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An Empirical Reexamination of the Solow Growth Model

by

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<Abstract>

The Solow growth model is studied empirically for 22 OECD and 28 non-OECD countries. We estimate a production function, a convergence process to steady state, and the steady state of the Solow growth model in order. First, a production function having a common parameter and different technological progress rates among countries is estimated by using cross-country and time series data from years 1965 to 1990. Second, we estimate a convergence process of income per effective labor by using estimates from production function estimation and get steady state income of each country. Finally, the determination of income per labor at the steady state is estimated in a devised manner. Each of the income of year 1990 and the steady state income estimated from a convergence process was used as the income of the steady state estimation. A devised manner is neither need to assume statistically initial technological level of each country nor estimate it directly. There are little violations of each of estimation results of production function, convergence and the steady state. Furthermore, there is a consistency of estimation results among a production function, convergence and the steady state except for a little difference of estimation results between convergence and the steady state.

1 Introduction

The Solow (1956) growth model has been a center of economic growth theory. However there has been a thorough reworking in past decade. The main reformulation is carried out through endogenous growth theory advocated by Romer (1986) and Lucas (1988). This theory can explain international differences in economic growth by factors such as human capital investment and R & D activities which are endogenous to the model.[1] In contrast, the Solow growth model describes economic growth by exogenous factors such as population growth and technological progress.

Mankiw, Romer and Weil (1992) investigated whether or not the Solow growth model can explain international differences of economic growth with cross-country data. They considered both the steady state and convergence to the steady state of each of the Solow model and an augmented Solow model including human capital. Although they did not argue the appropriateness of the steady state assumption, they concluded that an augmented Solow model is consistent with cross-country data. Barro and Sala-i-Martin (1992) also considered convergence to the steady state of neo-classical growth model and estimated it using cross-state data of U.S.A.. Their estimation results supported convergence of the states.

Related to the empirical studies of Mankiw, Romer and Weil (1992) and Barro and Sala-i-Martin (1992), many empirical studies of economic convergence were appeared. Levine and Renelt (1992) showed that a regression analysis using cross-country data to explain economic growth is sensible to a choice of explanatory variables and they investigated for the conditions to obtain economic convergence. Quah (1993a) explained analytically that

[1] Utilizing time series data of U.S.A., Jones (1995) tested an implication of endogenous growth theory that a permanent change of policy affects economic growth permanently. The estimation results were unfavorable to endogenous growth theory.

a growth regression using cross-section data such as Mankiw, Romer and Weil (1992) and Barro and Sala-i-Martin (1992) is not appropriate to prove convergence. Quah (1993a) showed empirically using a Markov chain model that a cross-country income distribution has a tendency not to converge but diverge.[2] To prove economic convergence, Evans and Karras (1993) and Evans (1997) considered a new estimation method not deriving from an economic theory and concluded that per capita consumption tends to diverge except for developed countries. Islam (1995) estimated convergence of the Solow model from panel data. Different from Mankiw, Romer and Weil (1992), it became possible to estimate an initial technological level of each country because of using not only cross-country but also time series data. A higher convergence speed than one of Mankiw, Romer and Weil (1992) was obtained and adding human capital became being unimportant. Goerz and Hu (1996) took notice of simultaneity between economic growth and human capital investment and found a higher contribution of human capital to economic growth and a higher convergence speed than ones of no simultaneity between them by using cross-state data of U.S.A..

Income per effective labor at time t can be approximated by income of both initial time and steady state, convergence speed and time t. On the previous research of economic convergence, income of the steady state appearing on convergence has been replaced by the steady state specification composed of saving rate, population growth rate and so on. This study estimates directly the steady state income of convergence using income dynamics.

It is difficult to estimate simultaneously a parameter of production function common among countries and both technological progress rates and the steady state income per effective labor different among countries. Therefore, we estimate a production function at first using annual data from years

[2] Quah (1993b) obtained the same conclusion.

1965 to 1990 and cross-country data of each of 22 OECD and 28 non-OECD countries.[3] Utilizing the estimates of both a parameter of production function and technological progress rates, the steady state income is estimated directly from income dynamics of each country. We derive the convergence process of the Solow model and explain the estimation procedure at section 2.1. The estimation results are reported at section 2.2.

Income per labor at the steady state can be explained by saving rate, technological level, population growth rate and so on. Because the data of technological level do not exist, Mankiw, Romer and Weil (1992) made a statistical assumption of technological level and Islam (1995) estimated it on convergence to the steady state by using panel data. This study replaces initial technological level with both output and capital per labor at an initial time by the restriction derived from a production function. The determination of steady state income not including initial technological level explicitly is explained at section 3.1. The steady state income of steady state specification is considered by each of the income of year 1990 and the steady state income estimated from convergence process. The steady state specification is estimated by the data of each of 22 OECD and 28 non-OECD samples. The estimation results are shown at section 3.2.

It can be seen from the estimation results of sections 3.2 and 4.2 whether or not there is a consistency of estimation results among a production function, convergence and the steady state and whether or not there is a violation of assumptions of the Solow model. The remaining research are considered at section 4.

[3] OECD and non-OECD samples were made different because there may be a structural change of a parameter of production function.

2 Convergence of the Solow growth model

2.1 A specification of the convergence process

The production function of ith country is the form of Cobb-Douglas and technological progress is the Harrod neutral type which improves only the efficiency of labor.[4] The parameter of α of the production function is assumed to be the same among countries.

$$Y_i(t) = K_i(t)^{\alpha}(A_i(t)L_i(t))^{1-\alpha}, \qquad 0 < \alpha < 1, \tag{1}$$

where $Y_i(t)$ is the output of ith country at time t, $K_i(t)$ is the capital of ith country at time t, $A_i(t)$ is the technological level of ith country at time t, and $L_i(t)$ is the labor of ith country at time t.

Each of labor and technological level grows at the rate of n_i and g_i respectively, *that is,*

$$L_i(t) = L_i(0)e^{n_i t}, \qquad A_i(t) = A_i(0)e^{g_i t}.$$

The dynamics of capital per effective labor becomes

$$\frac{d(\frac{k_i(t)}{A_i(t)})}{dt} = s_i(\frac{k_i(t)}{A_i(t)})^{\alpha} - (n_i + g_i + \delta_i)\frac{k_i(t)}{A_i(t)}, \tag{2}$$

where $k_i(t) \equiv \frac{K_i(t)}{L_i(t)}$ is the capital per labor, $s_i \equiv \frac{I_i(t)}{Y_i(t)}$ is the investment rate ($I_i(t)$ is the investment of ith country at time t), s_i is also the saving rate because of the equality between investment and saving, and δ_i is the depreciation rate of capital.

The convergence process of income per effective labor to the steady state can be approximated around the steady state, $\frac{d(\frac{k_i(t)}{A_i(t)})}{dt} = 0$.

$$\ln\frac{y_i(t)}{A_i(t)} = (1 - e^{-\lambda_i t})\ln(\frac{y_i}{A_i})^* + e^{-\lambda_i t}\ln\frac{y_i(0)}{A_i(0)}, \tag{3}$$

[4] The specification of the Solow growth model is the same to Mankiw, Romer and Weil (1992).

where $y_i(t) \equiv \frac{Y_i(t)}{L_i(t)}$ is the income per labor at time t where the income equals to the output, $(\frac{y_i}{A_i})^*$ and $\frac{y_i(0)}{A_i(0)}$ are respectively the income per effective labor at the steady state and the initial time, and $\lambda_i \equiv (n_i + g_i + \delta_i)(1-\alpha)$ is the convergence speed. The convergence speed is a positive value because of the assumption of $0 < (1-\alpha) < 1$ with the condition that the addition of labor growth rate, technological progress rate and depreciation rate, $(n_i + g_i + \delta_i)$, takes a positive value.

The income per effective labor at time t can be represented by the convex combination of the initial income and the steady state income. Both the weights of initial income and steady state income are determined by the convergence speed and time t. On the estimation of convergence, Mankiw, Romer and Weil (1992) and Islam (1995) used the following income determination at the steady state.

$$\ln(\frac{y_i}{A_i})^* = \frac{\alpha}{1-\alpha}(\ln s_i - \ln(n_i + g_i + \delta_i)). \tag{4}$$

The previous research put the steady state specification (4) into convergence (3). *That is*, the steady state specification of convergence was expressed explicitly and it was estimated on the estimation of convergence. This study considers the estimation of convergence by the time series data of income. We consider equation (3) at time $t+1$ as well as time t and subtract $\ln \frac{y_i(t)}{A_i(t)}$ from $\ln \frac{y_i(t+1)}{A_i(t+1)}$.

$$\ln \frac{y_i(t+1)}{A_i(t+1)} - \ln \frac{y_i(t)}{A_i(t)} = (-e^{-\lambda_i(t+1)} + e^{-\lambda_i t})(\ln(\frac{y_i}{A_i})^* - \ln \frac{y_i(0)}{A_i(0)}). \tag{5}$$

The growth rate of income per effective labor, $(\ln \frac{y_i(t+1)}{A_i(t+1)} - \ln \frac{y_i(t)}{A_i(t)})$, is computed from the difference between the steady state income and the initial income, $(\ln(\frac{y_i}{A_i})^* - \ln \frac{y_i(0)}{A_i(0)})$. The growth rate of income per effective labor takes the same sign to the difference between the steady state income and the initial income because of a positive value of the convergence speed. When the difference of income between the initial time and the steady state takes a positive value, the growth rate of income per effective labor takes a convex curve to the below and it converges to zero from a positive value by the form of exponential function. And when the difference of income between the initial time and the steady state takes a negative value, the growth rate of income per effective labor takes a convex curve to the above and it converges to zero from a negative value.

The dynamics of income per labor are transformed from the dynamics (5) of income per effective labor by the assumption of technological level.

$$\ln y_i(t+1) - \ln y_i(t) - g_i = (-e^{-\lambda_i(t+1)} + e^{-\lambda_i t})(\ln(\frac{y_i}{A_i})^* - \ln \frac{y_i(0)}{A_i(0)}),$$
$$t = 1, 2, \cdots. \tag{6}$$

The difference of income between the steady state and the initial time can be transformed to the following form by the assumption of technological level.

$$\ln(\frac{y_i}{A_i})^* - \ln \frac{y_i(0)}{A_i(0)} = \ln y_i^* - g_i t^* - \ln y_i(0), \tag{7}$$

where $y_i(t)^*$ and t^* are respectively the income per labor and time at the steady state and $\ln y_i^* - g_i t^*$ takes a constant value with respect to time.[5]

On the estimation of convergence (6) using equation (7), the data of income per labor, $y_i(t)$, exist and the population growth rate, n_i, and the depreciation rate, δ_i, can be computed from data. Therefore, the unknown parameters are the parameter of the production function, α, the technological progress rate, g_i, and the steady state income taking the technological progress rate into account, $\ln y_i^* - g_i t^*$.

Because it is very difficult to estimate simultaneously the parameter of the production function common among countries and both the technological

[5] Both $\ln y_i^*$ and t^* are the variables of time.

progress rates and the steady state income per labor taking the technological progress rates into account differed among countries, we consider the production function at first and obtain the estimates of the parameter of the production function and the technological progress rates.

The production function (1) is transformed to the log-form:

$$\ln y_i(t) - \ln A_i(t) = \alpha(\ln k_i(t) - \ln A_i(t)). \tag{8}$$

Subtracting (8) at time t from it at $t+1$ becomes

$$\ln y_i(t+1) - \ln y_i(t) = (1-\alpha)g_i + \alpha(\ln k_i(t+1) - \ln k_i(t)). \tag{9}$$

The equation (9) represents the relationship between the growth rates of output per labor and capital per labor. The growth rate of output per labor is made up of the different constant term of each country coming from the differences of technological progress rates and the growth rate of capital per labor having the common coefficient, α, which is the elasticity of capital to output. According to the division theory of marginal productivity, the common elasticity among countries means that the division rate of income to capital is α for all countries. Furthermore, because the growth rate of income per labor equals to the technological progress rate at the steady state of the Solow growth model being the Harrod neutral type, the different technological progress rates among countries imply that the growth rates of income per labor at the steady state differ among countries.

The parameter of the production function and the technological progress rate of each country can be estimated by the simultaneous estimation of the production function by using cross-country and time series data. Then, we can compute the steady state income per labor from the estimation of convergence (6) by using the estimates of α and g_i.

The estimation of the production function and convergence is explained and the estimation results are investigated in the next section.

2.2 Estimation of the convergence process

The estimation equation derived from the production function is specified as follows.

$$\Delta \ln y_i(t+1) = a_i + \alpha \Delta \ln k_i(t+1) + u_{i,t},$$

$$t = 0, 1, \cdots, T-1, \quad i = 1, 2, \cdots, N. \tag{10}$$

where $a_i \equiv (1-\alpha)g_i$, $u_{i,t}$ follows the serial correlation of first order, $u_{i,t} = \theta_i u_{i,t-1} + \epsilon_{i,t}$, $\epsilon_{i,t} \sim N(0, \sigma^2)$, and $\epsilon_{i,t}$ is mutually independent of any i and any t.

The estimation equation (10) is estimated across cross section and time series by the generalized least squares method and the estimates of α and a_i, $i = 1, 2, \cdots, N$, are obtained. The technological progress rates, g_i, $i = 1, 2, \cdots, N$, are computed from those estimates.

The data of GDP per labor, $y_i(t)$, and capital per labor, $k_i(t)$, were taken from the latest version Mark 5.6 of Penn World Table constructed by Summers and Heston (1991).[6] From the view of structural change of α across cross countries, 22 OECD countries called the OECD sample and 28 intermediate countries called the non-OECD sample were chosen to be estimated.[7] All the chosen intermediate countries were included in the intermediate sample of Mankiw, Romer and Weil (1992). *That is*, $N = 22$ in the OECD sample and $N = 28$ in the non-OECD sample. The time series data were taken as the annual data from years 1965 to 1990 by the availability, *that is*,

[6] The physical capital includes the residential construction and the transportation equipment.

[7] Although all of the OECD countries were considered, the data of more some countries were available in intermediate countries. Those countries were excluded because those were not included in the intermediate sample of Mankiw, Romer and Weil (1992) or the depreciation rate of capital could not be computed.

$T = 25$. The growth rates of GDP per labor and the capital per labor may not have a unit root on the annual data.

The SHAZAM (Version 8.0) was used as the statistical software. The estimation results of the OECD and the non-OECD samples are respectively shown in Tables 1.1 and 1.2. The estimates of technological progress rates are expressed in percent unit. The rejection region of the test of $\hat{\alpha} = 0$ was set at the upper side because of the assumption of $0 < \alpha$.

Table 1.1 Estimation results of the production function (10) (OECD sample)

country name	$\hat{g}_i$	country name	$\hat{g}_i$	country name	$\hat{g}_i$
Canada	1.338	U.S.A.	0.750	Japan	3.700
Austria	1.921	Belgium	2.028	Denmark	0.773
Finland	2.194	France	1.753	W. Germany	1.450
Greece	2.785	Ireland	3.004	Italy	2.945
Netherlands	1.245	Norway	2.583	Portugal	4.011
Spain	2.794	Sweden	0.537	Switzerland	0.603
Turkey	3.122	U.K.	1.404	Australia	1.109
New Zealand	−0.362				
$\hat{\alpha}$	0.223 (5.04**)				

Note: $\hat{g}_i$ and $\hat{\alpha}$ are respectively the estimates and $\hat{g}_i$ is expressed in percent unit. The figure in () is the asymptotic $t-$ value of the estimate and ** indicates to be significant at the 1% significance level. The estimate of standard error of error terms is 1.00. The sample sizes of cross country and time series are respectively 22 and 25.

Table 1.2 Estimation results of the production function (10) (non-OECD sample)

country name	$\hat{g}_i$	country name	$\hat{g}_i$	country name	$\hat{g}_i$
Ivory Coast	−1.624	Kenya	2.024	Malai	−0.388
Mauritius	1.370	Morocco	2.076	Nigeria	0.300
Sierra Leone	−2.294	Zambia	−1.379	Zimbabwe	1.287
Dominican Rep.	0.013	Honduras	0.518	Jamaica	−0.272
Mexico	1.009	Panama	0.386	Argentina	−0.942
Bolivia	0.263	Chile	0.035	Colombia	2.066
Ecuador	1.760	Venezuela	−2.372	Hong Kong	6.200
India	1.900	Israel	2.364	Korea, Rep.	5.742
Philippines	1.358	Syria	3.729	Thailand	3.657
Luxembourg	2.029				
$\hat{\alpha}$	0.297 (7.056**)				

Note: $\hat{g}_i$ and $\hat{\alpha}$ are respectively the estimates and $\hat{g}_i$ is expressed in percent unit. The figure in () is the asymptotic $t-$ value of the estimate and ** indicates to be significant at the 1% significance level. The estimate of standard error of error terms is 1.00. The sample sizes of cross country and time series are respectively 28 and 25.

The estimate of θ_i was significantly different from zero, *that is*, there was the serial correlation in some countries of each of the OECD and the non-OECD samples. The residuals taking the serial correlation into account seemed to be random.[8]

The estimate of parameter, α, took the value between zero and unity in accordance with the assumption of the Solow growth model. Following the marginal productivity theory of division, the division rate of income to capital

[8] The estimation results changed little under the assumption of no serial correlation of the error terms.

equals to the coefficient, α, of the Cobb-Douglas function and that rate is said to take about $\frac{1}{3}$ empirically. Therefore, the estimate, 0.223, is a little smaller than $\frac{1}{3}$. Concerning with the estimates of technological progress rates, each estimate of technological progress rates got the positive value of one figure percent except for the estimate of New Zealand. The estimate of New Zealand had the negative and small value. Two reasons are considered as follows. The one is that the production function of New Zealand differs from the other OECD countries and the true value of the technological progress rate is positive. Therefore, the estimate of the technological progress rate of New Zealand turned out to be negative because of the simultaneous estimation under the assumption of the same production function among the countries. The other is that the technological progress rate improving only the efficiency of labor of New Zealand is truly negative from years 1965 to 1990. The estimate of Japan, 3.7%, seemed to be high. This may be considered as one reason of high economic growth of Japan. The low estimates of technological progress rates of Canada, U.S.A., West Germany and U.K. may come from the low economic growth between years 1965 and 1990.

From the estimation results of the non-OECD sample, the estimate of parameter α took the value, 0.297, between zero and unity in accordance with the assumption. This estimate was a little higher than the one of the OECD sample and it was near to $\frac{1}{3}$. Therefore, this evidence accords with the empirical fact that the income division rates to the capital of developing countries are likely to take a little higher than those of developed countries. Concerning with the estimates of technological progress rates, the estimates were negative between 0 and -2% in the seven countries and the estimates of the other countries had the positive values of one figure percent. The similar reasons of the negative estimate of New Zealand can be considered for the seven countries. Furthermore, the estimates of Hong Kong and Korea of the

Asian NICs were very high as being near 6%. The high estimates can be considered as one reason of high economic growth of Hong Kong and Korea.

The pooling estimation of the OECD and the non-OECD samples was also conducted. The estimate of α, 0.266, was between the estimates of the OECD and the non-OECD samples. However the number of the negative estimates of technological progress rates was larger in the estimation of the pooling samples than that in the estimation of the split samples. This results may come from the structural change of α between the OECD and the non-OECD countries. Therefore, the specifications of convergence and the steady state are also estimated on each of the OECD and the non-OECD samples.

The convergence process including the income per labor at the steady state as the unknown parameter, std_i, is estimated by utilizing the estimates of $\hat{\alpha}$ and $\hat{g}_i$ obtained from the estimation of the production function.

$$\Delta \ln y_i(t+1) - \hat{g}_i = (-e^{-\hat{\lambda}_i(t+1)} + e^{-\hat{\lambda}_i t})(std_i - \ln y_i(0)) + u_{i,t},$$

$$t = 1, 2, \cdots, T-1, \quad i = 1, 2, \cdots, N. \tag{11}$$

where $\hat{\lambda}_i \equiv (n_i + \hat{g}_i + \delta_i)(1 - \hat{\alpha})$, $std_i \equiv \ln y_i^* - g_i t^*$ is the income per labor at the steady state taking the technological progress rate into account, the error terms, $u_{i,t}$, follows the serial correlation of first order, $u_{i,t} = \phi_i u_{i,t-1} + \epsilon_{i,t}$, $\epsilon_{i,t} \sim N(0, \sigma^2)$, and $\epsilon_{i,t}$ is mutually independent of any i and any t.

The estimation equation (11) of convergence can be rewritten as follows.

$$\Delta \ln y_i(t+1) - \hat{g}_i + (-e^{-\hat{\lambda}_i(t+1)} + e^{-\hat{\lambda}_i t}) \ln y_i(0)$$

$$= (-e^{-\hat{\lambda}_i(t+1)} + e^{-\hat{\lambda}_i t}) std_i + u_{i,t},$$

$$t = 1, 2, \cdots, T-1, \quad i = 1, 2, \cdots, N. \tag{12}$$

Equation (12) is the linear regression model which does not include the constant term and has the income per labor at the steady state, std_i, as the coefficient. The equation of each country is estimated by the time series data and each estimate of std_i is obtained.

We also consider the estimation which includes not only the income at the steady state but also the technological progress rate as the unknown parameters.[9]

$$\begin{aligned}&\Delta \ln y_i(t+1) + (-e^{-\hat{\lambda}_i(t+1)} + e^{-\hat{\lambda}_i t}) \ln y_i(0) \\ &= g_i + (-e^{-\hat{\lambda}_i(t+1)} + e^{-\hat{\lambda}_i t}) std_i + u_{i,t}, \\ &t = 1, 2, \cdots, T-1, \quad i = 1, 2, \cdots, N. \end{aligned} \tag{13}$$

where $\hat{\lambda}_i \equiv (n_i + \hat{g}_i + \delta_i)(1 - \hat{\alpha})$ and the error term, $u_{i,t}$, follows the assumption similar to that of equation (12).

Equation (13) is the linear regression model which the technological progress rate is the constant term and the income per labor at the steady state is the coefficient. The following hypothesis testing can be considered from the comparison between equations (12) and (13).

$$H: \quad g_i = \hat{g}_i \quad versus \quad A: \quad g_i \neq \hat{g}_i, \qquad i = 1, 2, \cdots, N. \tag{14}$$

The null hypothesis H implies that the parameter, g_i, of the technological progress rate equals to the estimate, $\hat{g}_i$, from the estimation of the production function. *That is*, we can estimate the income per labor at the steady state, std_i, not assuming the unknown parameter of technological progress rate but utilizing the estimate. The hypothesis testing (14) is conducted by the

[9] If the technological progress rate contained in the convergence speed was also assumed to be the unknown parameter, the convergence process would be the nonlinear regression model and the estimation would be difficult because of the exponential function and the small sample size. Therefore, the estimate of the technological progress rate is used in the convergence speed.

likelihood ratio test. The test statistic of likelihood ratio is distributed as χ^2 distribution with one degree of freedom under the null hypothesis.

Similar to the estimation of the production function, the data were taken from the Penn World Table (Mark 5.6) of Summers and Heston (1991). The convergence process of each country was estimated by the annual data from years 1965 to 1990. *That is*, the initial time 0 was year 1965 and the sample size of the estimation of each country was 24. The labor growth rate, n_i, was computed as follows. The labor, $\frac{Y_i(t)}{POP_i(t)} \cdot \frac{L_i(t)}{Y_i(t)} \cdot POP_i(t) = L_i(t)$, was computed at each year from years 1965 to 1990 where $POP_i(t)$ was the population in ith country at time t. The growth rates were calculated by the equation, $\ln L_i(t+1) - \ln L_i(t)$, and the average of the growth rates was set to be the data of n_i. Though the depreciation rate of each country was assumed to take the same value in Mankiw, Romer and Weil (1992) and Islam (1995), it could be computed from the data of the Penn World Table (Mark 5.6). The physical capital, $\frac{K_i(t)}{L_i(t)} \cdot L_i(t) = K_i(t)$, and the investment, $\frac{I_i(t)}{Y_i(t)} \cdot Y_i(t) = I_i(t)$, were calculated at each year from years 1965 to 1990.[10] The depreciation rate of each year was computed by the equation, $K_i(t+1) = K_i(t) + I_i(t) - \delta_i K_i(t)$, and the average of time series was taken to be the data of δ_i.[11]

The convergence process was estimated by the method of Beach and MacKinnon (1978) which is the maximum likelihood method under the assumption of serial correlation. The SHAZAM software (Version 8.0) was used and the repeated calculation for convergence of estimation was less than 50 times of each country. The estimation results of convergence (12) and the hypothesis testing (14) of the OECD and the non-OECD samples were re-

[10] The investment consists of both the private and the public investment.

[11] The averages of the depreciation rates of the OECD and the non-OECD countries were respectively 11.14% and 12.93 %. The depreciation rates may take rather high values because the elements of constitution of the investment data may be larger than the ones of the capital data.

spectively shown in Tables 2.1 and 2.2. The difference between the income of the steady state and the initial time, $dif_i \equiv \ln(\frac{y_i}{A_i})^* - \ln(\frac{y_i(0)}{A_i(0)}) = std_i - \ln y_i(0)$, was also computed on each country.

Table 2.1 Estimation results of convergence (11) (OECD sample)

country name	$\hat{std}_i$ (s.e.)	$\hat{dif}_i$ LR test	country name	$\hat{std}_i$ (s.e.)	$\hat{dif}_i$ LR test
Canada	10.093	0.083	U.S.A.	10.271	0.029
	(0.123)	0.01		(0.140)	0.11
Japan	9.275	0.375	Austria	9.804	0.281
	(0.128)	2.55		(0.094)	2.19
Belgium	9.977	0.190	Denmark	9.958	0.162
	(0.120)	2.00		(0.125)	0.00
Finland	9.691	0.149	France	9.978	0.235
	(0.179)	0.00		(0.094)	2.04
W. Germany	10.056	0.299	Greece	9.330	0.378
	(0.178)	1.16		(0.193)	5.44*
Ireland	9.442	0.200	Italy	9.757	0.198
	(0.180)	0.01		(0.141)	4.07*
Netherlands	10.194	0.260	Norway	9.734	−0.021
	(0.129)	1.37		(0.125)	1.62
Portugal	8.933	0.203	Spain	9.608	0.178
	(0.341)	0.13		(0.233)	0.74
Sweden	10.149	0.203	Switzerland	10.314	0.242
	(0.110)	0.05		(0.217)	0.00
Turkey	8.311	0.077	U.K.	9.820	0.100
	(0.208)	0.12		(0.135)	0.14
Australia	10.111	0.147	New Zealand	10.191	0.120
	(0.099)	0.84		(0.152)	0.20

Note: $\hat{std}_i$ and $\hat{dif}_i \equiv \hat{std}_i - \ln y_i(0)$ are respectively the estimates and the figure of () is the estimate of the standard error of $\hat{std}_i$. LR test indicates the value of likelihood ratio test of hypothesis testing (14) and it is distributed as χ^2 distribution with one degree of freedom under the null hypothesis. $*$ represents to be significant at the 5% significance level. The sample size of each country is 24.

Table 2.2 Estimation results of convergence (11) (non-OECD sample)

country name	$\hat{std}_i$	$\hat{dif}_i$	country name	$\hat{std}_i$	$\hat{dif}_i$
	(s.e.)	LR test		(s.e.)	LR test
Ivory Coast	8.424	0.532	Kenya	7.248	0.021
	(0.269)	0.04		(0.290)	1.99
Malai	7.051	0.310	Mauritius	8.649	−0.130
	(0.165)	1.01		(0.474)	0.65
Morocco	8.538	0.143	Nigeria	7.113	−0.188
	(0.182)	1.59		(0.599)	0.15
Sierra Leone	8.457	0.578	Zambia	8.396	0.352
	(0.306)	0.03		(0.230)	2.46
Zimbabwe	7.778	0.088	Dominican Rep.	8.980	0.558
	(0.468)	3.77		(0.278)	1.46
Honduras	8.413	0.215	Jamaica	8.906	0.324
	(0.285)	2.02		(0.286)	2.77
Mexico	9.672	0.319	Panama	9.202	0.499
	(0.320)	0.77		(0.439)	2.89
Argentina	9.963	0.504	Bolivia	8.761	0.466
	(0.216)	1.17		(0.190)	7.56**
Chile	9.266	0.039	Colombia	8.674	−0.024
	(0.423)	0.03		(0.209)	0.93
Ecuador	8.911	0.395	Venezuela	10.511	0.383
	(0.405)	1.69		(0.213)	0.68
Hong Kong	8.544	−0.236	India	7.758	0.267
	(0.190)	0.37		(0.137)	0.31

country name	$\hat{std}_i$ (s.e.)	$\hat{dif}_i$ LR test	country name	$\hat{std}_i$ (s.e.)	$\hat{dif}_i$ LR test
Israel	9.741	0.286	Korea, Rep.	8.200	0.176
	(0.255)	2.69		(0.214)	0.10
Philippines	8.374	0.264	Syria	9.327	0.387
	(0.306)	0.61		(0.378)	6.52*
Thailand	7.780	0.043	Luxembourg	9.970	0.006
	(0.185)	0.53		(0.260)	0.16

Note: $\hat{std}_i$ and $\hat{dif}_i \equiv \hat{std}_i - \ln y_i(0)$ are respectively the estimates and the figure of () is the estimate of the standard error of $\hat{std}_i$. LR test indicates the value of likelihood ratio test of hypothesis testing (14) and it is distributed as χ^2 distribution with one degree of freedom under the null hypothesis. $*$ and $**$ represents to be significant at the 5% and 1% significance levels respectively. The sample size of each country is 24.

The null hypothesis of no serial correlation was rejected for some countries in each of the OECD and the non-OECD samples. The residuals taking the serial correlation into account were likely seen to be random.

Concerning with the estimation results of the OECD countries, the estimate of income per labor at the steady state of each country, $\hat{std}_i$, took generally the value between 9 and 10. The estimate of standard error of $\hat{std}_i$ was nearly the value between 0.1 and 0.3. The estimate of steady state income per labor took the higher value as the income from years 1965 to 1990 were higher. For example, the estimates of steady state income of Canada, U.S.A. and Switzerland were larger than the others. The estimate of steady state income of Japan was not so high although Japan grew at a high speed. This may come from the fact that the income of initial time of convergence was low and the technological progress rate improving only the efficiency of labor was very high. The estimate of difference of income per effective labor between the steady state and the initial time, $\ln(\frac{y_i}{A_i})^* - \ln(\frac{y_i(0)}{A_i(0)})$, got nearly the value around 0.1 and 0.2. Only the estimate of Norway was negative. Because the sign of the difference of income per effective labor between the steady state and the initial time was unknown *a priori*, the rejection regions were set at the both sides. Then, the difference of income was significantly not zero at the 10% significance level on the seven countries. Therefore, the difference of income was not significant on many countries when the initial time of the convergence estimation was set to be year 1965. Because the estimate of the convergence speed was positive, the value of $(-e^{-\hat{\lambda}_i(t+1)} + e^{-\hat{\lambda}_i t})$ took the positive value. *That is*, the predicted value of growth rate of income per effective labor took the same sign to the difference of income per effective labor between the steady state and the initial time.[12] When the predicted values of growth rates of income per effective labor were plotted, the convex curve to the below was drawn and it converged to zero from a positive value except for the case of Norway. The curve of Norway which was convex to the above converged to zero from a negative value. It can be seen that the predicted values of growth rates of the OECD countries were near to zero around year 1990. This means that the OECD countries approached to the steady state around year 1990. The difference of income per effective labor of Japan between the steady state and the initial time of Japan took 0.375 which was higher than the other OECD countries because Japan grew rapidly from 1960s. On the other hand, the differences of Canada, U.S.A. and U.K. were about zero because these countries were already near to the steady state at year 1965. The null hypothesis, $H : g_i = \hat{g}_i$, of hypothesis testing (14) was rejected at the 1% significance level only at Greece and Italy. Therefore, it is not inappropriate to estimate the income per labor at the steady state using the estimate of technological progress rate from the production function.

Concerning with the estimation results of the non-OECD countries, the

[12] See the explanation of convergence (5).

estimate of income per labor at the steady state of each country, $\hat{std}_i$, took generally the value between 7 and 9. The estimate of standard error of $\hat{std}_i$ was nearly the value between 0.2 and 0.5. The estimates of steady state income of the non-OECD countries were likely to be lower than those of the OECD countries and the estimates of standard error of the non-OECD countries had a tendency to be larger than those of the OECD countries. Because the estimate of steady state income tended to take the higher value as the income from years 1965 to 1990 was higher, the estimates of the OECD countries were higher than those of the non-OECD countries. Furthermore, the estimate of steady state income of Venezuela was very high because the income per labor of Venezuela at 1960s were not much different from those of U.S.A.. Similar to the case of Japan, a possible reason of the low estimates of steady state income of Hong Kong and Korea may be that the income of those countries at 1960s were rather low and the technological progress rates improving the efficiency of labor were high. The estimate of difference of income per effective labor between the steady state and the initial time, $\ln(\frac{y_i}{A_i})^* - \ln(\frac{y_i(0)}{A_i(0)})$, got nearly the value between 0.2 and 0.5 except for the negative estimates of the four countries. The difference of income was significantly not zero at the 10% significance level on the eight countries. Therefore, similar to the OECD countries, the difference of income was not significant on many countries when the initial time was set to be year 1965 on the estimation. The differences of income of the non-OECD countries tended to be larger than those of the OECD countries. This may be considered that the OECD countries were nearer to the steady state than the non-OECD countries at the initial time. The null hypothesis, $H : g_i = \hat{g}_i$, of hypothesis testing (14) was rejected at the 5% significance level only at Bolivia and Syria. Similar to the OECD countries, it is not inappropriate to estimate the income per labor at the steady state using the estimate of technological progress rate from the production function.

We consider the determination of income per labor at the steady state in a devised manner at the next section. The income determination is estimated under each of the assumption that the income per labor at year 1990 is the steady state value and using the steady state income obtained from the convergence estimation.

3 Steady state of the Solow growth model

3.1 A specification of the steady state

Equation (4) shows the income determination at the steady state when the growth rate of capital per effective labor is zero, equivalently, the growth rate of income per effective labor is zero. If the economy stays at the steady state on time t, the income per labor at the steady state can be written as follows.[13]

$$\ln y_i(t) = \ln A_i(0) + g_i t + \frac{\alpha}{1-\alpha}[\ln s_i - \ln(n_i + g_i + \delta_i)], \qquad (15)$$

where $\ln A_i(0) + g_i t = \ln A_i(t)$ by the assumption of technological level.

The steady state income per labor at time t can be represented by the technological level at the initial time, the technological growth rate, the saving rate, the labor growth rate and so on. Because the data of technological level do not exist, Mankiw, Romer and Weil (1992) made the following statistical assumption to estimate the determination of income per labor at the steady state, (15), using cross-country data.

$$\ln A_i(0) = c + \epsilon_i,$$

[13] Equation (15) does not imply that the economy reaches at the steady state on time t.

where c is the constant term and $\epsilon_i \sim N(0, \sigma^2)$.

This assumption means that the technological level at the initial time of each country consists of the constant term common among countries and the error term being distributed as the normal distribution. Moreover, Mankiw, Romer and Weil (1992) made some assumptions that the economy stays at the steady state on year 1985 and the sum of both the technological progress rate and the depreciation rate of capital takes the same value among countries. Utilizing a battery of specification error tests, Nakamura (1995) found that the steady state estimation of Mankiw, Romer and Weil (1992) may have some specification errors.

Islam (1995) estimated the convergence process (3) including the steady state specification (15). Islam (1995) estimated directly the initial technological level of each country because not only cross-country data but also time series data at the intervals of 5 years were used.

We consider a devised method having to neither assume the initial technological level statistically nor estimate it directly. The technological progress rate can be expressed by both the income and the capital per labor with the restriction from the production function (1).

$$\ln A_i(t) = \frac{1}{1-\alpha}(\ln y_i(t) - \alpha \ln k_i(t)). \tag{16}$$

On the steady state specification, the initial technological level is replaced with the equation (16) at the initial time.

$$\ln y_i(t) = g_i t + \frac{1}{1-\alpha} \ln y_i(0) + \frac{\alpha}{1-\alpha}[\ln s_i - \ln(n_i + g_i + \delta_i) - \ln k_i(0)]. \tag{17}$$

The initial technological level is not appeared explicitly on the determination of steady state income. It is replaced by both the income and the capital per labor at the initial time.

3.2 Estimation of the steady state

We have to consider what the data of steady state income is used to estimate the determination of steady state income. Two methods are considered. The one method is to assume or approximate that the economy stays at the steady state on a year and to use the data of income per labor of that year. The other method is to use the estimates of steady state income obtained from the estimation of convergence at section 2.2.

First, we assume concretely that the OECD and the non-OECD countries stay at the steady state on year 1990 and use the GDP per labor of year 1990 as the data of $y_i(t)$ in equation (17). The assumption of the steady state on year 1990 may no be inappropriate on the whole as seen in the estimation results of convergence. When the initial time, 0, of equation (17) is taken to be year 1965, the time t is 25 in equation (17) because one year is assumed to be unity as the interval of time. The estimates of technological progress rates from the estimation of the production function are used in the estimation of the steady state. The steady state specification to be estimated is as follows.

$$\ln y_i(t) - \hat{g}_i t = \frac{1}{1-\alpha} \ln y_i(0) + \frac{\alpha}{1-\alpha} [\ln s_i - \ln(n_i + \hat{g}_i + \delta_i) - \ln k_i(0)] + \epsilon_i,$$

$$i = 1, 2, \cdots, N, \qquad (18)$$

where $t = 25$ and $\epsilon_i \sim N(0, \sigma^2)$.

All the data were also taken from the Penn World Table (Mark 5.6) of Summers and Heston (1991). The sample sizes, N, of the OECD and the non-OECD samples were respectively 22 and 28. The data of $\hat{g}_i$ in equation (18) was the estimate of technological progress rate from the estimation of production function. The GDP and the capital per labor of year 1965 were respectively used as the data of $y_i(0)$ and $k_i(0)$. The data of saving rate, s_i, was the annual average of investment rates from years 1965 to 1990. The labor growth rate, n_i, and the depreciation rate, δ_i, were the same to those of section 2.2.

The equation was estimated by the maximum likelihood estimation. The maximum likelihood estimation was conducted by the Davidon-Fletcher-Powell algorithm contained in SHAZAM (Version 8.0). The initial value of the parameter, α, was set to the value of the estimate from the production function. Some changes of the initial value did not affect the estimation results and the iteration for convergence of estimation was less than 50 times. The rejection region of the test of $0 < \alpha$ was set at the upper side. The estimation results of the OECD and the non-OECD samples were respectively shown in Tables 3.1 and 3.2.

Table 3.1 Estimation results of the steady state (18) (OECD sample)

$\hat{\alpha}$
0.217
(27.94**)

Note: $\hat{\alpha}$ is the estimate and the figure in () is the asymptotic $t-$ value. ** indicates to be significant at the 1% significance level. The estimate of standard error of error terms is 0.026 and the sample size is 22.

Table 3.2 Estimation results of the steady state (18) (non-OECD sample)

$\hat{\alpha}$
0.315
(24.17**)

Note: $\hat{\alpha}$ is the estimate and the figure in () is the asymptotic $t-$ value. ** indicates to be significant at the 1% significance level. The estimate of standard error of error terms is 0.073 and the sample size is 28.

The economic structure of each country represented by the parameter, α, is assumed to be able to be distinguished by the income per labor of year 1990. When the residuals of each of the OECD and the non-OECD samples were sorted from the assumption mentioned above, those were likely to have no tendency and to be random around zero.[14] The residuals of the non-OECD sample seemed to be larger than those of the OECD sample as could be also seen from the estimate of standard error of error terms. This means that the fitness of the data to the steady state specification of the OECD sample was better than that of the non-OECD sample.

First, we see the estimation results of the OECD sample. The estimate of the parameter, α, of the production function got the value, 0.217, with no violation of the assumption of $0 < \alpha < 1$. This estimate was highly significant and it was a little different from $\frac{1}{3}$ which is roughly the division rate of income to the capital. Furthermore, the estimate of the steady state specification was near to that of the production function. *That is*, there is no inconsistency of the estimation results between the production function and the steady state. The case of time t being the unkown parameter was also estimated. The estimate of time t was about 25 and there was little change of the estimate of α. Therefore, the assumption that the OECD countries stay at the steady state on year 1990 may not be inappropriate.

Next, the estimation results of the non-OECD sample are investigated. The estimate of the parameter, α, of the production function took the value ,0.315, near to $\frac{1}{3}$ and it was significantly larger than zero. Furthermore, this estimate was not significantly different from the estimate of the production function. The estimation of unknown parameters of t and α was also conducted. The hypothesis of $t = 25$ was not rejected and there was no significant change of α. Therefore, similar to the OECD sample, there was no

[14] The residuals should be sorted by a criterion to see the behavior of residuals because of the cross section data.

inconsistency between the estimation results of the production function and the steady state, and the assumption of the steady state of the non-OECD countries may not be so bad at year 1990. These results did not contradict to the estimation results of convergence that both the OECD and the non-OECD countries were likely to be the steady state at year 1990.

To check the robustness that the income per labor of year 1990 is taken as the explained variable of the steady state specification, we also take the income per labor of years 1988, 1989 and 1990 as the explained variable. *That is*, the variables of $y_i(t_j)$, $j = 1, 2, 3$, in equation (19) are respectively the income per labor of years 1988, 1989 and 1990 of ith country.

$$\ln y_i(t_j) - \hat{g}_i t_j = \frac{1}{1-\alpha} \ln y_i(0) + \frac{\alpha}{1-\alpha}[\ln s_i - \ln(n_i + \hat{g}_i + \delta_i) - \ln k_i(0)] + \epsilon_i,$$

$$i = 1, 2, \cdots, N, \quad j = 1, 2, 3, \tag{19}$$

where $t_1 = 23, t_2 = 24, t_3 = 25$ and $\epsilon_i \sim N(0, \sigma^2)$.[15]

The sample sizes of the OECD and the non-OECD samples were respectively 66 and 84 because of pooling the data of the three years. The equation (19) was estimated by the maximum likelihood estimation and the Davidon-Fletcher-Powell algorithm was used. The estimate of the production function was set to the initial value of this estimation and some changes of the initial value did no have any effect. The iteration of estimation was less than 50 times. The estimation results of the OECD and the non-OECD samples were respectively reported in Tables 4.1 and 4.2.

Table 4.1 Estimation results of the steady state (19) (OECD sample)

$\hat{\alpha}$
0.210
(24.51**)

[15] The variance of error terms are assumed to be equal among years 1988, 1989 and 1990.

Note: $\hat{\alpha}$ is the estimate and the figure in () is the asymptotic $t-$ value. ** indicates to be significant at the 1% significance level. The estimate of standard error of error terms is 0.050 and the sample size is 66.

Table 4.2 Estimation results of the steady state (19) (non-OECD sample)

$\hat{\alpha}$
0.311
(31.34**)

Note: $\hat{\alpha}$ is the estimate and the figure in () is the asymptotic $t-$ value. ** indicates to be significant at the 1% significance level. The estimate of standard error of error terms is 0.096 and the sample size is 84.

From the estimation results of the OECD sample, the estimate of α took the value, 0.210, and this is almost the same to the estimate of only the income per labor of year 1990 as the explained variable. The case of $t_j, j = 1, 2, 3$, assumed to be the unknown parameters was also estimated. Those estimates were respectively a little smaller than 23, 24 and 25 and the similar estimate of α to the estimate of equation (19) was obtained.[16] Therefore, it is not inappropriate to take the income per labor around 1990 as the explained variable of the steady state specification.

We can also ascertain the robustness of the assumption to stay at the steady state around year 1990 in the non-OECD sample. The estimate, 0.311, was near to the estimate of equation (18) and the estimate of the production function. When the time parameters were assumed to be the unknown parameters, the estimates of those parameters took the values nearer to the true values than those of the OECD sample.

[16] The estimates of time being nearer to the true values were obtained on the assumption of unequal variances of error terms.

Next, we consider the steady state specification using the estimate of the income per labor obtained from the estimation of convergence. *That is*, the explained variable is the estimate, $\hat{std}_i$, in the following equation.

$$\hat{std}_i = \frac{1}{1-\alpha}\ln y_i(0) + \frac{\alpha}{1-\alpha}[\ln s_i - \ln(n_i + \hat{g}_i + \hat{\delta}_i) - \ln k_i(0)] + \epsilon_i,$$

$$i = 1, 2, \cdots, N, \qquad (20)$$

where $\epsilon_i \sim N(0, \sigma^2)$.

There is only the difference of the explained variables of the steady state specifications between equation (18) and equation (20). The estimate, $\hat{std}_i$, obtained from the estimation of convergence (12) is used in the estimation of equation (20).

The estimation results of the OECD and the non-OECD samples were respectively shown in Tables 5.1 and 5.2. The estimation used the Davidon-Fletcher-Powell algorithm and the initial value of α was set to the estimate of the production function. There was no effect of some changes of the initial value and the iteration of estimation was less than 50 times.

Table 5.1 Estimation results of the steady state (20) (OECD sample)

$\hat{\alpha}$
0.301
(14.82**)

Note: $\hat{\alpha}$ is the estimate and the figure in () is the asymptotic $t-$ value. ** indicates to be significant at the 1% significance level. The estimate of standard error of error terms is 0.087 and the sample size is 22.

Table 5.2 Estimation results of the steady state (20) (non-OECD sample)

$\hat{\alpha}$
0.306 (6.99**)

Note: $\hat{\alpha}$ is the estimate and the figure in () is the asymptotic $t-$ value. ** indicates to be significant at the 1% significance level. The estimate of standard error of error terms is 0.240 and the sample size is 28.

When the residuals of each of the OECD and the non-OECD samples were sorted on the assumption that the economic structure at the steady state can be distinguished by the income per labor of the steady state, the residuals of the non-OECD sample may be larger than those of the OECD sample and the residuals of the non-OECD sample seemed to be biased a little to be positive. Furthermore, the residuals of both the OECD and the non-OECD samples were likely to be larger than those of the steady state specification (18) using the income per labor of year 1990. *That is,* the fitness of the steady state specification using the steady state income obtained from the estimation of convergence may be a little worse than that using the income per labor of year 1990.

We see the estimation results of the OECD sample at first. The estimate of α of equation (20) using the steady state income obtained from the convergence estimation was larger than that of equation (18) using the income of year 1990 although the estimate of equation (20) took the value near to $\frac{1}{3}$. The equality of α between equations (18) and (20) was tested by the likelihood ratio test. This equality was rejected at the 1% significance level. This means that the income per labor of year 1990 considering the technological progress rate, $\ln y_i(t) - \hat{g}_i t$, and the steady state income per labor estimated from the convergence process, $\hat{std}_i$, can not be equally treated on the estimation of the steady state. When those two were directly compared, the value of $\hat{std}_i$ seemed to be only a little larger than that of $\ln y_i(t) - \hat{g}_i t$.

Next, we see the estimation results of the non-OECD sample. The estimate of α took the value, 0.306, and this was similar to the estimate of equation (18) using the income per labor of year 1990. The hypothesis of equality of α between the equation (18) using the income per labor of year 1990 and the equation (20) using the steady state income obtained from the estimation of convergence was not rejected at all. This implies that both the income of year 1990 and the steady state income from the convergence estimation may be able to be treated equally. However, when the two explained variables, $\ln y_i(t) - \hat{g}_i t$ and $\hat{std}_i$, were directly compared, the value of $\hat{std}_i$ have a tendency to be larger than that of $\ln y_i(t) - \hat{g}_i t$ and this tendency seemed to be stronger than that of the OECD sample.

It was found that the residuals of equation (20) using the steady state income from the convergence estimation may be a little biased to be positive in the non-OECD sample and the estimate of steady state income, $\hat{std}_i$ has a tendency to be larger than the income per labor considering the technological progress rate, $\ln y_i(t) - \hat{g}_i t$, in both the OECD and the non-OECD samples. Therefore, we consider the steady state specification including the constant term although it is implied by not the Solow model but the empirical finding in this study.

$$\hat{std}_i = const + \frac{1}{1-\alpha}\ln y_i(0) + \frac{\alpha}{1-\alpha}[\ln s_i - \ln(n_i + \hat{g}_i + \delta_i) - \ln k_i(0)] + \epsilon_i,$$
$$i = 1, 2, \cdots, N, \tag{21}$$

where *const* is the constant term and $\epsilon_i \sim N(0, \sigma^2)$.

It is investigated whether or not the constant term is significantly different from zero and it affects the estimate of α. The estimation results of the OECD and the non-OECD samples by the maximum likelihood method were respectively reported in Tables 6.1 and 6.2. Because the sign of the constant term was not known *a priori*, the rejection regions of the null hypothesis of $const = 0$ were set at the both sides. In the estimation, the Davidon-Fletcher-Powell algorithm was used and the initial value of α was set to the estimate of the production function. There was no effect from changing the initial value and the iteration of convergence was less than 50 times.

Table 6.1 Estimation results of the steady state (21) (OECD sample)

$\hat{const}$	$\hat{\alpha}$
0.085	0.213
(2.88**)	(5.19**)

Note: $\hat{const}$ and $\hat{\alpha}$ were respectively the estimates of $const$ and α and the figure in () is the asymptotic $t-$ value. ** indicates to be significant at the 1% significance level. The estimate of standard error of error terms is 0.074 and the sample size is 22.

Table 6.2 Estimation results of the steady state (21) (non-OECD sample)

$\hat{const}$	$\hat{\alpha}$
0.166	0.213
(3.94**)	(4.07**)

Note: $\hat{const}$ and $\hat{\alpha}$ were respectively the estimates of $const$ and α and the figure in () is the asymptotic $t-$ value. ** indicates to be significant at the 1% significance level. The estimate of standard error of error terms is 0.193 and the sample size is 28.

When the residuals of each of the OECD and the non-OECD samples were sorted by the steady state income, the residuals seemed to be random around zero and to be a little smaller than those of equation (20) not including the constant term. Therefore, the fitness of the data to the steady state specification including the constant term may be better than that not including the constant term.

From the estimation results of the OECD sample, the estimate of the constant term seemed to be small but it was significantly not zero. The estimate of α took the value similar to that obtained from the estimation of equation (18) using the income of year 1990. The effect that $\hat{std}_i$ was likely to be a little larger than $\ln y_i(t) - \hat{g}_i t$ was appeared as the significant constant term of equation (21). When that effect was treated as the significant constant term, the estimate of α took the value very near to that of income per labor of year 1990. It was also tested whether or not the constant term was significant different from zero in equation (18) using the income per labor of year 1990. The constant term was not significant at all.

As seen from the estimation results of each of the production function, convergence and the steady state, there were little violations of the assumptions of the Solow growth model. Furthermore, there was a consistency of the estimation results among the production function, convergence and the steady state except for the effect that the estimate of the steady state income, $\hat{std}_i$, may be a little larger than the income per labor of year 1990 considering the technological progress rate, $\ln y_i(t) - \hat{g}_i t$. The estimate of the steady state income may be only a little larger than the true value. A reason may be considered that the convergence process to the steady state was estimated by the data from years 1965 to 1990 although the OECD countries could be already approximated to stay on the steady state before year 1990 or it was needed to estimate accurately the convergence process that an earlier initial time than year 1965 was taken.

From the estimation results of the non-OECD sample, the estimate of the constant term was rather large and it was significant. The estimate of α was a little different from that of equation (18) assuming year 1990 as the steady state. The estimate of the constant term of the non-OECD sample was larger than that of the OECD sample. The effect that $\hat{std}_i$ was likely to be a little larger than $\ln y_i(t) - \hat{g}_i t$ was appeared as the significant constant term of equation (21) and the significant constant term affected to the estimate of α. It was also tested whether or not the constant term was significantly different from zero in equation (18) using the income per labor of year 1990. The constant term was not significant at all.

The estimation results of each of the production function, convergence and the steady state had little violations of the assumptions of the Solow model and there was no much inconsistency of the estimation results among the production function, convergence and the steady state except for a little inconsistency of the estimation results between convergence and the steady state. A reason of that inconsistency may be considered that the estimate of the steady state differed a little from the true value because the data of income dynamics did not fit well to the convergence process implied by the Solow model or the sample size of time series was not be sufficient to estimate the convergence process.

4 Concluding remarks

This study estimated the production function, convergence and the steady state of the Solow model not including human capital by using the time series and the cross-section data. It may be found that the recent economic growth of the OECD countries can be explained approximately by the Solow model and there was a little inconsistency of the estimation results between convergence and the steady state on the non-OECD sample although the difference of the estimate, $\hat{\alpha}$, seemed to be small. It is the same to the results of Mankiw, Romer and Weil (1992) and Islam (1995) that the favorable results to the Solow model were obtained on the whole.

We should consider an augmented Solow model which includes human capital such as Mankiw, Romer and Weil (1992) and Islam (1995) and test whether or not there is a significant change in estimation results.[17] Furthermore, it should be investigated what no rejection of the Solow model implies to endogenous growth models.

[17] Mankiw, Romer and Weil (1992) and Islam (1995) considered both the Solow model and an augmented Solow model including human capital.

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