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Heterogeneity of Labor, the Phillips Curve, and Stagflation

by

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Since the appearance of Phillips article [20], it has been widely believed that there exists a simple and stable inverse relationship between the rate of unemployment and the rate of wage and price inflation, a relationship known as the Phillips curve. Nonetheless, recent experience seems to have discredited this inverse relationship in that in present economies high unemployment and high wage and price inflation often coexist, an occurrence known as stagflation. Many theoretical models have been presented to explain successfully the Phillips curve relationship. Nevertheless, these models have also failed to explain the phenomenon of stagflation.[1]

The purpose of this paper is to present an alternative theoretical foundation which is capable of generating both the Phillips curve relationship and the phenomenon of stagflation whihin the same framework. The distinguishing features of the present approach from the past literature include three assumptions. First, firms are monopolists in the product market and, in the labor market, they can select the amount of employment given exogenous wages; secondly, workers are heterogeneous both in their productivity and in their paid wages; and thirdly, the rate of wage inflation is subject to institutional but endogenous adjustments over time.

In Section I, I shall present the basic model giving a central role to the heterogeneity of labor. The first task is to derive the demand schedule for labor; and the second is to introduce alternative adjustment processes of wage inflation. In Section II, the Phillips curve is derived when the rate of wage inflation is adjusted to the potential real factor of the economy, that is, to the growth rate of the aggregate demand. The analysis of the factors determining the position and slope of the Phillips curve and the derivation of the 'Phillips-Lipsey loop' are also carried out. In Section III, the possibility of generating stagflation is analyzed when the rate of wage inflation is adjusted to the rate of price inflation. Section IV concludes the paper and discusses some policy recommendations.

I. The Basic Model

1. The Demand for Labor and the Rate of Unemployment

Consider a representative firm[2/] which employs workers and produces output as a monopolist. The workers are differentiated by skill, age, education, experience, etc., which are inclusively called 'quality' and the wages of the workers are exogenously and institutionally given to the firm. High quality workers can produce larger amount of output and are paid higher wages than low quality workers. For the sake of simplicity, workers are assumed to be represented by the continuous quality index n which lies between zero and unity, where a larger number represents higher quality. It is assumed that the quality index is uniformly distributed. In other words, workers are distributed evenly over all quality levels.[3/] Assume further that a firm employs workers by giving priority to the high quality workers[4/] and that it decides the number of employees so as to maximize profits.

Let y and p be the total amount of output and the price of output respectively; and let $w(n)$ and $f(n)$ be the wages of the worker of quality index n and the amount of output produced by such worker. Then, the problem of the representative firm is to maximize the expression

$$\int_U^1 [p(y)f(n) - w(n)]\, dn,$$

by choosing the optimal U $(0 < U < 1)$,[5/] where

$$y(U) = \int_U^1 f(n)\, dn.$$

Note that U (or 100U%) is nothing but the unemployment rate in the present context. Note also that unemployment is necessarily involuntary; workers are willing to accept job offers while it is firms that decide the amount of employment in order to maximize their profits.

The first-order condition follows immediately:

$$(1) \qquad p(y(U))(1 - \frac{1}{\eta})f(U) = w(U),$$

where η denotes the price elasticity of demand for output:

$$\eta = -\frac{p}{y}\frac{dy}{dp}.$$

The second-order condition must also be met, which demands the satisfaction of

$$(2) \qquad k(U) = \frac{Uf(U)}{\eta y(U)} + \epsilon_f - \epsilon_w > 0,$$

where

$$\epsilon_f = \frac{n}{f}\frac{df}{dn}\bigg|_{n=U}, \qquad \epsilon_w = \frac{n}{w}\frac{dw}{dn}\bigg|_{n=U}.$$

ϵ_f is the quality elasticity of productivity and ϵ_w is the quality elasticity of wages.[6/]

To look at conditions (1) and (2) in a more appealing manner, I shall define the ratio $q(n)$ by

$$(3) \qquad q(n) = \frac{p(y(U))(1-\frac{1}{\eta})f(n)}{w(n)} .$$

Then (1) means that U is determined so as to satisfy $q(U)=1$, while (2) is tantamount to saying that $q(U)$ is increasing in U as $k(U)$ is readily seen to denote the elasticity of $q(U)$ with respect to U. Therefore, the workers whose q is greater than unity are employed while those less than unity are unemployed. Namely, the equilibrium employment is determined at that critical level of labor quality at which the marginal revenue product (numerator of q) equals the marginal wage (denominator of q); and the second-order condition requires that the marginal revenue product be smaller than the wage at all lower labor qualities.[7/]

From (2), as long as the first term is sufficiently large (*e.g.*, η is small enough), the firm can still realize the maximal profits even when ϵ_f is smaller than ϵ_w. However, in view of human-capital theory and empirical findings, it seems most implausible that the quality elasticity of labor productivity is smaller than the quality elasticity of wages.[8/] Therefore, in what follows, I shall assume in my analysis that ϵ_f is greater than ϵ_w.

2. The Dynamic Path of the Rate of Unemployment

I shall introduce the time factor into the analysis and consider the intertemporal features of the model. At any instant in time, assume that the given spectrum of wages shifts at a rate of α_t, *i.e.*,

$$(4) \qquad \frac{\partial w_t(n)}{\partial t} = \alpha_t w_t(n).$$

Note that α_t does not depend on the quality index n and it is the same for all workers. In other words, the relative structure of wages does not change across time. Assume similarly that the output demand price or the demand for output shifts at a rate of β_t:

$$\frac{\partial p_t(y)}{\partial t} = \beta_t p_t(y). \tag{5}$$

Here again the demand functions over time are homothetic with respect to quantity.[9/] The intertemporal (proportional) changes in the productivity spectrum can certainly be incorporated into the following analysis without any difficulty.[10/]

Under these assumptions, the total differentiation of (1) with respect to time yields

$$\dot{U}_t = \frac{(\alpha_t - \beta_t)U_t}{k(U_t)}, \tag{6}$$

where a 'dot' denotes time derivative. (6) means that the direction of the instantaneous change of the unemployment rate depends on the relative magnitude of α_t and β_t; and it is increasing (or decreasing) as α_t is greater (or smaller) than β_t because $k(U_t)$ is positive. This describes the dynamic path of the unemployment rate.

3. Alternative Adjustment Processes of the Rate of Wage Inflation

Finally, I shall postulate the adjustment process of the rate of change of the entire wage structure to close the model. Consider the following two different adjustment processes:

(a) $\dot{\alpha}_t = \nu(\beta_t - \alpha_t)$,

(b) $\dot{\alpha}_t = \theta(\frac{\dot{p}_t}{p_t} - \alpha_t)$,

where ν and θ are positive constants. (a) and (b) attempt to describe the socio-economic or institutional bargaining processes between employers and employees in determingn the change of the rate of overall wage inflation.

Adjustment process (a) stresses a potential real factor of the eocnomy, that is, the growth rate of the demand for output. If the current growth rate of demand is higher than the current rate of wage inflation, the rate of wage inflation at the next bargaining table is raised. Proportional rightward shifts in the demand curve increase the sales perspective for the same amount of output which raises the amount of profits accruing to firms. In this situation firms, on the one hand, can afford a rise in wage payments while workers, on the other hand, may as well demand an upward wage push. Then the bargaining will certainly effect an increase in wages. Now it is immediate to extend this argument, with a strong support of the reality, to the rate of growth of each variable; then, (a) has been given its foundation.

Adjustment process (b) is a frequently told one and it stresses the rate of price inflation. If the current rate of price inflation is higher than the current rate of overall wage inflation, the wage increase in the next period is raised. A rationale for this adjustment process is usually atributed to the power of labor unions which press for the maintenance of real income.

One may as well postulate different adjustment processes with equal or more plausibility other than (a) and (b). This is because neither of them is clearly derived form the optimizing behavior of firms, or of workers, let alone of the society. For instance,

$$\text{(c)} \qquad \dot{\alpha}_t = \lambda[(\frac{\dot{p}_t}{p_t} + \beta_t) - \alpha_t],$$

$$\text{(d)} \qquad \dot{\alpha}_t = \nu(\beta_t - \alpha_t) + \theta(\frac{\dot{p}_t}{p_t} - \alpha_t),$$

may be other candidates. The new adjustment processes (c) and (d) may be obvious enough to be dispensed with their economic interpretations here. However, it turns out that (c) produces few substantial differences from (a) and (d), which reduces to (a) for $\theta=0$ and to (b) for $\nu=0$, generates qualitatively the same results as (b) does.

Now, I have presented the basic framework of the model. In Section II, I shall derive the Phillips curve relationship under adjustment process (a). In Section III, I shall analyze the possibility of generating stagflation under adjustment process (b).

II. The Phillips Curve

1. The Derivation of the Phillips Curve

When (6) and (a) are combined, I have the following system of simultaneous equations:11/

$$(A) \quad \begin{cases} \dot{\alpha} = \nu(\beta - \alpha), \\ \dot{U} = \dfrac{\alpha - \beta}{K(U)} U. \end{cases}$$

Under system (A), one can obtain the explicit relationship between α and U. This is because the reduced differential equation of system (A).

$$(7) \qquad \frac{d\alpha}{dU} = \frac{\dot{\alpha}}{\dot{U}} = -\frac{\nu k(U)}{U} < 0,$$

with reference to (2) and for the given initial conditions, U_0 and α_0, yields its solution in the following form:12/, 13/

$$(8) \qquad \alpha_t = \frac{\nu}{\eta} \log \frac{y(U_t)}{y(U_0)} - \nu(\epsilon_f - \epsilon_w)\log \frac{U_t}{U_0} + \alpha_0.$$

Eq. (8), which is nothing but a Phillips curve relation, is drawn as the PP curve for given constants U_0 and α_0 as depicted in Figure 1. From (7), the PP curve is downward sloping. In Figure 1, it is drawn to be convex toward the origin as empirical findings suggest. For this to be true in the present model, the additional condition

$$(9) \qquad \frac{d^2\alpha}{dU^2} = \frac{\nu}{\eta y^2} [\eta(\epsilon_f - \epsilon_w)y^2 - \epsilon_f Ufy - U^2f^2] > 0,$$

must hold. Thus, it is seen that the assumption, $\epsilon_f > \epsilon_w$, made earlier is necessary for a convex Phillips curve. In terms of the ratio $q(n)$, (9) is equivalent to the condition, $q''(U)q(U) < q'(U)^2$. Therefore, the concavity of $q(U)$ is a sufficient condition for the convex Phillips curve.

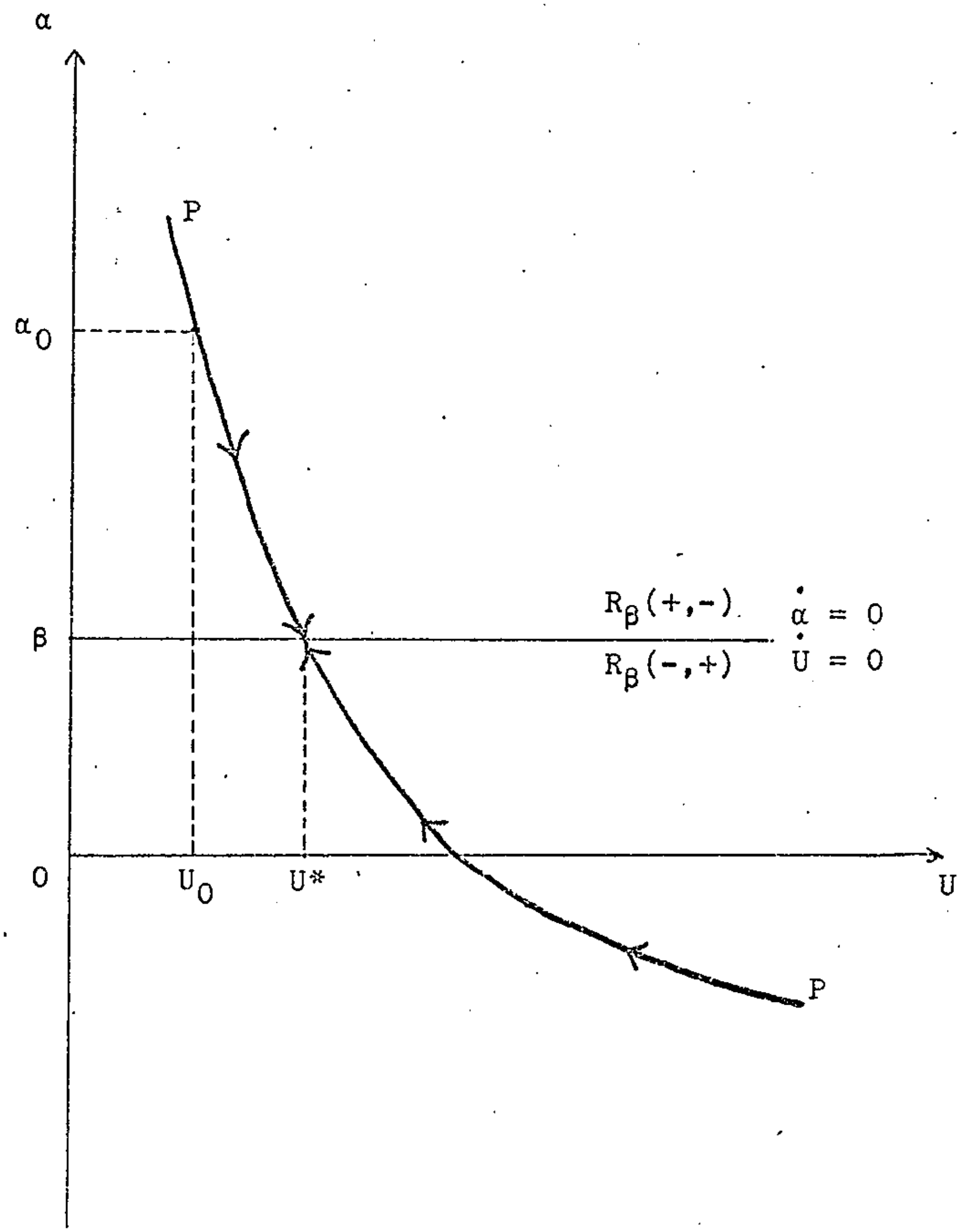

Figure 1

2. The Position and Slope of the Phillips Curve

From (8), the position and/or slope of a Phillips curve depends on the price elasticity of the demand for output η, two quality elasticities ϵ_f and ϵ_w, and the coefficient of the speed of adjustment ν as well as the initial values of U_0 and α_0. Interestingly enough but as it should be, β does not appear in (8), so that the Phillips curve is independently located of β.

Let P_x denote the partial derivative of α_t with respect to the parameter x in (8). P_η, P_{ϵ_f}, P_{ϵ_w}, and P_ν take the opposite sign depending on whether U_t is greater or smaller than U_0, implying that these parameters contribute to the slope of the Phillips curve. P_{U_0} and P_{α_0} exhibit the definite positive sign, which means that the initial conditions contribute to the position of the Phillips curve.[14/] The specific role played by each parameter is summarized in the top row of Table 1.

The result implies that a Phillips curve becomes steeper when: (i) the product market is less competitive (a smaller η); (ii) there is wide discrepancy in workers' productivities (a larger ϵ_f); (iii) there is little wage differentials (a smaller ϵ_w); and (iv) the rate of overall wage inflation is quickly adjusted (a larger ν). Of course, a Phillips curve becomes flatter when the converse is the case.

Factors (i)-(iii) make $k(U)$ or the elasticity of $q(U)$ with respect to U larger by reasons stated immediately below. Therefore, as (7) shows it analytically, the response of α to the same change in U becomes larger or the response of the optimal U to the same change in α becomes smaller, resulting in a steeper slope of a Phillips curve. Why, then, does the elasticity $K(U)$ get larger by these factors? Because, (i) a smaller η implies that a change in U and thereby in output causes a larger change in the output price which is a factor in the numerator of $q(U)$; (ii) a larger ϵ_f incurs a larger productivity change, which is again in the numerator, for one percent change in labor quality at especially $n=U$; and (iii) a smaller ϵ_w implies that one percent change in quality causes a smaller percent change in wages that are in the denominator of $q(U)$.

That the speed of adjustment of wage inflation affects the slope of a Phillips curve is in a sense apparent. For, although the rate of unemployment is a timeless variable, the rate of wage inflation has the dimension of $(\text{time})^{-1}$. To be more specific within the confines of system (A), the motion of α and that of U are not independent of each other since both adjust to the same disequilibrium, $(\beta - \alpha)$. In this situation, the Phillips curve is crucially affected by the relative speed of adjustment of the two, that is, ν and $-\frac{U}{K(U)}$. It is, then, clear that when ν is larger α moves to a certain extent ahead of U as compared to the case when ν is smaller, which steepens the slope.

An increase (or a decrease) in α_0 shifts the Phillips curve upward (or downward) by the same amount uniformly. Also, an increase (or a decrease) in U_0 shifts the Phillips curve to the right (or to the left) uniformly. In other words, different Phillips curves are drawn for the different initial rates of overall wage inflation and of unemployment. However, since α_0 is given historically and U_0 is determined, independently of α_0, so as to satisfy (1) at the initial time, there is only

		α_0	β	η	ϵ_1	ϵ_2	ν	U_0
position and/or slope		uniform upward shift	0	flatter	steeper	flatter	steeper	uniform shift to the right
U^*	$\alpha_0 > \beta$	+	-	+	-	+	-	+
	$\alpha_0 < \beta$			-	+	-	+	
$\bar{U}$ or U_0		0	0	-	+	-	0	X

Table 1[a]

a/. The effect of an increase in each parameter.

(+) increase (-) decrease (0) no effect

one Phillips curve that is intrinsic to a particular economy. Note that U_0 is dependent on the functional forms of the spectrum of productivity $f(n)$ and of wage differentials $w(n)$, as well as on the demand curve of output. Hence if they have constant elasticities, ϵ_f, ϵ_w, and η, respectively, then U_0 itself depends on these parameters. Therefore, the position of a Phillips curve as well as its slope is affected by these factors. The analysis of the factors affecting U_0 will be made in subsection 6.

Finally, the reason why a theoretically derived Phillips curve should be independently located of β, which the present theory correctly concludes, is explained. Suppose the contrary, then changes in β have impacts on the position and/or slope of the Phillips curve. But, as will be discussed later, the rate of change of the demand for output is likely subject to cyclical fluctuations. This implies that the trade-off relation tends to be highly volatile or that the stable Phillips curve ceases to exist. Therefore, the independence of the Phillips curve of the term β is necessary for an economy to observe an alledged stable inverse relationship between the rate of wage inflation and the unemployment rate.[15/]

3. The Stationary Unemployment Rate for a Constant β

Assume that β remains constant over time. Then, under system (A), $\dot{\alpha}=0$ when $\alpha=\beta$. Above the $\dot{\alpha}=0$ line, $\dot{\alpha}$ is negative whereas below the line $\dot{\alpha}$ is positive. Also, $\dot{U}=0$ when $\alpha=\beta$ or $U=0$; and, for $U > 0$, $\dot{U}$ is positive above the $\dot{U}=0$ line whereas below the line, $\dot{U}$ is negative.

Thus the direction of movement as time passes is depicted by arrows on the PP curve in Figure 1. If the initial pair of U_0 and α_0 belongs to the region $R_\beta(+,-)$ [or the region $R_\beta(-,+)$] , U increases (or decreases) and α decreases (or increases) as time passes, where the notation $R_\beta(+,-)$, for instance, should be understood as the region on the (U,α) plane with positive $\dot{U}$ and negative $\dot{\alpha}$ for a given β.

Under system (A), α_t approaches β and U_t approaches U^* from any initial state. U^* is obtained by equating α_t to β in (8):

$$(10) \qquad \beta = \frac{\nu}{\eta} \log \frac{y(U^*)}{y(U_0)} - \nu(\epsilon_f - \epsilon_w)\log \frac{U^*}{U_0} + \alpha_0 .$$

Therefore, the stationary unemployment rate U^* can be written as a function of the parameters listed below.

$$U^* = u(\alpha_0, \beta, U_0, \eta, \epsilon_f, \epsilon_w, \nu).$$

One can examine the influence of each parameter on U^* either by obtaining the partial derivatives of $u(.)$ from (10) or by manipulating Figure 1 with the findings on how the position and slope of a Phillips curve is affected by each parameter. The result is given in the middle row of Table 1.

Larger values of α_0 and U_0, and a smaller β make U^* larger. In other words, the stationary unemployment rate is higher when the initial rate of overall wage inflation and/or of unemployment is higher,16/ and when the grwoth rate of the demand for output is lower. The effects of the other parameters on U^* depend on the relative magnitude of α_0.

and β (or U^* and U_0). In the case of $\alpha_0 > \beta$, larger values of η and ϵ_w make U^* larger, while larger values of ϵ_f and ν make U^* smaller, and vice versa in the case of $\alpha_0 < \beta$. These results are explained in the following manner.

When $\alpha_0 > \beta$, U keeps increasing until α converges to β, i.e., until the stationary rate U^* is attained. During this convergent process, the magnitude of $\dot{U}$ or the extent to which U changes within a given time interval is commensurate to the coefficient of $(\alpha - \beta)$ in (6), that is, $\frac{U}{K(U)}$. Clearly, cetris paribus, the factors that make this coefficient larger and thereby make K(U) smaller contribute to raise U^*. These are larger η and ϵ_w, and smaller ϵ_f, as was explained earlier. Also, U^* is dependent on the speed with which α converges to β; the larger is ν the less time U has to increase. Hence, larger ν makes U^* smaller. By the symmetry of the argument, when $\alpha_0 < \beta$, the converse is true.

4. The Phillips-Lipsey Loop

So far, I have assumed that the parameters β, η, ϵ_f, ϵ_w, and ν are constant. Let us relax this assumption and assume instead that they take different values depending on the phase of business cycles. Then, one can readily assert the following.

First, it is almost tautological to say that β is positive and larger during expansionary periods while it becomes smaller and even negative during contractionary periods. In fact, in what follows. I may regard this as the definitional statement of business cycles. Secondly, under the present state of knowledge relying both on theory and on empirical studies, it is a firmly established consensus that wage differentials get narrowed during expansionary periods as opposed to during contractionary periods.[17/] Thus, ϵ_w becomes smaller (or larger) when $\dot{U} < 0$ (or $\dot{U} > 0$). Thirdly, it goes with one's intuition and may be supported by an empirical study that the spped of adjustment of wage inflation to the growth rate of the demand for output is faster during booms than during recessions. If so, ν is larger (or smaller) when $\dot{U} < 0$ (or $\dot{U} > 0$). Fourthly, as for the direction regarding the changes in η and ϵ_f, however, there seem to be neither firmly estabilised empirical findings nor convincing theories to predict it, so that I shall refrain myself from counting on these parameters in the following analysis and continue to assume that they are constant independently of the phases of business cycle.

Now, with these assertions combined, I shall construct a hypothetical situation where β takes discrete cyclical changes between a larger $\bar{\beta}$ and a smaller $\underline{\beta}$ after certain time intervals. It will be seen that this generates business cycles under system (A). Assume also that ϵ_w takes a smaller $\underline{\epsilon}_w$ when $\dot{U} < 0$ and a larger $\bar{\epsilon}_w$ when $\dot{U} > 0$, whereas ν takes a larger $\bar{\nu}$ when $\dot{U} < 0$ and a smaller $\underline{\nu}$ when $\dot{U} > 0$. Then, it follows immediately that a Phillips curve becomes steeper when the rate of unemployment is decreasing (during booms), while it gets flatter when the rate of unemployemnt is increasing (during recessions).

Assume that the economy is at T_0 in Figures 2a and 2b. Suppose β is $\bar{\beta}$ implying that T_0 lies in the region $R_{\bar{\beta}}(-,+)$, so that α starts increasing and U starts decreasing under system (A). According to the above observations, the pair (U,α) in this phase moves on the steeper Phillips curve P^-P^- (as $\dot{U} < 0$) towards T_1. Consider next that β jumps down to $\underline{\beta}$ at time T_1 [which also indicates the co-ordinate of (U,α)]; then, the point T_2 -- which is instantly realized by the assumption of discrete changes in ϵ_w and ν -- belongs now to the region $R_{\underline{\beta}}(+,-)$, so that α starts decreasing while U starts increasing along the flatter Phillips curve P^+P^+ (as $\dot{U} > 0$) to reach T_3 on the $\alpha=\underline{\beta}$ line at time T_3.

The observed pair (U_t,α_t) constitutes a counter-clockwise loop as in Figure 2a where α_0 is smaller than $\underline{\beta}$, whereas it makes a clockwise loop as in Figure 2b where α_0 is larger than $\bar{\beta}$. These loops scatter around the <u>average</u> Phillips curve P^0P^0 that would be the fitted curve by a least-squares estimation. If β jumps up to $\bar{\beta}$ again at T_3, a new loop will start.

The loops originally observed by Phillips [20] were counter-clockwise except in the most recent subperiod of his research. Thus it was natural that he (and his follower Lipsey [12]) concentrated mainly on explaining counter-clockwise loops. However, enough time has passed since then to render the direction of many loops in present economies clockwise, although counter-clockwise loops have by no means been all dead.[18/] The present theory may be capable of explaining this change in direction by a shift of the Phillips curve away from the origin. The position of the curve was shown to depend on the initial state (U_0,α_0), and hence it was uniquely determined. However, one can easily see that once the rate of wage inflation is subject to 'one-shot' changes that are exogenous to

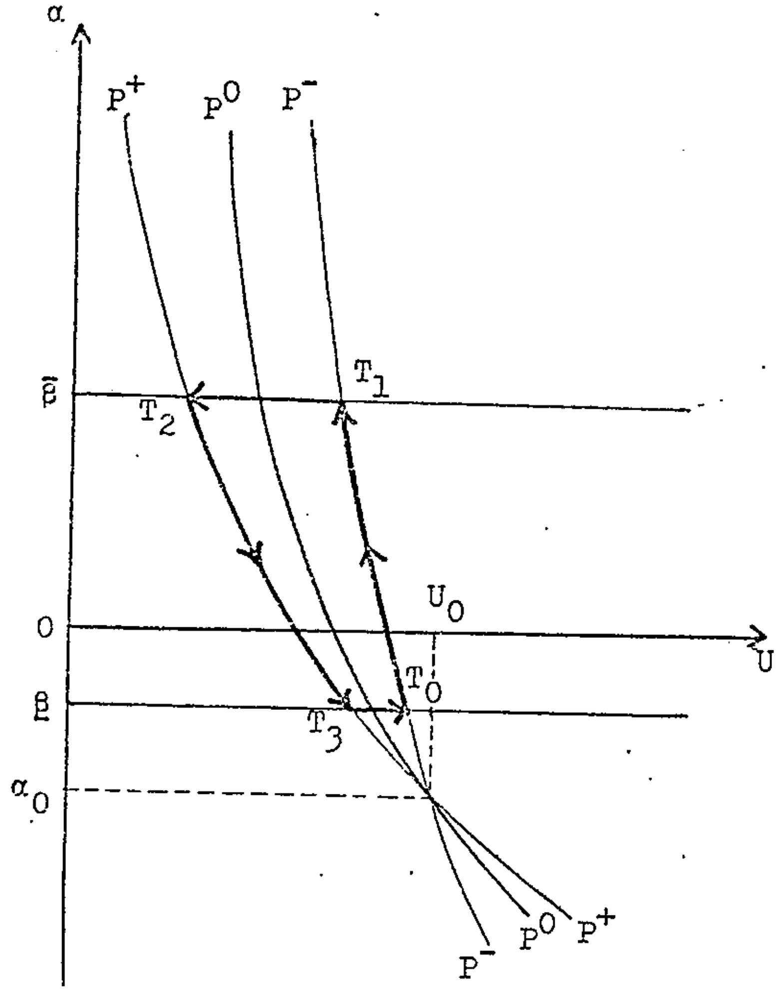

Figure 2a

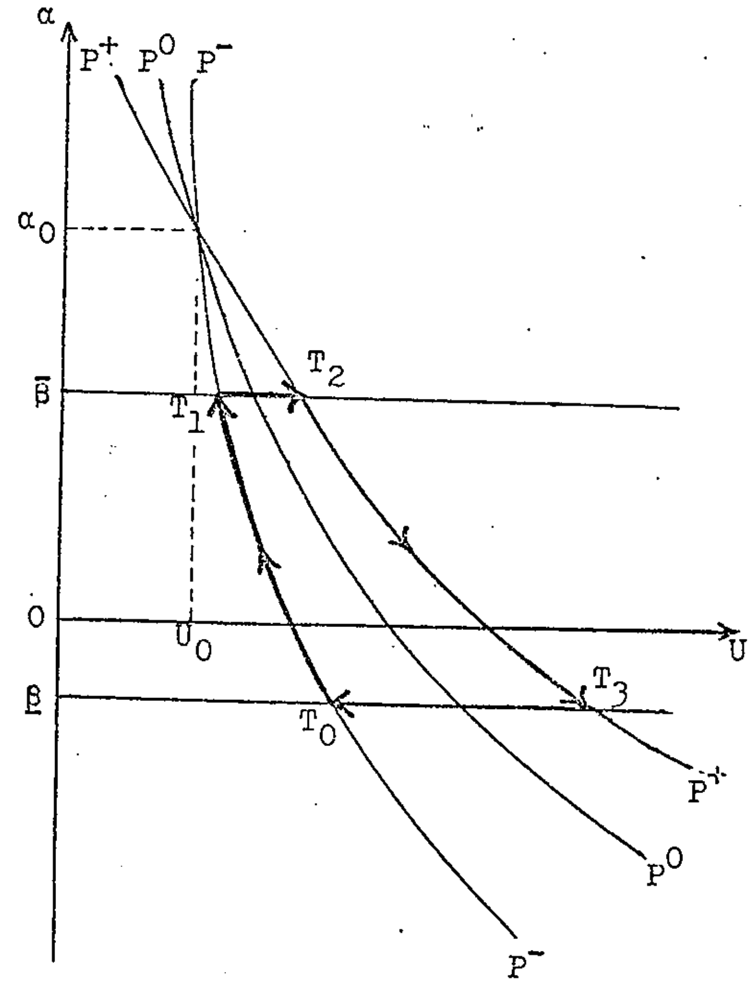

Figure 2b

system (A), the Phillips curve does shift. For instance, assume that α changes discretely and unsystematically by the amount $\Delta\alpha$ at time s, then the point $(U_s, \alpha_s + \Delta\alpha)$ is now regarded as the new initial state to system (A). Historically, presumably, these unsystematic one-shot changes in α have accumulated over time to shift the Phillips curve upwards and to the right. If so, then, one will observe the historical change from Figure 2a to Figure 2b or from counter-clockwise loops to clockwise loops.

5. Cyclical Changes in β: the Continuous Case

In deriving the Phillips-Lipsey loops, I assumed deliberately made changes in β in the previous subsection. However, this is not essential at all to the analysis. To show this, I shall introduce the cyclical movement of β in the following continuous way:

$$\beta_t = \beta_0 + a \sin(bt),$$

where a is the amplitude of the cycle, and the length of a cycle is seen to be $\frac{2\pi}{b}$. With this inserted into (a), I have

$$\dot{\alpha}_t + \nu\alpha_t = \nu[\beta_0 + a \sin(bt)],$$

which yields the solution

$$\alpha_t = (\alpha_0 - \beta_0 + \frac{ab\nu}{b^2+\nu^2})e^{-\nu t} + \beta_0 + \frac{a\nu}{(b^2+\nu^2)^{\frac{1}{2}}} \sin(bt-c),$$

where $c = \text{Arctan}(\frac{b}{\nu})$. Since $\nu > 0$, the first term vanishes as t increases. Hence α_t approaches the limit cycle

$$(11) \qquad \alpha_t = \beta_0 + \frac{a\nu}{(b^2+\nu^2)^{\frac{1}{2}}} \sin(bt-c)$$

for sufficiently large t.

From (11), α changes cyclically, as does U by (7) implying that the pair of (u,α) makes loops. Therefore, it has been shown that the analysis of the previous subsection is not weakened in its impact by the specific changes in β made there.

Meanwhile, the limit cycle (11) has interesting implications. Namely, the amplitude of the cycle of wage inflation depends on the speed of adjustment process (a), ν, the measure of length of the cycle of β, b, as well as the amplitude of the cycle of β, a. It is enlarged as (i) the speed of adjustment becomes faster; (ii) the amplitude of the cycle of β is enlarged; and (iii) the periodic cycle of β is prolonged. Also, it is seen that the cycle of α catches up with that of β after the time interval of c; and it is shorter the faster is the adjustment speed and the shorter is the periodic cycle of β.

These suggest that macroeconomic policy should counteract (i)-(iii) above in order to stabilize wage inflation. (Note that when the amplitude of α is smaller, the fluctuations of α and of U fall within narrower zones.) That is, the stabilization policy is asked to introduce some measure (e.g., income policy) to slow down the adjustment speed of wage inflation and to manage the aggregate demand by monetary and/or fiscal policy to stabilize β, effecting its amplitude smaller and its periodicity shorter.[19/]

6. A Special Case of $\nu = \infty$: the Vertical Phillips Curve

If the adjustment of the rate of wage inflation is so quick that it always reflects the growth rate of the demand for output, $\nu = \infty$ in adjsutment process (a) is the appropriate mathematical representation of such a case. Then $\alpha = \beta$ must hold, which means $\dot{U} = 0$ or U is constant over time. Let this constnat unemployment rate be $\bar{U}$; then one can draw the vertical Phillips curve at $U = \bar{U}$. Points on this vertical line correspond to various β. For a given β, the unique point $(\bar{U},\beta)$ is the only consistent equilibrium. The reason for the vertical Phillips curve when $\nu = \infty$ can be easily found in that, when $\alpha = \beta$, the ratio $q(n)$ in (3) does not change at all over time even the absolute levels of α and of β change, implying that the firm has no incentive to change its employment policy.

The constant unemployment rate $\bar{U}$ may be called the <u>intrinsic</u> rate or <u>structural</u> rate of unemployment. This $\bar{U}$ is determined by condition (1) and is apparently equal to the rate of unemployment that prevails at the initial time, *i.e.*, U_0 in the previous analysis. As mentioned earlier, U_0 and thereby $\bar{U}$ is dependent on the structural parameters of the product and the labor markets: η, ϵ_f, and ϵ_w. One can argue qualitatively the effects of these parameters on $\bar{U}$ in the following manner: (i) when η is smaller, the monopolistic firm can earn more profits by raising price and supplying less amount of output than under a larger η, which will reduce the employment of labor or raise $\bar{U}$; (ii) wider productivity discrepancies imply that relatively less skilled workers are required to produce the same amount of output than otherwise, so that a larger ϵ_f increases $\bar{U}$; and (iii) when ϵ_w is larger, $\bar{U}$ gets lower since wider wage differentials increase the number of hirable workers in descending order of quality for the given amount of wage payments.[20/]

In sum, the structural rate of unemployment $\bar{U}$, or, what is the same thing, the initial rate of unemployment U_0 in the previous analysis, is lower when the output market is more competitive, and when productivities are less elastic and/or wages are more elastic with respect to labor quality, and *vice versa*.[21/] These are summarized in the bottom row of Table 1.

III. Stagflation

1. The Dynamics under Adjustment Process (b)

Next, I shall analyze the dynamic features under adjustment process (b). Once (6) and (b) are combined, one has the following system of simultaneous equations:

$$(B)\quad \begin{cases} \dot{\alpha} = \theta(\frac{\dot{p}}{p} - \alpha), \\ \dot{U} = \frac{\alpha - \beta}{k(U)}\,U. \end{cases}$$

To begin with, note that the second equation [that is, (6)], once it is decomposed, is equivalently written as

$$(12) \qquad \frac{\dot{p}}{p} = \alpha - \beta - (\epsilon_f - \epsilon_w)\frac{\dot{U}}{U}.$$

Substituting (12) into (b), I obtain that $\dot{\alpha} = 0$ holds when

$$(13) \qquad \alpha = -\frac{Uf(U)}{\eta(\epsilon_f - \epsilon_w)y(U)}\,\beta,$$

so that

$$(14) \qquad \left.\frac{d\alpha}{dU}\right|_{\dot{\alpha}=0} = -\frac{f[(1+\epsilon_f)y + Uf]}{\eta(\epsilon_f - \epsilon_w)y^2}\,\beta.$$

Suppose $\dot{\alpha} = \frac{\dot{p}}{p}$ but β is positive (or negative). Then, the ratio $q_t(n)$ rises (or falls) implying that workers in general become more (or less) profitable to firms, so that the firms expand (or contract) the employment by decreasing (or increasing) the critical level of quality. Changes in $\dot{U}$ and thereby in output thus caused are necessarily accompanied by changes in the profit-maximizing price of output or the rate of price inflation, which in turn induce changes in the rate of wage inflation through adjustment process (b). (13) shows the exact condition under which the induced changes in $\frac{\dot{p}}{p}$ and α equal each other again at the next instant. It indicates a somewhat counter-intuitive consequence in that the rate of price inflation equals the rate of wage inflation when the latter (and thereby the former as well) is opposite in sign to the growth rate of the demand for output. However, if one notes that the rate of price inflation is zero when $\alpha = \beta$ (and at the origin) and it becomes negative (or positive) as $\alpha < \beta$ (or $\alpha > \beta$), the result is not too surprising.

For a positive (or negative) β, both (13) and (14) are negative (or positive). Hence, the $\dot{\alpha}=0$ curve, starting from the origin, is downward (or upward) sloping in the fourth (or the first) quadrant as drawn in Figure 3a (or in Figure 3b). Above the $\dot{\alpha}=0$ curve, $\dot{\alpha}$ is negative whereas below it $\dot{\alpha}$ is positive. The movement of U is the same as the one under system (A). Thus one can depict the phase diagram of system (B) as in Figure 3a (or in Figure 3b) when β is positive (or negative).

2. Interpretation of Figure 3a: $(\beta > 0)$

In Figure 3a, the given initial pair of (U,α) is designated as point T_0. As is seen from the Figure, the economy governed by system (B) tends to the stable full-employment equilibrium at the origin when the growth rate of the demand for output is positive.

In the region $R_\beta(+,-)$, one can observe a Phillips trade-off relation although the direction of movement is oneway. In this region, as α is greater than β, the relatively quickly increasing wage payments as opposed to the demand growth of output cause dismissal of workers with the critical level of quality who have begun to bring about losses to firms. The unemployment rate keeps increasing until the difference between α and β disappears. Behind this process, at the same time, the rate of overall wage inflation is adjusted to the rate of price inflation. Above the $\dot{\alpha}=0$ curve, which is the locus of (U,α) that insures the equality of the rates of price and wage inflation, the latter is lower than the former, forcing the downward adjustments for α. Eventually, α hits

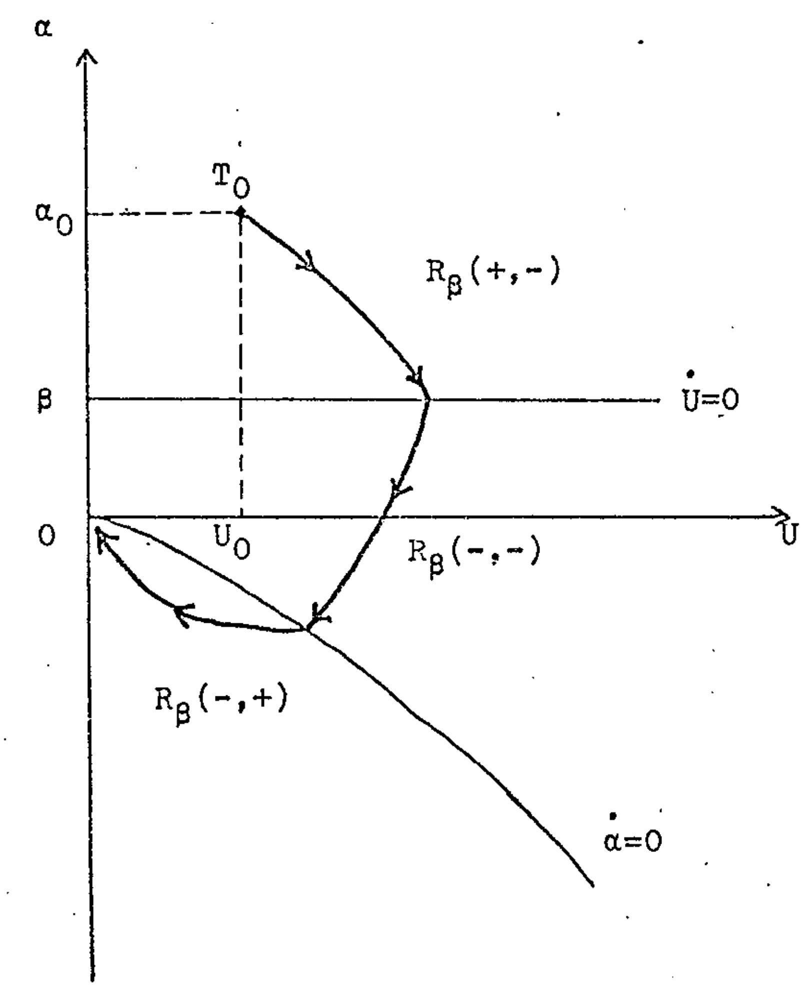

Figure 3a

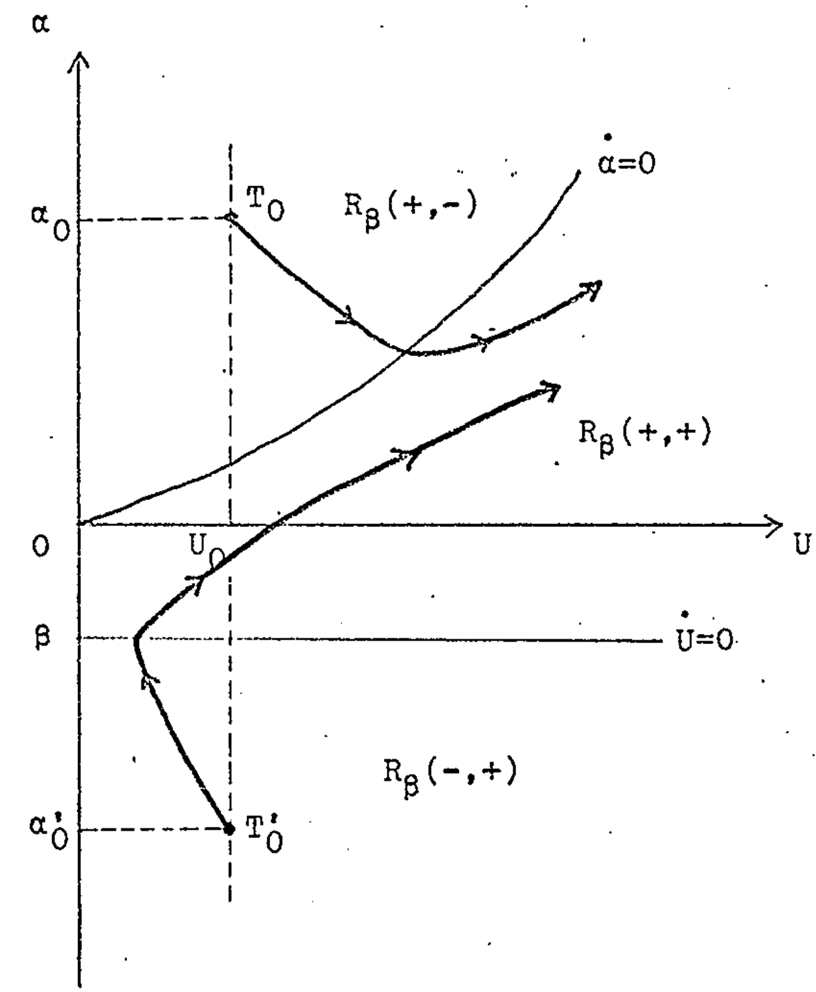

Figure 3b

on the critical rate β, and from then on, the story goes the reverse concerning the employment policy of the firms. The rate of price inflation, however, is still lower than the rate of wage inflation at this instant, so that, in the region $R_\beta(-,-)$, both the unemployment rate and the rate of wage inflation decrease -- the latter becomes even negative -- as time passes. The speedy wage deflation at last catches up with and surpasses afterwards the rate of price deflation on the occasion of passing the $\dot{\alpha}=0$ curve. Then, the economy enters into the region $R_\beta(-,+)$ to approach the full-employment equilibrium.

Note that the vertical axis is an asymptote for U as, from (6), $\dot{U}=0$ but $\frac{\dot{U}}{U} = \frac{\alpha - \beta}{k(0)}$ at $U=0$ and the latter does not vanish unless $\alpha=\beta$. Therefore, (U,α) never hits on the vertical axis. In other words, the full employment is never attained in a finite time. Note also that the region $R_\beta(-,+)$ may never be passed for some initial states and/or for some β. In such cases, (U,α) approaches the origin without going out of the first quadrant.

3. Interpretation of Figure 3b: $(\beta < 0)$

In Figure 3b, two hypothetical initial states are designated by points T_0 in the region $R_\beta(+,-)$ and T_0' in the region $R_\beta(-,+)$. The evolution of the economy in either case ends up with entering into the region $R_\beta(+,+)$ where both the unemployment rate and the rate of wage inflation keep increasing--an occurrence of stagflation. Thus, it is concluded that, when β is negative, the phenomenon of stagflation is likely to occur under system (B).

Since the important factor that determines the dynamic featrues of system (B) is the magnitude of α relative to β, and of α relative to $\frac{\dot{p}}{p}$, the qualitative aspects are not changed from the case where β is positive as long as α is sufficiently large in absolute value. Therefore, the regions $R_\beta(+,-)$ and $R_\beta(-,+)$ in Figure 3b exhibit the same features as the ones in Figure 3a. Different phenomenon does occur only in the reigon $R_\beta(+,+)$ in Figure 3b.

How then is stagflation generated? When the demand for output is decreasing, firms curtail their employment policy by raising the critical level of the hirable quality unless wage payments decrease more rapidly [which is the case in the region $R_\beta(-,+)$]. The increased unemployment pushes up the price of output set by firms causing an acceleration in price inflation which in turn pulls wage inflation unless the latter has so far been greater than the former [which is the case in the region $R_\beta(+,-)$]. Then, the ratio $q_t(n)$ keeps decreasing to further stimulate the dismissal of low quality workers. The vicious circle of stagflation will continue and aggravate itself as time passes.

4. Cyclical Changes in β

I shall introduce the cyclical changes in β. Assume for the sake of graphical representation that they take the form of discrete zigzag changes between a positive $\bar{\beta}$ and a negative $\underline{\beta}$. Suppose that initially β is given as $\bar{\beta}$, then from $\bar{\beta}$ it jumps down to $\underline{\beta}$ at times T_1 and T_3, and then jumps back from $\underline{\beta}$ up to $\bar{\beta}$ again at times T_2 and T_4. The combination of Figures 3a and 3b leads to Figure 4 as a possible locus of the pair (U,α) conrresponding to the above changes in β.

In Figure 4, T_0 belongs to the region $R_{\bar{\beta}}(+,-)$, so that (U,α) moves to T_1 as in Figure 3a; then at T_1, $\bar{\beta}$ falls down to $\underline{\beta}$, so that T_1 now belongs to the region $R_{\underline{\beta}}(+,+)$. Hence (U,α) starts moving to T_2, as in Figure 3b, at which point $\underline{\beta}$ jumps up to $\bar{\beta}$ with the result that T_2 now belongs to the region $R_{\bar{\beta}}(+,-)$. Then, (U,α) moves successively to T_3 and T_4 by two more jumps in β, and so on. Note that the observed locus of (U,α) necessarily exhibits clockwise loops.

The fact that the experience of stagflation is limited in rather recent periods explains that only a few loops have been recorded. Yet, available data seem to exhibit clockwise loops in countries that suffered from stagflation. Note also that the feature of the evolution of the economy crucially depends on the relative duration of the periods with positive β as opposed to those with negative β. The longer is the period with decreasing demand for output, the longer lasts stagflation.

In sum, the phenomenon of stagflation is shown to be possibly generated under system (B) when β is negative. Thus, the decreasing demand for output is a necessary condition for the economy to experience stagflation. In other words, the wage-price spiral postulated as adjustment process (b) alone is not sufficient to generate stagflation.[22/]

5. The Effects of Structural Changes

Under system (B), changes in the structural parameters of the economy have the following impacts: (i) Increases in η and ϵ_f and a decrease in ϵ_w make, as is clear from (13), the $\dot{\alpha}=0$ curve flatter, rendering

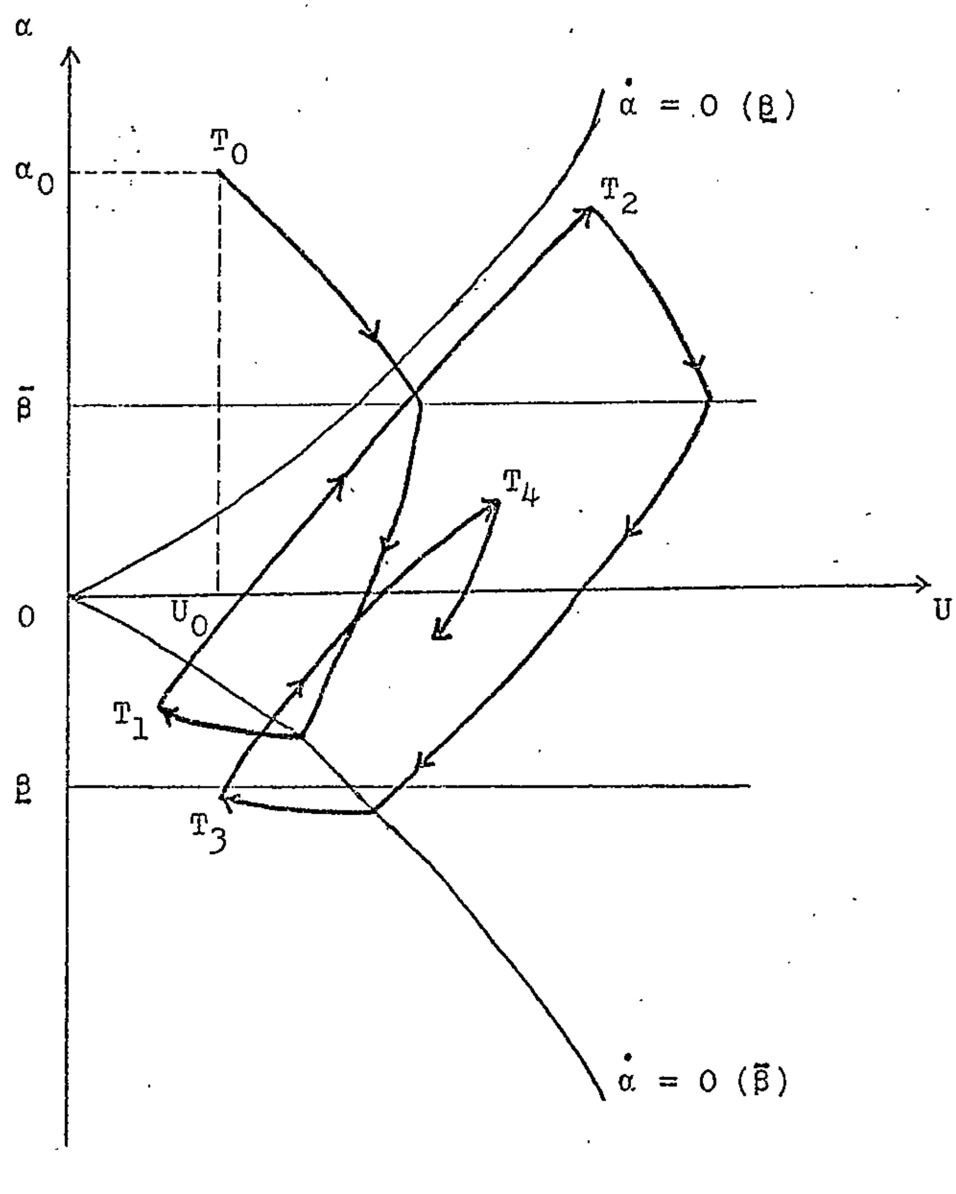
α
$\dot{\alpha} = 0$ ($\underline{\beta}$)
T_0
α_0
T_2
$\bar{\beta}$
T_4
0
U_0
U
T_1
$\underline{\beta}$
T_3
$\dot{\alpha} = 0$ ($\bar{\beta}$)

Figure 4

the loops in Figure 4 more horizontally shaped. These are again expalined by the force which lets the elasticity $k(U)$ larger, that is, a greater responsiveness of U to changes in the valuation of workers at the critical quality. (ii) An increase in θ, although it does not affect the slope of the $\dot{\alpha}=0$ curve at all, makes the loops more perpendicularly shaped since α changes to a larger extent than changes in U. Imagine, for example, an extreme case of $\theta=\infty$ when (U,α) moves on the $\dot{\alpha}=0$ curve, and of $\theta=0$ when (U,α) moves on some horizontal line.

Last, as a minor refinement, if $\underline{\epsilon}_w$ is greater than ϵ_f [but still (2) is satisfied], one can see that the economy is more likely to generate stagflation (even when β is positive).

IV. Concluding Remarks

The present analysis was based on a simple abstract model giving a special role to the heterogeneity of labor. The results derived from the model, however, turned out to be quite fruitful in understanding underlying mechanisms of the complex actual economies. The main conclusions reached were that (i) when the rate of inflation of the entire wage structure is adjusted to the growth rate of the demand for output, a Phillips curve relationship is derived, and (ii) when it is adjusted to the rate of price inflation, the phenomenon of stagflation is generated when the demand for output is decreasing.

Also I was capable of (iii) deriving a Phillips curve that relates involuntary unemployment and the rate of wage inflation; (iv) demonstrating that the Phillips curve is independently located of the demand condition; (v) clarifying the dependence of the position and slope of the Phillips curve on the structural parameters of both the product and the labor markets; (vi) generating both counter-clockwise and clockwise loops; and so on.

The policy implications can be summarized as follows. To stop stagflation, expansionary monetary and/or fiscal policies to increase the aggregate demand should be executed and, at the same time, some income-policy measure is needed to slow down wage-price spirals. Although income policies are less required (within the model) provided that the aggregate demand is steadily increasing, they may yet be helpful in curing the vicious circle of stagflation. Presumably, for the economy as a whole as well as to the policy maker, a stable trade-off relationship of the Phillips curve is preferable to the fear of suffering from stagflation. Then wage inflation should be led to reflect the potential growth of real factors, that is, the aggregate demand and the productivity of workers.

In order to shift inwards the Phillips curve, microscopic policies to improve the structure of the product and the labor markets is called for. These include regulating the monopoly power of firms and narrowing the quality differentials of workers in their skill or productivity by man-power policy and the like. Although it was demonstrated that widening wage differentials is favorable in many respects for the Phillips curve trade-off relationship per se, it may not be taken account of seriously due to other problems of equality and discrimination.

These policies are seen, at the same time, to flatten the Phillips curve. Usually the policy maker has a utility function that relates the unemployment rate and the rate of wage and price inflation. If so, the slope of the Phillips curve (and thereby the degree of these policy measures) is to be properly assessed.

A final remark concludes the paper. In the present model, the growth rate of the demand for output, β_t, was assumed to be exogenously given. To be sure, in more complete macroeconomic models, β_t may be determined endogenously within such models. For instance, β_t may in part depend on the rates of price and wage inflation. Although such a complication is inessential to the results of the Phillips curve relationship, it may introduce new aspects into the analysis of stagflation.

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Footnotes

*/. The author is much indebted to Katuhito Iwai, Kazuo Ueda, Hiroshi Yoshikawa, and other readers of the earlier draft for their valuable comments. Needless to say, however, the remaining errors are the exclusive responsibility of the author.

1/. See, among others, Lipsey [12], Hansen [8], and papers in Phelps et al. [19]. Laidler and Parkin [11], Goldon [3], and Frisch [5] may be reached as excellent surveys on the Phillips curve issues. One characteristic common to all literature is that the Phillips curve is a relation between structural or frictional unemployment and the rate of wage or price inflation.

Recently, the natural rate and rational expectations hypotheses have come to the forefront of the Phillips curve literature. Although I am well aware of the important roles played by expectations in shifting the (short-run) Phillips curve, I shall not attempt to place the present analysis within such a framework. Instead the present analysis should be best understood as an attempt to approach the Phillips curve issues from an alternative and equally important point of view.

2/. By adopting the concept of the Marshallian representative firm, I shall analyze both micro-foundation and macro-analysis simultaneously.

3/. This assumption is not so restrictive and is indeed easily extended to the cases of more general distributions. Also, I shall analyze only the case when the total supply of labor is fixed. Relaxation of this assumption, however, does not change the following results significantly.

4/. This assumption makes mathematical anlyses substantially easier and simpler. However, if the quality of workers is ordered by the value of $q(n)$ defined immediately below in (3), it is no longer necessary.

5/. Here I consider only an interior solution. The economic implication is that workers with highest quality bring about gains to the firm whereas those with the lowest quality cause losses. In other wordes, the firm can make profits by engaging in production but suffers from losses at full employment.

6/. Both of these are evaluated at that point which divides the employed from the unemployed, so that they in general depend on U. However, in the following discussion, for the sake of analytical convenience, I shall treat them as if they were independent of U (but not necessarily constant over time). The similar remark will also apply for η.

7/. Note that the elasticity of $q(n)$ with respect to U is different from the elasticity of $q(n)$ with respect to n evaluated at $n=U$. The latter equals $\epsilon_f - \epsilon_w$ whereas the former equals $k(U)$. This is because $q(n)$ defined in (3) contains U. In other words, the evaluation of an individual depends on the number of the total employed workers including himself. This is a kind of external effect to individuals in the labor market spilt over from the product market. Note also that, as the variable itself suggests, $q(n)$ shares certain similarity with Tobin's 'q', the ratio of the market valuation of capital to its replacement cost. (see, for instance, Tobin [24].)

8/. For example, firm-specificity of labor is on the side of causing larger ϵ_f, while the existence of minimum-wage laws is on the side of causing smaller ϵ_w. See, for instance Reder [21], Oi [15], Doeringer and Piore [2], and Williamson *et al.* [25].

9/. (4) and (5) [for instance (4)] can be rewritten as

$$w_t(n) = w_0(n)\exp\left(\int_0^t \alpha_s \, ds\right).$$

10/. If the entire spectrum of productivity shifts at a rate of γ_t, β_t in (6) ought only to be replaced by β_t', where $\beta_t' = \beta_t + (1-\frac{1}{\eta})\gamma_t$. The analysis in Section II needs no change at all by this redefinition of β_t. In Section III, however, a taxonomic analysis is needed if β_t and γ_t differ in sign, which adds only unnecessary complications without causing any significant change in results.

11/. Hereafter, for brevity, the time subscript t will again be omitted wherever it is unnecessary.

12/. Eq.(8) is written more generally as

$$\alpha_t = -\nu \log q_0(U_t) + \alpha_0.$$

Note that $q_0(U_t)$ is different from $q_t(U_t)$ which is unity for all t. The following holds between $q_0(n)$ and $q_t(n)$:

$$q_t(n) = q_0(n)\exp\left[\frac{1}{\nu}(\alpha_t - \alpha_0)\right].$$

13/. Under adjustment process (c), the Phillps curve becomes

$$\alpha_t = -\lambda(\epsilon_f - \epsilon_w)\log\frac{U_t}{U_0} + \alpha_0.$$

This is the same, so long as $\lambda=\nu$, as (8) except that the first term in (8) does not appear. Hence, adjustment process (a) with the competitive output market ($\eta = \infty$) and adjustment process (c) for any η end up with the same Phillips curve.

14/. One can easily obtain: $P_\eta = -\frac{\nu}{\eta^2} \log \frac{y(U_t)}{y(U_0)} \gtrless 0$, $P_{\epsilon_f} = -P_{\epsilon_w} = -\nu \log \frac{U_t}{U_0} \lesseqgtr 0$, $P_\nu = \frac{\alpha_t - \alpha_0}{\nu} \lesseqgtr 0$, as $U_t \gtrless U_0$; and $P_{U_0} = \nu \frac{k(U_0)}{U_0} > 0$, $P_{\alpha_0} = 1$.

15/. This point has not been clarified in the past literature of the Phillips curve. Of course, it is ascribed to the fact that the past literature dealt not with involuntary unemployment but with structural or frictional unemployment.

16/. That the stationary state is dependent on the initial state and the path the economy experienced is called 'hysteresis'. Phelps [18] points out such a possibility within a different framework concerning the natural rate of unemployment. The hysteresis phenomenon here is obtained from the fact that both $\dot{\alpha}$ and $\dot{U}$ adjust to the same disequilibirum $(\beta-\alpha)$ under system A.

17/. See, for instance, Reder [22], Rees [23], and empirical studies cited therein.

18/. Counter-clockwise loops are explained as an aggregation phenomenon (Lipsey [12]) while clockwise loops by adding an expectation term (Brechling [1]). If the response of unemployment (or changes in wage inflation) is slower in the short run than in the long run, counter-clockwise (or clockwise) loops are observable around the longer-run

Phillips curve (Holt et al. [9]). See also Kuska [10], Hansen [8], Mortensen [13], and Grossman [4]. Generally speaking, if α is a function of $\dot{U}$ as well as U and that if the partial derivative of α with respect to $\dot{U}$ is negative (or positive), counter-clockwise (or clockwise) loops are obtained.

Within the confines of the present model, if one calculates the current-weight (Paasch) average wages paid to the employed,

$$\omega = \frac{1}{1 - U}\int_U^1 w(n)\, dn,$$

then one can observe clockwise loops by tracing $(U, \frac{\dot{\omega}}{\omega})$ around the locus of (U,α) for even fixed parameter values of ν and ϵ_f. This is because low wages are included in the average during booms, but excluded during recessions. Meanwhile, however, this may not be relevant because empirical investigations are based on a fixed-weight (Laspeyres) average and, theoretically, it is equivalent to α here.

19/. The last prescription as to shorten the length of business cycles may not be readily confirmative to one's intuititon. It is explained as follows. The shorter is the length of a cycle of β, α is adjusted to the more rapidly moving target. Then, ups and downs of α tend to be offset by quick turns of β, forcing α stuck within a relatively narrow range.

20/. Analytically, the assmuption of constant elasticities η, ϵ_f, and ϵ_w implies that functions of $P(y)$, $f(n)$, and $w(n)$ are written, respectively, as $p(y) = \bar{p}y^{-\frac{1}{\eta}}$ ($\bar{p}$=const.), $f(n) = f(1)n^{\epsilon_f}$, and

$w(n) = w(1)n^{\epsilon_w}$. Hence, (1) is written as

$$\bar{p}y(\bar{U})^{-\frac{1}{\eta}}(1-\frac{1}{\eta})f(1)\bar{U}^{\epsilon_f} = w(1)\bar{U}^{\epsilon_w},$$

where

$$y(\bar{U}) = \frac{f(1)}{1+\epsilon_f}(1-\bar{U}^{1+\epsilon_f}).$$

Therefore, one obtains

$$\text{(F1)} \qquad \frac{\partial\bar{U}}{\partial\eta} = -\frac{\bar{U}}{\eta^2 k(\bar{U})}\left(\frac{\eta}{\eta-1} + \log y(\bar{U})\right),$$

$$\text{(F2)} \qquad \frac{\partial\bar{U}}{\partial\epsilon_f} = -\frac{\bar{U}}{k(\bar{U})}\left[\frac{1}{\eta(1+\epsilon_f)} + \left(1 + \frac{\bar{U}^{1+\epsilon_f}}{\eta(1-\bar{U}^{1+\epsilon_f})}\right)\log\bar{U}\right],$$

$$\text{(F3)} \qquad \frac{\partial\bar{U}}{\partial\epsilon_w} = \frac{\bar{U}}{k(\bar{U})}\log\bar{U}.$$

(F1) is negative if the unit of output is chosen in such a way that $f(1)$ is sufficiently large to let $y(\bar{U}) > 1$. The sign of (F2) is at first sight ambiguous. However, it is positive for $\bar{U}$ sufficiently close to zero. (F3) is seen to be always negative.

21/. In the United States economy, there is a unanimous agreement that the natural rate of unemployment (the structural rate, the full employment unemployment, etc.) has risen from 4% of 1960's to somewhere around 6% of nowadays. In the meantime, there are empirical studies that the secondary labor participants have increased significantly (Perry [17]) while wage differentials have shown a secular tendency to diminish despite afore-mentioned cyclical fluctuations (Reder [21], [22]). These may be boldly interpreted in the present model that ϵ_f has increased while ϵ_w has decreased in the United States. Then, as a first approximation, if $\bar{U}$ is identified with the natural rate, the theory is consistent with this increase in the natural rate of unemployment. See, however, for instance Hall [7] for other plausible causes. See also Pacher and Park [16] for an empirical finding that an increase in inter-industry wage distortion worsens the Phillips curve relationship (which is not incompatible with the present theory that focuses on inter-personal or occupational wage differentials.)

22/. See Nikaido and Kobayashi [14] for a possible generation of stagflation from wage-price spirals (alone), and Haberler [6] for another view of causing stagflation.