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Causes for the seemingly anomalous behavior of  
agricultural households: an alternative  
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by

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Abstract

In case off-farm wage employment is binding, agricultural households show several types of seemingly anomalous behavior including their output supply of negative price elasticity, mainly because their organization of farm production and their choice of consumption (labor supply) are to be jointly determined. Conventionally, their anomalous behavior is ascribed to the income and the commodity-factor cross substitution effects which are not readily amenable to the standard theory of microeconomic analysis. This paper proposes an alternative decomposition of their comparative statics in terms of the internal (shadow) rate of wage, which is readily amenable to the standard theory.

## 1. Introduction

Several reasons cause the decision making in agricultural households indecomposable<sup>1</sup> in the sense that their organization of farm production and their choice of consumption (labor supply) are to be jointly determined (see, e.g., Benjamin, 1992; Jacoby, 1993; Lopez, 1986). Constrained off-farm wage employment among others seems to be most prevailing in many industrialized and semi-industrialized countries including Japan, which is closely related to the practice of off-farm employers who offer “higher than equilibrium rate of wage” to leave sufficient unemployment as a worker discipline device (Shapiro and Stiglitz, 1984; Bulow and Summers, 1986). Since off-farm employment is constrained, household members are obliged to put the remainder of their labor supply into their own activity of farm production. It is not the market rate of wage but the “market rate of wage minus an amount of discount” they are prepared to allow that is relevant in determining their farm employment demand and their supply of labor or alternatively their demand for leisure. Thus, this rate of discounted wage may be referred to as an “internal rate of wage” instrumental in establishing an equilibrium of the labor market within the household, which by its nature is closely related to the virtual price (Neary and Roberts, 1980; Deaton and Muellbauer, 1980) and to the implicit price of labor or endowed time (Maruyama, 1984). At the equilibrium rate of internal wage, the residual profit imputable to farm production activity is maximized and furthermore no incentive for seeking additional employment is left to household members since their supply rate of wage is equated to this internal rate.

Indecomposability of their organization of farm production and their choice of consumption (labor supply) of agricultural households has several very significant effects on the comparative statics analysis of their behavior under constrained off-farm wage employment. The income effect and others inherent in the comparative statics of their consumption choice creep into those of their farm organization and render both their demands for production factors and their supply of output less elastic. In

extreme cases they give rise to portions of upward-sloping demand and downward-sloping supply curves. Conventionally, results of the comparative statics analysis of their behavior are naively attributed to the income and the commodity-factor cross substitution effects <sup>2</sup> besides the factor substitution and the output effects in their behavior of farm organization and to the commodity-factor cross substitution effect besides the commodity substitution and income effects in their choice of consumption (labor supply) as exemplified by Maruyama(1975) and Singh, Squire, and Strauss (1986). To the regret of many seriously concerned, these effects are very unique but are not readily amenable to the standard theory of microeconomic analysis, which may cause this so called “indecomposable model” of agricultural households less attractive in spite of its analytical realism. The present paper intends to propose an alternative method of decomposition in terms of the internal rate of wage without resort to the commodity-factor substitution effects in general and to the income effects specifically in their organization of farm production.

The internal rate of wage is an endogenous variable to be determined so as to maximize the welfare of household members, but it plays a distinct role in equating the farm employment demand and the supply of labor by household members, while other endogenous variables represent the quantities of commodities or production factors to be determined. At its equilibrium rate the residual profit imputable to farm production as well as the welfare of household members is maximized and their demand and supply of all commodities and factors are equated within the household, hence the constraint on off-farm employment ceases to be binding. This distinct role of the internal rate of wage leads the present authors to derive an alternative method for decomposing results of the comparative statics analysis of agricultural households into two classes, one due to the direct effects of changes in selected exogenous variables and the other due to the indirect effects of the changes in these exogenous variables via the changes they cause in the internal rate of wage which may be referred to as the “internal wage effect” .

The following section discusses the equilibrium of agricultural households under the

constrained off-farm wage employment so as to introduce the internal rate of wage. The third section addresses the comparative statics analysis of this equilibrium so as to propose an alternative decomposition of the results of these analyses. The fourth section gives an example as to how the proposed decomposition is applied to a specific model of agricultural households. The last section concludes the paper.

## 2. Equilibrium of the agricultural household under constrained off-farm employment <sup>3</sup>

It is assumed that a sufficient degree of consensus is observed among different members of the household, so that their welfare function exists. The welfare function  $W$  takes on the following form:

$$W = U(C_1, C_2, Z; G) \quad (1)$$

, where  $C_1$ ,  $C_2$ , and  $Z$  denote respectively in this order the amounts consumed of a home produced farm commodity, purchased commodities, and leisure.  $G$  represents a shift parameter of this function. The welfare function  $W$  is assumed to possess regular well-behaved properties.

The agricultural household is subject to a number of constraints, the first of which is the budget constraint.

$$pC_1 + p'C_2 + wZ \leq M \quad (2)$$

, where  $p$ ,  $p'$ , and  $w$  denote respectively in this order the prices of a farm commodity, purchased commodities, and the market rate of wage. <sup>4</sup>  $M$  denotes the full income (Becker, 1965) to be defined in the following way.

$$M = wTe + \pi + V \quad (3)$$

$$\pi = pX - wL_1 - qF - OC$$

, where  $X$ ,  $L_1$ ,  $Te$ ,  $q$ , and  $F$  denote respectively in this order the amount of a farm commodity produced, hours of farm labor, endowed time of this household, the price of current inputs, and their quantity.  $V$  and  $OC$  denote respectively unearned incomes and other costs which are assumed to be exogenous.

The household is assumed to produce the amount  $X$  of a commodity and consume  $C_1$  ( $< X$ ) of it within the household. The amount produced  $X$  is bounded by the production possibility. For simplicity, it is assumed that there is no uncertainty associated with the production technology.

$$X \leq f(L_1, F; K, T) \quad (4)$$

, where  $K$  and  $T$  denote respectively the real stock of capital and the area planted which are assumed to be fixed.

Finally, the household is assumed to be self-employing and the off-farm work hours  $L_2$  are bounded by the off-farm employment opportunities  $L$ . This constraint is understood to be closely related to the practice of off-farm employers who offer “higher than equilibrium rate of wage” to leave sufficient unemployment as a worker discipline device (see, e.g., Shapiro and Stiglitz, 1984; Bulow and Summers, 1986).<sup>5</sup>

$$L_2 = T_e - Z - L_1 \leq L = \text{const.} \quad (5)$$

It is assumed that this household is a price taker in all markets and that there is no uncertainty in these markets. Then, the problem it faces is to maximize its welfare  $W$  subject to the constraints (2)-(5). The Kuhn-Tucker-Lagrange conditions of optimality related to this problem imply the following relations, where  $\lambda \geq 0$  and  $\mu \geq 0$  denote respectively the Lagrange multipliers associated with the budget constraint (2) and the constraint (5) related to the off-farm employment opportunities, and  $f_i$  and  $U_j$  denote respectively the first derivatives of the functions  $f(\cdot)$  and  $U(\cdot)$ .

$$\lambda p f_1(L_1, F; K, T) - \lambda w + \mu \leq 0 \quad (6.1)$$

$$\lambda p f_2(L_1, F; K, T) - \lambda q \leq 0 \quad (6.2)$$

$$U_1(C_1, C_2, Z; G) - \lambda p \leq 0 \quad (6.3)$$

$$U_2(C_1, C_2, Z; G) - \lambda p' \leq 0 \quad (6.4)$$

$$U_3(C_1, C_2, Z; G) - \lambda w + \mu \leq 0 \quad (6.5)$$

$$-p C_1 - p' C_2 - w Z + M \geq 0 \quad (6.6)$$

$$Z + L_1 + L - T_e \geq 0 \quad (6.7)$$

The relations (6.1), (6.3), and (6.5) in equality for interior solutions imply the following relations.

$$pf_1(\cdot) = w - \mu/\lambda < w$$

$$p \frac{U_3(\cdot)}{U_1(\cdot)} = w - \mu/\lambda < w$$

The supply price of labor  $pU_3(\cdot)/U_1(\cdot)$  falls short of the market rate of wage  $w$  due to severe off-farm employment constraint and household members still seek additional employment at the rate of wage lower than the market rate  $w$ . The term  $\mu/\lambda$  represents the amount of discount they are prepared to allow. Since no additional off-farm employment is available, they cannot but put the remainder of their endowed time into their own farm production activity. Hence the marginal revenue product of farm labor or the demand rate of wage within this household  $pf_1(\cdot)$  in turn falls short of the market rate by the amount  $\mu/\lambda$ . The residual profit  $\pi$  imputable to their farm production activity is not maximized at the market rate  $w$ . However, such behavior of them is rational since the amount of other components of their full income can exceed the decrement of residual profit so that their welfare can be improved.

The inequalities (6.1)-(6.2) directly associated with the determination of production organization share the Lagrange multipliers  $\lambda$  and  $\mu$  with the inequalities (6.3)-(6.5) directly associated with that of consumption choice. Hence, the system of inequalities (6.1)-(6.7) is indecomposable (Maruyama, 1975), so that the organization of farm production and the choice of consumption are to be jointly determined. Indecomposability of this system has very significant effects on its comparative statics, so that the income effect and others inherent in the comparative statics of consumption choice creep into those of farm production organization and render both the supply of commodities and demand for factors less elastic. In extreme cases, they give rise to downward-sloping supply and upward-sloping demand curves.

In case the off-farm employment opportunities are not binding hence  $\mu$  vanishes, the system (6.1)-(6.7) turns out to be decomposable so that the determination of farm organization is independent of that of consumption choice. For the convenience of subsequent analyses, let us present the optimality conditions for this case. <sup>6</sup>

$$pf_1(L_1, F; K, T) - w \leq 0 \quad (6.1a)$$

$$pf_2(L_1, F; K, T) - q \leq 0 \quad (6.2a)$$

$$U_1(C_1, C_2, Z; G) - \lambda p \leq 0 \quad (6.3a)$$

$$U_2(C_1, C_2, Z; G) - \lambda p' \leq 0 \quad (6.4a)$$

$$U_3(C_1, C_2, Z; G) - \lambda w \leq 0 \quad (6.5a)$$

$$-pC_1 - p'C_2 - wZ + M \geq 0 \quad (6.6a)$$

$$Z + L_1 + L - T_e \geq 0 \quad (6.7a)$$

Thus, the hours of farm labor and the amount of current inputs hence the amount of a commodity produced are determined solely by the inequalities (6.1a)-(6.2a), hence they are independent of the other inequalities associated with the determination of consumption choice. For interior solutions, (6.1a), (6.3a), and (6.5a) imply that the supply price of labor  $pU_3(\cdot)/U_1(\cdot)$  is equated to the market rate of wage  $w$  so that household members now have no incentive to seek additional employment. The marginal revenue product of labor is also equated to its market rate  $w$  so that the residual profit imputable to the farm production activity is also maximized at this rate. Therefore the constraint (6.7a) associated with the off-farm employment opportunities now turns out to be redundant, though it is kept to show a clear correspondence with the system (6.1)-(6.7) above.

In case off-farm employment opportunities are severely constrained, it is clear from the preceding analysis that it is not the market rate of wage but the market rate of wage minus the amount of discount household members are prepared to allow which is relevant in determining the optimal demands both for farm labor and for their leisure consumption therefore the optimal supply of their labor. And at the latter rate the sum of on- and off-farm employment demands for labor and the demand for leisure by household members are equated to their endowed time. Or alternatively, the sum of on- and off-farm employment demands for labor are equated to the supply of labor by household members. Since off-farm employment offers greater remuneration than farm employment and it is taken first, the latter rate of discounted wage can be regarded as a kind of "internal rate of wage" which effects the equilibrium between



the sum of on- and off-farm employment demands for labor and the supply of labor by household members. It should be clear from its function that this rate of wage is closely related to the virtual price and the implicit price of labor or endowed time due to Neary and Roberts (1980), Maruyama (1984) and others. Using this rate of wage, the optimality conditions (6.1)-(6.7) above of this household under constrained off-farm employment are rewritten as follows:

$$w^* = w - \mu/\lambda \leq w, \quad w^* = w \quad \text{for } \mu = 0$$

$$pf_1(L_1, F; K, T) - w^* \leq 0 \quad (7.1)$$

$$pf_2(L_1, F; K, T) - q \leq 0 \quad (7.2)$$

$$U_1(C_1, C_2, Z; G) - \lambda p \leq 0 \quad (7.3)$$

$$U_2(C_1, C_2, Z; G) - \lambda p' \leq 0 \quad (7.4)$$

$$U_3(C_1, C_2, Z; G) - \lambda w^* \leq 0 \quad (7.5)$$

$$-pC_1 - p'C_2 - w^*Z + Y \geq 0 \quad (7.6)$$

$$Z + L_1 + L - Te \geq 0 \quad (7.7)$$

, where  $Y$  denotes the full income with endowed time evaluated at  $w^*$  and  $\pi^*$  the residual profit imputable to the farm production activity similarly evaluated.

$$Y = w^*Te + (w - w^*)L + \pi^* + V$$

$$\pi^* = pX - w^*L_1 - qF - OC$$

At this rate of wage  $w^*$  (more precisely at  $w^*$ ,  $q$  and  $p$ ) the residual profit imputable to farm production activity is maximized. Furthermore household members have no incentive to seek additional employment since their supply price of labor is equated to its demand price by their farm production activity. Thus, the constraint (7.7) on off-farm employment opportunities ceases to be binding and can be suppressed without a loss of rigor as the corresponding constraint (6.7a) above can be in the decomposable competitive case, in so far as the equilibrium values of endogenous variables are concerned. Thus, the formal equivalence of the two systems of optimality conditions has been established, which will be proved to be instrumental in highlighting the important roles played by the internal rate of wage  $w^*$  in the subsequent comparative statics analysis. However, a fundamental difference between the two still remains.

Here, the internal rate of wage  $w^*$  is an endogenous variable to be determined to equate the sum of on- and off-farm demands for labor to its supply by household members, while it is exogenous and is identically equated to its market rate in the decomposable competitive case.

The equilibrium of the internal labor market within this household is illustrated in Figure 17 to facilitate subsequent analysis. The demand (rate of wage) curve ABCD contains a horizontal straight line segment BC of the length equal to  $L$  at the height of  $w$ , which represents the off-farm wage employment. The supply (rate of wage) curve is based on the supply price of labor  $pU_3/U_1$  by household members. The equilibrium of this market is given by the intersection of these two curves at the single point E  $(L_1^*(1) + L + L_1^*(2), w_0^*)$  or equivalently  $(T_e - Z^*, w_0^*)$  on the  $(L_1 + L_2, w^*)$  plane, which symbolizes the indecomposable character of this case. In case off-farm employment opportunities are not constrained, household members face the demand (rate of wage) curve ABF. The equilibrium on-farm employment is given by the point B  $(L_1^*(1), w)$ , while the equilibrium supply of labor on the part of household members by the separate point E'  $(T_e - Z^*, w)$ , which symbolizes the decomposable character of the "decomposable" competitive case.

### 3. Response of the agricultural household to changes in selected exogenous variables

How does the household respond to changes in the prices of current inputs, purchased commodities, and a home-produced farm commodity under constrained off-farm employment? Its response can be examined by the comparative statics analysis of the optimality conditions (7.1)-(7.7) for interior solutions. For the convenience of comparison, a similar analysis is performed of the optimality conditions (6.1a)-(6.7a) for the decomposable competitive case as well. Results of these analyses are shown compactly in matrix expressions.

$$\begin{bmatrix} pf_{11} & pf_{12} & 0 & 0 & 0 & 0 & -1 \\ pf_{21} & pf_{22} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & U_{11} & U_{12} & U_{13} & -p & 0 \\ 0 & 0 & U_{21} & U_{22} & U_{23} & -p' & 0 \\ 0 & 0 & U_{31} & U_{32} & U_{33} & -w^* & -\lambda \\ 0 & 0 & -p & -p' & -w^* & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} dL_1 \\ dF \\ dC_1 \\ dC_2 \\ dZ \\ d\lambda \\ dw^* \end{bmatrix} = \begin{bmatrix} 0 & 0 & -f_1 \\ 0 & 1 & -f_2 \\ 0 & 0 & \lambda \\ \lambda & 0 & 0 \\ 0 & 0 & 0 \\ C_2 & F & C_1 - X \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} dp' \\ dq \\ dp \end{bmatrix}, \quad (8)$$

or equivalently

$$\begin{bmatrix} pf_{11} & pf_{12} & 0 & 0 & 0 & 0 \\ pf_{21} & pf_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & U_{11} & U_{12} & U_{13} & -p \\ 0 & 0 & U_{21} & U_{22} & U_{23} & -p' \\ 0 & 0 & U_{31} & U_{32} & U_{33} & -w^* \\ 0 & 0 & -p & -p' & -w^* & 0 \end{bmatrix} \begin{bmatrix} dL_1 \\ dF \\ dC_1 \\ dC_2 \\ dZ \\ d\lambda \end{bmatrix} + \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ -\lambda \\ 0 \end{bmatrix} dw^* = \begin{bmatrix} 0 & 0 & -f_1 \\ 0 & 1 & -f_2 \\ 0 & 0 & \lambda \\ \lambda & 0 & 0 \\ 0 & 0 & 0 \\ C_2 & F & C_1 - X \end{bmatrix} \begin{bmatrix} dp' \\ dq \\ dp \end{bmatrix}, \quad (9.1)$$

$$dL_1 + dZ = 0. \quad (9.2)$$

The equation (9.2) can be suppressed without a loss of rigor in so far as the equilibrium values of endogenous variables are concerned as noted above.

$$\begin{bmatrix} pf_{11} & pf_{12} & 0 & 0 & 0 & 0 \\ pf_{21} & pf_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & U_{11} & U_{12} & U_{13} & -p \\ 0 & 0 & U_{21} & U_{22} & U_{23} & -p' \\ 0 & 0 & U_{31} & U_{32} & U_{33} & -w \\ 0 & 0 & -p & -p' & -w & 0 \end{bmatrix} \begin{bmatrix} dL_1 \\ dF \\ dC_1 \\ dC_2 \\ dZ \\ d\lambda \end{bmatrix} = \begin{bmatrix} 0 & 0 & -f_1 \\ 0 & 1 & -f_2 \\ 0 & 0 & \lambda \\ \lambda & 0 & 0 \\ 0 & 0 & 0 \\ C_2 & F & C_1 - X \end{bmatrix} \begin{bmatrix} dp' \\ dq \\ dp \end{bmatrix}, \quad dw^* = dw = 0, \quad (9.1a)$$

In case the constraint (7.7) on the off-farm employment opportunities is not binding so that the associated Lagrange multiplier  $\mu$  vanishes, the internal rate of wage  $w^*$  is identically equated to its market rate hence the equations (9.1)-(9.2) degenerate into the system (9.1a). Since sources of the changes in endogenous variables are confined to those in  $p'$ ,  $q$  and  $p$ , the differential of internal rate of wage  $dw^*$  may be more

appropriately expressed in the following way.

$$dw^* = \frac{\partial w^*}{\partial p'} dp' + \frac{\partial w^*}{\partial q} dq + \frac{\partial w^*}{\partial p} dp$$

Then, the equations (9.1) and (9.1a) will be rewritten accordingly in a separated form with the equation (9.2) being suppressed on the ground given above.

$$\begin{bmatrix} pf_{11} & pf_{12} \\ pf_{21} & pf_{22} \end{bmatrix} \begin{bmatrix} dL_1 \\ dF \end{bmatrix} = \left\{ \begin{bmatrix} 0 & 0 & -f_1 \\ 0 & 1 & -f_2 \end{bmatrix} + \begin{bmatrix} \frac{\partial w^*}{\partial p'} & \frac{\partial w^*}{\partial q} & \frac{\partial w^*}{\partial p} \\ 0 & 0 & 0 \end{bmatrix} \right\} \begin{bmatrix} dp' \\ dq \\ dp \end{bmatrix}, \quad (10.1)$$

$$\begin{bmatrix} U_{11} & U_{12} & U_{13} & -p \\ U_{21} & U_{22} & U_{23} & -p' \\ U_{31} & U_{32} & U_{33} & -w^* \\ -p & -p' & -w^* & 0 \end{bmatrix} \begin{bmatrix} dC_1 \\ dC_2 \\ dZ \\ d\lambda \end{bmatrix} = \left\{ \begin{bmatrix} 0 & 0 & \lambda \\ \lambda & 0 & 0 \\ 0 & 0 & 0 \\ C_2 & F & C_1 - X \end{bmatrix} + \lambda \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{\partial w^*}{\partial p'} & \frac{\partial w^*}{\partial q} & \frac{\partial w^*}{\partial p} \\ 0 & 0 & 0 \end{bmatrix} \right\} \begin{bmatrix} dp' \\ dq \\ dp \end{bmatrix}. \quad (10.2)$$

$$\begin{bmatrix} pf_{11} & pf_{12} \\ pf_{21} & pf_{22} \end{bmatrix} \begin{bmatrix} dL_1 \\ dF \end{bmatrix} = \begin{bmatrix} 0 & 0 & -f_1 \\ 0 & 1 & -f_2 \end{bmatrix} \begin{bmatrix} dp' \\ dq \\ dp \end{bmatrix}, \quad dw^* = dw = 0, \quad (10.1a)$$

$$\begin{bmatrix} U_{11} & U_{12} & U_{13} & -p \\ U_{21} & U_{22} & U_{23} & -p' \\ U_{31} & U_{32} & U_{33} & -w^* \\ -p & -p' & -w^* & 0 \end{bmatrix} \begin{bmatrix} dC_1 \\ dC_2 \\ dZ \\ d\lambda \end{bmatrix} = \begin{bmatrix} 0 & 0 & \lambda \\ \lambda & 0 & 0 \\ 0 & 0 & 0 \\ C_2 & F & C_1 - X \end{bmatrix} \begin{bmatrix} dp' \\ dq \\ dp \end{bmatrix}, \quad dw^* = dw = 0. \quad (10.2a)$$

Thus, the effects of changes in these exogenous variables on the equilibrium values of endogenous variables other than the internal rate of wage  $w^*$  have been decomposed into two parts, one representing a direct effect of changes in these exogenous variables and the other an indirect effect through the changes in the internal rate of wage  $w^*$  caused by changes in the same exogenous variables. The latter effect is not present in the decomposable competitive case (10.1a)-(10.2a), where  $w^*$  is identically equated to the market rate of wage and is not directly affected by changes in these exogenous

variables. This indirect effect may be referred to as an “internal wage effect” by its construction, which is peculiar to the indecomposable case and plays an important role in coordinating changes in the organization of farm production and those in the choice of consumption (labor supply) by household members. The response of internal rate of wage  $w^*$  itself will be somewhat closely examined in relation to the working of the internal labor market later in this section.

Now, these results of the proposed decomposition will be compared with the corresponding results of the conventional one to characterize the former method. Conventionally, the decomposition has been directly applied to the equation (8) above without distinguishing the internal rate of wage  $w^*$  from other endogenous variables as shown by Maruyama (1975), Singh, Squire, and Strauss (1986) and others.<sup>8</sup> On the contrary, it is distinguished from other variables and furthermore it is given a distinct treatment in the proposed method so that its role in the comparative statics analysis is highlighted, while its role is submerged in the overall analysis in the conventional method where it is not distinguished from other variables. In the decomposable competitive case the internal labor market is fully open and is directly related to the corresponding external market so that no room is left for the internal rate of wage  $w^*$  to play its preassigned role. To see further details of the alternative methods of decomposition, the responses of farm employment  $L_1$  of labor to changes in selected exogenous variables will be examined. Similar responses of the leisure consumption  $Z$  will be examined for reference in the Appendix. The equations (8), (10.1), and (10.1a) respectively will be solved for  $dL_1$  to facilitate comparison, and their results will be presented in this order.

a) The response of farm employment  $L_1$  of labor to changes in the price  $p'$  of purchased commodities.

$$\frac{\partial L_1}{\partial p'} = \frac{1}{|A|} (\lambda A_{41} + C_2 A_{61}) \quad (11.1)$$

$$\frac{\partial L_1}{\partial p'} = \frac{1}{|B|} p f_{22} \frac{\partial w^*}{\partial p'} \quad (11.2)$$

$$\frac{\partial L_1}{\partial p'} = 0 \quad \text{and} \quad dw^* = 0 \quad (11.3)$$

, where A and B denote the matrices of coefficients on the left hand side (LHS for short) of the equations (8) and (10.1) respectively and  $A_{ij}$ 's the cofactors associated with the elements  $a_{ij}$ 's in  $|A|$ . The first term on the right hand side (RHS for short) of the first equation represents the commodity-factor cross substitution effect (Sasaki and Maruyama, 1966; Maruyama, 1975) related to the indirect substitution between farm employment of labor and consumption of purchased commodities mediated through the substitution between leisure and purchased commodities. The second term on the RHS of the same equation represents the income effect. Thus, these two effects are taken care of by a single internal wage effect in the proposed method of decomposition in the second equation. Changes in the price  $p'$  of purchased commodities has no effect on farm employment of labor in the decomposable competitive case, where its determination is independent of that of consumption choice on the part of household members.

- b) The response of farm employment  $L_1$  of labor to changes in the price  $q$  of current inputs.

$$\frac{\partial L_1}{\partial q} = \frac{1}{|A|} (A_{21} + F A_{61}) \quad (12.1)$$

$$\frac{\partial L_1}{\partial q} = \frac{1}{|B|} (-p f_{12} + p f_{22} \frac{\partial w^*}{\partial q}) \quad (12.2)$$

$$\frac{\partial L_1}{\partial q} = \frac{1}{|B|} (-p f_{12}) \quad \text{and} \quad dw^* = 0 \quad (12.3)$$

The first terms on the RHS's of these equations represent the factor substitution effects. The second term on the RHS of the first equation represents the income effect, while the similar term of the second equation the internal wage effect. Here, only the

income effect corresponds to the internal wage effect since the commodity-factor cross substitution effect does not arise in this case.

- c) The response of farm employment  $L_1$  of labor to changes in the price  $p$  of farm commodity.

$$\frac{\partial L_1}{\partial p} = \frac{1}{|A|} \{-f_1 A_{11} - f_2 A_{21} + \lambda A_{31} + (C_1 - X) A_{61}\} \quad (13.1)$$

$$\frac{\partial L_1}{\partial p} = \frac{1}{|B|} (-pf_1 f_{22} + pf_2 f_{12} + pf_{22} \frac{\partial w^*}{\partial p}) \quad (13.2)$$

$$\frac{\partial L_1}{\partial p} = \frac{1}{|B|} (-pf_1 f_{22} + pf_2 f_{12}) \quad \text{and} \quad dw^* = 0 \quad (13.3)$$

The first two terms on the RHS's of these equations represent the “output effects” (Ferguson and Gould, 1975). The third and fourth terms on the RHS of the first equation represent the commodity-factor cross substitution and the income effects respectively, while the third term on the RHS of the following equation the internal wage effect. Here, the pair of the commodity-factor cross substitution and the income effects corresponds to the internal wage effect as in its response to changes in the price of purchased commodities.

Thus, in all these equations relative to the response of farm employment of labor the pairs of the commodity-factor cross substitution and the income effects in the conventional decomposition correspond to the internal wage effects in the proposed one wherever all these effects are relevant. Furthermore, it is shown in the Appendix that the similar pairs in the conventional decomposition correspond to the internal wage and the income effects in the proposed decomposition of the responses of leisure consumption to changes in selected exogenous variables wherever all these effects are relevant. Now, it should be clear from these examinations that the proposed decomposition can do without both the commodity-factor cross substitution and the income effects in the analysis of changes in the organization of farm production and furthermore that it can do without the commodity-factor cross substitution effect in the

analysis of changes in the choice of consumption (labor supply) by household members which are not readily amenable to the standard theory of microeconomics. These facts may suffice to prove the desired facility of the proposed method of decomposition in terms of the internal wage effect over the conventional one as exemplified by Maruyama (1975), Singh, Squire, and Strauss (1986) in the analysis of the behavior of agricultural households under the constrained off-farm wage employment opportunities.

Now, it will be seen that the results of the preceding analyses have fully paved the way for examining the response of the internal rate of wage  $w^*$  itself to changes in selected exogenous variables.

$$\frac{\partial w^*}{\partial p'} = \frac{|A^{p'}|}{|A|} = \frac{A_{71}^{p'} + A_{75}^{p'}}{|A|} = -\frac{|B||C|}{|A|} \left( \left. \frac{\partial L_1}{\partial p'} \right|_{dw^*=0} + \left. \frac{\partial Z}{\partial p'} \right|_{dw^*=0} \right), \quad (14.1)$$

$$|A| > 0, |B| > 0, \text{ and } |C| < 0$$

$$\frac{\partial w^*}{\partial q} = \frac{|A^q|}{|A|} = \frac{A_{71}^q + A_{75}^q}{|A|} = -\frac{|B||C|}{|A|} \left( \left. \frac{\partial L_1}{\partial q} \right|_{dw^*=0} + \left. \frac{\partial Z}{\partial q} \right|_{dw^*=0} \right) \quad (14.2)$$

$$\frac{\partial w^*}{\partial p} = \frac{|A^p|}{|A|} = \frac{A_{71}^p + A_{75}^p}{|A|} = -\frac{|B||C|}{|A|} \left( \left. \frac{\partial L_1}{\partial p} \right|_{dw^*=0} + \left. \frac{\partial Z}{\partial p} \right|_{dw^*=0} \right) \quad (14.3)$$

, where  $A^k$ 's denote the matrices of coefficients on the LHS of the equation (8) above, the column of which associated with  $dw^*$  is replaced by the column of coefficients associated with  $dk$ ,  $k = p', q$ , and  $p$  on its RHS respectively.  $A_{ij}^k$ 's denote the cofactors associated with the elements  $e_{ij}$ 's in  $|A^k|$ . It should be clear from their construction that the first terms on the RHS's of these equations represent the responses of farm labor  $L_1$  to changes in selected exogenous variables with the internal rate of wage given constant, while the second terms the similar responses of leisure consumption  $Z$ . Therefore, the internal rate of wage  $w^*$  rises, falls, or is left invariant according as the sum of the changes in equilibrium values of farm labor and leisure consumption increases, decreases, or remains constant with the internal rate of wage given constant.



The response of the internal rate of wage  $w^*$  can be explained alternatively in terms of the shifts of demand and supply curves of labor on  $(L_1+L_2, w^*)$  plane. Since all the equations (11.2), (12.2), and (13.2) above imply that  $\partial L_1 / \partial w^* = pf_{22}|B|^{-1} < 0$ , the demand curve for farm labor proves to be downward-sloping. On the other hand, the supply curve of labor  $L_s^*$  by household members proves to be upward-sloping since  $L_s^* = Te - Z$  and all the equations (A1.2), (A2.2), and (A3.2) in the Appendix imply that  $\partial Z / \partial w^* = \lambda C_{33}|C|^{-1} < 0$  from the property of the bordered Hessian determinant  $|C|$ . The first and second terms in brackets on the RHS's of the equations (14.1)-(14.3) respectively represent the shifts of demand and supply (rate of wage) curves on the  $(L_1+L_2, w^*)$  plane, which have already been examined in the equations (11.3), (12.3), and (13.3) above and (A1.3), (A2.3), and (A3.3) in the Appendix. The associated change in the internal rate of wage is determined by the relative importance of these shifts.

#### 4. An example for the specific types of production and welfare functions

To give a more definite idea as to how it is applied to practical models of agricultural households under constrained off-farm wage employment, the proposed method of decomposition is applied to a household model composed of the specific types of production and welfare functions. The response of output supply  $X$  to its price  $p$  will be examined and furthermore it will be shown that the supply curve of commodity can have a part of downward slope.

$$\frac{\partial X}{\partial p} = f_1 \frac{\partial L_1}{\partial p} + f_2 \frac{\partial F}{\partial p}$$

From the equations (10.1) and (10.1a) above, it follows

$$\frac{\partial L_1}{\partial p} = \frac{1}{|B|} \left\{ p(f_2 f_{12} - f_1 f_{22}) + pf_{22} \frac{\partial w^*}{\partial p} \right\} = \frac{\partial L_1}{\partial p} \Big|_{dw^*=0} + \frac{pf_{22}}{|B|} \frac{\partial w^*}{\partial p}, \quad |B| > 0$$

$$\frac{\partial F}{\partial p} = \frac{1}{|B|} \left\{ -p(f_2 f_{11} - f_1 f_{21}) - pf_{21} \frac{\partial w^*}{\partial p} \right\} = \frac{\partial F}{\partial p} \Big|_{dw^*=0} - \frac{pf_{21}}{|B|} \frac{\partial w^*}{\partial p}, \quad |B| > 0$$

The first two terms on the first RHS's represent the output effects, while the third the internal wage effects peculiar to the behavior of agricultural households under constrained off-farm wage employment. Hence,

$$\begin{aligned}\frac{\partial X}{\partial p} &= f_1 \frac{\partial L_1}{\partial p} \Big|_{dw^*=0} + f_2 \frac{\partial F}{\partial p} \Big|_{dw^*=0} - \frac{p(f_2 f_{21} - f_1 f_{22})}{|B|} \frac{\partial w^*}{\partial p} \\ &= \frac{\partial X}{\partial p} \Big|_{dw^*=0} - \frac{\partial L_1}{\partial p} \Big|_{dw^*=0} \times \frac{\partial w^*}{\partial p} \\ \frac{\partial w^*}{\partial p} &= - \frac{|B||C|}{|A|} \left( \frac{\partial L_1}{\partial p} \Big|_{dw^*=0} + \frac{\partial Z}{\partial p} \Big|_{dw^*=0} \right), \quad |A| > 0, |B| > 0, \text{ and } |C| < 0\end{aligned}$$

A production function of the Cobb-Douglas type and a welfare function of the type underlying the linear expenditure system of consumption choice are chosen to be analyzed for their familiarity to many economists.

$$\ln X = \ln \beta_0 + \beta_1 \ln L_1 + \beta_2 \ln F + \beta_3 \ln K + \beta_4 \ln T$$

$$W = b_1 \ln(C_1 - a_1) + b_2 \ln(C_2 - a_2) + b_3 \ln(Z - a_3)$$

$$\beta_1 > 0, \quad \beta_2 > 0, \quad \beta_1 + \beta_2 < 1$$

$$b_1 > 0, \quad b_2 > 0, \quad b_3 > 0, \quad b_1 + b_2 + b_3 = 1$$

$$C_1 - a_1 > 0, \quad C_2 - a_2 > 0, \quad Z - a_3 > 0$$

Then,

$$\frac{\partial L_1}{\partial p} \Big|_{dw^*=0} = \frac{L_1}{(1 - \beta_1 - \beta_2)p} > 0$$

$$\frac{\partial F}{\partial p} \Big|_{dw^*=0} = \frac{F}{(1 - \beta_1 - \beta_2)p} > 0$$

$$\frac{\partial Z}{\partial p} \Big|_{dw^*=0} = \frac{b_3(X - a_1)}{w^*} > 0 \quad \text{for } C_1 < X$$

Therefore,

$$\left. \frac{\partial X}{\partial p} \right|_{dw^*=0} = \frac{X(\beta_1 + \beta_2)}{p(1 - \beta_1 - \beta_2)} > 0$$

$$\frac{\partial w^*}{\partial p} = \frac{\beta_1 X + b_3(1 - \beta_1 - \beta_2)(X - a_1)}{(1 - b_3)(1 - \beta_1 - \beta_2)(Z - a_3) + (1 - \beta_2)L_1} > 0$$

Thus, the supply curve of commodity can have a part of downward slope according as the internal wage effect exceeds the output effect.

## 5. Conclusion

Off-farm wage employment of agricultural households is constrained in many industrialized and semi-industrialized countries probably due to the practice of off-farm employers who offer “higher than equilibrium rate of wage” to leave sufficient unemployment as a worker discipline device. The remaining supply of labor by household members in excess of off-farm employment is obliged to put into farm employment. However, it is not the market rate of wage but the “market rate minus an amount of discount” they are prepared to allow which determines the farm employment and the supply of labor by household members and induces them to equate. Hence, the farm employment and the supply of labor by household members constitute a closed market within the household and the discounted rate of wage may be appropriately referred to as the “internal rate of wage” to be contrasted to its market rate. Thus their organization of farm production and their choice of consumption (labor supply) are to be jointly determined through the determination of the amount of discount they are prepared to allow.

The joint determination or the indecomposability of the two has very significant consequences on the comparative statics analysis of the behavior of agricultural households under constrained off-farm wage employment. The income effect and others inherent in the comparative statics of their consumption choice creep into their organization of farm production to render both their demand for production factors and their supply of output less elastic. In extreme cases, they yield portions of upward-

sloping demand and downward-sloping supply curves. Conventionally, the comparative statics analysis of the behavior of agricultural households under constrained off-farm wage employment has been directly applied to their conditions of optimality without paying a due attention to the difference in roles played by many endogenous variables. Results of this analysis are ascribed to the income effects and the commodity-factor cross substitution effects besides the standard factor substitution effects of selected exogenous variables in their organization of farm production and to the commodity-factor cross substitution effects besides the standard commodity substitution and the income effects in their choice of consumption (labor supply), which are not readily amenable to the standard theory of microeconomic analysis.

The internal rate of wage or one of its components, the amount of discount on the market rate of wage plays a distinct role in establishing the equality of the farm employment demand and the supply of labor by household members, while other endogenous variables except for the Lagrange multiplier  $\mu$  represent the quantities of commodities or production factors to be demanded. Its distinct role deserves a separate treatment of it in the comparative statics analysis of the behavior of agricultural households, which enables us to replace the income and the commodity-factor cross substitution effects by the internal wage effects in their organization of farm production and the commodity-factor cross substitution effects by the internal wage effects, the indirect effects via changes in the internal rate of wage caused by changes in selected exogenous variables.

Thus the comparative statics analysis of the behavior of agricultural households under constrained off-farm wage employment turns out to be more readily amenable to the standard microeconomic analysis and thereby enhances the tractability of the "indecomposable model of agricultural households" designed for analyzing their behavior under these circumstances.

Finally, it should be clear from the preceding exposition that a similar method of decomposition can be applied in analyzing the behavior of households, where they are subject to the constraint(s) on their output of home-produced commodity (commodities)

as addressed by Sicular (1986) in connection with their sales quota(s) and by Becker (1965) and others in connection with their home production with the corresponding "sales quota(s)" of them regarded as being equal to nil. In these cases the internal price(s)  $p^* = p(1+v/\lambda)$  take(s) the place of  $w^*$  in this paper, where  $v$  denote(s) the Lagrange multiplier(s) associated with the constraint(s) on the home-produced commodity (commodities).

**Appendix. Response of leisure consumption to changes in selected exogenous variables**

- a) The response of leisure consumption to changes in the price  $p'$  of purchased commodities.

$$\frac{\partial Z}{\partial p'} = \frac{1}{|A|}(\lambda A_{45} + C_2 A_{65}) \quad (A1.1)$$

$$\frac{\partial Z}{\partial p'} = \frac{1}{|C|}(\lambda C_{23} + \lambda C_{33} \frac{\partial w^*}{\partial p'} + C_2 C_{43}) \quad (A1.2)$$

$$\frac{\partial Z}{\partial p'} = \frac{1}{|C|}(\lambda C_{23} + C_2 C_{43}) \quad \text{and} \quad dw^* = 0 \quad (A1.3)$$

, where  $C$  denotes the matrix of coefficients on the LHS of the equation (10.2) and  $C_{ij}$ 's the cofactors associated with the elements  $c_{ij}$ 's in  $|C|$ . The first terms on the RHS's of these equations represent the commodity substitution effects, while the final terms represent the income effects. The second term on the RHS of the second equation represents the internal wage effect. The commodity-factor cross substitution effect does not arise in the first equation since both leisure consumption and the price of purchased commodities are directly related to the choice of consumption by household members.

- b) The response of leisure consumption to changes in the price  $q$  of current inputs

$$\frac{\partial Z}{\partial q} = \frac{1}{|A|}(A_{25} + FA_{65}) \quad (A2.1)$$

$$\frac{\partial Z}{\partial q} = \frac{1}{|C|}(\lambda C_{33} \frac{\partial w^*}{\partial q} + FC_{43}) \quad (A2.2)$$

$$\frac{\partial Z}{\partial q} = \frac{1}{|C|}(FC_{43}) \quad \text{and} \quad dw^* = 0 \quad (A2.3)$$

The final terms on the RHS's of these equations represent the income effects. The first

term on the RHS of the first equation represents the commodity-factor cross substitution effect, while the corresponding term of the second equation the internal wage effect. Thus, the pair of the commodity-factor cross substitution and the income effects in the conventional decomposition corresponds to the pair of the internal wage and the income effects in the proposed one.

c) The response of leisure consumption to changes in the price of farm commodity  $p$

$$\frac{\partial Z}{\partial p} = \frac{1}{|A|} \{-f_1 A_{15} - f_2 A_{25} + \lambda A_{35} + (C_1 - X) A_{65}\} \quad (A3.1)$$

$$\frac{\partial Z}{\partial p} = \frac{1}{|C|} \{\lambda C_{13} + \lambda C_{33} \frac{\partial w^*}{\partial p} + (C_1 - X) C_{43}\} \quad (A3.2)$$

$$\frac{\partial Z}{\partial p} = \frac{1}{|C|} \{\lambda C_{13} + (C_1 - X) C_{43}\} \quad \text{and} \quad dw^* = 0 \quad (A3.3)$$

The final terms on the RHS's of these equations represent the income effects, while the third term on the RHS of the first equation and first term on the RHS's of the second and third equations the commodity substitution effects. The first two terms on the RHS of the first equation represent the commodity-factor cross substitution effects, while the second term on the RHS of the second equation the internal wage effect.

Thus in all these equations relative to the response of leisure consumption the pair of the commodity-factor cross substitution and the income effects in the conventional decomposition corresponds to the pair of the internal wage and the income effects in the proposed one wherever all these effects are relevant.

## Footnotes

1. The term “decomposability” is due to Sasaki and Maruyama (1966) and Maruyama (1975), while other authors refer to this property as “block-recursive” or “separable” (see, e.g., Jorgenson and Lau, 1969; Singh, Squire, and Strauss, 1986).
2. The term “commodity-factor cross substitution effect” is due to Sasaki and Maruyama (1966) and Maruyama(1975), while Strauss refers to this effect as “a substitution-type (income) effect due to an induced change in the uncompensated virtual wage” in Singh, Squire, and Strauss (1986).
3. The model used in this paper is based on the agricultural household model due to Maruyama (1984), Singh, Squire, and Strauss (1986), and others.
4. For simplicity, the amounts taxed and saved are not included in this study.
5. For dependents, leisure hours are identified with their endowed time.
6. Equations for the decomposable competitive case are numbered in numerals with “a” in contrast to the corresponding ones for the indecomposable case.
7. For details of this illustration, see, e.g., Maruyama (1984).
8. Since the definition of  $w^*$  implies  $dw^* = -d(\mu/\lambda)$ , the expression (8) coincides with that in Singh, Squire, and Strauss (1986), p. 74 with the rate of wage evaluated at the internal rate of wage  $w^*$ .



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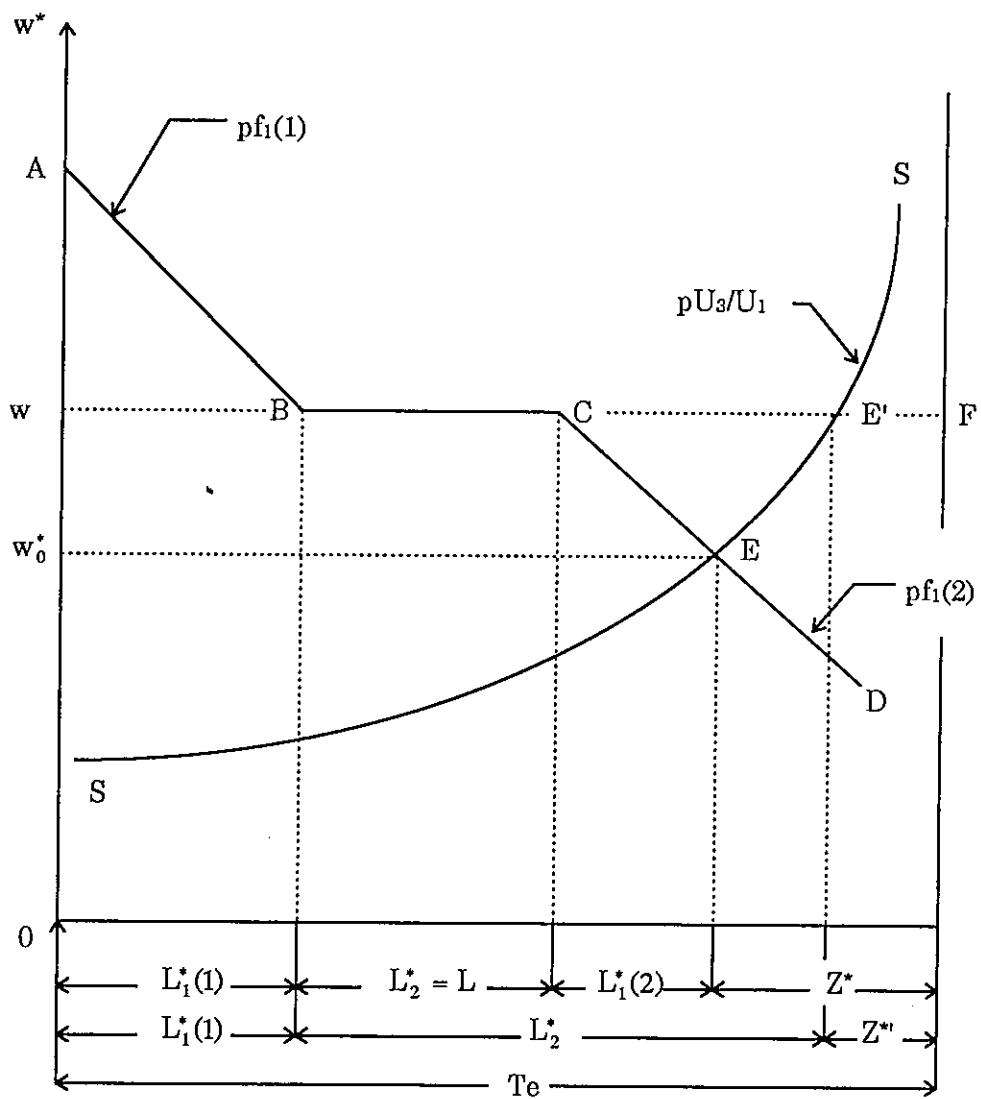


Figure 1. Equilibrium of the internal labor market within the household