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An Analysis of Inflation Targeting Policy in a Money-in-the-Utility-Function Framework

by

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Summary: This study investigates the possibility of successful use of inflation targeting in monetary policy. We construct a general equilibrium economic model which consists of identical infinitely-lived households whose utility depends not only on consumption but also on real money holdings. Our findings are that the government fails to control inflation in almost every case under the policy of a positive constant rate of money supply growth and that it is possible that there exists a determinate dynamic equilibrium path under the policy in which the government controls money supply so as to implement the target sequence for the price level. These results provide theoretical justification to monetary authorities that operates inflation targeting policy regime in practice.

Keywords: inflation targeting, money in the utility function, determinacy of equilibrium.

JEL **classification:** E31, E52

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1 Introduction

There is increasing interest in monetary policy, that is known as "inflation targeting" or "price-level targeting," to keep a target inflation rate or a target time path for the price level. According to Ammer and Freeman(1995) and Bernanke and Mishkin(1997), there are some monetary authorities that have made strong policy declarations that give increased emphasis on the control of the price level. Table 1 shows countries that have established specific inflation targets officially. Underlying such a policy, there is a notion that price stability is desirable.

The only study that provides a theoretical analysis of inflation targeting is Smith(1994) while there are some empirical research dealing with this policy.[1] The model of Smith is an economy with stationary, two-period lived overlapping generations under the constraint of a reserve requirement which forces money to be held. He examined various policies and reached the conclusion as follows: (a) under a policy in which the money supply grows at a constant rate, there is a high possibility that equilibria cannot implement target inflation rate, (b) indeterminacy of equilibrium may occur under inflation targeting policy regime using lump-sum transfer, and (c) no equilibrium may exist for some target price level path when the government uses open

[1] See, for example, Dueker and Fischer(1996).

Table 1: Targeted inflation rates among the industrial countries

	Targeted series	Target period	Target rate (percentage change)
New Zealand	CPI	Starting 1994	0-2
Canada	CPI	Through 1998	1-3
United Kingdom	RPI	Medium term	1-4
Sweden	CPI	Starting 1995	1-3
Finland	CPI	Starting 1995	2

Source: Ammer and Freeman(1995)

market operations to achieve targeted rate of inflation, and even if there is, it may be Pareto dominated by one supported by lump-sum transfer that implements the same price sequence. In other words, each policy examined in his paper has an undesirable property.

It is well known that there are several approaches that integrate monetary economic theory and price theory besides reserve requirement. One is "money in the utility function," which includes the level of real money balance as an argument of agents' utility function in addition to the level of consumption.[2] Another is "cash-in-advance constraint," which implies that money must be

[2] This is used by Sidrauski(1967), for example.

used to buy goods.[3] What happens when we apply one of these assumptions to the analysis of inflation targeting instead of reserve requirement? Are different results obtained? Or can the results of Smith be applied to a wider class of monetary models?

This paper, which analyzes an inflation targeting policy regime under the assumption of money-in-the-utility-function, gives one answer for these questions. We use a model of the economy that consists of infinitely-lived consumers and public-good-supplying government, and examine two kinds of policy: first a "constant-money-growth rule" which is simply to set the growth rate of money supply at a constant value, and second an "inflation targeting policy regime" which sets a target time path for the price level with the government controlling money supply so as to maintain it. The reason why the latter policy must be examined is that the former results in an undesirable consequence. In this paper, following Smith(1994), we use the term "inflation targeting" to indicate a policy target in which the rate of change in the price level is perpetually set to be constant.

The remainder of the paper proceeds as follows. In section 2 we construct a general equilibrium model with money. Section 3 considers equilibria under a constant-money-growth rule, while those of an inflation targeting policy

[3] Clower(1967) and Lucas and Stokey(1987) are representative studies that impose this assumption.

regime are examined in section 4. Section 5 concludes the paper.

2 The model

In this section, we construct a model of pure-exchange economy that consists of identical infinitely lived households.[4] The time flow is taken as discrete: $t = 1, 2, 3, \cdots$. It is assumed that there is only one kind of private commodity in this economy.

2.1 The government

The government is the sole agent that can issue money in this economy and we assume for simplicity that the cost of money creation is zero.

The government puts money into the economy by means of lump-sum transfer to the households and by purchasing a private good input, which is supplied to the households as a public good. A fraction $\phi \in [0, 1]$ of the newly printed money in the period is transferred in lump-sum fashion to the households, and the rest of it is used for purchasing the private good, where the parameter ϕ is a constant value. In addition, we treat the government to act to maintain zero financial balance in every period. Then the budget

[4] This model is based on Azariadis(1993, section 26.3). The government that supplies public good is newly introduced. Its merit will be discussed briefly in section 5.

constraints of the government at period t are written as follows:

$$\tau_t = \phi(M_t - M_{t-1}), \tag{1}$$

$$p_t g_t = (1-\phi)(M_t - M_{t-1}), \tag{2}$$

where M_t, τ_t, g_t, p_t are nominal money balance, lump-sum transfer, government purchase of the private good and price level respectively, for the period t. We assume that one unit of the private good produces one unit of the public good which is supplied free to the households.

2.2 The households

The representative agent's utility depends not only on the level of consumption but also on his real money holdings. In this study, the instantaneous utility of the agent at period t is assumed to be represented by the following additively separable function:

$$u(c_t) + v(z_t) + q(g_t),$$

where $c_t \in \mathbf{R}_+$ is his consumption level and z_t is his real money balance, both at period t. We suppose that functions u and v are strictly monotone and convex: $u' > 0$, $v' > 0$, $u'' < 0$, $v'' < 0$, and, in addition, satisfy the following properties:[5]

$$u'(0) = \infty, \quad v'(0) = \infty, \tag{3}$$

$$v'(\infty) = 0. \tag{4}$$

We also assume that $u'(a)$, $v'(a)$, $u''(a)$ and $v''(a)$ have finite values for any $a > 0$.[6] The representative agent's lifetime utility is expressed as:

$$U = \sum_{t=1}^{\infty} \delta^{t-1}\{u(c_t) + v(z_t) + q(g_t)\}, \tag{5}$$

where $\delta \in (0, 1)$ denotes his subjective discount factor. All households receive the endowment $e > 0$ in each period. So the households at period t face the budget constraint

$$c_t + z_t = e + \frac{\tau_t}{p_t} + \frac{p_{t-1}}{p_t} z_{t-1}. \tag{6}$$

The representative household determines the sequence $\{c_t, z_t\}_{t=1}^{\infty}$ in order to maximize his lifetime utility (5) subject to the constraint (6).[7] Now we can obtain the first-order condition for his lifetime utility maximization:

$$u'(c_{t+1}) = \frac{1}{\delta} \frac{p_{t+1}}{p_t} \{u'(c_t) - v'(z_t)\} \quad \text{for all } t \geq 1. \tag{7}$$

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[5] These assumptions are used in Brock(1975), for instance.

[6] This implies that $u''(0) = v''(0) = -\infty$.

[7] The household cannot choose $\{g_t\}_{t=1}^{\infty}$ because the government determines them according to the policy rule.

2.3 Market clearing conditions

The commodity market clearing condition at period t can be written as

$$c_t + g_t = e. \tag{8}$$

When (8) is satisfied, the money market is also in equilibrium by Walras' law, i.e., if we use m_t to express M_t/p_t, the real money balance at t, the following relation holds.

$$m_t = z_t \tag{9}$$

2.4 Dynamic equilibrium under perfect foresight

Finally, we define a dynamic equilibrium for this economy under perfect foresight.

Definition Given a sequence $\{M_t\}_{t=0}^{\infty}$, *a perfect-foresight dynamic equilibrium* is a sequence $\{\tau_t,\ g_t,\ c_t,\ z_t,\ p_t,\ m_t\}_{t=1}^{\infty}$ satisfying the budget constraints (1) and (2) for the government, the budget constraint (6) for the representative household, the first-order condition (7) for lifetime utility maximization of the household, market clearing conditions (8) and (9), the non-negativity of nominal money balance $M_t \geq 0$, and the transversality condition

$$\lim_{T\to\infty} \delta^T \frac{p_t}{p_{t+T}} u'(c_{t+T}) = 0 \tag{10}$$

for all $t \geq 1$.

3 Price level under a constant-money-growth rule

In this section, we analyze fluctuations of the price level under the policy to increase the money supply at a constant rate, that is commonly examined in literatures studying the dynamic property of a monetary economy. Specifically, the money supply grows according to the following relation:

$$M_{t+1} = (1+\theta)M_t \quad \text{for all } t \geq 0, \tag{11}$$

with $\theta \geq 0$ and $M_0 > 0$. Our main purpose here is to investigate whether there are any perfect foresight dynamic equilibria, and, if so, whether the price level also changes at the rate $(1+\theta)$, i.e., whether the government can use this policy to maintain a constant rate of inflation.

We can derive

$$\frac{p_{t+1}}{p_t} = (1+\theta)\frac{m_t}{m_{t+1}} \tag{12}$$

from (11), and

$$g_t = x m_t \tag{13}$$

from (2), where

$$x \equiv \frac{(1-\phi)\theta}{1+\theta} \in [0,1).$$

Then substituting (12) and (13) into (7), we can obtain a first-order difference equation with respect to the real money balance m_t:

$$a(m_{t+1}) = \frac{1+\theta}{\delta} b(m_t), \tag{14}$$

where $a(m) \equiv mu'(e - xm)$ and $b(m) \equiv m\{u'(e - xm) - v'(m)\}$. Both functions $a(m)$ and $b(m)$ are defined on $0 \leq m < e/x$. It is easily verified that $a(0) = 0$, $a(e/x) = \infty$, $a' > 0$ and $b(e/x) = \infty$. In addition, we obtain the next relationship on $b(0)$:

$$\lim_{m_t \to +0} m_t v'(m_t) \left\{ \begin{array}{c} > \\ = \end{array} \right\} 0 \quad \Leftrightarrow \quad b(0) \left\{ \begin{array}{c} < \\ = \end{array} \right\} 0.$$

We define $\hat{m}$ as the maximum value of m satisfying $b(m) = 0$ and apply the implicit function theorem to (14) for any $m_t \in (\hat{m}, e/x)$, then the existence and the uniqueness of a function f such that $m_{t+1} = f(m_t)$ are guaranteed. The slope of f is calculated as

$$\frac{dm_{t+1}}{dm_t} = \frac{1+\theta}{\delta} \frac{u'(e - xm_t) - v'(m_t) - xm_t u''(e - xm_t) - m_t v''(m_t)}{u'(e - xm_{t+1}) - xm_{t+1} u''(e - xm_{t+1})} > 0,$$

i.e., the phase curve f is upward-sloping.[8]

The monetary steady state of this economy is given as $\bar{m} \in (\hat{m}, e/x)$ which satisfies

$$u'(e - x\bar{m}) - yv'(\bar{m}) = 0, \quad y \equiv \frac{1+\theta}{1+\theta-\delta} > 1. \tag{15}$$

[8] This is because $b(m_t) > 0$ for any m_t in that interval. Here it is equivalent to $u'(e - xm_t) - v'(m_t) > 0$.

We should notice that the left-hand side of the first equation of (15) is increasing in $\bar{m}$. When $\hat{m} = 0$, $\lim_{m \to +0} \{u'(e - xm) - yv'(m)\} < 0$ holds by assumption (3), while $u'(e - x\hat{m}) - yv'(\hat{m}) < 0$ holds if $\hat{m} > 0$.[9] Moreover, (3) also says that $\lim_{m \to e/x - 0} \{u'(e - xm) - yv'(m)\} > 0$. These facts assure that there exists $\bar{m}$ and that it is unique.

Finally, we should examine the stability of this steady state. It is apparent that the phase curve f intersects the 45° line from below if $\hat{m} > 0$. On the other hand, in the case of $\hat{m} = 0$, we can evaluate the slope of f at the origin as[10]

$$\lim_{m_t \to +0} \frac{m_{t+1}}{m_t} = \lim_{m_t \to +0} \frac{1 + \theta}{\delta} \frac{m_t \{u'(e - xm_t) - v'(m_t)\}}{u'(e - xm_{t+1})} = 0.$$

Hence, the phase curve always intersects the diagonal from below even in the case of f passing through the origin. Then we can say that the only monetary steady state of this economy is unstable whether $\hat{m} > 0$ or $\hat{m} = 0$.

Proposition 1 *Our economy expressed by equation (14) under the policy regime of constant money supply growth has a monetary steady state which is unique and unstable.*

Now consider possibilities on the shape of the phase curve for this economy.

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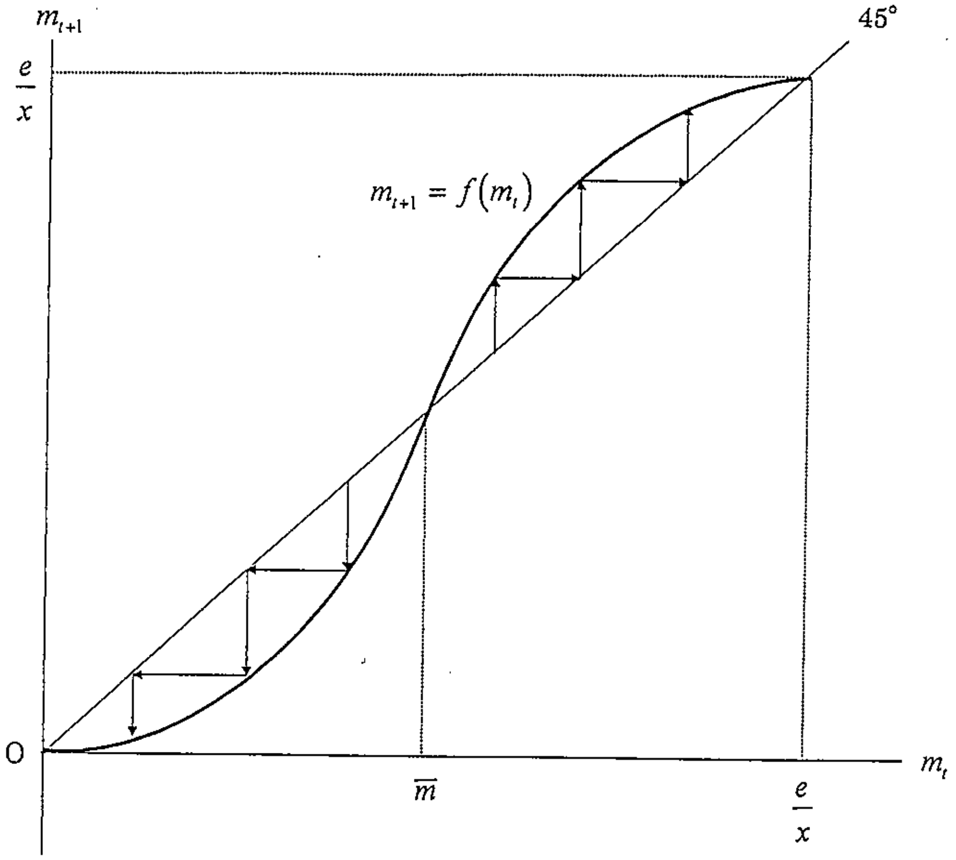
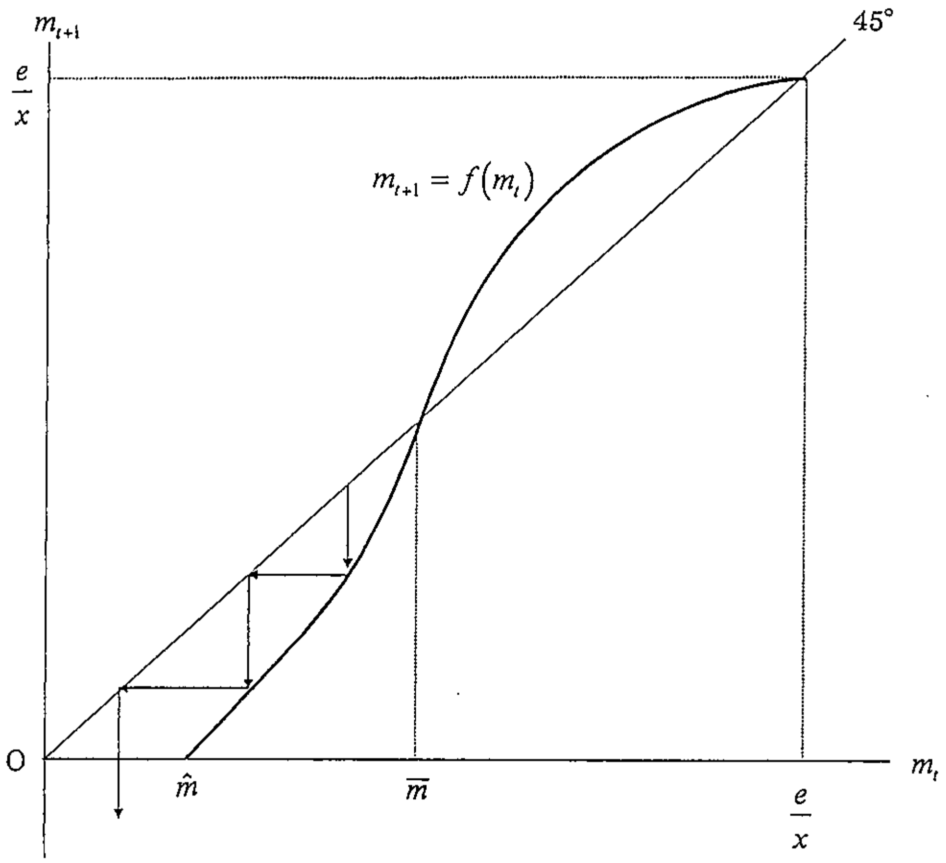
[9] This is because $u'(e - x\hat{m}) - v'(\hat{m}) = 0$ from the definition of $\hat{m}$, and $y > 1$.

[10] Remember that $\lim_{m_t \to +0} m_t v'(m_t) = 0$ when $\hat{m} = 0$.



(a) Target rate of inflation will not be implemented unless $m_1 = \overline{m}$.

Figure 1: Motion of the real money balance under constant-money-growth rule



(b) There is no dynamic equilibrium when $m_1 < \overline{m}$.

Figure 1: (continued)

Case A: $\hat{m} = 0$. See figure 1(a). If the initial value of the real money balance m_1 is less than $\bar{m}$, m_t converges to zero as $t \to \infty$. On the other hand, m_t approaches to e/x as time goes on when $m_1 > \bar{m}$. Although those sequences $\{m_t\}$ are dynamic equilibria, m_{t+1}/m_t is not a constant value in each case, i.e., the government will fail to support a constant rate of inflation.[11] Then we conclude that there exists a perfect-foresight dynamic equilibrium in which the rate of inflation remain constant only if $m_1 = \bar{m}$ holds by chance.

Case B: $\hat{m} > 0$. Figure 1(b) shows this possibility. When $m_1 < \bar{m}$, the real money balance will be negative in a finite time period. Such a sequence $\{m_t\}$ cannot be a dynamic equilibrium.

Under the policy of a positive constant rate of money supply growth, there exists a perfect-foresight dynamic equilibrium in which the rate of inflation is kept constant *only if it happens to be that* $m_1 = \bar{m}$. In almost every occasion, things end up in undesirable states, i.e., a dynamic equilibrium does not exist, or, even if it does, it cannot support the target rate of inflation. These results suggest that we should consider another policy rule to succeed in inflation

[11] In the case of $\phi = 1$, $e/x = \infty$ holds and m_t gets infinitely large as $t \to \infty$ when $m_1 > \bar{m}$. But such a sequence $\{m_t\}$ cannot be a dynamic equilibrium because it does not satisfy the transversality condition (10).

targeting.

4 Inflation targeting policy regime

This section examines a policy regime in which the government fixes the price level path in advance and controls money supply to implement it. Specifically, we denote this target time path of the price level as $\{p_t^*\}$. Moreover, we define the fixed inflation factor $\rho > 0$ such that

$$p_{t+1}^* = \rho p_t^* \quad \text{for all } t \geq 1. \tag{16}$$

In other words, the aim of the government here is to control money supply so as to maintain the inflation factor at a fixed level ρ.

Substituting (16) into the government budget constraint (2) and using (8), we will obtain the expression

$$e - c_{t+1} = (1 - \phi)\left(m_{t+1} - \frac{1}{\rho}m_t\right). \tag{17}$$

In addition, we can rewrite the first-order condition of the representative household (7) as

$$u'(c_{t+1}) = \frac{\rho}{\delta}\{u'(c_t) - v'(m_t)\}. \tag{18}$$

Our task here is to analyze this dynamic system represented by these two equations with two state variables m_t and c_t.

First, suppose that this economy has a steady state (m^*, c^*) and discuss its properties. It must satisfy the following relations, which can be obtained by substituting $m_{t+1} = m_t = m^*$ and $c_{t+1} = c_t = c^*$ for (17) and (18).

$$e - c^* = (1 - \phi)\frac{\rho - 1}{\rho} m^* \tag{19}$$

$$v'(m^*) = \left(1 - \frac{\delta}{\rho}\right) u'(c^*) \tag{20}$$

We must note that it is necessary that $\rho \geq 1$ for (19) to hold since $0 \leq c^* \leq e$. Namely, the government must not expect deflation to guarantee the existence of the steady state. We should remember that monetary authorities that operates inflation targeting policy regime in practice listed in table 1 set nonnegative values to the target rate of inflation. And it is worth pointing out that our model is consistent with the fact that they target *nonnegative* rate of inflation, i.e., $\rho \geq 1$. In this paper, we hereafter consider only inflationary economy in which $\rho \geq 1$ holds.[12]

Next we examine the dynamics of this economy using a phase diagram on (m_t, c_t) plane. From (18) we obtain

$$\begin{aligned} c_{t+1} \geq c_t \;\; &\Leftrightarrow \;\; u'(c_{t+1}) = \frac{\rho}{\delta}\{u'(c_t) - v'(m_t)\} \leq u'(c_t) \\ &\Leftrightarrow \;\; \left(1 - \frac{\delta}{\rho}\right) u'(c_t) \leq v'(m_t). \end{aligned}$$

[12] $\rho > \delta$ is also necessary for (20) to hold because $u' > 0$ and $v' > 0$, but it is always satisfied whenever $\rho \geq 1$.

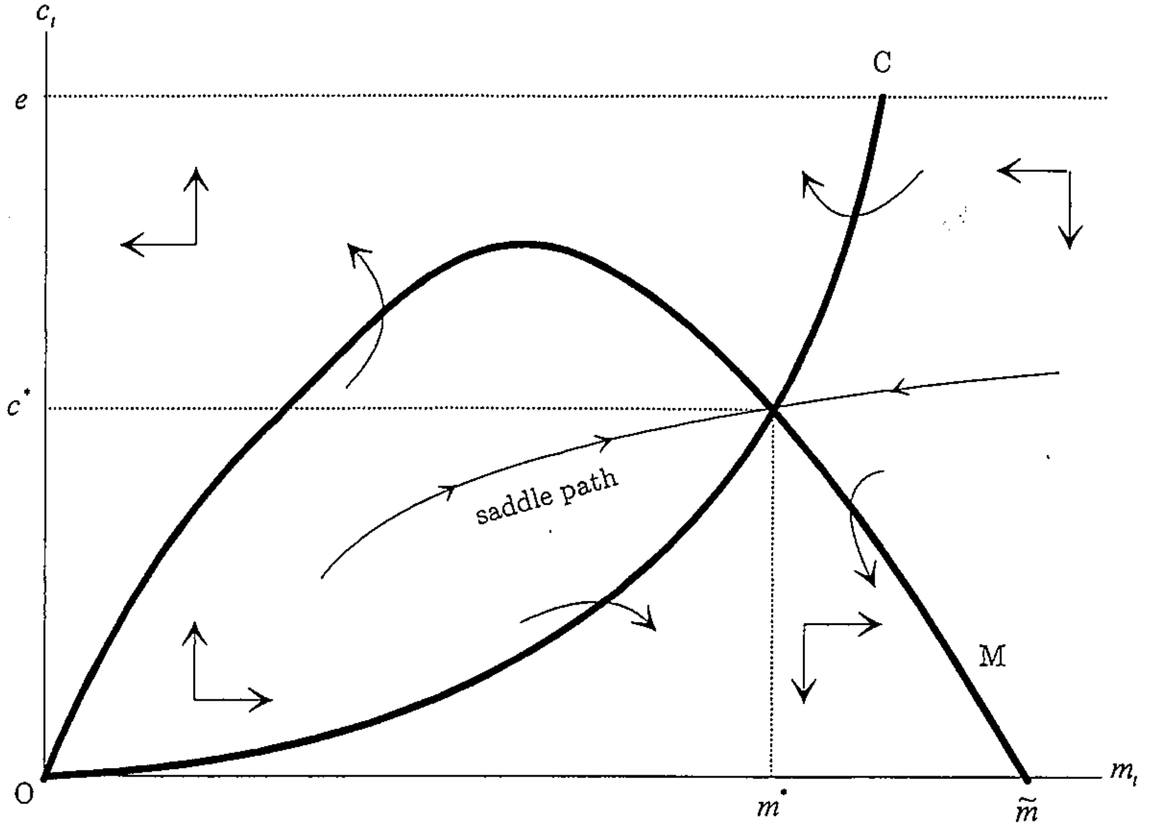


Figure 2: Phase diagram under inflation targeting policy regime

Then the condition

$$\left(1 - \frac{\delta}{\rho}\right) u'(c_t) = v'(m_t)$$

which is described as the phase curve OC in figure 2 is equivalent to $c_{t+1} = c_t$. We can see also that $c_{t+1} > c_t$ to the left of OC and $c_{t+1} < c_t$ to the right. The phase curve OC is upward-sloping since

$$\frac{dc_t}{dm_t} = \frac{v''(m_t)}{\left(1 - \frac{\delta}{\rho}\right) u''(c_t)} > 0,$$

and using assumption (3) we have $c_t \to 0$ as $m_t \to 0$, which implies that OC passes through the origin.

Next we will derive another phase curve that corresponds to $m_{t+1} = m_t$.

Equations (17) and (18) yield

$$
\begin{aligned}
m_{t+1} \geq m_t \quad &\Leftrightarrow \quad e - (1-\phi)\left(m_{t+1} - \frac{1}{\rho}m_t\right) \leq e - (1-\phi)\frac{\rho - 1}{\rho}m_t \\
&\Leftrightarrow \quad u'(c_{t+1}) \geq u'\left[e - (1-\phi)\frac{\rho-1}{\rho}m_t\right] \\
&\Leftrightarrow \quad u'(c_t) \geq \frac{\delta}{\rho}u'\left[e - (1-\phi)\frac{\rho-1}{\rho}m_t\right] + v'(m_t).
\end{aligned}
$$

Then $m_{t+1} = m_t$ is equivalent to

$$
u'(c_t) = \frac{\delta}{\rho}u'\left[e - (1-\phi)\frac{\rho-1}{\rho}m_t\right] + v'(m_t),
$$

which is drawn as the curve OM in figure 2. And $m_{t+1} > m_t$ on the lower side of that curve while $m_{t+1} < m_t$ on the upper. In addition, the right-hand side of the previous equation diverges to infinity as $m_t \to +0$, and then c_t must converge to $+0$, which means that OM passes through the origin. Besides, the right-hand side also goes to infinity as

$$
m_t \longrightarrow \frac{\rho e}{(1-\phi)(\rho - 1)} \equiv \tilde{m},
$$

which implies that this phase curve passes through $(\tilde{m}, 0)$. We can compute the slope of OM as follows.

$$
\frac{dc_t}{dm_t} = -\frac{\delta(1-\phi)(\rho-1)u''\left[e - (1-\phi)\frac{\rho-1}{\rho}m_t\right] - \rho^2 v''(m_t)}{\rho^2 u''(c_t)}
$$

When we evaluate this at the origin and at $(\tilde{m}, 0)$, it reduces to

$$
\left.\frac{dc_t}{dm_t}\right|_{(m_t, c_t)=(0,0)} = -\frac{\delta(1-\phi)(\rho-1)u''(e) - \rho^2 v''(0)}{\rho^2 u''(0)} \geq 0
$$

and

$$\left.\frac{dc_t}{dm_t}\right|_{(m_t,c_t)=(\tilde{m},0)} = -\frac{\delta(1-\phi)(\rho-1)u''(0)-\rho^2 v''(\tilde{m})}{\rho^2 u''(0)} < 0,$$

respectively. These are evidences that the phase curve OM is upward-sloping at the origin, while it is downward-sloping at the point $(\tilde{m}, 0)$.

It is apparent from figure 2 that there exists a unique monetary steady state (m^*, c^*) in this economy,[13] and appendix 1 provides a proof that this steady state has the saddle point property.[14]

When the initial state of this economy is on the upper side of the saddle path, the sequence of (m_t, c_t) goes into the infeasible area, hence such a solution cannot be a dynamic equilibrium. Contrarily, when the initial point is on the lower side, the sequence of (m_t, c_t) converges to $(\tilde{m}, 0)$. But, in fact, such a solution is not a dynamic equilibrium as well since it violates the transversality condition (10).[15] Then we can say that there exists a unique path of perfect-foresight dynamic equilibrium which converges to (m^*, c^*) along the saddle path in this economy.

We state our findings in the following proposition.

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[13] We can easily ascertain that the slope of OC is greater than that of OM at (m^*, c^*). This guarantees uniqueness of the monetary steady state.

[14] The sequence $\{m_t, c_t\}_{t=1}^{\infty}$ which converges to this steady state along the saddle path satisfies the transversality condition (10). See appendix 2 for detail.

[15] It is also shown in appendix 2.

Proposition 2 *It is possible that there exists a determinate perfect-foresight dynamic equilibrium which converges to the monetary steady state* (m^*, c^*) *along the saddle path in an economy expressed by equations* (17) *and* (18) *under inflation targeting policy regime, provided that the inflation factor is set to be greater than or equal to unity.*

5 Concluding remarks

In this study, we have investigated the possibility whether the government succeeds in the inflation targeting policy regime in a monetary economy with infinitely-lived households whose utility depends not only on consumption but also on real money holdings.

In section 3, we have examined a simple policy regime which is to increase money supply at a constant rate in every period, and found that there may not exist any perfect-foresight dynamic equilibrium. Even if there is one, the targeted rate of inflation will not be implemented.

In section 4, then, we analyzed an inflation targeting policy regime in which the government controls money supply so as to support the target inflation rate. We sum up the main results of the analysis in the following:

1. There exists no dynamic equilibrium with a deflationary target rate. The existence of a perfect-foresight dynamic equilibrium with price-

level targeting requires a constant rate of inflation by government's resolution on a rise in the price level.

2. When a dynamic equilibrium exists, the path of solutions which implement the target time path for the price level is unique since the only monetary steady state is saddle-node stable. In other words, the perfect-foresight dynamic equilibrium of this economy is determinate.

These are positive results providing theoretical justification to monetary authorities that operates inflation targeting policy regime in practice. In the model of Smith(1994), by contrast, inflation targeting policy regime via lump-sum transfer resulted in indeterminacy of equilibrium, or in a negative result. Thus we have to say that the conclusion of Smith was not robust depending on monetary economic models.

The results in sections 3 and 4 of our study do not depend on ϕ, the ratio of money transferred to households in lump-sum fashion to newly created money, but this does not imply that the introduction of a public good supplied by the government to our model is meaningless. Public-good-supplying policy may be considered as a kind of fiscal policy, so we should interpret our result as follows. Once the government targets a constant rate of inflation, then monetary and fiscal policies are no longer independent each other.

One of the most important problems would be that our analysis does not deal with the open market operations, which is one of the common measures to control money supply. Do the results obtained in this study change when we consider the open market operations? Are there any policies more desirable than one examined in section 4? We intend to answer these questions in our future study.

Appendix 1.

The only monetary steady state has the saddle point property. (Section 4)

The existence of a saddle point is checked by calculating the Jacobian matrix

$$J = \begin{pmatrix} \partial m_{t+1}/\partial m_t & \partial m_{t+1}/\partial c_t \\ \partial c_{t+1}/\partial m_t & \partial c_{t+1}/\partial c_t \end{pmatrix}$$

at the steady state. Substituting (17) into (18) yields

$$u'\left[e-(1-\phi)\left(m_{t+1}-\frac{1}{\rho}m_t\right)\right]=\frac{\rho}{\delta}\left\{u'(c_t)-v'(m_t)\right\}.$$

From this, we can compute partial derivatives of m_{t+1} as follows:

$$\frac{\partial m_{t+1}}{\partial m_t}=\frac{\rho}{(1-\phi)\delta}\frac{v''(m_t)}{A}+\frac{1}{\rho},\quad \frac{\partial m_{t+1}}{\partial c_t}=-\frac{\rho}{(1-\phi)\delta}\frac{u''(c_t)}{A},$$

where

$$A \equiv u'' \left[e - (1-\phi) \left(m_{t+1} - \frac{1}{\rho} m_t \right) \right].$$

Furthermore, from (18), partial derivatives of c_{t+1} will be obtained as

$$\frac{\partial c_{t+1}}{\partial m_t} = -\frac{\rho}{\delta} \frac{v''(m_t)}{u''(c_{t+1})}, \quad \frac{\partial c_{t+1}}{\partial c_t} = \frac{\rho}{\delta} \frac{u''(c_t)}{u''(c_{t+1})}.$$

The Jacobian matrix evaluated at the steady state reduces to

$$J^* = \begin{pmatrix} \frac{\rho}{(1-\phi)\delta} \frac{v''(m^*)}{A^*} + \frac{1}{\rho} & -\frac{\rho}{(1-\phi)\delta} \frac{u''(c^*)}{A^*} \\ -\frac{\rho}{\delta} \frac{v''(m^*)}{u''(c^*)} & \frac{\rho}{\delta} \end{pmatrix},$$

where

$$A^* \equiv u'' \left[e - \frac{(1-\phi)(\rho-1)}{\rho} m^* \right].$$

Then the trace and the determinant of J^* are

$$T \equiv \text{tr } J^* = \frac{\rho}{\delta} \left\{ 1 + \frac{v''(m^*)}{(1-\phi)A^*} \right\} + \frac{1}{\rho} > 0, \quad D \equiv \det J^* = \frac{1}{\delta} > 0.$$

Eigenvalues of J^* are given as roots λ of the characteristic polynomial

$$s(\lambda) = \lambda^2 - T\lambda + D = 0.$$

Let Δ be the discriminant of $s(\lambda)$, that is,

$$\Delta = T^2 - 4D = \left[\frac{\rho}{\delta} \left\{ 1 + \frac{v''(m^*)}{(1-\phi)A^*} \right\} - \frac{1}{\rho} \right]^2 + \frac{4}{\delta} \frac{v''(m^*)}{(1-\phi)A^*} > 0,$$

which means that $s(\lambda) = 0$ has two different real roots $\lambda = \lambda_1, \lambda_2$. Here we assume that $\lambda_1 < \lambda_2$ without loss of generality. It is necessary that every λ

satisfying $s(\lambda) = 0$ are positive because both T and D are positive. Moreover, value of the characteristic polynomial evaluated at $\lambda = 1$ is

$$s(1) = 1 - \frac{\rho}{\delta}\left\{1 + \frac{v''(m^*)}{(1-\phi)A^*}\right\} < 0$$

implying $(1-\lambda_1)(1-\lambda_2) < 0$. This means that $\lambda_1 < 1 < \lambda_2$ must hold. Therefore, the steady state $(m^*,\ c^*)$ of the dynamic system of equations (17) and (18) is a saddle node.

Appendix 2.

Does a sequence $\{m_t,\ c_t\}_{t=1}^{\infty}$ converging to $(m^*,\ c^*)$ or to $(\tilde{m},\ 0)$ satisfy the transversality condition? (Section 4)

Here it is sufficient to examine whether the relation obtained by substituting (16) for (10), i.e.,

$$\lim_{T\to\infty}\left(\frac{\delta}{\rho}\right)^T u'(c_{t+T}) = 0, \tag{21}$$

holds or not. Iterating (18) forward, we have the following.

$$u'(c_{t+T}) = \left(\frac{\rho}{\delta}\right)^T u'(c_t) - \sum_{i=1}^{T}\left(\frac{\rho}{\delta}\right)^i v'(m_{t+T-i})$$

Substituting this into the left-hand side of (21) yields

$$\begin{aligned}&\lim_{T\to\infty}\left[\left(\frac{\delta}{\rho}\right)^T\left\{\left(\frac{\rho}{\delta}\right)^T u'(c_t) - \sum_{i=1}^{T}\left(\frac{\rho}{\delta}\right)^i v'(m_{t+T-i})\right\}\right]\\ &= u'(c_t) - \lim_{T\to\infty}\sum_{i=1}^{T}\left(\frac{\delta}{\rho}\right)^{T-i} v'(m_{t+T-i}). \end{aligned}\tag{22}$$

Let us consider a sequence which converges to the steady state (m^*, c^*). We have

$$\begin{aligned} \text{equation}(22) &= u'(c^*) - v'(m^*) \lim_{T\to\infty} \sum_{i=1}^{T} \left(\frac{\delta}{\rho}\right)^{T-i} \\ &= u'(c^*) - v'(m^*) \lim_{T\to\infty} \frac{1-\left(\frac{\delta}{\rho}\right)^T}{1-\frac{\delta}{\rho}} \\ &= u'(c^*) - \frac{\rho}{\rho-\delta} v'(m^*) \end{aligned}$$

for sufficiently large t, but we know that this is equal to zero using relation (20). Therefore, this sequence satisfies equation (21).

On the other hand, for any sequence which converges to the point of $(\tilde{m}, 0)$,

$$\text{equation}(22) = u'(0) - \frac{\rho}{\rho-\delta} v'(\tilde{m}) = \infty \neq 0$$

holds for sufficiently large t, i.e., such a sequence does not satisfy (21).

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