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Rational Expectations and the Effectiveness of Monetary Policy with a Special Reference to the Barro-Fischer Model

by

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Abstract

Monetary policy is sometimes alleged to be ineffective when the rational-expectations hypothesis is imposed on macroeconomic models. Barro and Fischer once developed a simple macroeconomic model in order to explain such a claim in this journal (1976). However, their conclusion depends on a specific rule employed for the future course of the money supply. It is shown that their model embodies an important factor which generally renders monetary policies effective, rational expectations notwithstanding. It is suggested that this property holds in more general macroeconomic frameworks.

1. Introduction

In their survey article of monetary policy published in this journal (1976), Barro and Fischer (hereafter BF) constructed a simple macroeconomic model to show that an anticipated monetary expansion results in the equi-proportionate rise in the price level. Apparently, this analysis was intended to provide the readers with a heuristic explanation of a yet controversial proposition — claimed by, e.g., Sargent and Wallace (1975) — that no anticipated monetary policy has effects on real variables such as output.

The main purposes of this note are to question the generality of this conclusion and to remind the readers of the fact that different policy-rules do matter, even in the very world of BF, in stabilizing output. To be more precise, I shall argue that the claimed property holds because the money supply follows a specific rule. This is the case when the money supply changes according to pure random walk.

In Section 2, the BF model is briefly explained. In Section 3, the BF model with a different money-supply rule is analyzed and monetary policy is shown to be effective in influencing output, rational expectations notwithstanding. Section 4 explains the essential factor in the BF model which generally renders monetary policies effective and suggests the validity of the results obtained in Section 3 in more general macroeconomic frameworks.

2. The BF model

BF developed the following heuristic macroeconomic model in Section 8:

$$y_t^s = \alpha[P_t - E_t(P_{t+1})] + u_t^s, \tag{1}$$

$$y_t^d = \beta[M_t - E_t(P_{t+1})] + u_t^d. \tag{2}$$

Eq.(1) is aggregate supply function embodying a speculative behavior in supplying output ($\alpha>0$). P_t is the actual price of output at period t and $E_t(P_{t+1})$ denotes the rational expectation, formed at period t, of the price level that will prevail next period, t+1.

Eq.(2) is aggregate demand function with the positive real-balance effect ($\beta>0$) where M_t is the nominal money supply. u_t^s and u_t^d are random terms (with means zero) that shift supply and demand through unanticipated changes in productivity, tastes, and so on. All the variables are in logarithmic form and they are deviations from "normal" levels so that constnat terms are omitted.

Given the nominal stock of money and the rational expectation of the future price level, the actual price level and output are determined by a market-clearing condition equating aggregate supply and demand; i.e., (1) and (2) yield the "pseudo" reduced-form equation

$$P_t = (1-\frac{\beta}{\alpha})E_t(P_{t+1}) + \frac{\beta}{\alpha}M_t + \frac{1}{\alpha}u_t, \tag{3}$$

where $u_t = u_t^d - u_t^s$. Eq.(3) is called pseudo-reduced-form equation because the term $E_t(P_{t+1})$ is not truly exogenous but determined simultaneously with P_t.

In order for the economic agents to compute the rational expectation $E_t(P_{t+1})$ the money-supply rule has to be known. BF specify the random-walk rule for M_t (and for u_t^s and u_t^d as well) and obtain

$$P_t = E_t(P_{t+1}) = M_t + \frac{1}{\beta}u_t , \tag{4}$$

and accordingly

$$y_t = (1-\beta)u_t^d + \beta u_t^s, \tag{5}$$

as the market-clearing price level and output.

The implication of (4) and (5) is straightforward. Namely, from (4), the equilibrium price level changes equiproportionately as the nominal money supply; consequently, from (5), output appears to be determined independently of the money supply. It is clear that these conclusions per se are not to be blamed in any way. However, the random-walk rule for the money supply is too specific a case to deny altogether the effectiveness of all the anticipated stabilization policies.

3. Feedback Money-Supply Rule

In showing that monetary policy in general can exert a stabilizing power on output even within the BF model, suffice it here to consider a simple feedback money-supply rule against the previous price level:

$$M_t = -\gamma P_{t-1}. \tag{6}$$

I shall allow both counter-cyclical $(\gamma>0)$ and pro-cyclical $(\gamma<0)$

stances of the authority, provided that doing so does not violate several conditions being imposed below. However, one may well follow the analysis by regarding γ as nonnegative. Eq.(6) is an example of the "proportional" feedback policy of Phillips (1954).

Substituting (6) into (3), one obtains

$$P_t = (1-\frac{\beta}{\alpha})E_t(P_{t+1}) - \frac{\beta\gamma}{\alpha} P_{t-1} + \frac{1}{\alpha}u_t. \tag{7}$$

Forwarding j ($j \geq 1$) periods ahead of t and taking conditional expectations as of period t, one gets from (7)

$$(1-\frac{\beta}{\alpha})E_t(P_{t+j+1}) - E_t(P_{t+j}) - \frac{\beta\gamma}{\alpha} E_t(P_{t+j-1}) = -\frac{1}{\alpha}u_t, \tag{8}$$

where the random-walk assumption for u_t^s and u_t^d has been retained, i.e., $E_t(u_{t+j}) = u_t$ for all $j \geq 1$. This is a second-order difference equation with the initial conditon $E_t(P_t) = P_t$ for j=1.

The mathematical solution for (8) can be written as

$$E_t(P_{t+j}) = a_1\lambda_1^j + a_2\lambda_2^j + \frac{1}{\beta(1+\gamma)} u_t, \tag{9}$$

where λ_i (i=1,2) are the two distinct roots of the characteristic equation

$$f(\lambda) \equiv (1-\frac{\beta}{\alpha})\lambda^2 - \lambda - \frac{\beta\gamma}{\alpha} = 0, \tag{10}$$

and a_i are to be determined by the "economic theory" and the initial condition.

The theory of rational expectations prescribes that the rational-expectations path should be unique and convergent (or nondivergent in a weaker sense) in the long run.[1/] In order for (8) to have the unique convergent long-run rational-expectations path, the characteristic equation (10) must have one and only one root with $0 < |\lambda| < 1$. (Therefore, it must be a real and simple root!) The other root must be either $\lambda = 0$ or $|\lambda| \geq 1$. The condition $|\lambda| < 1$ is needed for convergence and the condition for one and only one root to be so (excluding $\lambda = 0$) is needed for uniqueness because there is only one initial condition.[2/] The evolution of $E_t(P_{t+j})$ governed by $|\lambda_i| > 1$ is to be given zero weight $(a_i=0)$ in (9) and the one governed by $\lambda_i=0$ raises no problem.

The desired property discussed above follows when one has $f(1)f(-1) < 0$ or

$$0 < 1+\gamma < 2\frac{\alpha}{\beta}. \tag{11}$$

In what follows, I shall assume the satisfaction of (11). This implies that the authority does not attempt to execute too drastic counter- or pro-cyclical changes in the money supply, given fluctuations of the past price level.[3/]

The long-run rational expectations path is now written as

$$E_t(P_{t+j}) = a\lambda^{*j} + \frac{1}{\beta(1+\gamma)} u_t, \tag{12}$$

where λ^* denotes the unique characteristic root with $0 < |\lambda^*| < 1$. Using the initial condition $E_t(P_t) = P_t$ as the case of $j=0$ in (12), the unknown coefficient a is obtainable. Thus, for $j \geq 1$.

$$E_t(P_{t+j}) = \lambda^{*j} P_t + \frac{1-\lambda^{*j}}{\beta(1+\gamma)} u_t. \tag{13}$$

The long-run rational-expectations path is seen to converge to $\frac{1}{\beta(1+\gamma)} u_t$ as j increases. It is interesting to note that the current price level looses its weight geometrically as the predictor of the future price level.

Substituting (13) into (7), one obtains

$$P_t = \lambda^* P_{t-1} + \frac{1-\lambda^*}{\beta(1+\gamma)} u_t, \tag{14}$$

where relation (10) has been freely made use of in simplifying the expressions. Eq.(14) is consistent with (13), as it should be so. The elimination of P_{t-1} from (14) by using (6) yields

$$P_t = -\frac{\lambda^*}{\gamma} M_t + \frac{1-\lambda^*}{\beta(1+\gamma)} u_t. \tag{15}$$

A little computation reveals that $0 < -\frac{\lambda^*}{\gamma} < 1$,4/ implying that changes in the actual price level only partially reflect changes in the nominal money supply although both of them move in the same direction.

Consequently, monetary policy can have an impact on output as one obtains from (2) and (13)

$$y_t = \frac{\beta}{\gamma} (\gamma+\lambda^{*2}) M_t + \frac{\gamma+\lambda^{*2}}{1+\gamma} u_t^d + \frac{1-\lambda^{*2}}{1+\gamma} u_t^s. \tag{16}$$

The money multiplier (or elasticity, to be precise) is not null but positive. Moreover, it is greater than β, the short-run multiplier in (2). This proves that monetary policy is effective in influencing output, even though it follows a known rule and it is rationally anticipated.

The point that monetary policy can stabilize output can be more clearly seen by analyzing the variance of output in a stochastic stationary-state. This is because, in the above analysis, the money supply is not freely chosen by the authority in each period and thereby the meaning of stabilization is ambiguous. Under the money-supply rule (6), the value of γ is the only policy variable. Therefore, the effectiveness of monetary policy can be best understood by finding a relationship between the value of γ and the variance of output. For this task, I shall reassume to the end of this section that the random terms u_t^d and u_t^s are independent trendless white-noise disturbances with the stationary variances σ_d^2 and σ_s^2, respectively. This is because random-walk processes cannot have stationary variances. The main thread of the analyses are kept intact by this change.

However, under the new setup, the right-hand side of (8) becomes zero and accordingly the second terms of the right-hand side of (12) and (13) also vanish. Eq.(14) now reads as

$$P_t = \lambda^* P_{t-1} - \frac{\lambda^*}{\beta\gamma} u_t, \tag{17}$$

and, from (1) and the new (13), one obtains

$$y_t = \alpha(1-\lambda^*)P_t + u_t^s. \tag{18}$$

Since $|\lambda^*| < 1$, there exists a stochastic stationary-state. From (17):

$$\mathrm{Var}(P) = \frac{1}{1-\lambda^{*2}} \left(\frac{\lambda^*}{\beta\gamma}\right)^2 (\sigma_d^2+\sigma_s^2),$$

$$\mathrm{Cov}(P,u^s) = \frac{\lambda^*}{\beta\gamma}\sigma_s^2,$$

so that from (18)

$$\mathrm{Var}(y) = \alpha^2(1-\lambda^*)^2\mathrm{Var}(P) + 2\alpha(1-\lambda^*)\mathrm{Cov}(P,\ u^s) + \sigma_s^2$$

$$= \frac{1-\lambda^*}{1+\lambda^*}\eta^2\sigma_d^2 + [\frac{1-\lambda^*}{1+\lambda^*}\eta^2 + 2(1-\lambda^*)\eta + 1]\sigma_s^2,$$

where $\eta = \alpha\lambda^*/\beta\gamma$.

Clearly, the stationary variance of output depends on the policy rule (i.e., the value of γ) both directly and indirectly through λ^*. There is an optimal γ in the sense that it minimizes Var(y). In the simple model analyzed here in which no past variable affects the current equilibrium, this optimal γ equals zero. This is because the policy rule (6) with $\gamma \neq 0$ introduces the past disturbances into the current equilibrium as additional shocks. However, in a more general model in which random terms are serially correlated or the past output enters into (1) or (2) as an autocorrelated term, the optimal policy sholud call for $\gamma \neq 0$. This is to be expected because such a policy partially or entirely offsets the spill-over of the past disturbances into the current equilibrium. This is an exercise left to the interested readers, as the main purpose of the present analysis is not finding the optimal policy but (as a prerequisite to such an inquiry) merely showing the effectiveness of monetary policies.

4. The Source of Effectiveness

Figure 1 illustrates the determination of the equilibrium price level in the $(E_t(P_{t+1}),\ P_t)$ plane. The upward-sloping AA line describes

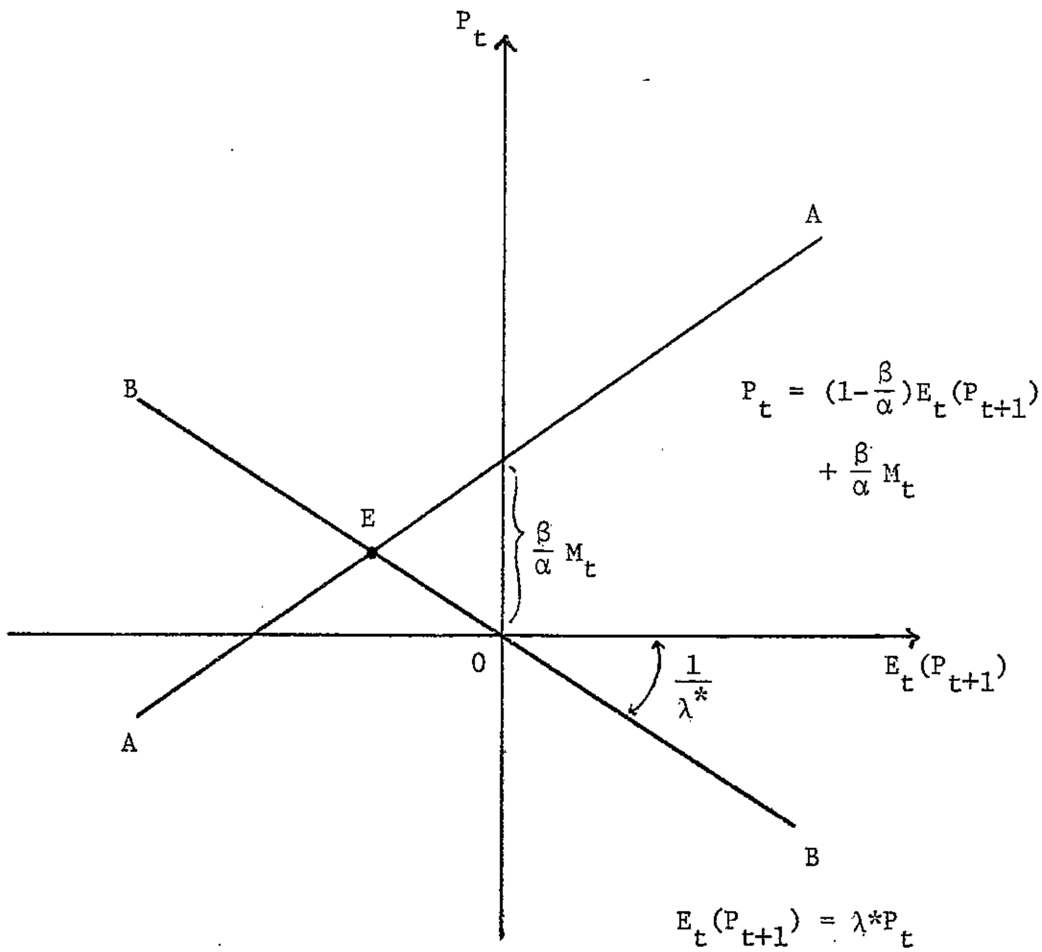
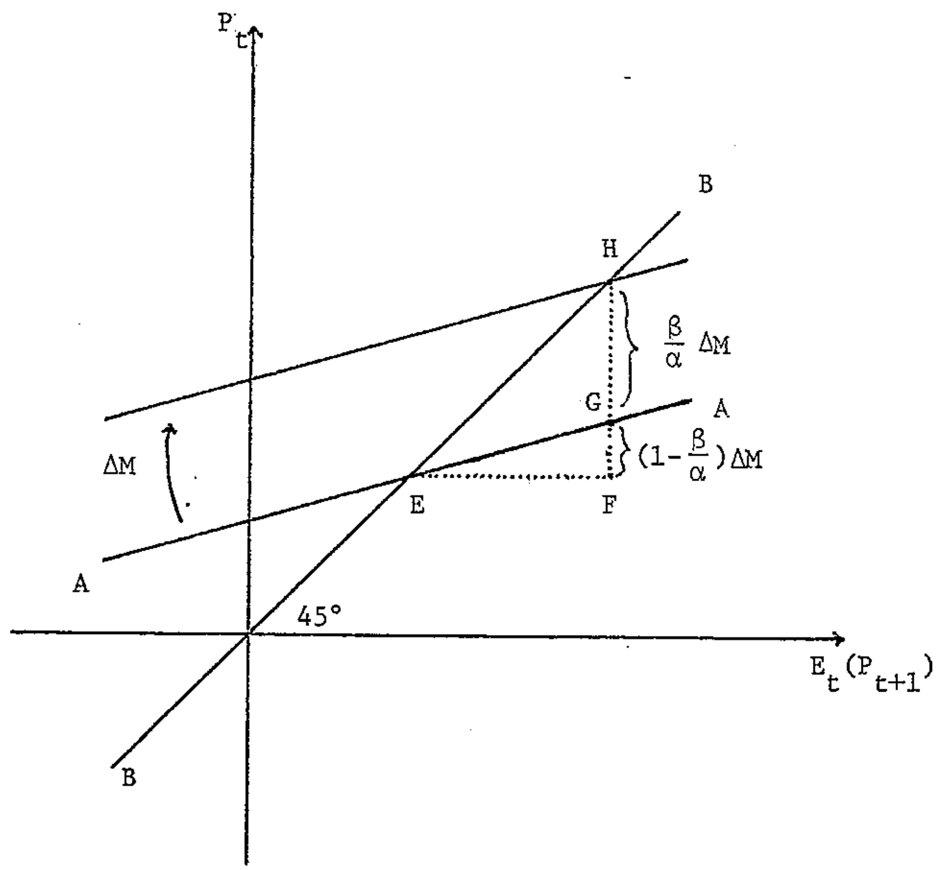

P_t
A
B
$P_t = (1-\frac{\beta}{\alpha})E_t(P_{t+1})$
$+ \frac{\beta}{\alpha} M_t$
E
$\frac{\beta}{\alpha} M_t$
0
$\frac{1}{\lambda^*}$
$E_t(P_{t+1})$
A
B
$E_t(P_{t+1}) = \lambda^* P_t$

Fig. 1



P_t
B
H
$\frac{\beta}{\alpha}\Delta M$
G
A
ΔM
$(1-\frac{\beta}{\alpha})\Delta M$
E
F
A
45°
$E_t(P_{t+1})$
B

Fig. 2

the pseudo-reduced-form equation (3) for the given money supply whereas the downward-sloping BB line describes the rational-expectation formation (13). It is assumed that the realized random terms happen to be nil at period t in drawing the figure. The actual, equilibrium price level is determined at the cross of the two lines, E. The position of the AA line depends on the policy rule (the value of γ) as it determines the money supply at period t. On the other hand, the slope of the BB line is dependent of γ through λ^*. Therefore, the equilibirum price level cannot generally be free from γ or the money-supply rule.

In the case of the random-walk rule for the moeny supply, the BB line of Figure 1 becoms the 45-degree line as drawn in Figure 2 because $P_t = E_t(P_{t+1})$ holds. An increase in the money supply by the amount ΔM shifts upwards the AA line by the amount $\frac{\beta}{\alpha}\Delta M$ or $\overline{GH}$. The total increase in P_t [and that in $E_t(P_{t+1})$ as well], $\overline{FH}$, equals the sum of this $\overline{GH}$ and $\overline{FG}$ which is equal to $(1-\frac{\beta}{\alpha})\overline{EF}$ as the slope of the AA line is $1-\frac{\beta}{\alpha}$. Then, since

$$\overline{FH} = \frac{\beta}{\alpha}\Delta M + (1-\frac{\beta}{\alpha})\ \overline{EF} = \overline{EF},$$

the increase in P_t, $\overline{FH}$, has to equal ΔM. This is the graphical explanation of the exact proportionate relationship between the money supply and the actual price level.

A little reflection reveals that the above property holds only when the slope of the BB line is 45-degree. The random-walk rule analyzed by BF leads to this special case.[5/]

In the BF model, there is an expectation term which utilizes all the information available (up to and) at the current period, t. The economic agents observe the current, actual price level in forecasting the future price levels. In other words, the rational expectation of the future price level depends on the actual price level. Meanwhile, the actual price level is dependent on the expected future price level, as is clear from (3). The "equilibrium" price level as a "fixed-point" of these two relationships, therefore, is deprived of a certain degree of freedom and becomes "rigid" or "partially flexible". Then, this loss of freedom in price flexibility creates room for monetary policies to have stabilizing powers on output, as has been demonstrated in different contexts by Phelps and Taylor (1977) and Fischer (1977).

The effectiveness of monetary policies is not the characteristics specific to the BF model. Any macroeconomic model in which the available information set includes the current observation of economic data leads to the same conclusion even the rational-expectations hypothesis is imposed together with the "market-clearing" specification. In other words, if X_t is the vector of endogenous variables in a macroeconomic model and if $E_t(X_{t+1})$ is an important factor in determining X_t, then alternative policy rules will generally influence the stabilization of X_t.[6/]

References

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Footnotes

1/. If the rational-expectations path is not unique, no further analysis should be conducted [Phelps and Taylor (1977)]. The long-run divergence is precluded in order to rule out "speculative bubbles" [Sargent and Wallace (1975)]. Muth (1961) starts from the beginning finding unique, real, and bounded sloution.

2/. Note that (8) is defined for $j \geq 1$. The initial condition is $E_t(P_t) = P_t$. In (7), P_{t-1} is given as the past data at period t. However, it is only used in determining P_t.

3/. When (11) is not satisfied and when two of the characteristic roots are with $|\lambda| > 1$, two alternative interpretations are possible. First, the long-run consistent rational-expectations path cannot simply exist. Expectations are doomed to be irrational in the long run even though rational in the short run. Second, both a_i are to be set zero, or the long-run rational-expectations path is stationary. In this case monetary policies are effective as it corresponds to "static" expectations.

4/. The condition

$$-\frac{\lambda^*}{\gamma} = [\{\alpha^2+4(\alpha-\beta)\beta\gamma\}^{\frac{1}{2}} - \alpha]/2(\alpha-\beta)\gamma \gtrless 1,$$

is equivalent to $(\alpha-\beta)^2\gamma^2(1+\gamma) \lesseqgtr 0$ when $\alpha>\beta$. That $-\frac{\lambda^*}{\gamma} > 0$ is obvious.

5/. When $\gamma \to -1$ (and if $\alpha < 2\beta$), $\lambda^* \to 1$ so that the BB line is again 45-degree line. In this case one can obtain

$$E_t(P_{t+j}) = P_t + \frac{j}{2\beta-\alpha} u_t,$$

for all $j \geq 1$ as the long-run rational-expectations path, which is apparently divergent. (However, the money supply per se will be divergent as $M_t = P_{t-1}$.)

6/. See Asako (1979) for more in detail about general discussions on the source of the effectiveness of stabilization policies.