No. 720

Collusive Price Leadership under Repeated Games

by

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April 1997

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Abstract

In this paper we shall try to formulate a model to explain price leadership in the framework of a repeated game. In each component game of our model, the order of firms in chooseing standard strategic variables will be assumed to be endogenous. Assuming the identical technology (cost function) and symmetric information among firms, we will show that firms' collusion can be sustained by means of price leadership even under the situation where their collusion cannot be sustained when firms simultaneously choose their standard strategies. One framework to enable price leadership equilibrium plausible is that all firms become the leader in turns (rotating price leadership) or different firms become the leader in different markets (divided price leadership).

*I am especially grateful to my advisor, Prof essor Yoshihiko Otani for his help and encouragement. I would also like to thank Professors Hiroyuki Odagiri, Hitoshi Matsushima and Yoshikatsu Tatamitani for their helpful comments. Any remaining errors are my own.

1 Introduction

In this paper we shall attempt to formulate a model to explain price leadership in the framework of a repeated game. Price leadership is known to have various types in practice. We shall not explain all of them. The type we shall consider in this paper is the one in which one firm, i.e., the leader sets a price which is followed by accommodating price settings by other firms. Moreover we shall confine our analysis to a case of collusive price leadership among firms with symmetric technology and information.

Price leadership is often observed in oligopoly markets. It is traditionally discussed in industrial organization literature that in such a market a price leader would emerge among firms "in lieu of overt collusion". There leader's price is considered as a signal to communicate the collusive price to other firms.[1] Rotemberg and Saloner (1985,1990) also study collusive price leadership in the framework of a repeated game. Following the traditional consideration, their argument heavily depends on asymmetric information among firms. More concretely, they assume that firms face stochastic market demand and one firm has better information on the demand state than the

[1]See Scherer and Ross (1990).

other. *If information is sufficiently asymmetric, the less informed firm prefers to follow the better informed firm, so the leader can emerge endogenously (1990, p.93).* Suppose that both firms agree to form such a leader-follower relationship in their model. In each period, at the time when the better informed firm makes its strategic decision, the less informed firm can observe the action chosen by the better informed firm but, as it cannot observe the demand state accurately, does not know whether the better informed firm deviates from its collusive strategy. Nevertheless their collusion will be sustained as long as the less informed firm believes that the better informed firm sets the collusive price and the less informed firm charges the same price set by the better informed firm. Then the leader (the better informed firm) will be able to secure more profit than the follower (the less informed firm).

In this paper we suppose symmetric information. Under symmetric information we show that leader-follower relationship may emerge. Moreover we argue that price leadership has another aspect on the collusion, i.e., to reduce the firms' incentive of deviation from the collusion. In this paper we regard a price leader as a firm which fixes its price and a market share before other firms' choice of strategies. Price and share are supposed to be fixed during the period once a firm chooses them. Therefore, if a leader firm

deviates, the other firms, the contrary to Rotemberg and Saloner, notice the deviation and can punish the leader in the period. Thus a leader firm gets no gain from deviation. Hence it will not deviate. The fact that the leader will not deviate in turn provide other firms incentive not to deviate.

In addition to providing a theoretical explanation on why price leadership emerges, our model enables us to explain interesting methods of implementing price leadership. They are rotating price leadership and divided price leadership. These types of price leadership have often been observed and have usually been considered to be the ways to camouflage cartels against the antitrust law.[2] However we shall explain them as schemes to stabilize and facilitate implicit collusion.

We shall set up the following model. The component game of the repeated game has three stages. In addition to the standard strategic variables (in this paper, price or share), at stage 0 in each period, firms are supposed to select the stage (1 or 2) in which they make strategic decisions. In other words, the order of firms' play is endogenous. We call a firm choosing the first stage "the leader" and the other "the follower". Under this setting we shall

[2]See e.g. Machlup (1952).

consider two different types of strategy combinations. One type is that both firms select the same stage. The other is that one firm becomes the leader and the others followers. We call the former Simultaneous Strategy and the latter Leadership Strategy.[3] We shall argue that the collusive outcome can be sustained by means of the Leadership Strategy even if the discount factor is too low to collude in using the Simultaneous Strategy.

However this result is with a proviso that the allocation to be sustained is one such that the followers are more profitable than the leader. Hence it may be difficult to justify why a particular firm would willingly take the position of the leader. We shall consider rotating price leadership and divided price leadership as means to evade such difficulty. Rotating price leadership is a scheme in which firms become the leader in turn and divided price leadership is one in which different firms act as the leader in different markets. We interpret both types of price leadership as schemes which equalize the leader's disadvantage among firms and stabilize their collusion. We shall give the detailed explanation of these phenomena in the text.

Our argument proceeds as follows. Section 2 provides a basic model. In

[3]We shall give more precise definitions of these strategies in the text.

section 3 we obtain the condition that Simultaneous Strategy is enforceable. In section 4 we obtain the condition that Leadership Strategy is enforceable and compare it to Simultaneous Strategy. We consider rotating price leadership and divided price leadership in section 5 and 6 respectively. We have concluding remarks in section 7.

2 Basic Model

There are n firms in the industry, where $n \geq 2$. They will be assumed to play an infinitely repeated game. Each period t of the repeated game is made up of the component game which has the following properties. Each firm produces a homogeneous good. The market demand function is $x = D(p)$, and its inverse function is denoted by $p = P(x)$ where $xP(x)$ is concave in x. Cost functions obey constant returns to scale and firm i's marginal cost is denoted by c_i. In this paper we shall assume an identical cost, i.e., $c_i = c$ for all i. Each firm chooses its price p_i and its market shares $s_i \in [0, 1]$ such that $\sum_{i=1}^{n} s_i = 1$ when they collude. Otherwise m firms charging the lowest price get the whole demand and each of these firms gets $\frac{1}{m}$ of the whole demand. Now we shall introduce the following additional assumption.

Assumption

Each component game has three stages. At stage 0 each firm can choose whether it chooses its action pair (p_i, s_i) at stage 1 or 2. If a firm chooses the pair at stage 1, the firm cannot change it at stage 2. If a firm chooses it at stage 2, the firm can observe the actions chosen at stage 1. Each firm cannot choose its action pair at the stage not chosen at stage 0. Each firm chooses both the price and share at the same stage.[4]

The underlined part of the assumption may not be applicable to price competition, since implies that price and share have some commitment effects. Price is usually considered to be adjustable and thus we may need some justification on the fixty of price. We shall consider this point in the concluding remarks and proceed our argument on this assumption.

As a result of this assumption, each firm has a chance to reveal its price and share to other firms. The decision on selecting the order of decision making will be denoted by $o \in \{L, F\}$. L (F) means that the firm chooses its price and share at stage 1 (stage 2) respectively. Hence an action triple which firm i selects in each period will be expressed as (p_i, s_i, o_i). The firm

[4]For example, a firm cannot set its price at stage 1 and choose its share at stage 2 and so forth.

selecting L (F) shall be called the leader (the follower) respectively.

This component game has a unique static subgame perfect equilibirum, in which all firms choose F and charge c. Setting prices slightly below leaders' price is the best response provided that leader's price is greater than c. Thus choosing L and the price more than c is not the best response to any firms. Thus all firms choose F and we end up with the usual Bertrand equilibrium at the price c. Note that each firm has the zero profit in this equilibrium.

Now we consider the repeated game where the component game we described is replicated infinite times. Let δ denote the discount factor which is assumed to be common to all firms where $\delta \in [0, 1)$.

3 Simultaneous Strategy

In this section we consider a particular collusive strategy of our infinitely repeated game in which firms set their prices and choose their share simultaneously in each period. Each firm will not choose L voluntarily since the follower has a chance to get the whole demand by under-cutting leader's price. Hence the case that all firms choose F is natural. Thus we shall consider the following strategy.

Simultaneous Strategy (SS)

Let us define the collusive rule as follows. Each firm chooses F and sets a collusive price p and chooses a collusive share s_i such that $\sum_{i=1}^{n} s_i = 1$. All firms obey this rule in period 1. At stage 0 in period t each firm chooses F independently of actions of all firms in periods 0 to $t-1$. At stage 2 in period t each firm sets the collusive price and chooses the collusive share as long as all firms obey the collusive rule in every period preceding t and choose F at stage 0 in period t. Otherwise all firms set their prices as $p = c$ forever.

In this strategy firm i's per-period collusive profit is $s_i\pi(p)$, where $\pi(p) = (p-c)D(p)$. $\pi(p)$ is increasing on p in the interval $[0, p^{max}]$ and decreasing above p^{max}, where we use p^{max} by denoting the monopoly price. In using SS each firm has the possibility that it deviates at stages 0 or 2. However each firm has no incentive to deviate at stage 0 in obeying SS, because it does not gain by doing so. Thus it suffices to check whether a firm deviates at stage 2 in any period to show the sustainability of SS. Therefore a necessary and sufficient condition that SS is self-enforceable becomes as follows:

for all i,

$$(1 - s_i)\pi(p) \leq \frac{\delta}{1-\delta}\{s_i\pi(p)\}. \tag{1}$$

The left-hand side is the gain from deviation. A deviating firm can get the whole demand in this market by under-cutting its price. Here its optimal deviation is to undercut its price slightly and get demand $(1-s_i)D(p)$. [5] On the other hand the right-hand side is the discounted sum of collusive payoffs for a firm when all firms continue to maintain the collusive price and share. We can reduce these inequalities to the following:

$$\delta \geq 1 - s_i. \tag{2}$$

Note that this condition does not depend on the collusive price p. Thus we may suppose for simplicity that firms always charge p^{max} if they collude.

The inequality (2) says that the collusion is sustained if and only if $\delta \geq \max_i\{1-s_i\}$. Thus there are many combinations of firms' collusive shares. However it is plausible that $s_i = \frac{1}{n}$ for all i because of symmetric technology and simultaneous move. Thus we may proceed our argument on the assumption that the collusive equilibrium is $p = p^{max}$ and $s_i = \frac{1}{n}$ for all i in using SS. Then the collusion is sustained if and only if $\delta \geq \frac{n-1}{n}$. In

[5] More precisely the left-hand side is

$$\sup_{\epsilon}\{(p-\epsilon)D(p-\epsilon)\} - s_i\pi(p) = (1-s_i)\pi(p).$$

other words, if $\delta < \frac{n-1}{n}$, then it is impossible for firms to collude as long as firms are following SS.

4 Leadership Strategy

In this section we introduce a particular leadership strategy of our infinitely repeated game. This strategy will be suitably modified in the later section.

Even in such a case as $\delta < \frac{n-1}{n}$ where the collusion under SS can not be sustained, we shall argue that firms could sustain their collusion provided that they use the following leadership strategy.

Leadership Strategy (LS)

(i) Before playing the game, all firms select a firm which chooses L in every period, that is, a firm acting as the leader. Let us define the collusive rule as follows. A firm, which is selected as the leader, chooses L, sets a collusive price p^{max}, and chooses a collusive share s_L. Other firms choose F, set the collusive price p^{max} and the collusive share $\frac{1-s_L}{n-1}$. All firms obey this collusive rule in period 1.

(ii) The firm which acts as the leader chooses L at stage 0 in period t as long as all firms obey the collusive rule in every period preceding t. Otherwise it

reverts to the static subgame perfect equilibrium from period t forever. it chooses (p^{max}, s_L) at stage 1 in period t only when it chooses L and others choose F at stage 0 in period t. Otherwise it sets its price as $p = c$ at stage 1 in period t and continues to play the static subgame perfect equilibrium from period $t+1$ forever.

(iii) Each firm which acts as the follower chooses F at stage 0 in every period. On the premise that all firms continued to obey the collusive rule in every period preceding t, it chooses $(p^{max}, \frac{1-s_L}{n-1})$ at stage 2 in period t only when the leader firm chooses (L, p^{max}, s_L) in period t and other firms choose F at stage 0 in period t. Otherwise it sets its price $p = c$ at stage 2 in period t and plays the static subgame perfect equilibrium from period $t+1$ forever.

The reason why the follower's share is assumed to be $\frac{1-s_L}{n-1}$ is that all the followers have symmetric technology and simultaneous move.

If all firms follow LS, we can see that the leader will have no incentive to deviate this collusive state because it can get no gain from deviation. Since the leader's deviation yield zero profit even within the deviating period. There is no possibility that the follower bothers to choose L as written in part (ii) since the leader sets its price as $p = c$ in stage 1 of the same period. Thus

it yields no gain. Hence, in order to show that LS is enforceable, we need only check whether the follower deviates at stage 2 in each period. Therefore LS is enforceable if and only if

$$(1 - \frac{1 - s_L}{n - 1})\pi(p^{max}) \leq \frac{\delta}{1 - \delta}\frac{1 - s_L}{n - 1}\pi(p^{max}). \tag{3}$$

The left-hand side is the gain from deviation and the right-hand side is the long-run losses. From this inequality we directly get the next proposition.

Proposition 1

Suppose that all firms follow the strategy LS. Then the collusion is sustainable if and only if

$$\delta \geq \delta^{LS}(s_L, n) \equiv 1 - \frac{1 - s_L}{n - 1}. \tag{4}$$

Now we confirm that $\delta^{LS}(s_L, n)$ has the following characteristics:

(i) for any s_L^1 and s_L^2 such that $1 \geq s_L^1 \geq s_L^2 \geq 0$, $\delta^{LS}(s_L^1, n) \geq \delta^{LS}(s_L^2, n)$,

(ii) $\delta^{LS}(\frac{1}{n}, n) = \frac{n - 1}{n}$.

LS as well as SS is enforceable when $\delta \geq \frac{n-1}{n}$, though the use of LS would be more interesting when $\delta < \frac{n-1}{n}$. But s_L needs to be in the range $[0, \frac{1}{n})$ so that using LS makes firms collude when $\delta < \frac{n-1}{n}$. Note that the value of

$\delta^{SL}(s_L, n)$ will lie in $[1 - \frac{1}{n-1}, 1 - \frac{1}{n})$ if $s_L \in [0, \frac{1}{n})$. We rewrite this range as follows:

$$\delta^{LS} \in [\frac{n-2}{n-1}, \frac{n-1}{n}). \tag{5}$$

Price leadership can emerge when $\delta \geq \delta^{LS}$. Thus we can state the followings.

1. Even if $\delta < \frac{n-1}{n}$, using LS may make a collusion possible.

2. However $s_L < \frac{1}{n}$. In other words, the leader has less share and less profit than followers to sustain the collusion.

Thus we have shown that the introduction of LS enables firms to collude and get higher profits than using SS even if the discount factor is sufficiently low. However, finding a leader would be difficult, since the leader will have to expect less profit than followers. This motivates us consider a rotation system as a way to evade this difficulty in the next section.

Price leadership is more likely to be observed in concentrated industries. In our model we get the conclusion which is similar to this observation. In the inequality (4), if s_L is fixed, the range of the discount factor which LS is enforceable shrinks as the number of firms n increases. Moreover the range

(5) shrinks as n increases.[6] The range (5) is the additional one in which collusion becomes to be sustained by swithcing from SS to LS.

5 Rotating Price Leadership

In this section we consider the rotating price leadership. This rotation system is often observed in practice when firms in an industry, as we assume in this paper, have similar scales or technologies. As examples we may mention the cases of the cigarette industry in USA during 1920s and 1930s or the carrent beer industry in Japan.

In order to have the plausibility of collusion under a price leader, we will consider the next strategy.

Rotating Leadership Strategy (RS)

(i) Let us define the collusive rule as follows. A firm, which is selected *randomly* at the beginning of each period, chooses L, sets the collusive price p^{max}, and the collusive share s_L, where s_L is assumed to be identical among

[6] From (5) we get

$$\frac{n-2}{n-1} - \frac{n-1}{n} = \frac{1}{n(n-1)}.$$

leaders in all periods. [7] Other firms choose F, set p^{max}, and choose the share $\frac{1-s_L}{n-1}$. All firms obey this collusive rule in period 1.

(ii) In period t, a firm, which is selected *randomly* at the beginning of the period, behaves the same way as in part (ii) of the definition of LS. Other firms behaves the same way as in part (iii) of the definition of LS.

The distinctive feature of this strategy is that all firms potentially assume the leadership in turn. On this ground, the rotation system may be considered as a rule to evade the disadvantage of leader firms.

Like LS it suffices to check whether the follower deviates at stage 2 in every period in order to show that firms collude in using RS. Thus RS is enforceable if and only if

$$(1-\frac{1-s_L}{n-1})\pi(p^{max}) \leq \frac{\delta}{1-\delta}\{\frac{1}{n}s_L\pi(p^{max})+\frac{n-1}{n}\frac{1-s_L}{n-1}\pi(p^{max})\}. \quad (9)$$

The left-hand side is the gain from deviation as in the inequality (3). In RS each firm becomes the leader with the probability $\frac{1}{n}$ and the follower with the probability $\frac{n-1}{n}$. Thus the right-hand side is the long-run loss.

From the inequality (9) we get the next proposition directly.

[7] "randomly" means drawing lots in which each firm gets a winning number with probability $\frac{1}{n}$.

Proposition 2

Suppose that all firms follow the strategy RS. Then collusion is sustainable if and only if

$$\delta \geq \delta^{RS}(s_L, n) \equiv \frac{1}{1 + \frac{n-1}{n(n+s_L-2)}}. \tag{10}$$

The derivation of this inequality is as follows:

$$(9) \Leftrightarrow \quad (1 - \frac{1-s_L}{n-1}) \leq \frac{\delta}{1-\delta}(\frac{1}{n}s_L + \frac{n-1}{n}\frac{1-s_L}{n-1}) \tag{11}$$

$$\Leftrightarrow \quad n(n+s_L-2) \leq \frac{\delta}{1-\delta}(n-1) \tag{12}$$

$$\Leftrightarrow \quad n(n+s_L-2) \leq \delta\{n-1+n(n+s_L-2)\} \tag{13}$$

$$\Leftrightarrow \quad \delta \geq \frac{n(n+s_L-2)}{n-1+n(n+s_L-2)}. \tag{14}$$

$\delta^{RS}(s_L, n)$ also has the same characteristics as $\delta^{LS}(s_L, n)$ in section 4. Thus s_L needs to be in the range $[0, \frac{1}{n})$ so that using RS makes firms collude when $\delta < \frac{n-1}{n}$. If $s_L \in [0, \frac{1}{n})$, the value of $\delta^{RS}(s_L, n)$ will lie in the range as follows:

$$\delta^{RS} \in [\frac{n(n-2)}{n^2-n-1}, \frac{n-1}{n}). \tag{15}$$

The lower bounds of the ranges (5) and (15) are different and we confirm

from simple calculations that for any $n \geq 2$,

$$\frac{n-2}{n-1} \leq \frac{n(n-2)}{n^2-n-1}. \tag{16}$$

In other words, in using LS firms can collude when the discount factor is lower than in using RS. The reason is as follows. From the inequality (3) we see that decreasing s_L not only decreases the gain from deviation but also increases the per-period collusive profit when firms obey LS. On the other hand from the inequality (12) we see that the ratio of the per-period collusive profit to the gain from deviation is $n-1$ to $n(n+s_L-2)$, which is the second term of the denominator of the right-hand side in the inequality (10). Thus we see that decreasing s_L has the only effect decreasing the gain from deviation when firms obey RS. Hence the collusive range in using RS is smaller than in using LS.

We may state the following economic implications about proposition 2.

1. Collusion is sustained even if the discount factor δ is lower than $\frac{n-1}{n}$.

2. Each firm's *ex-ante* profit is the same among all firms.

3. $s_L < \frac{1}{n}$ if $\delta < \frac{n-1}{n}$.

The implication 1 means that firms can collude by obeying RS even under a

situation where their collusion can not be sustained by obeying SS. This is similar to the case for firms obeying LS. However the range of the discount factor where price leadership is sustainable is smaller in using RS than in using LS. Implications 2 and 3 mean that while adopting RS insures the industry-wide profit of the monopoly level and the equality of each firm's expected profit, each leader is required to reduce its share to sustain the collusion. That is, a leader will incur short-run loss of assuming leadership, but the loss of being a leader can be compensated by being followers.

The rotation system has been considered to be a method in which firms camouflage a cartel. However this phenomenon seems to be observed independent of the existence of a cartel. For example, in the Japanese beer industry, the Fair Trade Commission has never disclosed any cartel, though the rotating price leadership by Supporo and Asahi has been observed. Thus it may be more natural to consider the rotating price leadership as one form of implicit collusion. Proposition 2 seems to provide a satisfactory explanation of these observed phenomena. We shall consider another way to evade the difficulty in which the leader firms less profit than others in the next section.

6 Divided Price Leadership

Divided price leadership is a case which firms in an industry sell in multiple markets and different firms act as leaders in different markets. For example, in American glass container industry this type of price leadership has been observed.[8] Another example may be the cigarette industry in USA after World War II.[9]

To explain these facts we may use a model of multimarket contact. [10] In a model of multimarket contact markets do not have interdependency each other. Now we shall modify the per-period demand and cost function in our model as follows. There are two firms, i and j, in this industry. Both firms produce two kinds of goods a and b. We assume that the demands for good a and b are not related to each other. Thus the demand curves for good a and b are respectively denoted by $D^a(p^a)$ and $D^b(p^b)$ where p^a and p^b denote respectively prices of good a and b. For simplicity we suppose that both demand functions are symmetrical, that is, $D^a(p) = D^b(p) = D(p)$ for any p.

[8]See Machlup, p.500-501.

[9]See the detail in Schere and Ross, p.251

[10]Bernheim and Whinston (1986) and Matsushima (1993) study the relationship between multimarket contact and implicit collusion.

Both firms have identical cost functions. Cost functions of both goods obey constant returns to scale and the marginal costs of producing both goods are the same. The marginal cost is denoted by c.

Under the above setting, a necessary and sufficient condition that SS is self-enforceable is as follows:

for all i,

$$(1-s_i^a)\pi(p^a)+(1-s_i^b)\pi(p^b) \leq \frac{\delta}{1-\delta}\{s_i^a\pi(p^a)+s_i^b\pi(p^b)\}, \qquad (17)$$

where $\pi(p) \equiv (p-c)D(p)$. Again , by the symmetry, we shall assume that $s_i^a = s_j^a$ and $s_i^b = s_j^b$. Then we reduce the inequality (17) reduces to:

$$\delta \geq \frac{1}{2}.$$

SS is not enforceable when $\delta < \frac{1}{2}$. Then firms may use LS or RS enabling firms collude even when $\delta < \frac{1}{2}$. However in this setting we can find a better strategy.

Consider the following strategy.

Divided Leadership Strategy (DS)

(i) Let us define the collusive rule as follows: Firm i chooses L^a and F^b and firm j chooses F^a and L^b at stage 0. Firm i sets the collusive price p^a and the collusive share s_L^a and firm j sets the collusive price p^b and the collusive

share s_L^b at stage 1. Firm i sets the price p^b and the share $1 - s_L^b$ and firm j sets the price p^a and the share $1 - s_L^a$ at stage 2. All firms obey this collusive rule in period 1.

(ii) At stage 0 in period t, firm i chooses L^a and F^b and firm j chooses F^a and L^b as long as all firms obey the collusive rule in every period proceeding t. Otherwise both firms play the static subgame perfect equilibrium in both markets from period t forever.

(iii) On the premise that all firms continued to obey the collusive rule in every period proceeding t, at stage 1 in period t firm i sets p^a and s_L^a and firm j sets p^b and s_L^b only when firm i chooses L^a and F^b and firm j chooses F^a and L^b at stage 0 in period t. Otherwise both firms sets their price as $p^a = p^b = c$ and play the static subgame perfect equilibrium in both markets from period $t + 1$ forever.

(iv) On the premise that as all firms obey the collusive rule in every period proceeding t and at stage 0 in period t, at stage 2 in period t firm i sets p^b and $1 - s_L^b$ and firm j sets p^a and $1 - s_L^a$ only when firm i sets the collusive price p^a and the collusive share s_L^a and firm j sets the collusive price p^b and the collusive share s_L^b at stage 1 in period t. Otherwise both firms sets their prices as $p = c$ and play the static subgame perfect equilibrium in both

markets from period $t+1$ forever.

In part (i) of DS, the actions which both firms choose in each period as long as they collude are stated. Part (i) says that firm i acts as the leader in market a and the follower in market b while firm j acts as the leader in market b and the follower in market a. From part (ii) and (iii), it is clear that firms can not gain from deviation when they deviate either at stage 0 or 1. Thus we need only find a condition firms do not deviate just at stage 2.

For simplicity we assume that $p^a = p^b = p^{max}$ and $s_L^a = s_L^b = s_L$. Then a necessary and sufficient condition that DS is self-enforceable is as follows: for all i,

$$(1 - s_L)\pi(p^{max}) \leq \frac{\delta}{1-\delta}\{s_L\pi(p^{max}) + (1 - s_L)\pi(p^{max})\}. \qquad (18)$$

The left-hand side is the gain from deviation. When using DS, each firm does not gain from deviation in the market where the firm acts as the leader because the leader is punished by the follower at stage 2. Hence each firm gets the gain from deviation only in the market where it acts as the follower. The right-hand side is the long-run loss when each firm is punished in both markets in the long-run. From the inequality (18) we get the following

proposition.

Proposition 3

Suppose that both firms follow the strategy DS. Then their collusion is sustainable if and only if

$$\delta \geq \delta^{DS}(s_L) \equiv \frac{s_L}{1+s_L}. \tag{19}$$

$\delta^{DS}(s_L)$ has the following characteristics:

(i) for any s_L^1 and s_L^2 such that $1 \geq s_L^1 \geq s_L^2 \geq 0$, $\delta^{DS}(s_L^1) \geq \delta^{DS}(s_L^2)$,

(ii) $\delta^{DS}(1) = \frac{1}{2}$.

Noting these characteristics, we may state the following economic implications of proposition 3.

1. Collusion is sustainable even if the discount factor is lower than $\frac{1}{2}$.

2. Each firm has the same profit.

3. A particular firm continues to be the leader in a particular market.

4. The leader's share may be bigger than the follower's share when $\delta^{DS} \in (\frac{1}{3}, \frac{1}{2})$.

The implication 1 is the same as Propositions 1 and 2. The implication 2 says that being the leader does not make firms disadvantagejous either in the

long-run or in the short-run while firms incur some losses of being the leader in LS and RS.

The word "price leadership" often invokes an impression that a special firm continues to be the leader and gain more profit than others. Implication 3 and 4 may explain part of this impression. However our model do not explain the type of price leadership that a firm acts as the leader *of a whole industry*. This case may typify the image of "price leadership".

Nevertheless our model appears to explain the case of the cigarette industry in USA after World War II when American Tabacco acted as the leader in nonfilter cigarettes while Reynolds was the leader in filter tips. Moreover in the market where one company was the leader, it had a wider share than the other and each acted as the follower in the other market.[11] This observation fits well the characterization of the DS equilibrium in our model.

7 Concluding Remarks

In this paper we have considered the effect of price leadership on firms' collusive behavior. We have shown that price leadership enables firms collude

[11]See in detail Schere and Ross, p251.

even if the discount factor is too low to sustain their collusion without price leadership and firms gain more collusive profits. However at the collusive equilibrium, the leader is less advantageous than followers. To avoid this problem we have investigated cases of rotating price leadership and divided price leadership.

In practice the phenomena that a special firm continues to be the leader of a whole industry are often observed. Probably the leader firm is considered to have special power which other firms do not have. Asymmetric information may provide a firm with such special power, as Rotemberg and Saloner have shown. Their assumption that a firm has more information on the market demand may not be persuasive in justifying collusive price leadership because such asymmetric information may disappear soon among firms which have similar technology and scale. Hence it remains a problem whether or not we can justify the presence of persistent asymmetric information in the long-run in a oligopolistic market.

The origin of leader's power may include technical advantage. However we have limited technology to be symmetric in this paper. If we allow asymmetric technologies, then we run into a difficulty in specifying a plausible asymmetric equilibirum point in the collusion. We would like to pursue fur-

ther investigation to allow the presence of asymmetry in our model.

Finally we would like to comment on our assumption in section 2 that price has some commitment effects. Commitment enables firms more likely to collude and the systems of rotating price leadership and divided price leadership strengthen firms' commitment. It is true that we may have little justification of our assumption on price commitment. However the fact that our model appears to explain some real phenomena may be our best justification. Among them there may be something more suitable for this thinking than price leadership.

References

[1] Bernheim, D. and M. Whinston (1990) "Multimarket Contact and Collusive Behavior." *Rand Journal of Economics*, Vol. 21, pp. 1-25.

[2] Machlup, F (1952) *TheEconomics of Seller's Competition*, Baltimore: The Johns Hopkins Press.

[3] Matsushima, H (1994) "Multimarket Contact, Imperfect Monitoring and Implicit Collusion." mimeo.

[4] Rotemberg, J.J. and G. Saloner (1985) "Price Leadership." Mimeo, Massachusetts Institute of Technology.

[5] Rotemberg, J.J. and G. Saloner (1990) "Collusive Price Leadership." *Journal of Industrial Economics*, Vol. 39, pp. 93-111.

[6] Scherer, F. and D. Ross (1990) *Industrial Market Structure and Economic Performance*, Third Edition, Houghton Mifflin.

[7] Tirole, J. (1988) *The Theory of Industrial Organization*, MIT press.