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Collusion under Financial Constraints: Collusion or Predation when the discount factor is near one?

by

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Collusion or Predation when the discount factor is near one?*

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Abstract

In this paper the possibility of collusion between financially constrained firms is considered. It is found that they cannot collude when the discount factor is sufficiently close to one and their collusion emerges at lower discount factors.

Keywords: Collusion; Predation; Repeated game; Financial constraint

JEL classification: L10; L13

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1 Introduction

Folk theorems claim that firms are likely to cooperate when the discount factor is sufficiently high (e.g., Friedman, 1971; Rubinstein, 1979; Fudenberg and Maskin, 1986). On the contrary, established firms with high discount factors are known to have the incentive to predation (e.g., Milgrom and Roberts, 1982; Benoit, 1986). In an attempt to explain this apparent contradiction, we shall incorporate predation into the repeated "prisoner's dilemma" game. One way to accomplish this attempt is to assume that all firms in the market face some financial constraints.[1] In this framework we consider the possibility of collusion and find that financially constrained firms cannot collude when the discount factor is sufficiently close to one and their collusion emerges at lower discount factors.

2 The Model

First we define the one-shot simultaneous game $g(s_1, s_2) = (\{1,2\}, \{A(s_i)\}_{i=1,2}, \pi)$, in which actions available for players depend on state variables s_1 and s_2. $A(s_i)$ is the set of the available actions a_i for firm i defined as follows:

$$A(s_i) = \begin{cases} \{C, F, E\} & \text{if } s_i \leq T_i \\ \{E\} & \text{if } s_i \geq T_i + 1, \end{cases}$$

for a given $T_i \in \mathbb{N} \cup \{\infty\}$.[2] The actions "$C$" and "$F$" mean "cooperating" and "finking" in the market while "E" means "exiting" from the market. When $s_i \geq T_i + 1$, firm i is bankrupt

[1] McGee (1958, 1980), Bork (1978), and Easterbrook (1981) suggest that predatory pricing is unlikely to occur when firms do not face financial constraints.

[2] In this paper $\mathbb{N}$ and $\mathbb{R}$ stand for the sets of natural numbers and real numbers respectively.

and must continue to exit from the market. The payoff function π: $A(s_i) \times A(s_j) \to \mathbb{R}^2$ is defined by the following payoff matrix.[3]

Firm 2

Firm 1		C	F	E
	C	$\pi^c - K, \pi^c - K$	$-K, 2\pi^c - K$	$M - K, 0$
	F	$2\pi^c - K, -K$	$-K, -K$	$M - K, 0$
	E	$0, M - K$	$0, M - K$	$0, 0$

We assume that $M > \pi^c > K > 0$. Note that each firm must pay K as a fixed cost as long as it chooses C or F.

We next define the extensive game $G(T_1, T_2; \delta)$ where at each stage $t = 1, 2, \cdots$, firms play $g(s_{1t}, s_{2t})$ defined according to the levels of state variables. The motion of the state variable s_i is as follows: At stage 1, $s_{i1} = 0$. If firm i obtains a negative profit (i.e., $-K$) at stage t, $s_{i,t+1} = s_{it} + 1$; and otherwise $s_{i,t+1} = s_{it}$. A strategy of firm i, σ_i, is a function which associates an available action to every possible information set. Let $a_{it}(\sigma)$ denote the realized action of firm i at stage t when both firms act according to $\sigma = (\sigma_1, \sigma_2)$. Thus the payoff function of $G(T_1, T_2; \delta)$ is as follows:

$$V_i(\sigma; \delta) = \sum_{t=1}^{\infty} \delta^{t-1} \pi_i(a_{1t}(\sigma), a_{2t}(\sigma)).$$

Our formulation described above intends to capture the situation that firms in the market face financial constraints. Specifically in our model, if firm i has a negative profit $T_i + 1$ times, it goes into bankruptcy and must exit. Thus T_i means the maximum length of stages which

[3]When the action space is $\{E\}$, the payoff matrix should be interpreted similarly.

firm i can withstand a predation or a price war in the market. When $T_1, T_2 = \infty$, $G(T_1, T_2; \delta)$ is reduced to a simple repeated game. In this paper we shall assume that $0 < T_1 \leq T_2$.

3 The Possibility of Collusion

In the literature of repeated games we can find that collusion is more likely to occur as the discount factor becomes higher. However in this model, where firms face financial constraints, we can show that their collusion does not emerge when the discount factor is near one.

Proposition 1: *Let $\theta_2(M-K)$ be the average per-period profit of firm 2 under collusion for $\theta_2 \in (0,1)$. Then for any $\theta_2 \in (0,1)$ there exists δ' such that for all $\delta > \delta'$ firm 2 has an incentive to predation.*

Proof: First note that firm 2 continues to choose F instead of a collusive action in order to make the opponent firm exit from the market and thereby enjoy monopoly power in the future. The minimum payoff which firm 2 gets through predation is:

$$\begin{aligned} & -K + \delta(-K) + \cdots + \delta^{T_1}(-K) + \delta^{T_1}(M-K) + \delta^{T_1+1}(M-K) + \cdots \\ &= \frac{1}{1-\delta}(-K) + \frac{\delta^{T_1+1}}{1-\delta}M. \end{aligned}$$

Hence firm 2 has an incentive to predation if

$$\frac{1}{1-\delta}(-K) + \frac{\delta^{T_1+1}}{1-\delta}M > \frac{\theta_2(M-K)}{1-\delta}.$$

We can reduce this inequality to:

$$\delta > (\frac{\theta_2 M + (1-\theta_2)K}{M})^{\frac{1}{T_1+1}}.$$

Q.E.D

Taking this result into consideration, we next consider the possibility of collusion or find the condition of collusion, using trigger strategy.

Trigger Strategy: *σ_i^{TR} is a sequence of σ_{it}^{TR} such that $\sigma_{i1}^{TR} = C$ and for $t = 2, 3, \cdots$,*

$$\sigma_{it}^{TR} = \begin{cases} C & \textit{if both firms have chosen } C \textit{ in every stage preceding } t \\ F & \textit{if firm } j \neq i \textit{ is the first one to deviate from } C \textit{ and } F \in A(s_{it}) \\ E & \textit{if firm } i \textit{ is the first one to deviate from } C \textit{ or } \{E\} = A(s_{it}). \end{cases}$$

The necessary and sufficient conditions for $(\sigma_1^{TR}, \sigma_2^{TR})$ to be a subgame perfect Nash equilibrium of $G(T_1, T_2; \delta)$ are as follows :[4]

$$\pi^c - K + \delta(\pi^c - K) + \delta^2(\pi^c - K) + \cdots \geq 2\pi^c - K \tag{1}$$

and for $i = 1, 2$,

$$0 \geq -K + \delta(-K) + \cdots + \delta^{T_i - 1}(-K) + \delta^{T_i}(M - K) + \delta^{T_i + 1}(M - K) + \cdots. \tag{2}$$

Inequality (1) asserts that a firm cannot gain by deviating from C. If $T_1 = \infty$, (1) is sufficient for σ_i^{TR} to be subgame perfect. When T_1 is finite, however, the subgame perfectness of σ_i^{TR} breaks down. The condition which guarantees the subgame perfectness for a finite T_1 is inequality (2). We reduce inequalities (1) and (2) to the following, respectively:

$$\delta \geq \frac{\pi^c}{2\pi^c - K} \equiv \underline{\delta} \tag{3}$$

and for $i = 1, 2$,

$$\delta \leq \left(\frac{K}{M}\right)^{\frac{1}{T_i}} \equiv \overline{\delta}(T_i). \tag{4}$$

Noting that $\overline{\delta}(T_1) < \overline{\delta}(T_2)$, we get the following proposition.

[4]When i=1, the inequality (2) is the incentive constraint for firm 2 and vice versa.

Proposition 2 : $(\sigma_1^{TR}, \sigma_2^{TR})$ *is a subgame perfect Nash equilibrium of* $G(T_1, T_2; \delta)$ *if and only if*

$$\overline{\delta}(T_1) \geq \delta \geq \underline{\delta}.$$

Note that this Proposition includes the standard result of folk theorem (e.g., Fudenberg and Maskin, 1986) since $\overline{\delta}(\infty) = 1$.

4 Conclusion

In Proposition 1 we claim that when δ is near one if T_1 is finite, the firm with financial advantage (firm 2) surely forces a price war if the rival (firm 1) stays in the market and therefore firms cannot collude. Thus this credible threat of predation forces firm 1 to exit and then firm 2 enjoys monopoly power.

In Proposition 2 we consider the possibility of collusion when we restrict our attention to trigger strategies. When $\overline{\delta}(T_1) \geq \delta \geq \underline{\delta}$, firm 1 stays in the market because the threat of predation is not credible. Thus their collusion will emerge.

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