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by

Hideaki Takagi, Ken-ichi Sakamaki
and Tohru Miyashiro

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Hideaki Takagi
Institute of Policy and Planning Sciences, University of Tsukuba
1-1-1 Tennoudai, Tsukuba-shi, Ibaraki 305, Japan
phone: (0298)53-5003 fax: (0298)55-3849
e-mail: takagi@shako.sk.tsukuba.ac.jp (corresponding author)

Ken-ichi Sakamaki †
Doctoral Program in Policy and Planning Sciences, University of Tsukuba

and
Tohru Miyashiro ‡
College of Policy and Planning Sciences, University of Tsukuba

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We propose and analyze a traffic model of a cellular radio communication network with an arbitrary cell connection and arbitrary probabilistic movement of mobiles between the cells. Our analytic model consists of birth-and-death processes for individual cells connected by the numerical adjustment of hand-off rates. This approximation is validated by simulation. We evaluate the probabilities of the immediate loss, the completion, and the forced termination during hand-off for an arbitrary call in the network. Our numerical examples reveal the cases in which the increase in the generation rate of new calls results in the increase in the loss probability without affecting much the probability of forced termination in a limited service area.

Key words: mobile communication, cellular network, hand-off, performance evaluation, call loss, forced termination

1. Introduction

In cellular radio communication networks, the entire service area is divided into smaller subareas called *cells* in order to reuse the same frequency band in distant cells. While this technique enhances the efficiency in the bandwidth utilization given to the service area, an inherent problem is that mobiles crossing the cell boundary cannot continue to use the same frequency channel because different channels are assigned to neighboring cells to avoid the radio interference. Therefore, a process called *hand-off* is enforced when a mobile moves to another cell by which a new channel is given from that cell. If no channels

†Present address: Multimedia Technology Center, NTT Data Corporation, Kowa Kawasaki Nishiguchi Building, 66-2 Horikawa-cho, Saiwai-ku, Kawasaki-shi, Kanagawa 210, Japan

‡Present address: Advanced Information Network Service Sector, NTT Data Corporation, Toyosu Center Building, 3-3-3 Toyosu, Koto-ku, Tokyo 135, Japan

are available, the call suffers from forced termination. Thus, in cellular networks, calls involved in a cell consists of new ones initiated in the cell and hand-off calls brought by the mobiles moving in from neighboring cells. It is therefore essential to incorporate the hand-off calls in the traffic modeling of cellular networks, which are absent in one-hop communication systems such as wireless local area networks and satellite communications.

In this paper we present a traffic model of a cellular communication network in which an arbitrary topology of cell connection can be handled. We assume that hand-off calls can wait for a new channel from the cell that it is entering by continuing to use the channel of the cell that it is leaving while it resides in the boundary of the two cells called *hand-off area*. However they are forcibly terminated if no channels become available before passing the hand-off area.

In this model, we are interested in the performance measures from the user's viewpoint. Namely, we will evaluate the probability Pb that a new call is blocked at once, the probability Pc that a call is completed until the conversation ends, and the probability $Pf = 1 - Pb - Pc$ that a call once accepted is terminated during hand-off. These probabilities are computed given the generation rate of new calls, the mean duration of each call, the mean residence times of a mobile in the cell and in the hand-off area, and the matrix of mobile movement probabilities between the cells. Our analytic model consists of birth-and-death processes for individual cells that are connected by the numerical adjustment of hand-off traffic rates between the cells. We also conduct simulation to validate our approximation procedure.

Several Markovian models have been proposed for studying the traffic characteristics in the cellular communication systems. In [5], the hand-off rate is computed from the probabilistic analysis of the movement of a mobile, and the call loss probability is calculated for the models in which hand-off calls have priority over new calls in channel access and they can wait in the hand-off area. In [3], it is shown that the channel usage time in a cell has exponential distribution by analysis and simulation. These and other studies [1, 4, 10] are only concerned with a single cell. A hierarchical model is given in [6], but the movement of users from cell to cell is not considered.

The rest of this paper is organized as follows. In Section 2, we give a birth-and-death process model for each cell, and show how to calculate the hand-off rates from other system parameters. In Section 3, we obtain the probabilities Pb, Pc, and Pf for an arbitrary call generated in the network. The simulation model is described in Section 4. In Section 5, we present the numerical results for three example networks to which our modeling has been applied. We conclude in Section 6 with a summary of the present study and a few remarks on the future research subjects.

2. Analytic Model

Our analytic traffic model consists of birth-and-death processes for individual cells connected by numerically adjusted hand-off rates. Let us assume that n cells in the network are identified as cells 1 through n. Section 2.1 describes a birth-and-death process model for cell i, assuming that the hand-off rate flowing into cell i is a Poisson process. Section 2.2 shows an iterative procedure for calculating the hand-off rate for each cell by aggregating the flow rates coming out of its neighboring cells.

2.1. A birth-and-death process model for each cell

In our traffic model for each cell, we distinguish three ways in which calls associated with the cell disappear. Namely, they disappear (1) when the conversation is completed, (2) when the mobiles go out of the cell, and (3) when the waiting hand-off calls are terminated because they are not given a channel before passing the hand-off area. The rates of these events in cell i are denoted by $\mu_1^{(i)}$, $\mu_2^{(i)}$, and $\mu_3^{(i)}$, respectively. The intervals between these successive events are assumed to be exponentially distributed. In [5], the rates $\mu_1^{(i)}$ and $\mu_2^{(i)}$ are combined into a single parameter. In this paper we calculate the hand-off rates by distinguishing $\mu_1^{(i)}$ and $\mu_2^{(i)}$. If V denotes the average speed of a mobile and R the radius of a cell, the mean time that the mobile resides in cell i is given by $1/\mu_2^{(i)} = 2R/V$. If L denotes the diameter of the hand-off area, the mean residence time in the hand-off area is given by $1/\mu_3^{(i)} = L/V$.

Let $\lambda_1^{(1)}$ and $\lambda_2^{(1)}$ be the rates at which new and hand-off calls are generated according to Poisson processes, respectively. It is assumed that each cell has C fixed channels, namely, at most C calls can be accepted at a time. Out of C channels, new calls can be assigned only up to $C - g$ channels, where g is the number of guard channels dedicated to hand-off calls. Thus hand-off calls are given cutoff priority over new calls [1]. Blocked new calls are simply lost. Each blocked hand-off call waits for a channel of the new cell by continuing to use a channel of the previous cell while it resides in the hand-off area at the boundary of the two cells. If such a call passes the hand-off area before a channel of the new cell becomes available, it is lost resulting in the forced termination of the hand-off call.

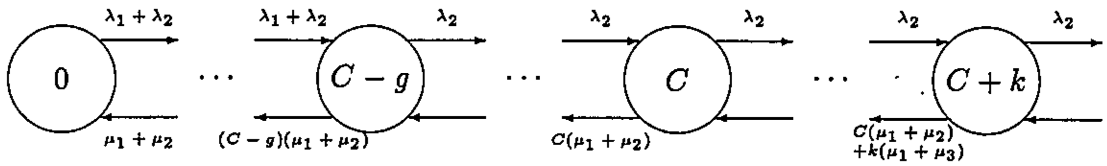


Figure 1. State transition rate diagram for a single cell.

Based on the above-mentioned model, we can construct a birth-and-death process with respect to the number of calls either using or waiting for channels of cell i. In this process, the birth rate in state k is $\lambda_1^{(i)} + \lambda_2^{(i)}$ for $0 \le k \le C - g$ and $\lambda_2^{(i)}$ for $k > C - g$, because new calls are lost when $k > C - g$. The death rate in state k is $k(\mu_1^{(i)} + \mu_2^{(i)})$ for $1 \le k \le C$ and $C(\mu_1^{(i)} + \mu_2^{(i)}) + (k - C)(\mu_1^{(i)} + \mu_3^{(i)})$ for $k > C$. The contribution $(k - C)(\mu_1^{(i)} + \mu_3^{(i)})$ comes from the rate that each mobile waiting in the hand-off area either completes the call at rate $\mu_1^{(i)}$ or expires the residence time at rate $\mu_3^{(i)}$.

The state transition diagram for this process is shown in Figure 1. If we denote by $P^{(i)}(k)$ the steady-state probability that there are k calls in cell i, a set of local balance equations is given by

$$
\begin{aligned}
&k(\mu_1^{(i)} + \mu_2^{(i)})P^{(i)}(k) = (\lambda_1^{(i)} + \lambda_2^{(i)})P^{(i)}(k-1) && 1 \le k \le C - g \\
&k(\mu_1^{(i)} + \mu_2^{(i)})P^{(i)}(k) = \lambda_2^{(i)} P^{(i)}(k-1) && C - g < k \le C \\
&\left[C(\mu_1^{(i)} + \mu_2^{(i)}) + (k - C)(\mu_1^{(i)} + \mu_3^{(i)})\right] P^{(i)}(k) = \lambda_2^{(i)} P^{(i)}(k-1) && k > C
\end{aligned}
\tag{1}
$$

The solution to these equations is given by

$$P^{(i)}(k) = \begin{cases} \left(\frac{\lambda_1^{(i)} + \lambda_2^{(i)}}{\mu_1^{(i)} + \mu_2^{(i)}}\right)^k \frac{1}{k!} P^{(i)}(0) & 1 \le k \le C-g \\ \frac{(\lambda_2^{(i)})^{k-(C-g)}(\lambda_1^{(i)} + \lambda_2^{(i)})^{C-g}}{(\mu_1^{(i)} + \mu_2^{(i)})^k} \frac{1}{k!} P^{(i)}(0) & C-g < k \le C \\ \frac{(\lambda_2^{(i)})^{k-(C-g)}(\lambda_1^{(i)} + \lambda_2^{(i)})^{C-g}}{C!(\mu_1^{(i)} + \mu_2^{(i)})^C \prod_{j=1}^{k-C}[C(\mu_1^{(i)} + \mu_2^{(i)}) + j(\mu_1^{(i)} + \mu_3^{(i)})]} P^{(i)}(0) & k > C \end{cases} \tag{2}$$

where $P^{(i)}(0)$ is determined from the normalization condition

$$\sum_{k=0}^{\infty} P^{(i)}(k) = 1 \tag{3}$$

Thus we can calculate $\{P^{(i)}(k); k = 0, 1, 2, \ldots\}$ for each i.

2.2. Determination of the hand-off generation rates

When we consider a network of cells, it is natural to believe that the hand-off generation rate $\lambda_2^{(i)}$ for cell i depends on the parameters of its neighboring cells. Let us assume that a call from cell i moves to cell j with probability γ_{ij} $(1 \le j \le n)$, and moves outside the service area with probability γ_{ie} so that

$$\sum_{j=1}^{n} \gamma_{ij} + \gamma_{ie} = 1 \qquad 1 \le i \le n \tag{4}$$

We employ the fixed-point method to determine the values of $\lambda_2^{(i)}$ by the following procedure in which each step is executed for all i:

1. Let $\lambda_2^{(i)} = 0$, and set the values for $\lambda_1^{(i)}$, $\mu_1^{(i)}$, $\mu_2^{(i)}$, and $\mu_3^{(i)}$.
2. Calculate $\{P^{(i)}(k); k = 0, 1, 2, \ldots\}$ by equations (2) and (3).
3. Compute
$$\lambda_2^{(i)*} = \sum_{k=1}^{\infty} P^{(i)}(k) \min[k, C]\mu_2^{(i)} \tag{5}$$
4. Compute
$$\lambda_2^{(i)} = \sum_{j=1}^{n} \lambda_2^{(j)*} \gamma_{ji} \tag{6}$$
5. Repeat steps 3, 4, and 5 until $\lambda_2^{(i)}$ converges.

Note that in step 3 the factor $\min[k, C]\mu_2^{(i)}$ accounts for the rate of hand-off calls going out of cell i when there are k calls in cell i.

3. Call Loss and Forced Termination Probabilities

A user of a mobile will be interested in the probability Pb that his new call is blocked, the probability Pc that his call is completed, and the probability $Pf = 1 - Pb - Pc$ that his call is once accepted but terminated before completion. We evaluate these probabilities for an arbitrary call in the network, based on the traffic model given in Section 2. In Section 3.1, we show the expressions for several probabilities related to individual cells. In Section 3.2, they are used to calculate the probabilities Pb, Pf, and Pc through recursive consideration. In Section 3.3, we derive simple expressions for the three probabilities in the special case in which all cells have the same system parameters.

3.1. Probabilities for individual cells

Let us focus on cell i, and evaluate several probabilities related to cell i, based on the birth-and-death process model for each cell given in Section 2.1.

We first consider the probability $Pb^{(i)}$ that a new call generated in cell i is blocked and lost. Recall that a new call is blocked if $C - g$ or more channels are used when it is generated. Because of the assumption that new calls are generated according to a Poisson process, the probability distribution of the system state at the time of call generation is the same as that at an arbitrary time in the steady state. Therefore we have

$$Pb^{(i)} = \sum_{k=C-g}^{\infty} P^{(i)}(k) \tag{7}$$

We next find the probability $Ph^{(i)}$ that a hand-off call entering cell i is terminated (hand-off failure). It is given as the fraction of the hand-off calls that cannot get channels while waiting in the hand-off area:

$$Ph^{(i)} = \frac{1}{\lambda_2^{(i)}} \sum_{k=C+1}^{\infty} P^{(i)}(k)(k-C)\mu_3^{(i)} \tag{8}$$

where the factor $(k - C)\mu_3^{(i)}$ is the rate that mobiles pass the hand-off area when there are k calls in cell i.

Let us proceed to consider a new or hand-off call that has been successfully started in cell i. This call leaves cell i in one of the following ways:

(i) It is completed in cell i before moving out of cell i. This case occurs with probability

$$Pe^{(i)} = \frac{\mu_1^{(i)}}{\mu_1^{(i)} + \mu_2^{(i)}} \tag{9}$$

(ii) It moves to cell j before completion, and the hand-off to cell j is successful. This case occurs with probability

$$P_j^{(i)} = \frac{\mu_2^{(i)}}{\mu_1^{(i)} + \mu_2^{(i)}} \gamma_{ij}(1 - Ph^{(j)}) \qquad 1 \le j \le n \tag{10}$$

(iii) It moves out of cell i to another cell but the hand-off is unsuccessful or it moves outside the service area. This case occurs with probability

$$1 - Pe^{(i)} - \sum_{j=1}^{n} P_j^{(i)} = \frac{\mu_2^{(i)}}{\mu_1^{(i)} + \mu_2^{(i)}} \left[\sum_{j=1}^{n} \gamma_{ij} Ph^{(j)} + \gamma_{ie} \right] \tag{11}$$

This case results in the forced termination of the call.

3.2. Probabilities for an arbitrary call in the network

We are now in the position to calculate the probabilities Pb, Pc, and Pf for a call generated in an arbitrary cell in the network. They are given in terms of the probability π_i that a call currently in cell i is completed successfully somewhere in the network. By a recursive consideration it is clear that the set $\{\pi_i; 1 \leq i \leq n\}$ satisfies the equations

$$\pi_i = Pe^{(i)} + \sum_{j=1}^{n} P_j^{(i)} \pi_j \tag{12}$$

The probability that a call in cell i is terminated forcibly before completion is then given by $1 - \pi_i$. Thus a new call generated in cell i is

(i) blocked and lost with probability $Pb^{(i)}$, or

(ii) accepted and completed with probability $(1 - Pb^{(i)})\pi_i$, or

(iii) accepted but later terminated with probability $(1 - Pb^{(i)})(1 - \pi_i)$.

By taking the average of these probabilities with the weight of the generation rate of new calls in each cell, we obtain

$$Pb = \sum_{i=1}^{n} \frac{\lambda_1^{(i)}}{\Lambda_1} Pb^{(i)} \tag{13}$$

$$Pc = \sum_{i=1}^{n} \frac{\lambda_1^{(i)}}{\Lambda_1} (1 - Pb^{(i)})\pi_i \tag{14}$$

$$Pf = \sum_{i=1}^{n} \frac{\lambda_1^{(i)}}{\Lambda_1} (1 - Pb^{(i)})(1 - \pi_i) = 1 - Pb - Pc \tag{15}$$

where

$$\Lambda_1 = \sum_{i=1}^{n} \lambda_1^{(i)} \tag{16}$$

is the generation rate of new calls in the entire network.

3.3. Probabilities in a symmetric network

If every parameter is the same for all the cells in the network, we can derive simple expressions for the probabilities Pc and Pf. In this case, let us omit the superscript for cell identification in all the notation, and use γ for γ_{ie}. Arguments similar to the one below are given in [5, 8]. The change in the state of each call is depicted in Figure 2.

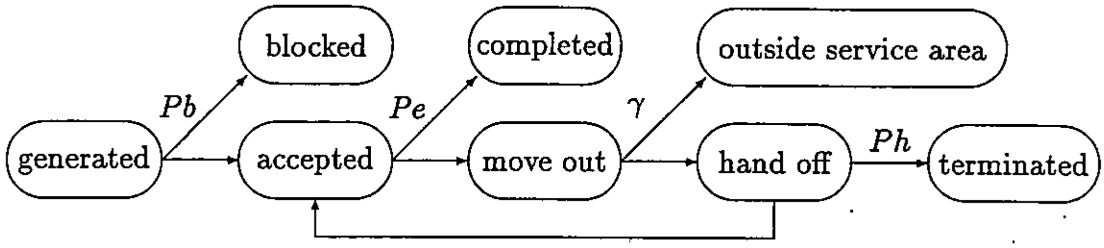


Figure 2. State of a call in a symmetric network.

Figure 2 shows that a call generated in a cell is blocked with probability Pb; if accepted it is completed in that cell with probability $Pe = \mu_1/(\mu_1 + \mu_2)$; otherwise it moves out of the service area with probability γ; otherwise the hand-off is successful with probability $1 - Ph$, or unsuccessful with probability Ph. Thus the probability Pc of call completion is given by

$$\begin{aligned} Pc &= (1 - Pb) \sum_{k=0}^{\infty} [(1 - Pe)(1 - \gamma)(1 - Ph)]^k Pe \\ &= \frac{(1 - Pb)Pe}{1 - (1 - Pe)(1 - \gamma)(1 - Ph)} = \frac{(1 - Pb)\mu_1}{\mu_1 + [Ph + \gamma(1 - Ph)]\mu_2} \end{aligned} \tag{17}$$

The probability Pf of forced termination is given by

$$\begin{aligned} Pf &= (1 - Pb) \sum_{k=0}^{\infty} [(1 - Pe)(1 - \gamma)(1 - Ph)]^k (1 - Pe)[\gamma + (1 - \gamma)Ph] \\ &= \frac{(1 - Pb)(1 - Pe)[\gamma + (1 - \gamma)Ph]}{1 - (1 - Pe)(1 - \gamma)(1 - Ph)} = \frac{(1 - Pb)[Ph + \gamma(1 - Ph)]\mu_2}{\mu_1 + [Ph + \gamma(1 - Ph)]\mu_2} \\ &= 1 - Pb - Pc \end{aligned} \tag{18}$$

The same result may be obtained by substituting

$$\sum_{j=1}^{n} P_j = (1 - Pe)(1 - \gamma)(1 - Ph) \tag{19}$$

into (12) to yield

$$\pi = \frac{Pe}{1 - (1 - Pe)(1 - \gamma)(1 - Ph)} = \frac{\mu_1}{\mu_1 + [Ph + \gamma(1 - Ph)]\mu_2} \tag{20}$$

Then, from (14) and (15), we get (17) and (18), respectively.

If no mobiles go outside the service area, we have $\gamma = 0$ and then

$$Pc = \frac{(1 - Pb)\mu_1}{\mu_1 + Ph\mu_2} \quad ; \quad Pf = \frac{(1 - Pb)Ph\mu_2}{\mu_1 + Ph\mu_2} \tag{21}$$

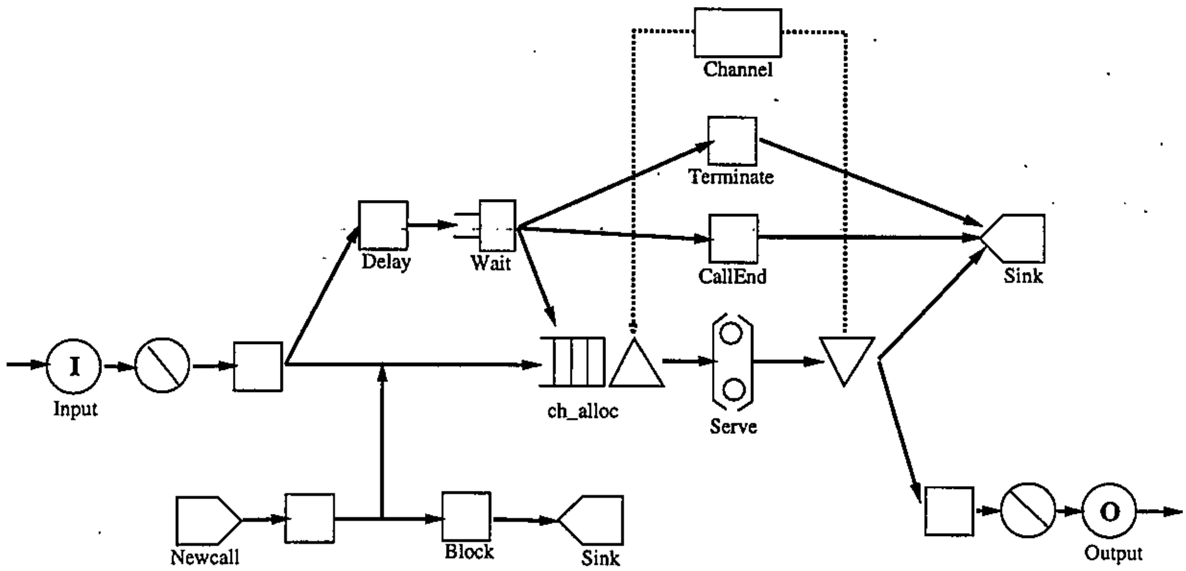


Figure 3. A simulation model for a cell.

4. Simulation Model

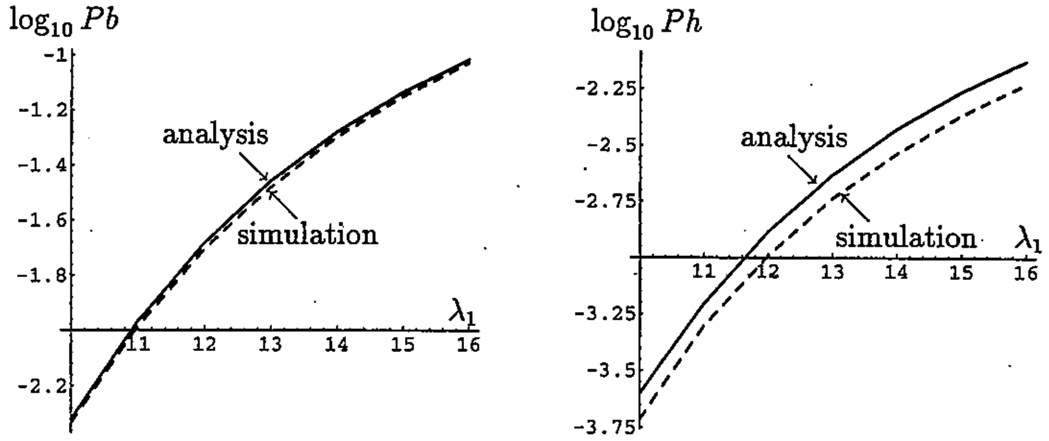
In parallel with the analytic model, we have constructed a simulation model by using the discrete event simulation software RESQ ME on OS/2 from IBM Research [2]. Figure 3 depicts a RESQ model for a single cell, and we connect such models according to the topology of the network.

In this figure, each new call is generated at `Newcall`, receives a channel at `ch_alloc` if more than g channels are available, and gets served at `Serve`. Otherwise it goes to the `Sink` node through `Block`. Note that a square node such as `Block` counts the number of passing calls. Each hand-off call comes in through `Input`, receives a channel at `ch_alloc` if any channel is available, and gets served at `Serve`. Otherwise it waits for a channel at `Wait` (this corresponds to the time when a mobile is in the hand-off area). There are three ways to go after `Wait`: (1) If a channel becomes available, the call receives a channel at `ch_alloc`, and then gets served at `Serve`. (2) If the call is completed while waiting, it goes to the `Sink` node through `CallEnd` where the number of completed calls is counted. (3) If the residence time in the hand-off area expires, the call goes to the `Sink` node through `Terminate` where the number of forced terminations is counted. After a call is served at `Serve`, it returns a channel to `Channel`, and then either goes to the `Sink` node (call completion) or goes to `Output` which means the hand-off to a neighboring cell.

We obtain the values of Pb, Pc, and Pf as the following ratios:

$$Pb = \frac{\sharp[\texttt{Block}]}{\sharp[\texttt{Newcall}]} \tag{22}$$

$$Pc = \frac{\sharp[\texttt{Serve}] - \sharp[\texttt{Output}] + \sharp[\texttt{CallEnd}]}{\sharp[\texttt{Newcall}]} \tag{23}$$



(a) Loss probability. (b) Probability of hand-off failure.

Figure 4. Performance of a pair of identical cells.

$$Pf = \frac{\sharp[\texttt{Output}] - \sharp[\texttt{Input}] + \sharp[\texttt{Terminate}]}{\sharp[\texttt{Newcall}]} \tag{24}$$

where $\sharp$[Block], for example, denotes the number of calls that pass the Block node. We have $Pb + Pc + Pf = 1$ because of the conservation relationship

$$\sharp[\texttt{Newcall}] + \sharp[\texttt{Input}] = \sharp[\texttt{Block}] + \sharp[\texttt{Serve}] + \sharp[\texttt{CallEnd}] + \sharp[\texttt{Terminate}] \tag{25}$$

In the numerical examples given in Section 5, we have generated 1,000,000 calls before terminating the simulation run in each setting of the parameter values.

5. Numerical Examples

We have used both the analytic and simulation models to evaluate the probabilities Pb, Pc, and Pf for several networks with various topologies and system parameters. The numerical values for these probabilities from the analytic and simulation models generally agree well, which validates our analytic approach. The results also reveal some interesting characteristics in these probabilities.

Below we show three example networks among others we have considered; the other examples are found in [7]. In all the examples, we have assumed $C = 20$ and $g = 1$, and truncated the infinite sum with respect to the system states such as in equations (3) and (5) by $k = 50$.

5.1. A pair of identical cells

The first example is a system of two statistically identical cells [8]. Hand-off occurs between the two cells and the mobiles do not go outside the system. This is a symmetric network considered in Section 3.3 with $n = 2$ and $\gamma = 0$. We assume that $\mu_1 = 1$, $\mu_2 = 2$, and $\mu_3 = 10$. This setting corresponds to the case in which the mean duration of a call is

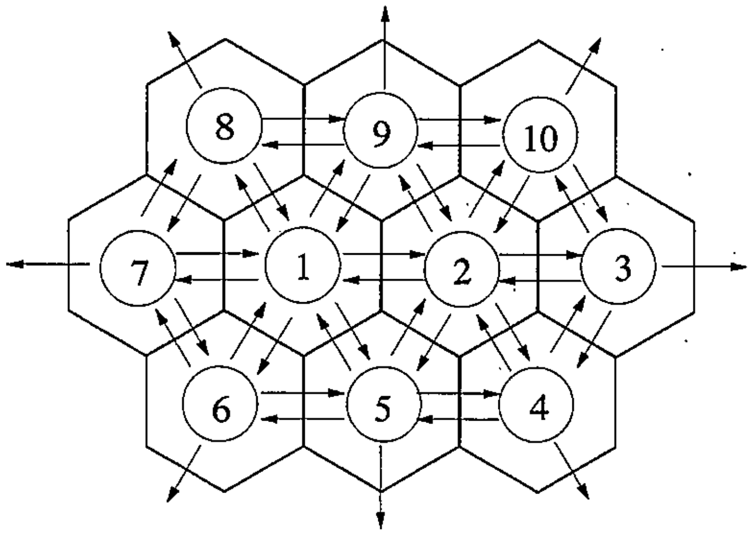


Figure 5. A finite service area with hexagonal cells.

1 minute, mobiles are on the cars that pass microcells of diameter 500m with the speed of 60km/h, and the diameter of the hand-off area is 50m.

Figures 4(a) and (b) plot the probability Pb of call blocking and the probability Ph of hand-off failure as functions of the generation rate λ_1 of new calls from the analytic and simulation models. They both increase as λ_1 as expected. It is also seen that $Ph \ll Pb$, which is desirable because the user would usually prefer to get a busy tone on dialing rather than the forced termination in the middle of his conversation. The agreement between simulation and analysis is good, although the analysis overestimates both Pb and Ph in this particular example.

5.2. A finite service area with hexagonal cells

For the second and third examples, we consider a finite service area that consists of 10 hexagonal cells as shown in Figure 5. While hand-off may occur between neighboring cells, calls of the mobiles going outside the service area are inevitably terminated.

In the second example, we assume that $\mu_1^{(i)} = 1$, $\mu_2^{(i)} = 1$, and $\mu_3^{(i)} = 10$ for all cell i as in the first example. We also assume that the movement of mobiles are isotropic, namely, homogeneous in direction. Thus we have

$$\gamma_{ij} = \begin{cases} \frac{1}{6} & \text{if cells } i \text{ and } j \text{ are neighbors} \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

and γ_{ie} is given by equation (4). Figures 6(a)–(c) plot the probabilities Pb, Pf, and Pc, respectively, when $\lambda_1^{(i)}$'s are uniformly increased. The solid lines show the values from the analysis and the dotted lines those from the simulation. Figure 6(d) displays their relative values as the destiny of an arbitrary call. From this figure, we see that the call loss probability Pb is negligible and the probability Pf of forced termination and the probability Pc of call completion are constant until the traffic intensity $\lambda_1^{(i)}/\mu_1^{(i)}$ of new calls reaches the cell capacity $C = 20$. The reason for $Pf > 0$ even when $\lambda_1^{(i)} \approx 0$ is that mobiles are assumed to go outside the service area with a certain probability. After $\lambda_1^{(i)}/\mu_1^{(i)}$ exceeds C, the probability Pb increases, Pc decreases, and Pf decreases slightly.

In the third example, we assume different traffic parameters for each cell as given in Table 1. The different values for $\mu_2^{(i)}$ imply the different sizes of the cell. We also assume that the movement of mobiles are not isotropic so that the matrix (γ_{ij}) is given Table 2.

cell i	1	2	3	4	5	6	7	8	9	10
$\lambda_1^{(i)}/\lambda_1^{(1)}$	1	1	2/3	1/2	2/3	2/3	3/4	1/2	3/4	2/3
$\mu_1^{(i)}$	1	1	1	1	1	1	1	1	1	1
$\mu_2^{(i)}$	2	2	1	1	1	1	1	1	1	1
$\mu_3^{(i)}$	10	10	10	10	10	10	10	10	10	10

Table 1. Parameters for the network of 10 cells.

$i\backslash j$	1	2	3	4	5	6	7	8	9	10	e
1	0	1/4	0	0	1/6	1/6	1/6	1/12	1/6	0	0
2	1/2	0	1/6	1/12	1/12	0	0	0	1/12	1/12	0
3	0	1/2	0	1/3	0	0	0	0	0	1/12	1/12
4	0	1/3	1/12	0	1/2	0	0	0	0	0	1/12
5	1/3	1/3	0	1/12	0	1/12	0	0	0	0	1/6
6	1/3	0	0	0	1/4	0	1/4	0	0	0	1/6
7	1/2	0	0	0	0	1/4	0	1/6	0	0	1/12
8	1/4	0	0	0	0	0	1/3	0	1/4	0	1/6
9	1/3	1/5	0	0	0	0	0	2/15	0	2/15	1/5
10	0	1/3	1/4	0	0	0	0	0	1/4	0	1/6

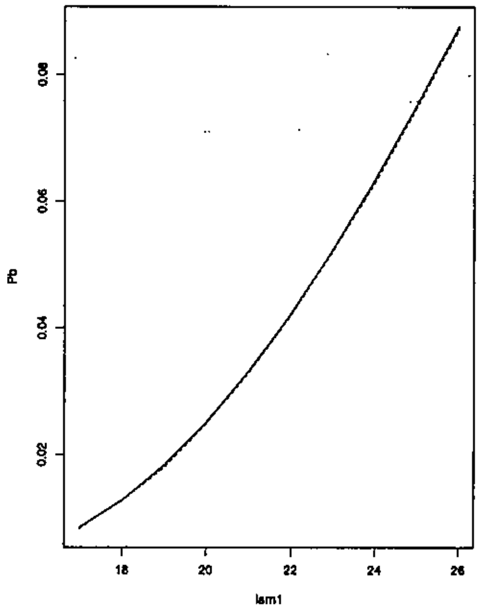
Table 2. Matrix of probabilities for the movement of mobiles.

Figures 7(a)-(d) plot the probabilities Pb, Pf, Pc, and their relative values, respectively, when $\lambda_1^{(1)}$ is increased. The tendency in this example is similar to that in the second example shown in Figure 6, except that the probability Pf of forced termination increases very slightly in some range of λ_1. However, the overall destiny of an arbitrary call shown in Figure 7(d) is that Pf is almost constant while the increase in Pb results in the decrease in Pc after $\lambda_1^{(1)}/\mu_1^{(1)}$ exceeds C. In another example network in which mobiles do not go outside the service area (not shown in this paper; see [7]), we have also observed that Pf does not increase much as $\lambda_1^{(1)}$ increases while it is zero at $\lambda_1^{(1)} = 0$.

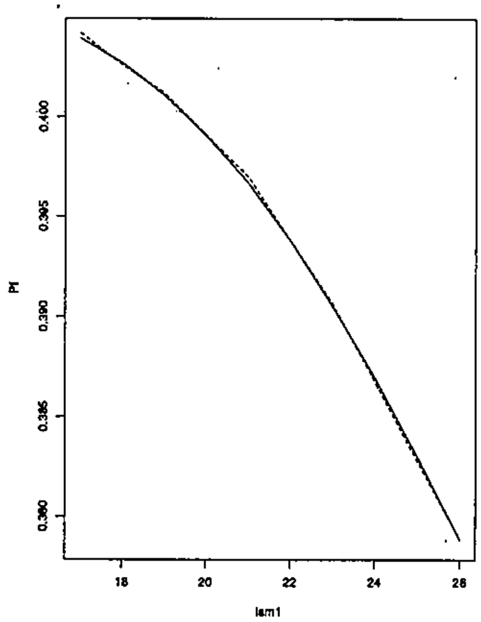
6. Summary and Comments

In this paper we have presented a traffic model for cellular communication networks with arbitrary cell connection and arbitrary probabilistic movement of mobiles between the cells, and evaluated the probabilities of call loss, completion, and forced termination. The accuracy of our approximate analysis has been validated by simulation. In example networks with finite service area, we have observed that the call loss probability is negligible until the traffic intensity reaches the cell capacity, and that the probability of forced termination does not increase much as the traffic intensity increases further.

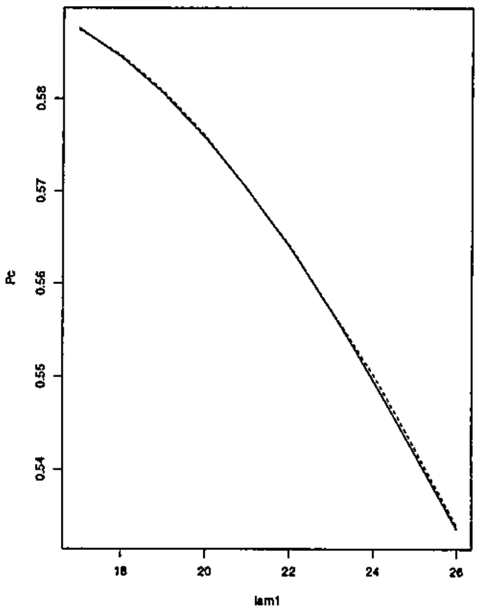
Our analytic model can be applied to networks of any number of cells with different parameters. However, the computation of hand-off rates by the procedure given in Section



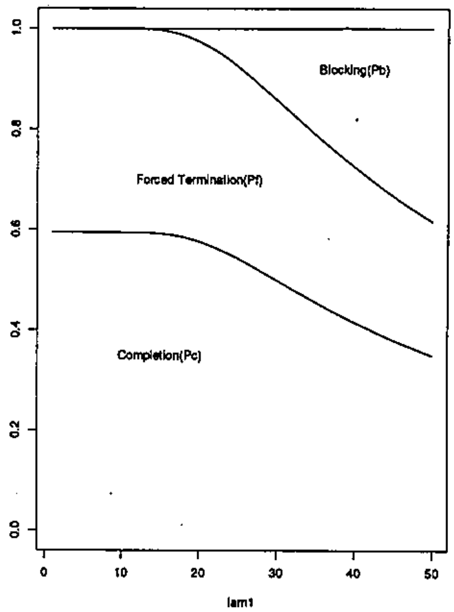
(a) Pb



(b) Pf

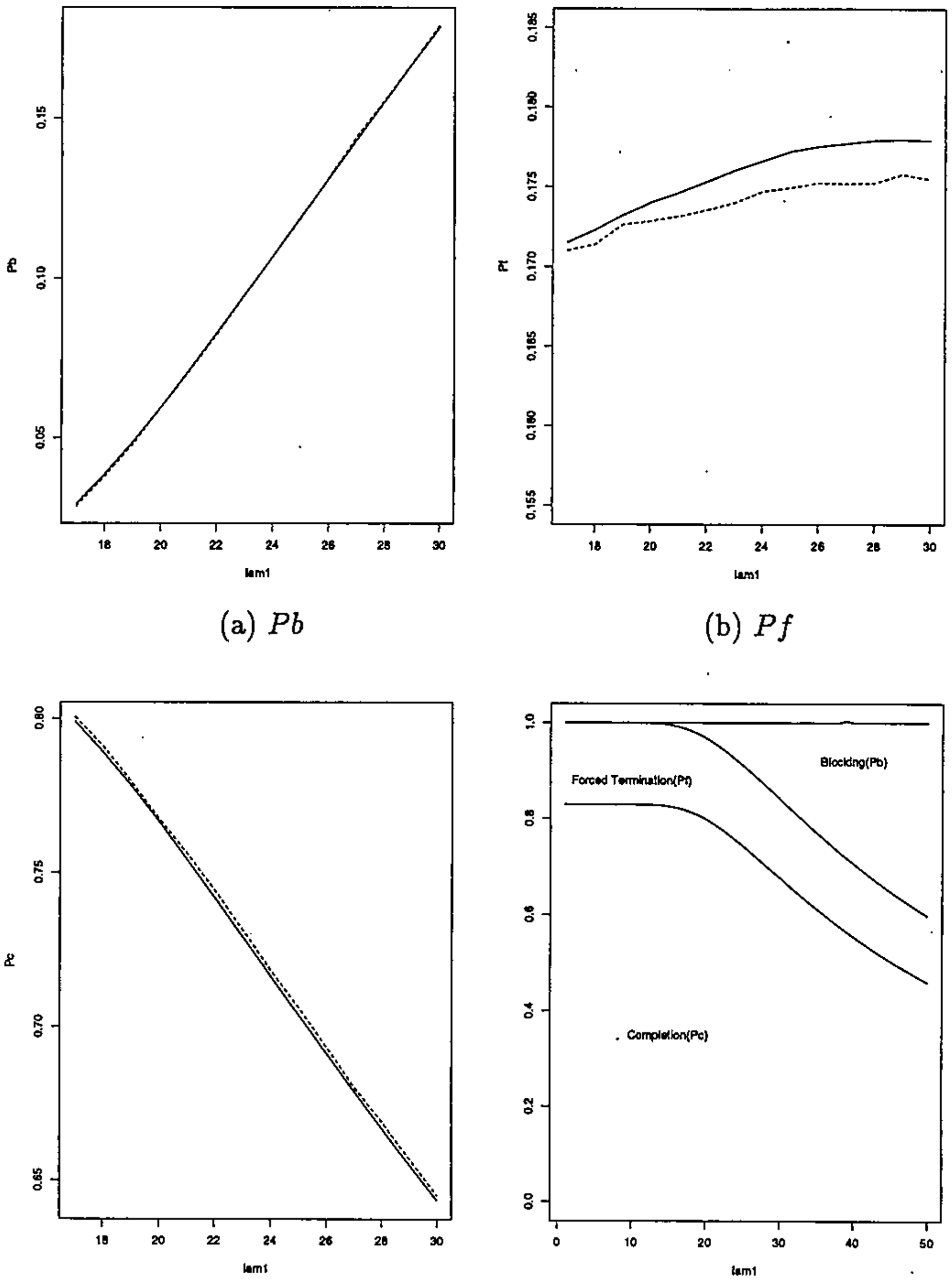


(c) Pc



(d) Destiny of a call

Figure 6. Performance of a network of 10 cells with isotropic movement of mobiles.



(a) Pb

(b) Pf

(c) Pc

(d) Destiny of a call

Figure 7. Performance of a network of 10 cells with different parameters.

2.2 takes the time in proportion to the number of cells in the network, because each step is executed for every sell. It is left to the future research to develop a technique that can be applied to realistic networks with thousands of cells.

A recursive approach to finding the probabilities of destiny of an arbitrary call generated in the network, given in Section 3.2, may be applied to cellular networks of other types of channel assignment mechanism than the fixed assignment assumed in this paper. For example, it would be interesting to compare the probabilities of destiny of a call between the networks with fixed channel assignment and those with dynamic channel assignment (DCA) [9]. A similar approach may also handle the cellular networks with code division multiple access (CDMA) once the method for evaluating the probabilities of call loss and hand-off failure is established for individual cells in the network.

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