\circ$, the revenue of ith government gets lower because of the U-shaped structure. When $\tau = p_{i-1}^\circ$, its revenue is $p_{i-1}^\circ L$. Following the system (2), we have $p_{i-1}^\circ = 4p_i^\circ - p_{i+1}^\circ - L \geq 3p_i^\circ - L$. On the other hand, its revenue in the unconstrained Nash equilibrium is $2(p_i^\circ)^2$. From $L/2 < p_i^\circ < L$ in the bounds (5), we have $p_{i-1}^\circ L - 2(p_i^\circ)^2 \geq -2(p_i^\circ)^2 + 3Lp_i^\circ - L^2 = -(2p_i^\circ - L)(p_i^\circ - L) > 0$.

In the last case, when $\tau = p_{M-1}^\circ$, the revenue of Mth government gets lower because p_{M-1}° is not the best reply against p_{M-1}° and p_{M+1}°. When $\tau = p_{M-2}^\circ$, its revenue is $p_{M-2}^\circ L$. Based on the system (2), we have $4p_{M-1}^\circ = L + p_{M-2}^\circ + p_M^\circ$ and $4p_M^\circ = L + p_{M-1}^\circ + p_{M+1}^\circ$. These together with $p_{M-1}^\circ = p_{M+1}^\circ$ yields $p_{M-2}^\circ = 7p_M^\circ - 3L$. On the other hand, its revenue in the unconstrained Nash equilibrium is $2(p_M^\circ)^2$. Making use of $L/2 < p_i^\circ < L$, we have $p_{M-2}^\circ L - 2(p_M^\circ)^2 = -2(p_M^\circ)^2 + 7Lp_M^\circ - 3L^2 = -(p_M^\circ - 3L)(2p_M^\circ - L) > 0$. □

Example 1

Take the case of $N = 3$, $\gamma = 1$, $L_1 = L$, $L_2 = 3L$, $L_3 = 0$, and $\overline{p} = \infty$. It follows from Proposition 2 and the system (2) that for $7L/12 \leq \underline{p} \leq 7L/6$, the equilibrium tax rates are $p_1^* = L + \underline{p}/7$, $p_2^* = L + 2\underline{p}/7$, and $p_3^* = \underline{p}$. So we have $p_3^* < p_1^* < p_2^*$ and the number of cross-border shoppers from second government to first one is proportional to $p_2^* - p_1^* = \underline{p}/7$. Thus we see that as $\underline{p}$ increases, so does this number.

On the other hand, take the case of $N = 3$, $\gamma = 1$, $L_1 = L_2 = L$, $L_3 = 9L/7$, and $\underline{p} = 0$. It follows from Proposition 2 and the system (2) that for $L \leq \overline{p} \leq 5L/6$, the equilibrium tax rates are $p_1^* = (5L + \overline{p})/7$, $p_2^* = (3L + 2\overline{p})/7$, and $p_3^* = \overline{p}$. So we have $p_3^* < p_1^* < p_2^*$. The number of cross-border shoppers from first government to second one is proportional to $p_1^* - p_2^* = (2L - \overline{p})/7$. Thus we see that as $\overline{p}$ decreases, this number increases.

Example 2

Take the case of $N = 2$, $\gamma = 1$, $L_1 \geq 4L_2$ and $\overline{p} = \infty$. It follows from Proposition 2 that there exists a unique Nash equilibrium. Based on the system (2), the equilibrium tax rates are

$$p_1^* = \begin{cases} (2L_1 + L_2)/3, & \underline{p} \leq (L_1 + 2L_2)/3; \\ (L_1 + \underline{p})/2, & (L_1 + 2L_2)/3 \leq \underline{p} \leq L_1; \\ \underline{p}, & L_1 \leq \underline{p}, \end{cases}$$
$$p_2^* = \begin{cases} (L_1 + 2L_2)/3, & \underline{p} \leq (L_1 + 2L_2)/3; \\ \underline{p}, & (L_1 + 2L_2)/3 \leq \underline{p}. \end{cases}$$

Hence, we have

$$\pi_2^* = \begin{cases} (L_1 + 2L_2)^2/9, & \underline{p} \le (L_1 + 2L_2)/3; \\ \underline{p}(L_1 + 2L_2 - \underline{p})/2, & (L_1 + 2L_2)/3 \le L_1; \\ \underline{p}L_2, & L_1 \le \underline{p}. \end{cases}$$

This function is plotted in Figure 9. This Figure makes it clear that imposing $\underline{p}$ with $(2/3)(L_1 + 2L_2) < \underline{p} < (1/9)(L_1 + 2L_2)^2/L_2$ harms the second government.

Solution Method

For solving the system (2), i.e., finding a fixed point of G, several algorithms have been developed: for example, see Dai and Yamamoto(1994). Although such algorithms may run efficiently, making the program for such algorithms is very complicated. Therefore, rather than using these algorithms, we propose a simple algorithm for finding the solution to the system (2).

Define the matrix A, the vectors x and b by

$$A = \begin{bmatrix} 1 & -\frac{1}{2} & 0 & \dots & 0 & 0 \\ -\frac{1}{4} & 1 & -\frac{1}{4} & 0 & \ddots & 0 \\ 0 & -\frac{1}{4} & 1 & -\frac{1}{4} & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & 0 \\ 0 & \ddots & 0 & -\frac{1}{4} & 1 & -\frac{1}{4} \\ 0 & 0 & \dots & 0 & -\frac{1}{2} & 1 \end{bmatrix}, \quad \mathrm{x} = \begin{bmatrix} p_1^* \\ p_2^* \\ \vdots \\ p_N^* \end{bmatrix}, \quad \mathrm{b} = \begin{bmatrix} \frac{\gamma}{2}L_1 \\ \frac{\gamma}{4}L_2 \\ \vdots \\ \frac{\gamma}{2}L_N \end{bmatrix}.$$

Let $A_{J_1 \cup J_2}$ denote the matrix which arises from A by replacing ith row with unit vector for $\forall i \in J_1 \cup J_2$, and let $\mathrm{b}(\underline{q}, \overline{q})_{J_1, J_2}$ denotes the vector which arise from b by replacing ith element with $\underline{q}$ for $\forall i \in J_1$, and by replacing the jth element with $\overline{q}$ for $\forall j \in J_2$. It follows from Proposition 1 that for $\forall \underline{p}$, $\forall \overline{p}$, the system (2) has a unique solution, so there have to exist $J_1 \subset I$ and $J_2 \subset I$ such that the solution to the system (2) and $\mathrm{x} = A^{-1}_{\underline{J_1} \cup \overline{J_2}} \mathrm{b}(\underline{p}, \overline{p})_{\underline{J_1}, \overline{J_2}}$ coincide. Therefore, it suffices to find the sets J_1 and J_2 satisfying $\underline{\mathrm{p}} \le A^{-1}_{J_1 \cup J_2} \mathrm{b}_{J_1, J_2} \le \overline{\mathrm{p}}$. The solution technique is summarized below.

1. Set $\underline{q} = 0$, $\overline{q} = \infty$, $J_1 = \phi$ and $J_2 = \phi$. Calculate $A^{-1}_{J_1 \cup J_2} \mathrm{b}(\underline{q}, \overline{q})_{J_1, J_2}$.
2. Decrease $\overline{q}$ by $\delta > 0$ small and carry out Step 4 until $\overline{q} = \overline{p}$.
3. Increase $\underline{q}$ by $\delta > 0$ and do Step 4 until $\underline{q} = \underline{p}$.
4. Compute $A^{-1}_{J_1 \cup J_2} \mathrm{b}(\underline{q}, \overline{q})_{J_1, J_2}$. If there exists $i \notin J_1$ with $p_i^* < \overline{q}$, then update J_1 by $J_1 \cup \{i\}$. If there exists $j \notin J_2$ such that $p_j^* > \underline{q}$, then update J_2 by $J_2 \cup \{j\}$.

5. Stop.

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References

[1] Anderson, S.P., A. de Palma, and J.-F. Thisse, 1992, *Discrete Choice Theory of Product Differentiation* (MIT Press, Cambridge).

[2] Christiansen, V., 1994, Cross-border shopping and the optimum commodity tax in a competitive and a monopoly market, *Scandinavian Journal of Economics, 96*, 329-341.

[3] Dai, Y, and Y. Yamamoto, 1994, A continuous deformation algorithm for variational inequality problems on polytopes, *Mathematical Programming, 64*, 103-122.

[4] d'Aspremont C., J.J. Gabszewicz and J.-F. Thisse, 1979, On Hotelling's stability in competition, *Econometrica, 47*, 1145-1150.

[5] Edwards, J and M. Keen, 1996, Tax competition and Leviathan, *European Economic Review, 40*, 113-134.

[6] Gabszewicz, J.J. and J.-F. Thisse, 1992, Location, in: R.J.Aumann and S.Hart, eds., *Handbook of Game Theory with Economic Applications* (North-Holland, Amsterdam), 281-304.

[7] Hitiris, T., 1988, *European Community Economics* (Harvester Wheatsheaf, Hertfordshire).

[8] Kanbur, R. and M. Keen, 1993, Tax competition and tax coordination when countries differ in size, *American Economic Review, 83*, 877-892.

[9] Ohsawa, Y., 1998, Cross-border shopping and tax competition among governments, *Regional Science and Urban Economics* (to appear).

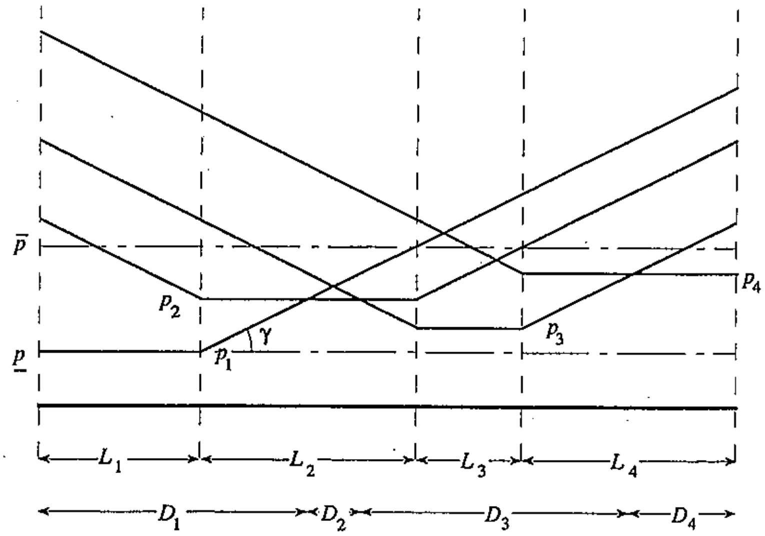

Figure 1: Market

[10] Ortega, J.M. and W.C. Rheinboldt, 1970, *Iterative Solution of Nonlinear Equations in Several Variables* (Academic Press, New York).

[11] Radaelli, C.M., 1997, *The Politics of Corporate Taxation in the European Union* (Routledge, London).

[12] Robson, P., 1998, *The Economics of International Integration* (Routledge, London).

[13] Sinn, H.W., 1990, Tax harmonization and tax competition in Europe, *European Economic Review, 34*, 489-504.

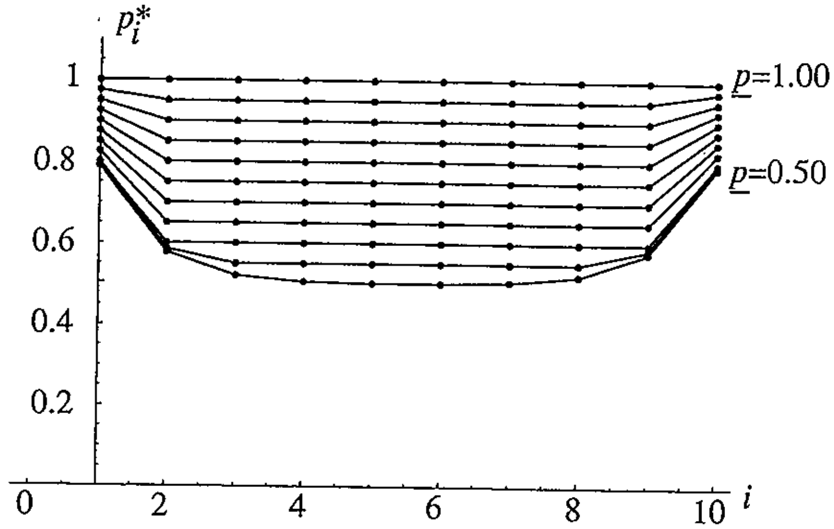

Figure 2: Equilibrium tax rates for $\underline{p}$ ranging from 0.5 to 1.0 at 0.05 intervals

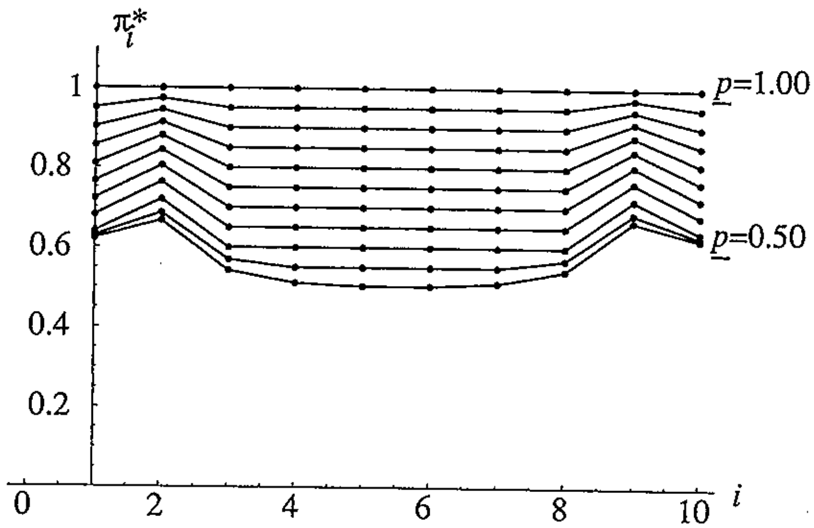

Figure 3: Equilibrium revenues for $\underline{p}$ ranging from 0.5 to 1.0 at 0.05 intervals

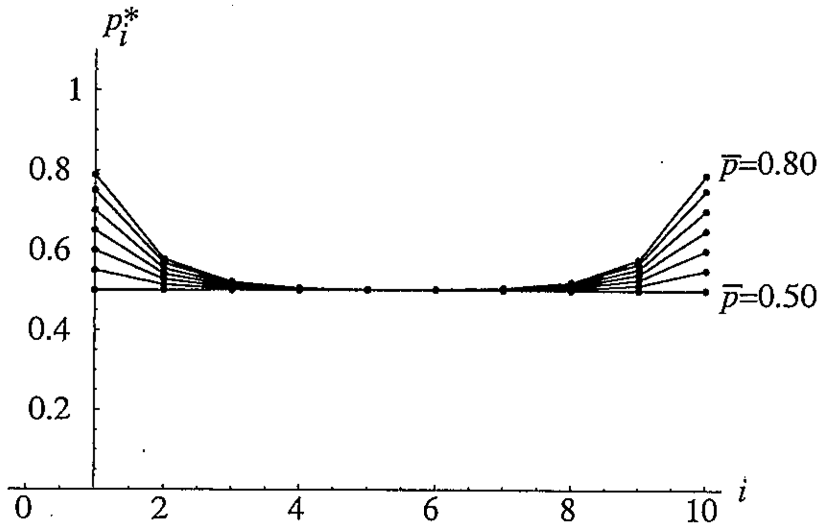

Figure 4: Equilibrium tax rates for $\overline{p}$ ranging from 1.0 to 0.5 at 0.05 intervals

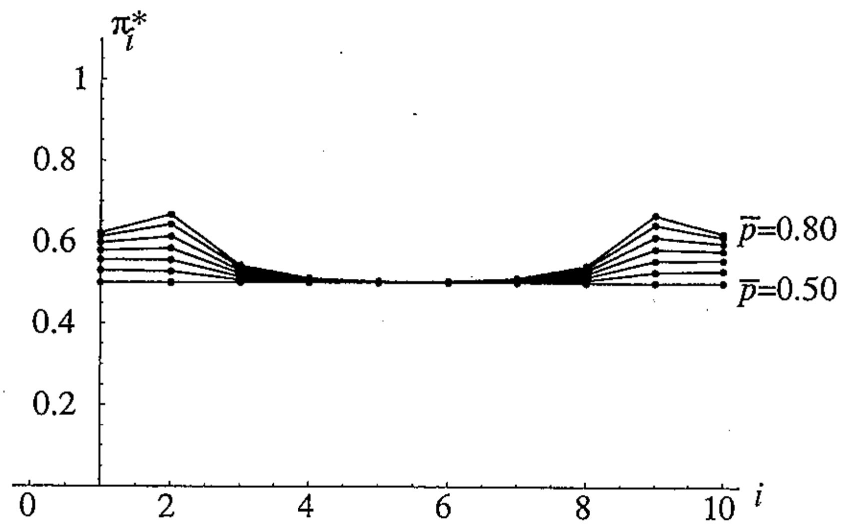

Figure 5: Equilibrium revenues for $\overline{p}$ ranging from 1.0 to 0.5 at 0.05 intervals

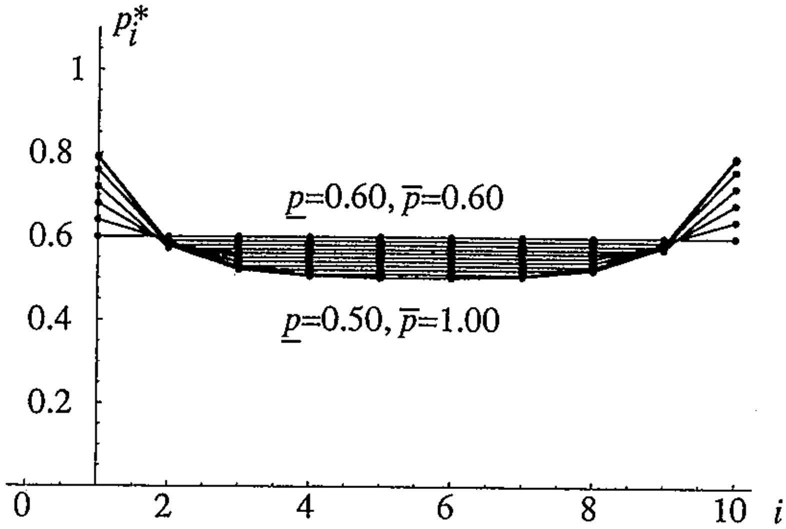

Figure 6: Equilibrium tax rates for $\underline{p}$ ranging from 0.5 to 0.6 at 0.01 intervals and $\overline{p}$ ranging from 1.0 to 0.6 at 0.04 intervals

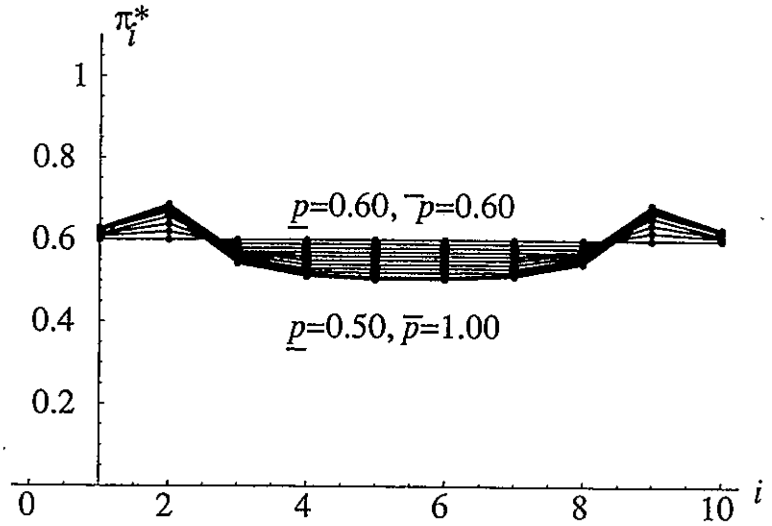

Figure 7: Equilibrium revenues for $\underline{p}$ ranging from 0.5 to 0.6 at 0.01 intervals and $\overline{p}$ ranging from 1.0 to 0.6 at 0.04 intervals

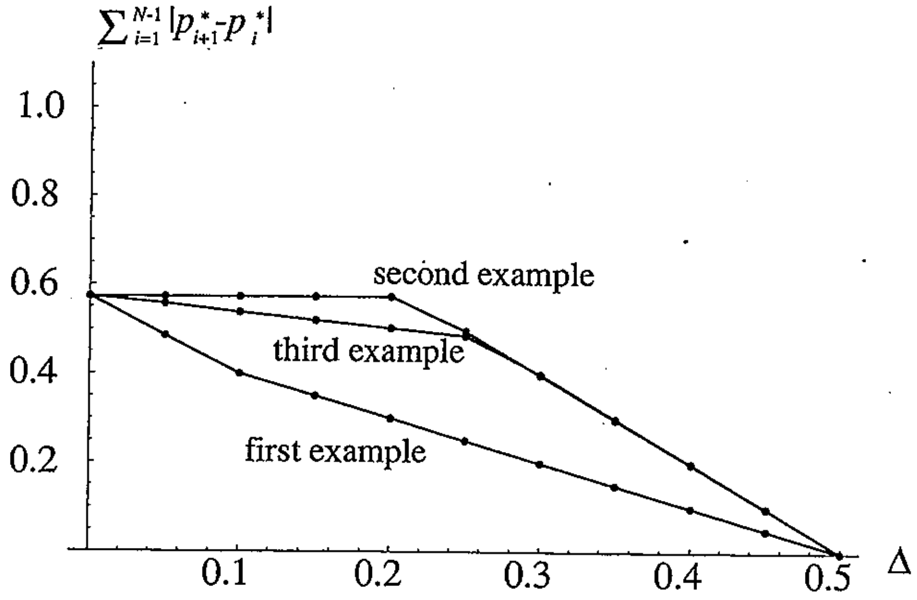

Figure 8: The number of all cross-shoppers in equilibrium

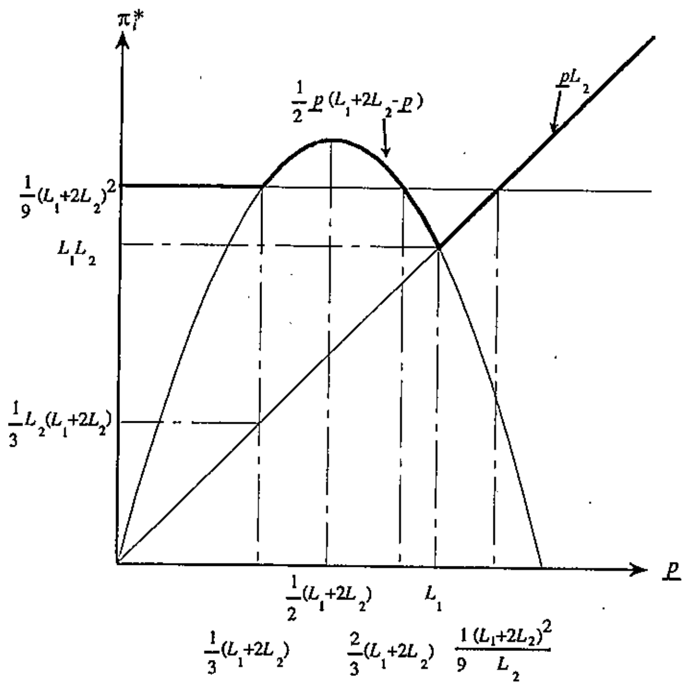

Figure 9: Equilibrium revenue of the second government