No. 714

A New Statistical Inference.

(Part A)

---Case of the Uniform Distribution U[θ, θ+1).---

by

Yoshika Nogami

February 1997

A New Statistical Inference.
(Part A)
----Case of the Uniform Distribution $U[\theta,\theta+1)$.---

by
Yoshika[*] Nogami
January, 1997

Institute of Socio-Economic Planning·Institute of Policy and Planning Sciences·University of Tsukuba

Abstract.

In this paper we use the uniform distribution with density $f(x|\theta)=1$ for $\theta\leqq x<\theta+1$; $=0$, otherwise for real θ. The purpose of this paper is to propose a test based on an optimal confidence sets $\{S^*(y)\}$ for testing the Hypothesis $H_0:\theta=\theta_0$ versus the alternative H_1: $\theta\neq\theta_0$.

(*) "Yoshika" is a pen name of the author's first name, Yoshiko.

§ 1. Introduction.

Let $I_A(x)$ be an indicator function such that $I_A(x)=1$ for $x \in A$; $=0$ for $x \notin A$. Throughout this paper the underlined distribution is the uniform with density $f(x|\vartheta)=I_{(\vartheta,\vartheta+1)}(x)$ for real ϑ. We also let $X_{(i)}$ $(i=1,2,\ \ldots\ ,n)$ denote the i-th smallest observation of $X_1,\ \ldots\ ,X_n$, taken randomly from the population with density $f(x|\vartheta)$. Let α be a real number such that $0<\alpha<1$. To test the hypothesis $H_0:\vartheta=\vartheta_0$ against an alternative $H_1:\vartheta\neq\vartheta_0$ at a size α, we use an unbiased estimator $Y \doteq (X_{(1)}+X_{(n)}-1)/2$ and construct uniformly most acculate (UMA) unbiased family of confidence sets $\{S^*(y)\}$ for ϑ at (confidence) level $1-\alpha$.

For notational convensions we denote the defining property by $\doteq$ and also let $h'(x) \doteq dh(x)/dx$.

§2. The Optimal Confidence Sets $\{S^*(y)\}$ at level $1-\alpha$.

In this section we shall construct the optimal confidence sets $\{S^*(y)\}$ at level $1-\alpha$.

Let us call $\{S(y)\}$ the family of confidence sets at level $1-\alpha$, when

(1) $$P_\vartheta\ [S(Y)\ni\vartheta]=1-\alpha, \qquad \text{for all } \vartheta\in(-\infty,\infty).$$

We consider, for each ϑ_0, the problem of testing $H_0:\vartheta=\vartheta_0$ versus any alternative K at size α. Let $A(\vartheta_0)$ be the acceptance region of a size-α (nonrandomized) test ψ of testing $H_0:\vartheta=\vartheta_0$ versus K;

(2) $$\psi(y)=\begin{cases}1, & \text{if } y\notin A(\vartheta_0),\\ 0, & \text{if } y\in A(\vartheta_0)\end{cases},$$

where

$$P_{\vartheta_0}\ [Y\in A(\vartheta_0)]=1-\alpha.$$

From Nogami(1997), we obtain the uniformly most powerful unbiased test ϕ^* of size α for testing $H_0:\theta=\theta_0$ versus $H_1:\theta\neq\theta_0$ as follows:

(3) $$\phi^*(y)=\begin{cases}1, & \text{if } y\notin A^*(\theta_0)\\ 0, & \text{if } y\in A^*(\theta_0),\end{cases}$$

where

(4) $$A^*(\theta_0)=\{y: \quad \theta_0-r<y<\theta_0+r\}$$

and

(5) $$r=(1-\alpha^{1/n})/2.$$

Define

(6) $$S^*(y)\doteq\{\theta: \quad y\in A^*(\theta)\}=\{\theta: \quad y-r<\theta<y+r\}.$$

Then, by Theorem 5.8.2 of Ferguson(1967) $\{S^*(y)\}$ is UMA unbiased family of confidence sets at level $1-\alpha$.

References

1). Ferguson, T. S.(1967). Mathematical Statistics---Decision Theoretic Approach, Academic Press.

2) Nogami, Y.(1997). "A New View on Statistical Inference.(Part I)--- the Case of the Uniform Distribution U[θ,θ+1)---" Discussion Paper Series No. 713, pp. 1-4.