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by

Manabu Asai

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Manabu Asai*

Doctoral Program in Policy and Planning Sciences, University of Tsukuba, Tsukuba, Ibaraki 305-8573 Japan

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Abstract

This note develops a regression-based testing procedure for serial correlation in the presence of stochastic volatility. We investigated finite sample properties of our test and found that our test is robust to stochastic volatility in terms of both size and power performances.

Keywords: Serial correlation tests, Stochastic Volatility, Log-GARCH models

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*e-mail: masai@sk.tsukuba.ac.jp

1 Introduction

Changes in asset return variance or volatility over time, may be modeled using either the GARCH class models pioneered by Engle (1982), or stochastic volatility (SV) models; see, *e.g.*, Taylor (1994). A test for the presence of serial correlation is routinely carried out as a test for efficiency in financial markets. Most of the existing tests of autocorrelation, however, require homoskedastic errors and hence may not be robust to heteroskedasticity. There also exists tests of autocorrelation literature that take account of heteroskedasticity, such as Diebold (1986), Wooldridge (1991), and Bollerslev and Wooldridge (1992). These tests are, however, designed to have good performance only under conditional heteroskedasticity. The purpose of this note is to develop a regression-based testing procedure for serial correlation in the presence of stochastic volatility.

2 The Testing Procedure

Consider the model

$$y_t = x_t\beta + u_t, \qquad t = 1, \ldots, T \tag{1}$$

where y_t is a dependent variable, $x_t = [x_{1t}, \ldots, x_{kt}]$ is a $1 \times k$ vector of variables, which may include stochastic and non-stochastic variables, lagged regressors and lagged values of y_t, and β is a $k \times 1$ vector of unknown parameters, and u_t follows the stationary AR(p) process

$$u_t = \rho_1 u_{t-1} + \cdots + \rho_p u_{t-p} + e_t \qquad t = 1, \ldots, T \tag{2}$$

where $\rho_1, \ldots, \rho_p$ are unknown autoregressive parameters. In order to ensure the stationarity of (2), we assume that roots of $1 - \rho_1 L - \cdots - \rho_p L^p = 0$, where L is the lag operator, lie outside the unit circle. The term e_t is assumed to follow a simple stochastic volatility process

$$
\begin{aligned}
e_t &= \sqrt{h_t}\eta_t, \quad \eta_t \sim \text{i.i.d.N}(0,1), & (3)\\
\ln h_t &= \gamma + \phi \ln h_{t-1} + \sigma_\nu \nu_t, \quad \nu_t \sim \text{i.i.d.N}(0,1), & (4)
\end{aligned}
$$

where ν_t is generated independently of η_t. This model may be estimated by GMM, Asai's (1998) QML method via a log-GARCH approach, or Bayesian Markov chain Monte Carlo by Jacquire, Polson, and Rossi (1994).

We wish to test the null hypothesis, $\text{H}_0 : \rho_1 = \cdots = \rho_p = 0$, against the alternative hypothesis $\text{H}_1 :$ Not all $\rho_j = 0$, $(j = 1, \ldots, p)$.

Asai (1998) showed that a simple SV process in e_t can be interpreted as a log-GARCH(1,1) model in e_t

$$
\begin{aligned}
e_t &= \sigma_t z_t, & (5)\\
\ln \sigma_t^2 &= \alpha_0 + \alpha_1 \ln e_{t-1}^2 + \beta_1 \ln \sigma_{t-1}^2, & (6)
\end{aligned}
$$

where $\alpha_0 = \gamma + (1-\phi)c_\eta + (1-\theta)\ln c_z^2$, $\alpha_1 = \phi - \theta$, $\beta_1 = \theta$, and $c_\eta = E(\ln \eta_t^2) \simeq -1.27$,

$$
\begin{aligned}
\theta &= \frac{1}{2\phi}\left[1 + \phi^2 + \frac{2\sigma_\nu^2}{\pi^2} - \sqrt{\left(1 - \phi^2 + \frac{2\sigma_\nu^2}{\pi^2}\right)^2 + \frac{8\phi^2\sigma_\nu^2}{\pi^2}}\right],\\
c_z &= \exp\left[\frac{\sigma_\nu^2}{4(1-\theta^2)} - \frac{c_\eta}{2} - \frac{(\theta-\phi)\theta}{2(1-\theta)}\psi\left(\frac{1}{2}\right)\right.\\
&\qquad \left. + \frac{1}{2}\sum_{i=1}^{\infty}\left[\ln\Gamma\left((\theta-\phi)\theta^i + \frac{1}{2}\right) - \ln\Gamma\left(\frac{1}{2}\right)\right]\right].
\end{aligned}
$$

z_t is a weak stationary process, odd-order moments of which are all zero, and $E(z_t^2) = 1$ and $E(z_t z_{t-j})$ for $j \geq 1$. The forth order moment exists if $|\phi| < 1$ and $\sigma_\nu > 0$, and is larger than three. σ_t is measurable with respect to the time $t-1$ information set. This representation enables us to test for serial correlation in the presence of SV, using Wooldridge's (1991) robust tests.

Wooldridge's (1991) ARCH-corrected LM test is robust for testing H_0 in time series models with correctly specified conditional mean. The construction of the LM statistic involves the following step:

1. Obtain the fitted value denoted here by $\ln \hat{\sigma}_t^2$, $t = 1, \ldots, T$ from the linear regression

$$\ln \hat{u}_t^2 = a_0 + a_1 \ln \hat{u}_{t-1}^2 + \cdots + a_m \ln \hat{u}_{t-m}^2 + \epsilon_t, \quad t = 1, \ldots, T.$$

2. Define $x_t^* = x_t / \hat{\sigma}_t$ and $\tilde{u}_t = \hat{u}_t / \hat{\sigma}_t$, $t = 1, \ldots, T$.
3. Save the $1 \times p$ vector of residuals, say $\tilde{r}_t$, from the regression of each of the $\tilde{\lambda}_t$ on x_t^*, where $\tilde{\lambda}_t = (\tilde{u}_{t-1}, \ldots, \tilde{u}_{t-p})$.
4. Compute (T-SSR), where SSR is the sum of squared residuals from the regression of 1 on $\tilde{u}_t \tilde{r}_t$. (T-SSR) $\sim \chi^2(p)$ asymptotically under H_0.

Remaining problem is how to choose m. Indeed, $m = 10$ is enough in empirical analysis. A log-GARCH(1,1) process in e_t can be interpreted as a ARMA(1,1) process in $\ln e_t^2$, and thus a AR(∞) process in $\ln e_t^2$;

$$\ln e_t^2 = ((1 - \phi)c_\eta + \gamma)/(1 - \theta) + (\phi - \theta) \sum_{j=1}^{\infty} \theta^j \ln e_{t-j}^2 + \epsilon_t. \tag{7}$$

If, for example, $\phi = 0.90$ and $\sigma_\nu = 0.363$, which were used in Monte Carlo simulations of Jacquire, Polson, and Rossi (1994), then $\theta = 0.817$ and the coefficient of $\ln y_{t-11}^2$ is about 0.01.

3 Size and poer in finite samples

This section examines finite-sample properties of our test through simulation experiments. All the work described subsequently was conducted using the GAUSS programming language.

The true model in our experiments is given by a linear regression model

$$y_t = x_t \beta_0 + u_t \quad \text{for } t = -20, \ldots, T$$

with $x_t = (1, x_{1t})$ and the true vector $\beta_0 = (1, 1)'$. The expression $\{x_{1t} : t = -19, \ldots, T\}$ is generated from an AR(1) process

$$x_{1t} = 0.5x_{1,t-1} + \xi_t \quad \text{with } \xi_t \sim \text{i.i.d.N}(0, 1)$$

with $X_{-20} = 0$. And $\{u_t : t = -19, \ldots, T\}$ is generated from an AR(1)-SV process

$$u_t = \rho u_{t-1} + e_t \quad \text{with } u_{-20} = 0$$

where e_t is generated by equations (3) and (4). The data are creating $T + 20$ observations, and discarding the first 20 observations to remove the effect of the initial conditions. Samples of size T =250, 500, 1000, 2000, and 4000 are used in the experiments. A sample of 1000–4000 is not uncommon in studies using daily or weekly data. As for the parameters in equations (3) and (4), we take $(\gamma, \phi, \sigma_\nu)$ =$\{(-0.7360, 0.90, 0.3629)$, $(-0.3680, 0.95, 0.2600)$, $(-0.1472, 0.98, 0.1657)\}$. These parameter values are also used by Jacquire, Polson, and Rossi (1994), among others. As we noted in previous section, we set $m = 10$.

We first consider the size of the tests. The null hypothesis specifies $\rho = 0$. We can see in table 1 that the tests have approximately correct size even for $T = 250$. We next consider power of the tests. Table 2 presents simulation results giving the powers of the test for ρ =± 0.1, ± 0.5, ± 0.8. We find appreciable power except when ρ =± 0.1 and T is not large. For ρ =± 0.1, power increases as T increases. When T is larger than 1000, powers for 5 percent level are large enough. We may conclude that our test of 5 percent level is robust to stochastic volatility in terms of both size and power performances for sample sizes in studies using daily or weekly data.

References

[1] Asai, M., 1998, A new method to estimate stochastic volatility models: a log-GARCH approach, forthcoming in: Journal of the Japan Statistical Society 28.

[2] Bollerslev, T. and J.M. Wooldridge, 1992, Quasi-maximum likelihood estimation and inference in dynamic models with time-varying covariances, Econometric Reviews, 11, 143–172.

[3] Diebold, F.X., 1986, Testing for serial correlation in the presence of ARCH, Proceedings of the American Statistical Association, 323-328.

[4] Jacquier, E., N.G. Polson, and P.E. Rossi, 1994, Bayesian analysis of stochastic volatilities, Journal of Business and Economic Statistics 12, 371–389.

[5] Taylor, S.J., 1994, Modeling stochastic volatility: a review and comparative study, Mathematical Finance 4, 183–204.

[6] Wooldridge, J.M., 1991, On the application of robust, regression-based diagnostics to models of conditional means and conditional variances, Journal of Econometrics 47, 5–46.

Table 1: Size of the tests (5000 replications)

T	5 percent level			1 percent level		
	$\phi = 0.90$	$\phi = 0.95$	$\phi = 0.98$	$\phi = 0.90$	$\phi = 0.95$	$\phi = 0.98$
250	0.048	0.047	0.049	0.0066	0.0080	0.0074
500	0.048	0.053	0.047	0.0078	0.0102	0.0090
1000	0.055	0.048	0.052	0.0090	0.0074	0.0088
2000	0.049	0.042	0.051	0.0110	0.0086	0.0102
4000	0.049	0.052	0.047	0.0110	0.0088	0.0064

Note: "$\phi = 0.90$" denotes the SV parameter setting $(\gamma, \phi, \sigma_\nu)$=$(-0.7360, 0.90, 0.3629)$.

Table 2: Power of the tests (5000 replications)

		5 percent level			1 percent level		
T	ρ	$\phi = 0.90$	$\phi = 0.95$	$\phi = 0.98$	$\phi = 0.90$	$\phi = 0.95$	$\phi = 0.98$
250	0.8	0.998	0.999	0.996	0.996	0.997	0.990
	0.5	0.999	1.000	0.999	0.998	0.999	0.996
	0.1	0.214	0.214	0.209	0.070	0.073	0.067
	-0.1	0.262	0.270	0.258	0.099	0.102	0.095
	-0.5	1.000	1.000	1.000	0.999	0.999	0.996
	-0.8	1.000	1.000	0.999	0.999	0.999	0.997
500	0.8	0.999	0.999	0.997	0.998	0.998	0.994
	0.5	1.000	1.000	1.000	1.000	1.000	1.000
	0.1	0.416	0.424	0.401	0.195	0.202	0.188
	-0.1	0.449	0.459	0.445	0.218	0.230	0.222
	-0.5	1.000	1.000	1.000	1.000	1.000	1.000
	-0.8	1.000	1.000	0.999	0.999	1.000	0.997
1000	0.8	1.000	1.000	1.000	0.999	0.999	0.999
	0.5	1.000	1.000	1.000	1.000	1.000	1.000
	0.1	0.704	0.709	0.695	0.473	0.477	0.453
	-0.1	0.721	0.738	0.727	0.496	0.508	0.496
	-0.5	1.000	1.000	1.000	1.000	1.000	1.000
	-0.8	1.000	1.000	1.000	1.000	1.000	1.000
2000	0.8	1.000	1.000	0.999	1.000	1.000	0.999
	0.5	1.000	1.000	1.000	1.000	1.000	1.000
	0.1	0.938	0.947	0.938	0.823	0.839	0.832
	-0.1	0.946	0.948	0.947	0.835	0.852	0.845
	-0.5	1.000	1.000	1.000	1.000	1.000	1.000
	-0.8	1.000	1.000	1.000	1.000	1.000	1.000
4000	0.8	1.000	1.000	1.000	1.000	1.000	0.999
	0.5	1.000	1.000	1.000	1.000	1.000	1.000
	0.1	0.999	1.000	0.999	0.993	0.995	0.992
	-0.1	0.999	0.999	0.999	0.993	0.993	0.993
	-0.5	1.000	1.000	1.000	1.000	1.000	1.000
	-0.8	1.000	1.000	1.000	1.000	1.000	1.000

Note: "$\phi = 0.90$" denotes the SV parameter setting $(\gamma, \phi, \sigma_\nu) = (-0.7360, 0.90, 0.3629)$.