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Effects of the Shadow Rate of Wage on Output Supply : A Structural Estimation for Japanese Rice Farmers

by

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Abstract

The shadow rate or "internal rate" of wage plays a crucial role in the models of agricultural households with constrained or lacking off-farm wage employment. To verify its empirical relevance in the comparative statics analysis, whole parameters of the production and welfare functions are estimated for Japanese rice farmers. It is inferred from this structural estimation that their off-farm wage employment is actually constrained and that the "internal wage effects" through changes in the internal rate of wage significantly influence their responses of rice supply.

Keywords: internal rate of wage, output supply response, structural estimation

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1. Introduction

Analytical realism of the models of agricultural households with constrained or lacking off-farm wage employment have invited many authors to employ them in studying the behavior of these households both in developed and in developing economies where this is the case for some reason or other. It is well known that these models are indecomposable [1] or nonrecursive in the sense that the determinations of their production organization and their consumption choice are to be jointly made. Problems posed by their indecomposability have been tackled mainly in two distinct ways.[2] In one the demand and supply of labor of these households are specified for estimation in linear functions of the exogenous variables relevant both to their production organization and their consumption choice (e.g., Arayama, 1986; Kang and Maruyama, 1992) However, these efforts cannot evade another problem associated with reduced form estimations. In the other their production function is estimated first to obtain their demand rate of wage or marginal revenue product of labor, which is equated to their supply rate in equilibrium. These two rates are not often distinguished but are jointly referred to as their shadow rates of wage by many authors. Then their supply of labor is specified for estimation in a log-linear function of their estimated rate of shadow wage, their full income evaluated at this rate of wage and other relevant variables (e.g., Jacoby, 1993; Skoufias, 1994).

In these efforts, however, the shadow rate of wage or the internal rate of wage [3] which induces the demand and supply of labor to equate in the internal market within the household (Sonoda and Maruyama, 1996) and typically characterizes the agricultural

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[1] This terminology is due to Sasaki and Maruyama (1966) and Maruyama (1975) who refer to it as indicating the said property of a system of simultaneous equations, while other authors refer to this property as not "block-recursive" or not "separable" for other reasons (see, e.g., Jorgenson and Lau, 1969; Singh, Squire, and Strauss, 1986).

[2] Lopez (1984) estimated various elasticities related to the behavior of self-employing agricultural households in Canada by use of a kind of indecomposable model, but he imposed some strong assumptions including constant returns to scale.

[3] This rate of wage is closely related to the "virtual price" and "implicit price" of labor or endowed time due to Neary and Roberts (1980), Maruyama (1984) and others.

household models with constrained or lacking off-farm wage employment in contrast to those without is not clearly identified. Hence, the mechanisms both how it is affected by changes in the exogenous variables and how changes in it in turn affect other endogenous variables are not analyzed to the sufficient extent. Furthermore, it is not deniable that analyzing their supply of labor is of limited relevance to the households concerned in case their off-farm employment is constrained or lacking. This paper attempts to claim more immediate relevance to them by analyzing their supply of output, since they should be more concerned about employing the rest of their labor to produce as much livelihood as possible for members of their household. For this purpose a full-fledged structure of agricultural households carrying both their production and welfare functions will be specified for estimation so that the standard theory of comparative statics can be fully utilized. Following Jacoby (1993) and Skoufias (1994), their production function will be estimated first to obtain their demand rate of wage or marginal revenue product of labor, which is to be equated to their supply rate of wage in equilibrium. Then this rate will be utilized to estimate the whole parameters of their welfare function by means of a system of expenditure functions similar to the linear expenditure system.

It is to be noted that the bindingness of their off-farm employment is not a priori assumed, but it is tested by comparing their estimated internal rate with their estimated market rate of wage. If their internal rate is estimated to be significantly lower than their market rate, their off-farm employment is inferred to be binding. Otherwise, it is not. For the case of Japanese rice farmers in the period 1982-91, their internal rate of wage is estimated to be significantly lower than their market rate, hence their off-farm wage employment is inferred to be binding. Furthermore, the direct effect on output supply of changes in its price proves to be dominated by the indirect effect through changes in the internal rate of wage, i.e., the internal wage effect (Sonoda and Maruyama, 1996) caused by the same changes in its price. Therefore, their supply function of rice turns out to be downward-sloping at the mean values of relevant variables.

The following section introduces a model of agricultural household with constrained off-farm wage employment and discusses the properties of its internal rate of wage which induces its demand to equate to its supply of labor in excess of external employment. The third section attempts a structural estimation of relevant parameters of the model and discusses the implications of their estimates. The fourth section concludes the paper.

2. The Model

It is assumed that a sufficient degree of consensus is observed among different members of a household, so that their welfare function exists. The welfare function $U(\cdot)$ takes on the following form:

$$W = U(C_1, C_2, Z; G), \tag{1}$$

where C_1, C_2, and Z denote respectively in this order the amounts consumed of a home produced farm commodity, purchased commodities, and leisure. G represents a shift parameter of this function. The welfare function $U(\cdot)$ is assumed to possess regular well-behaved properties.

The household allocates its endowed time Te among hours of farm work L_1, off-farm work L_2, and leisure Z but it does not hire labor from external sources as is the case with a majority of Japanese rice farmers.

$$L_1 + L_2 + Z = Te. \tag{2}$$

The household must satisfy the budget constraint.

$$pC_1 + p'C_2 + wZ + S + TAX \leqq M, \tag{3}$$

where p, p', and w denote respectively the prices of a farm commodity, purchased commodities, and the market rate of wage. S and TAX respectively represent the amounts saved and taxed. M denotes its full income (Becker, 1965) to be defined in the following way:

$$M \equiv wTe + \pi + V, \qquad \pi \equiv pX - wL_1 - qF - OC, \tag{4}$$

where X, q, and F denote respectively the amount produced of a farm commodity, the price of current inputs, and their quantity. V and OC respectively represent unearned incomes and other costs which are assumed to be exogenous.

The household is assumed to produce the amount X of a farm commodity and consume C_1 (< X) of it within the household. The amount produced X is bounded by the production possibility.

$$X \leqq f(L_1, F; K, T, \psi, TT), \tag{5}$$

where K, T, ψ, and TT denote respectively in this order the real stock of capital, the total area planted, the intensity rate of set-aside program [4] , and an index indicating the technological level represented by the time trend in this study. K and T are assumed to be fixed. The rate ψ is coercively determined by the government in exchange for the guaranteed minimum price as is the case with Japanese rice farmers. The production function $f(\cdot)$ is assumed to possess regular well-behaved properties.

The household is assumed to be a price taker in all markets, then the problem it faces is to maximize its welfare W subject to the constraints (2)-(5). The Kuhn-Tucker-Lagrange conditions of optimality associated with this problem imply the following relations, where $\lambda \geqq 0$ denotes the Lagrange multiplier associated with the budget constraint (3), and f_i and U_j respectively denote the first derivatives of the functions $f(\cdot)$ and $U(\cdot)$.

$$pf_1(L_1,F;K,T,\psi,TT)-w \leqq 0, \tag{6.1}$$

$$pf_2(L_1,F;K,T,\psi,TT)-q \leqq 0, \tag{6.2}$$

$$U_1(C_1,C_2,Z;G)-\lambda p \leqq 0, \tag{6.3}$$

$$U_2(C_1,C_2,Z;G)-\lambda p' \leqq 0, \tag{6.4}$$

$$U_3(C_1,C_2,Z;G)-\lambda w \leqq 0, \tag{6.5}$$

$$-pC_1-p'C_2-wZ-S-TAX+M \geqq 0. \tag{6.6}$$

For interior solutions, the inequalities (6.1), (6.3), and (6.5) imply that both the demand rate $pf_1(\cdot)$ and the supply rate $pU_3(\cdot)/U_1(\cdot)$ of wage should be equated to its market rate w.

$$pf_1(\cdot) = w = pU_3(\cdot)/U_1(\cdot). \tag{7}$$

However, Arayama (1986) and Kang and Maruyama (1992) point out that rice farmers in Japan are likely to face constrained off-farm employment. This constraint seems to be closely related to the practice of off-farm employers who offer "higher than equilibrium rate of wage" and use the resulting excess supply of labor as a worker

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[4] The Japan Ministry of Agriculture, Forestry, and Fishery has been employing the "set-aside program" to induce rice growers to convert a part of their paddy to other uses since 1969. The practically coercive rate of intensity of this program on the average has reached nearly 30% in recent years. To compensate the associated reduction of their income, the Ministry has been paying compensatory subsidy to farmers

discipline device (see, e.g., Shapiro and Stiglitz, 1984; Bulow and Summers, 1986).[5]

$$L_2 = Te - Z - L_1 \leqq L = \text{constant}. \quad (8)$$

The household is now subject to this constraint in addition to the budget constraint. Then the optimality conditions associated with this new problem imply the following augmented set of relations:

$$w^* \equiv w - \mu/\lambda \leqq w, \quad w^* \equiv w \quad \text{for } \mu = 0, \quad (9)$$

$$pf_1(L_1,F;K,T,\psi,TT) - w^* \leqq 0, \quad (10.1)$$

$$pf_2(L_1,F;K,T,\psi,TT) - q \leqq 0, \quad (10.2)$$

$$U_1(C_1,C_2,Z;G) - \lambda p \leqq 0, \quad (10.3)$$

$$U_2(C_1,C_2,Z;G) - \lambda p' \leqq 0, \quad (10.4)$$

$$U_3(C_1,C_2,Z;G) - \lambda w^* \leqq 0, \quad (10.5)$$

$$-pC_1 - p'C_2 - w^*Z - S - TAX + Y \geqq 0, \quad (10.6)$$

$$Z + L_1 + L - Te \geqq 0, \quad (10.7)$$

where μ denotes the Lagrange multiplier associated with the constraint (8), and Y and π^* stand for the full income and the residual profit imputable to the farm production activity respectively evaluated at the rate of wage w^*.

$$Y \equiv w^*Te + (w - w^*)L + \pi^* + V, \quad \pi^* \equiv pX - w^*L_1 - qF - OC.$$

For interior solutions, it follows from the inequalities (10.1), (10.3), and (10.5) that

$$pf_1(\cdot) = w^* = pU_3(\cdot)/U_1(\cdot). \quad (11)$$

Thus, in case off-farm employment is binding, it is not the market rate w but the discounted rate w^* of wage that is relevant in determining the household's organization of production and its choice of consumption. Hence, it may be appropriately referred to as the "(equilibrium) internal rate of wage", which induces to equate within the household the sum of its farm employment L_1 and leisure Z to its endowed time Te in excess of off-farm wage employment L as shown in the condition (10.7). For further details, see, e.g., Sonoda and Maruyama (1996). The relations (7) and (11) offer a convenient device to test whether its off-farm employment opportunities are binding or not. In case its demand or

who participated in this program, although this subsidy has been cut much in recent years.

[5] For dependents, leisure hours are equated to their endowed time.

supply rate of wage is estimated to be significantly lower than the market rate, its off-farm employment opportunities are inferred to be binding. Otherwise, they are not.

The inequalities (10.1) and (10.2) directly associated with the determination of its production organization share its internal rate w^* of wage with the inequalities (10.3)-(10.7) directly associated with that of its consumption choice. Hence, the system of inequalities (10.1)-(10.7) is indecomposable in the sense that its organization of farm production and its choice of consumption are to be jointly determined. Indecomposability of this system has very significant effects on its comparative statics. The "internal wage effects" inherent to the agricultural household model under the binding or constrained off-farm employment opportunities render both its supply of commodity and demand for factors less elastic. In extreme cases, it gives rise to downward-sloping supply and upward-sloping demand functions.

Supply of Commodity to Be Related to the " Internal Rate of Wage"

In reference to the optimality conditions (10.1)-(10.7), let us examine the responses of commodity supply to changes in selected exogenous variables. To facilitate the planned estimation in the following section, let us specify the welfare and production functions of the following types:

$$U(\cdot) = b_1 \ln(C_1 - a_1) + b_2 \ln(C_2 - a_2) + b_3 \ln(Z - a_3), \quad b_1 + b_2 + b_3 = 1, \tag{12}$$

$$\ln f(\cdot) = \ln A + \beta_1 \ln L_1 + \beta_2 \ln F + \beta_3 \ln K + \beta_4 \ln T + \beta_5 \ln \psi + \beta_6 TT. \tag{13}$$

In so far as the equilibrium of this household is concerned, the inequality (10.7) is always satisfied, hence it need not be considered without loss of analytical rigor. Then the formal equivalence between the optimality conditions (10.1)-(10.6) and those for the competitive decomposable model of agricultural household (6.1)-(6.6) enables us to derive its supply function of commodity by means of the standard practice of microeconomic analysis. Substituting the production function (13) into the optimality conditions (10.1) and (10.2) in equality gives its supply function of commodity in a form involving its internal rate w^* of wage.

$$X = A_X \left\{p^{(\beta_1+\beta_2)} w^{*(-\beta_1)} q^{(-\beta_2)} K^{\beta_3} T^{\beta_4} \psi^{\beta_5} \exp(\beta_6 TT)\right\}^{1/(1-\beta_1-\beta_2)}, \tag{14}$$

where Ax stands for a constant term. Hence, the elasticity of its commodity supply with respect to an exogenous variable s can be expressed in the following form:

$$\frac{\partial \ln X}{\partial \ln s} = \left.\frac{\partial \ln X}{\partial \ln s}\right|_{dw^*=0} + \frac{\partial \ln X}{\partial \ln w^*}\frac{\partial \ln w^*}{\partial \ln s}. \tag{15}$$

This decomposition of elasticities in terms of the internal rate w^* of wage follows the method proposed by Sonoda and Maruyama (1996). The first term on the right hand side of this equation represents a direct effect of changes in the exogenous variable, while the second term an indirect "internal wage effect" through the changes in the internal rate w^* of wage caused by changes in the same exogenous variable. The direct effects and the elasticities of X with respect to w^* constituting a part of the indirect effects are readily evaluated by use of the equation (14). Then all we have yet to do in evaluating the elasticities of commodity supply is to evaluate the elasticities $\partial \ln w^*/\partial \ln s$ of the internal rate of wage with respect to various exogenous variables. This task is easily carried out by solving the following equations associated with the comparative statics analysis of the optimality conditions (10.1)-(10.7) above.

$$\begin{bmatrix} pf_{11} & pf_{12} & 0 & 0 & 0 & 0 & -1 \\ pf_{21} & pf_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & U_{11} & U_{12} & U_{13} & -p & 0 \\ 0 & 0 & U_{21} & U_{22} & U_{23} & -p' & 0 \\ 0 & 0 & U_{31} & U_{32} & U_{33} & -w^* & -\lambda \\ 0 & 0 & -p & -p' & -w^* & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} dL_1 \\ dF \\ dC_1 \\ dC_2 \\ dZ \\ d\lambda \\ dw^* \end{bmatrix} = \begin{bmatrix} -f_1 & 0 & -pf_{13} & -pf_{14} & -pf_{15} & 0 & 0 & 0 \\ -f_2 & 1 & -pf_{23} & -pf_{24} & -pf_{25} & 0 & 0 & 0 \\ \lambda & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \lambda & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ y_1 & F & -pf_3 & -pf_4 & -pf_5 & C_2 & -1 & y_2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} dp \\ dq \\ dK \\ dT \\ d\psi \\ dp' \\ dV \\ dL \end{bmatrix},$$

where $y_1 = C_1 - X$ and $y_2 = w^* - w$. Now, unearned incomes V are divided into two parts, i.e., the subsidy VS on set-aside areas and others VO, so that the effect of this program is readily recognized. Similarly, the total area planted is divided into two parts, i.e., rice paddy TP and others TO which are not subject to the set-aside program. Since $VS = \eta\psi TP$ (η : per area subsidy on set-aside areas), the case $dVO = dTP = 0$ implies $dV = (\psi d\eta + \eta d\psi)TP$.

In case the welfare and production functions are specified in the forms (12) and (13) respectively, responses of the internal rate w^* of wage to changes, e.g., in the price p of farm commodity is evaluated as follows:

$$\frac{\partial w^*}{\partial p} = \frac{b_3(1-\beta_1-\beta_2)(X-a_1)+\beta_1 X}{(1-b_3)(1-\beta_1-\beta_2)(Z-a_3)+(1-\beta_2)L_1} > 0 \quad \text{for } X > C_1. \tag{16}$$

Then the responses of internal rate of wage thus evaluated and the supply function (14) are substituted into the equation (15) to obtain the elasticities of commodity supply with respect to various exogenous variables. As an important example of these elasticities, the price elasticity of commodity supply will be examined in some detail:

$$\frac{\partial \ln X}{\partial \ln p} = \frac{\beta_1+\beta_2}{1-\beta_1-\beta_2} + \frac{-\beta_1}{1-\beta_1-\beta_2}\frac{\partial \ln w^*}{\partial \ln p}. \tag{17}$$

The first term on the right hand side represents the direct "output effect" (e.g., Ferguson and Gould, 1975) while the second term the indirect internal wage effect of changes in the commodity price. The first term coincides with the regular price elasticity of commodity supply in the competitive decomposable model of agricultural household and has a positive effect on the commodity supply since β_1, $\beta_2 > 0$ and $1-\beta_1-\beta_2 > 0$. Whereas the equation (16) shows that the second internal wage effect is of a negative sign. Hence the sign of the combined effects depends on the relative importance of these two effects. In case the second effect dominates the first, the supply function of commodity will have a downward slope.

Thus changes in the internal rate w^* of wage play an important role in the comparative statics analysis of the agricultural household under constrained off-farm employment opportunities. Changes in it are found to be caused by the interaction of the demand and supply functions of labor within the household. Let us first note that the slopes of these functions prove to be downward- and upward-sloping respectively.

$$\frac{\partial L_D}{\partial w^*} = -\left(\frac{1-\beta_2}{1-\beta_1-\beta_2}\right)\frac{L_1}{w^*} < 0, \qquad \frac{\partial L_S}{\partial w^*} = \frac{(1-b_3)(Z-a_3)}{w^*} > 0, \tag{18}$$

where $L_D \equiv L_1+L$ and $L_S \equiv Te-Z$. Then, a rise in the commodity price shifts the demand function to the right, while it shifts the supply function to the left.

$$\left.\frac{\partial L_D}{\partial p}\right|_{dw^*=0} = \frac{L_1}{(1-\beta_1-\beta_2)p} > 0, \qquad \left.\frac{\partial L_S}{\partial p}\right|_{dw^*=0} = -\frac{b_3(X-a_1)}{w^*} < 0 \quad \text{for } X > C_1.$$

These shifts of the demand and supply functions of labor in turn cause the internal rate w^*

of wage to rise as shown in the relation (16), which in turn has a contractive effect on commodity supply as shown in the equation (17). A similar method of evaluation applies to the elasticities of commodity supply with respect to other exogenous variables, which may be summarized in the following formula for the convenience of subsequent reference.

$$\frac{\partial w^*}{\partial s} = \frac{(1-\beta_1-\beta_2)w^*}{(1-b_3)(1-\beta_1-\beta_2)(Z-a_3)+(1-\beta_2)L_1}\left(\left.\frac{\partial L_D}{\partial s}\right|_{dw^*=0} - \left.\frac{\partial L_S}{\partial s}\right|_{dw^*=0}\right), \tag{19}$$

where $s = p, q, \ldots, L$.

3. Estimation and Its Results Examined

The Data

Data used in this study are adapted from the *Survey of Farm Households Economy by Types of Farm Households* (FHET for short) and the *Statistics of Prices and Wages in Rural Areas* (PWRA for short) published by the Japan Ministry of Agriculture, Forestry, and Fishery and from the *Annual Report on the Consumer Price Index* (RCPI for short) published by the Japan Management and Coordination Agency for the period 1982-91. In each year of this period, average data for seven scales of paddy in eight domestic regions excluding Hokkaido are utilized, but these data do not constitute a complete time series-cross section since FHET misses several kinds of relevant information.[6] Hence the number of observations amounts to 283. The patterns of preference ordering and technological structure are estimated for households of monocultural rice farming in FHET. Price indices for current inputs and capital goods are adapted from PWRA and those for purchased commodities are adapted from RCPI. Main variables used in this study are generated in the following way.

[6] Only classes containing more than five households are used for estimation. Scales of paddy here refer to those of planted one, and scale classes 1 to 7 stand respectively for 0.5-1.0ha, 1.0-1.5ha, 1.5-2.0ha, 2.0-2.5ha, 2.5-3.0ha, 3.0-5.0ha, and 5.0ha and over. Data for households of paddy scale less than 0.5ha are not used since their output of rice does not constitute an important source of their revenue and their marketed surplus of it amounts to a little more than 10% of its national total.

First, the price of rice is estimated by dividing the production revenue from rice (yen) by its output X (kg). The wage rate for off-farm work w is estimated by dividing the sum of salaries, wages, and other compensations by the corresponding off-farm work hours L_2. The price index q of current inputs (seeds and seedlings; fertilizers; feed; agricultural chemicals; fuel, light, heat, and processing materials) is estimated by weighting individual prices with their respective shares in the total expenditure. The price index of capital stock (buildings; agricultural motor vehicles; agricultural implements) is similarly estimated by weighting individual prices with their respective shares in the total values of capital stock.

Then, the amount F of current inputs is estimated by dividing by q the sum of expenditures for them. The amount of real capital stock K is similarly estimated. The amount consumed C_1 of rice is estimated by dividing by p the production revenue in excess of cash revenue from rice. The numbers of on-farm workers N_1 and off-farm workers N_2 are respectively adapted from FHET to estimate the endowed time of households at $Te \equiv 24\times365\times(N_1+N_2)$. Leisure hours Z are estimated from endowed time Te in excess of the sum L_1+L_2 of on- and off-farm work hours. The intensity rate ψ of set-aside program is estimated by dividing the set-aside areas by the paddy areas under cultivation. Estimation of the values of other variables should be clear by their definitions.

Finally, several remarks on the labor supply of Japanese rice farmers should be in order. The above data show that the mean on- and off-farm work hours are estimated respectively at 1487.9 and 3384.2 hours per household, amounting to 30.5% and 69.5% of the mean total work hours with their coefficients of variation estimated at 45.3% and 21.7%. Since off-farm wage employment does not allow much flexibility, it appears farmers are obliged to adjust their total work hours mainly in the area of on-farm work. This feature of their labor supply has also been observed by several other authors who use other sources of data of Japanese agriculture (see, e.g., Arayama, 1986; Kang and Maruyama, 1992). These observations may suggest that Japanese farmers face constrained off-farm wage employment and recommend a line of researches to test the validity of this constraint and to analyze their behavior under this constraint, which will be addressed in the rest of this paper.

Procedures for Estimation

Following Jacoby (1993) and Skoufias (1994), the production function is first estimated to obtain the demand rate of wage or the marginal revenue product of farm labor. Although the data used in this study do not constitute a complete panel, the production function to be estimated is specified in the following way to control time invariant unobserved factors.

$$\ln X = \ln A + \sum_{i=1}^{5} \beta_i \ln X_i + \beta_6 TT + \sum_{j=1}^{7} \gamma_j RD_j + \sum_{k=1}^{6} \delta_k SD_k + u, \quad (20)$$

where X_i's (i = 1,...,5) denote the four factor inputs (L_1, F, K, and T) and ψ with RD_j (j = 1,...,7) and SD_k (k = 1,...,6) respectively representing regional and scale dummies, and u an error term.

Both Jacoby (1993) and Skoufias (1994) estimate the production function without imposing restrictions associated with the optimality conditions for variable inputs, which is not appropriate in case all the structural parameters need to be estimated in a consistent way. In the present specification of the production function, the ratio of the expenditure for current inputs to the production revenue qF/pX should be equal to β_2.

$$qF / pX = \beta_2 + v, \quad (21)$$

where v denotes an error term. u and v are assumed to be independent of error terms in consumption choice. Furthermore, the number of households N_{jk} included in each class differs greatly among different regions as well as among different scales of paddy field. Thus, the equations (20) and (21) are estimated simultaneously by the method of three stage least squares after multiplying both sides of these equations by the square root of N_{jk}. To obtain more efficient estimates of β_i's, dummies whose coefficients are not significant at 10% level are excluded in the final estimation. The two estimates of β_2's will be tested for equality by use of an LR (likelihood ratio) type test proposed by Gallant and Jorgenson (1979).

Two sets of instrumental variables are used for this estimation. One consists of the constant, seven regional dummies, six scale dummies, the exogenous variables in the production function (K, T, ψ, and TT), and other exogenous variables (p, p', q, η, TAX, and OC), which will be referred to as set I. The other consists of the interaction and squared terms among p, p', q, η, TAX, and OC in addition to the instrumental variables in the set I. For the second set, all the variables in this set are first regressed on the endogenous variables on the right hand sides of the equations (20) and (21), $\ln X_1$ (= $\ln L_1$) and $\ln X_2$ (= lnF). Then, the interaction and squared terms whose coefficients are significant at 5% level in these regressions are added to the variables in the set I to form the set II of instrumental variables.

Now, the internal rate of wage is estimated by use of the relation $w^* = \hat{\beta}_1 p\hat{X}/L_1$ ($\hat{X}$: fitted value of X) and used to estimate parameters associated with preference ordering. The specification (12) of the household welfare function and the optimality conditions (10.3)-(10.6) in equality yield a system of three expenditure equations similar to the linear expenditure system with the rate of wage w^* and the full income Y' being endogenous. For the present case, effects of time trend TT as well as those of time invariant unobserved factors are controlled and the following two equations are estimated.

$$pC_1 = a_1p + b_1(Y' - a_1p - a_2p' - a_3w^*) + \sum_{j=1}^{7} c_j RD_j + \sum_{k=1}^{6} d_k SD_k + \tau_1 TT + u_1, \tag{22}$$

$$p'C_2 = a_2p' + b_2(Y' - a_1p - a_2p' - a_3w^*) + \sum_{j=1}^{7} g_j RD_j + \sum_{k=1}^{6} h_k SD_k + \tau_2 TT + u_2, \tag{23}$$

where Y' = Y−S−TAX, and u_1 and u_2 are error terms. Furthermore, to allow for different composition of members among different households, the number NF of family members and the share FSHARE of farm workers N_1 in all workers N_1+N_2 are used as shift factors of the welfare function in the following way:

$$a_i = a_{i0} + a_{i1}NF + a_{i2}FSHARE \quad (i = 1,2,3). \tag{24}$$

Since w^* and Y' are endogenous, the equations (22) and (23) with the relations (24) incorporated are estimated simultaneously by the method of nonlinear three stage least squares after multiplying both sides of these equations by the square root of N_{jk}. Sets of instrumental variables used are the set I above and another set constructed in a similar way to the set II above, where w^* and Y' are dependent variables regressed to select the significant interaction and squared terms.

The Estimated Internal Rate of Wage

Estimated coefficients of the production function (20) and the equation (21) associated with the optimal choice of current inputs are shown in Table 1 for the two sets of instrumental variables, where estimated coefficients of dummy variables are not shown. The LR-type test statistics in this table are compared with the respective critical values of χ^2 distribution with one degree of freedom to show that the equality of the estimates of β_2's in the equations (20) and (21) is not rejected at 5% level of significance. The coefficients β_1-β_4 of factor inputs are positive and all of them are significant at 10% level for the set II. The coefficient β_5 of the intensity rate ψ of set-aside program is significantly negative, which implies that a rise in ψ reduces the output of rice with other things being equal.

Among these coefficients, the elasticity β_1 of output with respect to farm labor plays the most important role in determining the level of the internal rate w^* of wage. Comparison of these estimates with other similar elasticities may be in order. Shintani (1983) estimated the value of this elasticity at 0.26 for the periods 1969-71 and 1977-79 by use of a production function of the Cobb-Douglas type, while Kusakari (1985) estimated the values of the same elasticity between 0.15-0.23 for the period 1967-82 by use of a translog production function for rice in Japan. In the light of these estimates, the estimates 0.23 and 0.25 of β_1 seem to be reasonable.

Table 2 presents estimates of the coefficients of the expenditure equations (22) and (23) with the relations (24) incorporated. Most of them are significantly estimated and concavity of the welfare function is satisfied at the mean values of independent variables for the two sets of instruments. The estimated coefficients of shift factors of the welfare function indicate that increases in the number NF of family members raise the household's subsistence levels of all commodities, while increases in the share FSHARE of farm workers have an effect of lowering them.

From the estimated coefficients in Table 1, the estimates of the internal rates of wage are obtained. Their means and standard deviations along with their market rates are classified for all scale classes and for their sum in Table 3, which reveal a large discrepancy between the two rates. The existence of a large discrepancy between them should suggest a conjecture that the off-farm wage employment is severely constrained. To confirm this conjecture, the significance of the estimated discrepancy will be tested somehow or other. We follow Skoufias (1994) in estimating the equation $\ln w^* = a + b \ln w$ to test the joint hypothesis that a = 0 and b = 1. Since such instruments as ages of family members and years of their education are not available in the data used in this study, only OLS can be applied. Skoufias (1994) shows that the OLS estimation results in a very similar test to that based on the instrumental variable estimation, though it may involve some measurement errors. Hence it can offer us some inferences. Estimation of the said equation for the two series of the internal rate of wage results in:

$\ln w^* = 9.238 - 0.440 \ln w$, Ftest = 1803.1,

$\ln w^* = 9.113 - 0.436 \ln w$, Ftest = 2154.2,

where Ftest denotes a F statistic to test the joint hypothesis that a = 0 and b = 1. These test statistics are sufficiently large to reject the said hypothesis at any reasonable level of significance.

Now, it may be in order to test whether the mean internal rate of wage is significantly lower than the mean market rate. Since neither the internal nor the market rate is normally distributed, their logarithms $Z_{1i} = \ln w_i^*$ and $Z_{2i} = \ln w_i$ (i = 1,...,283) are utilized for this purpose. Actually, the normality of either Z_1 or Z_2 is not rejected at 1% significance level in terms of the Jarque-Bera test[7], while that of both rates themselves is rejected at any reasonable level of significance. Since Z_{1i} may not be independent of Z_{2i}, it is assumed that the pair (Z_{1i}, Z_{2i}) is a random sample from a bivariate normal distribution with its mean $\mu = (\mu_1, \mu_2)$ and covariance matrix $\Sigma = (\sigma_{jk})$ (j, k = 1,2). Then a statistic to test the null hypothesis H_0: $\mu_1 = \mu_2$ against an alternative H_1: $\mu_1 < \mu_2$ proves to be $\tau = n^{1/2}\{(\overline{Z}_1 - \overline{Z}_2) - (\mu_1 - \mu_2)\}/(s_{11} + s_{22} - 2s_{12})^{1/2}$, where $\overline{Z}_j$ and s_{jk} respectively denote the sample mean of Z_{ji} and the sample covariance between Z_{ji} and Z_{ki} (i = 1,...,283; j, k = 1,2), and τ has a t-distribution with n−1 degrees of freedom. τ is found to be equal to −40.5 and −44.5 for the two series of the internal rate of wage, which implies that H_0 is rejected at any reasonable level of significance. Hence, the mean of $\ln w^*$ proves to be significantly lower than that of $\ln w$.

Both the internal and market rates of wage may include some measurement errors, so that sufficient care must be taken in interpreting the result of this test. Since it is not

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[7] This is a simple test of normality based on the estimated skewness and kurtosis of the variable in

the wage rates themselves but the difference between their logarithms that have been analyzed in this test, measurement errors may have been offset by each other to some extent. Furthermore, the data used in this study are aggregate, hence the problem may not be as serious as in case where household level data are utilized.

Estimated Elasticities of Rice Supply and the Internal Rate of Wage with Respect to Selected Exogenous and Fixed Variables

Estimated values of the elasticities of rice supply and the internal rate of wage are evaluated at the mean values of relevant variables and are compared with the corresponding values for the competitive decomposable case of the agricultural household model in Table 4. The values for the competitive decomposable case are estimated from those for the indecomposable case by setting $dw^* = 0$, since the internal rate of wage is identically equated to its market rate in case all relevant labor markets are competitive. Since the estimates of these elasticities for the two instrument sets are similar and those for the set II exhibit a better performance as shown in Tables 1 and 2, only the estimates for the set II will be examined. In determining the commodity supply of the agricultural household under constrained off-farm wage employment, the effect of the working of the internal labor market or simply the internal wage effect plays an extremely important role. Hence the elasticities associated with the working of the internal labor market will be examined first.

By use of the relations (18), the elasticity of internal demand for labor with respect to the internal rate of wage is estimated at –0.422, while the similar elasticity of (internal) supply of labor is estimated at 0.220 at the mean values of the variables concerned. Therefore, the (internal) supply function stands relatively steep on the (L_1+L_2, w^*) plane. Such an inelastic supply of labor on the part of Japanese rice farmers seems to reflect the nature of the labor market surrounding them, since they supply a major part of their labor to off-farm wage employment which does not allow much flexibility as noted above in the preceding subsection.

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question. See, e.g., Davidson and Mackinnon (1993).

On the other hand, the elasticities of the internal rate of wage with respect to p, T, and L are estimated to be relatively high, which implies that changes in them can cause large shifts of the internal demand and/or supply functions of labor. The elasticities of shifts of the internal demand function caused by changes in p, T, and L respectively are equal to $L_1/(1-\beta_1-\beta_2)L_D$, $\beta_4 L_1/(1-\beta_1-\beta_2)L_D$, and L/L_D, whose values are estimated respectively at 0.512, 0.171, and 0.695. Whereas similar elasticities of the (internal) supply function respectively are equal to $-b_3 p(X-a_1)/w^* L_S$, $-b_3\beta_4 pX/w^* L_S$, and $b_3(w^*-w)L/w^* L_S$, whose values respectively are estimated at −0.878, −0.307, and −0.905. Then, the equation (19) rewritten in elasticities helps us evaluate how much each of these shifts contributes to the estimated elasticities of the internal rate w^* of wage. The shifts of the internal demand function for labor caused by changes in p, T, and L contribute 37, 36, and 43% respectively, while those of its (internal) supply function similarly caused 63, 64, and 57% of the overall elasticities of w^*. Thus, it has been made clear that these relatively large shifts of the supply function of labor are the prime causes of high elasticities of the internal rate of wage observed on Japanese rice farm for the period under study.

Now, responses of rice supply itself will be examined in reference to these elasticities. Its elasticity with respect to its own price p is estimated at −0.180, which may be conveniently decomposed into two parts, one reflecting the output effect and the other the internal wage effect. The former is estimated at 0.677 while the latter at −0.857, which dominates the former in absolute value. Hence the corresponding supply function of rice slopes downward. Thus, its downward slope has mainly been caused by the shifts both of the internal demand and supply functions of labor, which in turn have been caused by the changes in the price of rice. Next, its elasticity with respect to the intensity rate ψ of set-aside program is estimated at −0.107. The internal wage effect is estimated to be small in this case, however it can render the supply of rice less elastic. A relatively small estimated absolute value of this elasticity suggests the existence of many loopholes which enable rice farmers to evade the anticipated effects of this program. Therefore an extremely high intensity rate will be required to attain its proposed objectives. Furthermore, the elasticity of rice with respect to the amount η of subsidy on set-aside areas is estimated to be negligible, hence an isolated change in this subsidy can hardly affect the supply of rice. Finally, its elasticity with respect to the off-farm employment opportunities L is estimated at -0.952. Thus, a large amount of rice supply will be reduced as off-farm wage employment opportunities expand. This relatively high elasticity is solely due to the internal wage effect, which has been caused by the large shifts of the demand and supply functions of labor, which in turn have been caused by expansion of the off-farm employment opportunities.

4. Concluding Remarks

Agricultural households very often face constrained or lacking off-farm wage employment for some reasons or other both in developed and in developing economies. In case they do, their behavior, e.g., their supply of output and their demand for factors, is significantly influenced by their internal rate of wage which equates their demand to their supply of labor in excess of their off-farm employment. Hence, it is extremely important in analyzing their behavior to have a consistent estimate of all parameters that govern the mechanisms by which their internal rate of wage is influenced by other variables and by which it in turn influences other endogenous variables. A method of structural estimation by use of a system of expenditure functions similar to the linear expenditure system is proposed and applied to analyze the behavior of rice farmers in Japan, where it has been shown by previous authors that they are likely to face constrained off-farm wage employment.

Actually, their internal rate of wage is estimated to be significantly lower than their market rate. Hence their off-farm wage employment is inferred to be constrained, verifying the finding of previous authors. Changes in their price of rice, their total planted area, and their off-farm employment are found to significantly influence both their demand and supply of labor within the household therefore their internal rate of wage. The indirect effect through changes in their internal rate of wage caused by changes in their price of rice is estimated to dominate the direct effect of the same changes, consequently their supply function of rice is inferred to be downward-sloping at the mean values of relevant variables. Whereas, changes in such policy variables as the intensity rate of set-aside program and the associated per area subsidy hardly influence either their demand or supply of labor within the household hence their supply of rice. These results may suffice to endorse the usefulness of the proposed method in analyzing the behavior of agricultural household both in developed and developing economies where their off-farm wage employment is constrained or lacking for some reason or other.

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Table 1. Estimated Coefficients of the Production Function

	Instrument Set I		Instrument Set II	
β_1	0.2518	(3.521)	0.2281	(3.586)
β_2	0.1756	(118.0)	0.1756	(118.1)
β_3	0.0563	(1.421)	0.0736	(1.926)
β_4	0.1571	(1.276)	0.3338	(3.641)
β_5	−0.0855	(3.062)	−0.1150	(4.574)
β_6	0.0108	(3.624)	0.0113	(4.126)
R^2_X	0.9997		0.9997	
R^2_F	0.9371		0.9371	
LR	0.9475	[0.330]	1.5942	[0.207]

Note: R^2_X and R^2_F denote the coefficients of determination for the equation (20) and (21) respectively. LR denotes an LR-type statistic to test the equality of estimated β_2's in the equations (20) and (21). Absolute values of t-statistics are shown in parentheses ( ), while the upper tail area for $\chi^2(1)$ in brackets [ ].

Table 2. Estimated Coefficients of the Equations (22) and (23) with the Relations (24) Incorporated

	Instrument Set I		Instrument Set II	
a_{10}	−3277.2	(2.572)	−685.3	(3.012)
a_{11}	739.2	(3.531)	289.7	(5.556)
a_{12}	710.9	(0.913)	−966.4	(3.465)
a_{20}	−74516.2	(2.573)	−85089.9	(3.199)
a_{21}	24455.7	(5.186)	26769.9	(6.479)
a_{22}	−108721.0	(5.907)	−127924.0	(6.511)
a_{30}	−18511.2	(1.801)	−19904.0	(2.272)
a_{31}	9165.1	(3.632)	9932.4	(4.699)
a_{32}	−35296.3	(3.360)	−40116.6	(4.111)
b_1	0.0257	(3.312)	0.0111	(4.288)
b_2	0.3166	(6.342)	0.3614	(7.803)
b_3	0.6578	(12.748)	0.6275	(13.305)
τ_1	−339.6	(0.155)	−5284.5	(6.870)
τ_2	64175.0	(3.814)	69148.9	(4.706)

Note: The values of b_3 is estimated by use of the relation $b_1+b_2+b_3 = 1$. Absolute values of t-statistics are shown in parentheses ( ).

Table 3. Comparison of the Estimated Internal Rate with the Market Rate of Wage (Yen/hour)

Scale Class	Internal Rate of Wage Ⅰ	Internal Rate of Wage Ⅱ	Market Rate of Wage
1	320.7 (61.2)	290.8 (55.9)	1481.7 (261.2)
2	416.6 (94.0)	378.0 (83.1)	1375.8 (223.6)
3	485.0 (83.2)	440.2 (75.4)	1260.0 (294.5)
4	559.0 (71.7)	507.9 (67.1)	1112.5 (223.9)
5	602.7 (100.1)	545.2 (91.4)	1069.7 (219.0)
6	678.3 (108.5)	618.7 (100.2)	1106.2 (192.3)
7	724.9 (73.1)	613.5 (64.0)	1229.1 (126.0)
All	467.2 (150.1)	422.8 (134.6)	1300.7 (283.7)
Minimum	213.8	194.1	725.6
Maximum	923.9	847.7	2155.2

Note: Internal rates of wage Ⅰ and Ⅱ are derived respectively from the estimations for the instrument sets Ⅰ and Ⅱ. Standard deviations are shown in parentheses ().

Table 4. Estimated Elasticities of Rice Supply and the Internal Rate of Wage with Respect to Selected Exogenous and Fixed Variables

	Instrument Set I		Instrument Set II	
Model	Decomposable Case	Indecomposable Case	Decomposable Case	Indecomposable Case
∂lnX/∂lnp	0.746	−0.136	0.677	−0.180
∂lnX/∂lnq	−0.307	−0.166	−0.294	−0.158
∂lnX/∂lnK	0.098	0.047	0.123	0.059
∂lnX/∂lnT	0.274	0.131	0.560	0.266
∂lnX/∂lnψ	−0.149	−0.092	−0.193	−0.112
∂lnX/∂lnp'	0.000	0.188	0.000	0.091
∂lnX/∂lnη	0.000	−0.020	0.000	−0.020
∂lnX/∂lnL	0.000	−0.948	0.000	−0.952
∂lnw*/∂lnp	0.000	2.007	0.000	2.240
∂lnw*/∂lnq	0.000	−0.321	0.000	−0.356
∂lnw*/∂lnK	0.000	0.117	0.000	0.170
∂lnw*/∂lnT	0.000	0.327	0.000	0.769
∂lnw*/∂lnψ	0.000	−0.131	0.000	−0.212
∂lnw*/∂lnp'	0.000	−0.427	0.000	−0.238
∂lnw*/∂lnη	0.000	0.047	0.000	0.054
∂lnw*/∂lnL	0.000	2.156	0.000	2.490

Note: Elasticities for the competitive decomposable case are estimated by setting dw* = 0.