

No. **676**

HAS LIVABILITY OF JAPAN GOTTEN BETTER
FOR 1956 - 1990? : A DEA APPROACH

by

AKIHIRO HASHIMOTO AND MIGAKU KODAMA

April 1996

AKIHIRO HASHIMOTO¹ AND MIGAKU KODAMA²

HAS LIVABILITY OF JAPAN GOTTEN BETTER FOR 1956–1990?:
A DEA APPROACH

¹*Institute of Socio–Economic Planning, University of Tsukuba and* ²*Master's Program in Management Sciences and Public Policy Studies, University of Tsukuba, Tsukuba, Ibaraki 305, Japan*

I. INTRODUCTION

This paper examines whether livability of Japan has gotten better for the period 1956–1990 by observing its transition. As measures for livability, we use social indicators, each of which seems to reflect some aspect of society. This is a relative evaluation of Japan's livability in terms of data on social indicators for the period 1956–1990.

In order to grasp livability appropriately, we should evaluate multiple aspects of society comprehensively, implying the simultaneous use of many social indicators. In this multi-dimensional evaluation, we generally use the indicators' weighted sum as an integrated measure. But it is difficult to define such an *a priori* weighting because of the complexity and variety of human preference. If we employed this type of weighting system, resulting discussions might lead to uniform evaluation of societies with varying characteristics. This would not be appropriate because livability is personal, subjective and/or sensible matter.

In this paper, we seek a non-uniform evaluation of livability using *DEA* (*Data Envelopment Analysis*), developed first by Charnes *et al.* (1978) for measuring the relative efficiency of *DMUs* (*Decision Making Units*) with multiple *inputs* and multiple *outputs* [see, e.g., Boussofiane *et al.* (1991), Charnes *et al.* (1994) for overviews]. (Mathematical models for DEA are presented in the Appendix.) We here perform a time series DEA analysis, treating each year as a separate DMU.

II. DEA TIME SERIES ANALYSIS OF LIVABILITY

DEA examines how efficiently DMUs convert multiple inputs into multiple outputs. That is, any DMU producing more outputs with fewer inputs is judged relatively efficient (*DEA efficient*). However, DEA models do not necessarily assume such organic relationships between inputs and outputs as those in production (see Appendix). Therefore, replacing inputs with *negative* (the smaller the value, the better) evaluation items and outputs with *positive* (the greater the value, the better) evaluation items, yields a combined evaluation of these items. This is a comprehensive evaluation different from traditional ones in which it replaces a uniform evaluation using an *a priori* weighting system with a flexibly defined weighting system corresponding to each DMU.

In this study, we use DEA to evaluate *livability* of Japan for the period 1956–1990, defining each year as a separate DMU. As inputs and outputs in DEA, we apply negative and positive social indicators, respectively. Therefore, any year with greater positive and smaller negative indicators than others is judged relatively livable (*DEA livable*).

From a methodological point of view, this study explores a field application of DEA beyond the standard DEA efficiency analysis. Although a great number of applications of DEA exist, we found but a few "non-standard" applications. These included computer printer comparison (Doyle and Green, 1991), ranked voting systems analysis (Cook and Kress, 1990; Hashimoto, 1995), prefectures' livability assessment (Hashimoto and Ishikawa, 1993), baseball batters evaluation (Hashimoto, 1993a), and examination applicants selection (Hashimoto, 1993b). For time series DEA analyses treating each year as a DMU, we could find only Cooper *et al.* (1995) as a DEA efficiency analysis. In the current paper, we focus on DEA as a *non-uniform, multi-dimensional and relative* evaluation tool, and seek its application to a time series analysis of Japan's livability.

III. DATA

As data for evaluating livability of Japan, we apply the following eight social indicators:

Health

Life expectancy (Life expectancy at birth)

Suicide (Suicides per total population)[†]

Safety

Crime (Criminal cases recognized by police per total population)[†]

Traffic accidents (Persons killed in road traffic accidents per total population)[†]

Economy

National income (Per capita national income deflated by consumer price index)

Unemployment (Ratio of totally unemployed persons to labor force)[†]

Environment

Forest area (Per capita area of forest)

Water service (Diffusion rate of water service)

([†] Negative indicator)

We employed this indicator system referring to OECD (1976), listing nine aspects for measuring social well-being. Considering the basic and universal aspects that would not be influenced by the times nor society strata, we selected and integrated the nine aspects into four: health, safety, economy and environment. The individual components were chosen based on the following reasons: We adopted life expectancy, crime, national income and forest area as ones which directly reflect the levels of the above-stated four aspects, respectively. Next, we added another social indicator to each aspect as one measuring the level of other side of the aspect. Here, we employed suicide as one reflecting an entirely negative quality of life, and water service as one for housing conditions.

It is noteworthy that we may choose negative and positive social indicators without considering organic relationships between them. Further, we need not always select the same number of negative and positive indicators. Although there can no outputs without inputs in production, in this study, it is possible, for example, to have only negative or only positive social indicators.

As with all the social indicators, we employ normalized scores as the means are 50 and the variances are 100. DEA/AR analysis mentioned later requires such normalization though we do not necessarily need it for the standard DEA computation.

IV. DEA EVALUATION OF LIVABILITY TRANSITION

Applying data for the period 1956–1990 on four negative and four positive indicators, the model in the Appendix finds that 20 years have a DEA measure of $h_{jo} = 1$. That is, 20 of the total 35 years are judged DEA livable and the remaining 15 are DEA unlivable. Table I (the left end column) shows DEA measures for the 35 years (see also Figure 1).

It is noteworthy that considerably many years attain the maximum measure 1, i.e., each of them ranks top in terms of its own optimal weights. This is because DEA evaluates each year in terms of a flexible weighting system that can vary by year. We can here see a property peculiar to DEA vs other such comprehensive evaluation methods that evaluate DMUs with various featured characteristics uniformly using an *a priori* weighting system.

We find more than half the DEA livable years in the second half of the 1960's to the 1970's, and many of the DEA unlivable years in the period up to the first half of the 1960's

and in the 1980's. That is, we cannot simply say that livability of Japan has gotten either better or worse for the period 1956–1990. Especially, we should here note that the last decade, the 1980's, is not generally DEA livable.

Virtual Indicator Values

We now examine with what weights all the years might attain their DEA measures using *virtual indicator values* (Boussofiane *et al.*, 1991), the products of social indicator value and the corresponding optimal weight. Virtual indicator values convey information on the importance a year attaches to particular social indicators in order to attain its maximum DEA measure. They are used instead of social indicator weights since the actual weights are dependent on the scale of the associated social indicators. That is, virtual indicator values are normalized weights, by which we can see feature indicators of a year.

Table I shows the virtual indicator values for all the years. Here, the sum of virtual negative indicator values is 1 and the sum of virtual positive indicator values is equal to DEA measure [see model (A.2)], so that the individual virtual negative/positive indicator values show the contribution in negative/positive indicators to attaining the maximum DEA measure. For example, year 1970 attains DEA measure 1 being evaluated in terms of unemployment and national income, while year 1980 achieves this in terms of water service, unemployment and traffic accidents. Year 1990 attains DEA measure 1 in terms of suicide (negative indicators) and national income (positive indicators), while year 1959's maximum DEA measure cannot reach 1 (0.9442) though it is evaluated in terms of forest area (positive indicators) and traffic accidents and crime (negative indicators).

In this way, all the years are evaluated on a variety of attributes, and the 20 years with various featured characteristics can attain the maximum measure 1. However, we should also note that the DEA model may yield alternative optimal solutions, which would lead to alternative virtual indicator values.

Tendency along Time Passage

Table I shows that years 1956–1968 are evaluated in terms of forest area most in positive indicators, and years 1982–1989 are evaluated in terms of traffic accidents most in negative indicators. Since this is a time series analysis, data on social indicators do not seem to change so drastically year by year. There must be some general tendency along the passage of time, though feature indicators of each year can vary.

Table II chronologically shows the DEA unlivable years with their *reference sets* and *combination coefficients*, optimal solutions, λ_j , to model (A.3). The reference set of DEA unlivable year j consists of those years that have a DEA measure 1 in terms of the weights optimal for year j . That is, the reference set composes a part of the *frontier* that involves a *reference point* comparison of the DEA unlivable year. For example, year 1975 is compared with the frontier facet composed by its reference set, years 1973, 1974 and 1979. The DEA measure 0.9907 is compared to 1.0, the supposed value of the reference point, and the combination coefficients indicate that the reference point of year 1975 is near the point of year 1974 on the frontier.

In Table II, we obviously see a tendency along time passage. The reference sets of DEA unlivable years seem to appear chronologically, and so do the greatest combination

coefficients for respective DEA unlivable years.

DEA/AR Analysis

DEA is able to define a weighting system for inputs and outputs corresponding to a target DMU. On this basis, the 20 years were judged DEA livable. This approach is in sharp contrast to the uniform evaluation of using an *a priori* weighting system. A compromise between these two approaches is represented by *DEA/AR* (*DEA/ Assurance Region*) analysis (Thompson *et al.*, 1986). In DEA models, the ratio of weights v_i (u_r) to negative (positive) social indicators is equal to the ratio of shadow prices for the negative (positive) indicators (see Appendix). Therefore, we can discriminate the importance of social indicators by bounding the ratios of weights. DEA/AR analysis aims at a more realistic analysis by incorporating experiences and expert opinions in the shape of constrained weight systems.

In this study, we perform two DEA/AR analyses focusing on a trade-off between economy and environment: DEA/AR1 = the case in which the environment takes precedence to the economy, i.e., precedence order is assumed that environment, [health and safety (indifferent)] and economy; DEA/AR2 = the opposite precedence order case. Concretely, for AR1, we bound the ratios of weights v_i , u_r to social indicators as follows:

$$\begin{aligned}v[\text{Suicide}] &\geq v[\text{Unemployment}], \\v[\text{Crime}] &\geq v[\text{Unemployment}], \\v[\text{Traffic accidents}] &\geq v[\text{Unemployment}], \\u[\text{Forest area}] &\geq u[\text{Life expectancy}] \geq u[\text{National income}], \\u[\text{Water service}] &\geq u[\text{Life expectancy}].\end{aligned}$$

For AR2, we reverse the direction of all the inequality signs. Table III and Figure 1 show results of the DEA/AR analyses with the DEA measures.

The number of DEA/AR2 livable years is reduced to less than half the number of DEA livable years, while the number of DEA/AR1 livable years is scarcely reduced (Table III). It seems to be unadvisable for many of the DEA livable years if the environment is lightly weighted, while it seems to be regardless if the economy is lightly weighted.

Figure 1 shows that the DEA/AR2 measures downward shift more from the DEA measures than the DEA/AR1 measures. Referring to the virtual indicator values in Table I, the main causes of DEA/AR2 measure reduction appear to be the relative importance decrease of forest area for years up to the first half of the 1960's and those of traffic accidents and water service for the 1980's. On the other hand, almost all years would not be evaluated in terms of only economy aspect, so that the DEA/AR1 measures are not reduced from the DEA measures.

Table III shows that eight years are judged both DEA/AR1 and DEA/AR2 (therefore, naturally also DEA) livable. Since each of these years can rank top in both of the DEA/AR analyses, it would be endowed with aspects of both environment and economy. We should here consider the eight years' livability. These years except for 1990 are all in the period the second half of the 1960's to the 1970's. Therefore, this period would be better-balanced than the periods before and after.

V. SUMMARY AND CONCLUSIONS

This study performed a DEA time series analysis of Japan's livability for the period

1956–1990 regarding each year as a separate DMU. We found that 20 of the 35 years are DEA livable, and that, among them, the eight years, most of which are in the period 1968–1980, can be considered best-balanced. However, as shown in the results of analysis, we cannot simply say that livability of Japan has gotten better for 1956–1990.

While the idea of comparing years by taking a weighted sum of their attributes is commonplace, the idea that each year may have the freedom to choose its own optimal weights is not commonplace. DEA has a flexible weighting system that can vary by DMU, so that it can avoid indiscriminately unified comparisons as well as uniform evaluations by *a priori* weighting. Therefore, DEA can be a non-uniform, multi-dimensional and relative evaluation tool with distinct advantages over alternative models.

APPENDIX

Charnes *et al.* (1978) showed that the relative efficiency (DEA measure) of target DMU j_0 , h_{j_0} ($0 \leq h_{j_0} \leq 1$), can be obtained by solving the following fractional programming problem:

$$\text{Maximize } h_{j_0} = \frac{\sum_{r=1}^t u_r y_{rj_0}}{\sum_{i=1}^m v_i x_{ij_0}} \quad (\text{A.1})$$

$$\text{subject to } \frac{\sum_{r=1}^t u_r y_{rj}}{\sum_{i=1}^m v_i x_{ij}} \leq 1, \quad j = 1, \dots, n,$$

$$u_r, v_i > 0, \quad r = 1, \dots, t, \quad i = 1, \dots, m,$$

where y_{rj} = the amount of output r from DMU j ; x_{ij} = the amount of input i to DMU j ; u_r = the weight given to output r ; v_i = the weight given to input i ; n = the number of DMUs; t = the number of outputs; m = the number of inputs.

DEA measures of all the DMUs can be found by solving problem (A.1) n times, setting each DMU as target DMU j_0 in turn. Here, DMUs j_0 with maximum $h_{j_0} = 1$ are judged DEA efficient, while the other DMUs j_0 with $h_{j_0} < 1$ are DEA inefficient.

The fractional programming problem (A.1) can be converted into the following linear programming formulation:

$$\text{Maximize } h_{j_0} = \sum_{r=1}^t u_r y_{rj_0} \quad (\text{A.2})$$

$$\text{subject to } \sum_{i=1}^m v_i x_{ij_0} = 1,$$

$$\sum_{r=1}^t u_r y_{rj} - \sum_{i=1}^m v_i x_{ij} \leq 0, \quad j = 1, \dots, n,$$

$$u_r, v_i \geq \varepsilon, \quad r = 1, \dots, t, \quad i = 1, \dots, m,$$

where ε = a positive non-Archimedean infinitesimal.

Of course, instead of problem (A.2), we may solve the dual:

$$\text{Minimize} \quad \theta - \varepsilon \left(\sum_{r=1}^t s_r^+ + \sum_{i=1}^m s_i^- \right) \quad (\text{A.3})$$

$$\text{subject to} \quad \sum_{j=1}^n y_{rj} \lambda_j - s_r^+ = y_{rj_0}, \quad r = 1, \dots, t,$$

$$x_{ij_0} \theta - \sum_{j=1}^n x_{ij} \lambda_j - s_i^- = 0, \quad i = 1, \dots, m,$$

$$\lambda_j, s_r^+, s_i^- \geq 0, \quad j = 1, \dots, n, \quad r = 1, \dots, t, \quad i = 1, \dots, m,$$

$$(\theta \text{ unconstrained}),$$

where θ, λ_j = dual variables; s_r^+, s_i^- = slack variables.

Actually, we can obtain the solution by solving problem (A.3) in terms of the two-phase optimization method not dealing with ε as any concrete number (Charnes *et al.*, 1986).

ACKNOWLEDGMENTS

The authors would like to thank Mr. Kuniaki Kawai, College of Socio-Economic Planning, University of Tsukuba (currently working at Daidoh Steel Co. Ltd), for his assistance in executing this study.

REFERENCES

- Boussofiane, A., Dyson, R. G. and Thanassoulis, E.: 1991, 'Applied data envelopment analysis', *European Journal of Operational Research* 52, pp. 1-15.
- Charnes, A., Cooper, W. W., Lewin, A. Y. and Seiford, L. M. (eds.): 1994, *Data Envelopment Analysis: Theory, Methodology, and Application* (Kluwer Academic Publishers, Boston).
- Charnes, A., Cooper, W. W. and Rhodes, E.: 1978, 'Measuring the efficiency of decision making units', *European Journal of Operational Research* 2, pp. 429-444.
- Charnes, A., Cooper, W. W. and Thrall, R. M.: 1986, 'Classifying and characterizing efficiencies and inefficiencies in data envelopment analysis', *Operations Research Letters* 5, pp. 105-110.
- Cook, W. D. and Kress, M.: 1990, 'A data envelopment model for aggregating preference

- rankings', *Management Science* 36, pp. 1302–1310.
- Cooper, W. W., Kumbhakar, S., Thrall, R. M. and Yu, X.: 1995, 'DEA and stochastic frontier analyses of the 1978 Chinese economic reforms', *Socio-Economic Planning Sciences* 29, pp. 85–112.
- Doyle, J. R. and Green, R. H.: 1991, 'Comparing products using data envelopment analysis', *OMEGA International Journal of Management Science* 19, pp. 631–638.
- Hashimoto, A.: 1993a, 'Evaluating baseball batters using DEA' (in Japanese), *Communications of the Operations Research Society of Japan* 38, pp. 146–153.
- Hashimoto, A.: 1993b, 'A DEA selection system for selective examinations', *Institute of Socio-Economic Planning Discussion Paper Series 542* (University of Tsukuba, Tsukuba).
- Hashimoto, A.: 1995, 'A ranked voting system using DEA/AR exclusion model: A note', *Institute of Socio-Economic Planning Discussion Paper Series 626* (University of Tsukuba, Tsukuba).
- Hashimoto, A. and Ishikawa, H.: 1993, 'Using DEA to evaluate the state of society as measured by multiple social indicators', *Socio-Economic Planning Sciences* 27, pp. 257–268.
- OECD: 1976, *Measuring Social Well-Being: A Progress Report on the Development of Social Indicators* (Paris).
- Thompson, R. G., Singleton, F. D. Jr., Thrall, R. M. and Smith, B. A.: 1986, 'Comparative site evaluations for locating a high-energy physics lab in Texas', *Interfaces* 16, pp. 35–49.

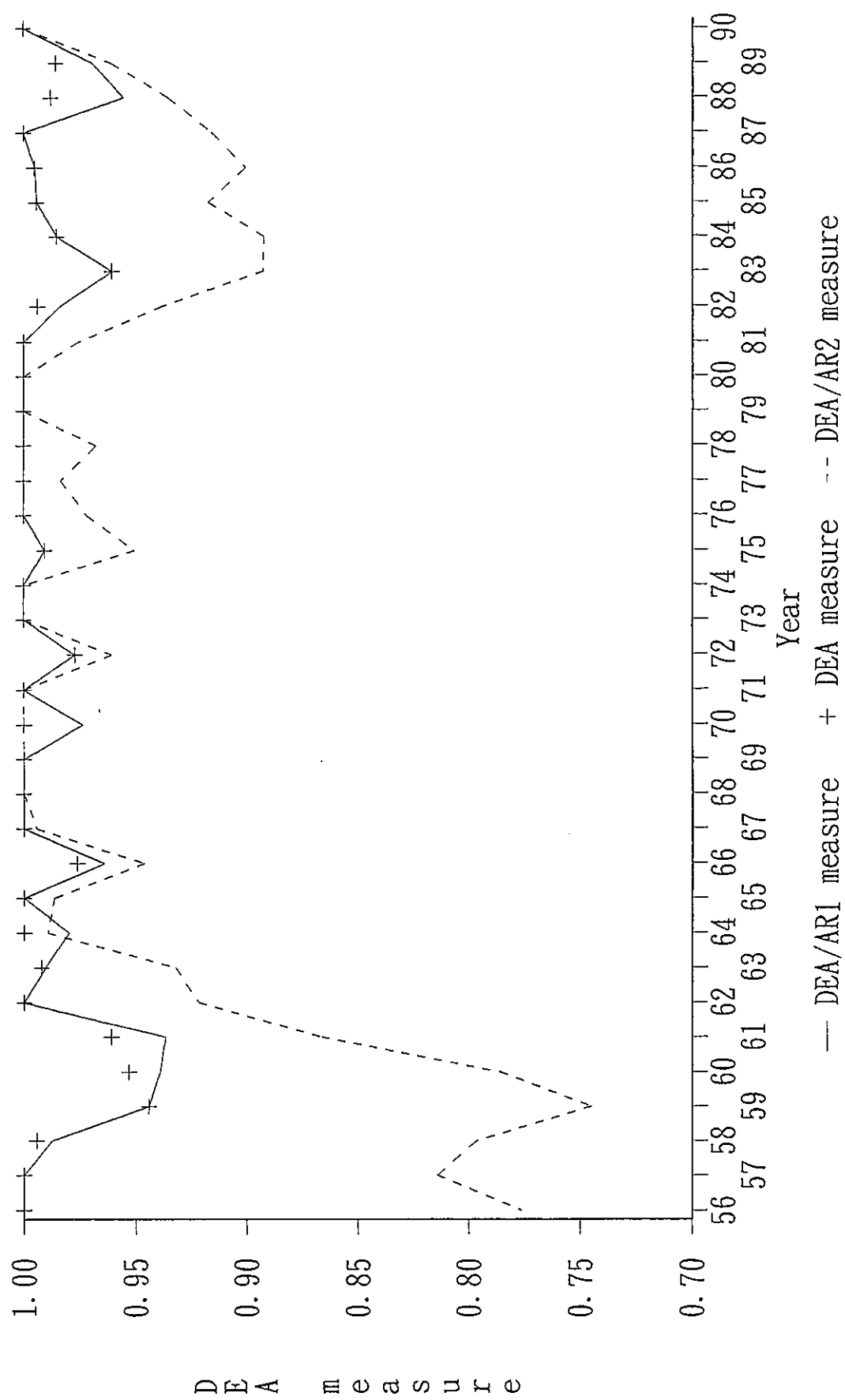


Fig. 1 Transitions of DEA and DEA/AR measures

TABLE I
DEA measures and virtual indicator values

DEA measure	Year	Negative indicator				Positive indicator			
		Suicide	Crime	Traffic acci- dents	Un- employ- ment	Life expect- ancy	Na- tional income	Forest area	Water ser- vice
1	1956	0	0	1.000	0	0.209	0	0.791	0
1	1957	0	0	0.475	0.525	0	0	1.000	0
0.9943	1958	0	0.553	0.321	0.126	0.036	0	0.958	0
0.9442	1959	0.173	0.318	0.371	0.138	0	0	0.944	0
0.9531	1960	0	0.383	0.327	0.290	0	0	0.953	0
0.9609	1961	0	0.330	0.225	0.445	0	0	0.961	0
1	1962	0	0	0.460	0.540	0	0	1.000	0
0.9923	1963	0.305	0	0.650	0.045	0	0	0.992	0
1	1964	0	0	0	1.000	0.011	0	0.989	0
1	1965	0.205	0	0.641	0.154	0	0	0.691	0.309
0.9762	1966	0	0.315	0.212	0.473	0	0	0.976	0
1	1967	0.807	0.193	0	0	0	0	1.000	0
1	1968	0	0.168	0.312	0.520	0	0	0.625	0.375
1	1969	0.793	0	0	0.207	0	0	0	1.000
1	1970	0	0	0	1.000	0	0.824	0.176	0
1	1971	0.216	0.135	0	0.649	1.000	0	0	0
0.9770	1972	0.591	0.409	0	0	0.230	0	0.234	0.513
1	1973	0	0.177	0	0.823	0	1.000	0	0
1	1974	0	0.068	0.441	0.491	0	0	0.410	0.590
0.9907	1975	0	0.981	0.019	0	0	0	0.162	0.829
1	1976	0.390	0.387	0.223	0	0	0	0.518	0.482
1	1977	0.078	0.369	0.459	0.094	0	0	0.866	0.134
1	1978	0.362	0	0.638	0	0	0	0.689	0.311
1	1979	0	0.553	0.447	0	0	1.000	0	0
1	1980	0.110	0	0.343	0.547	0	0	0	1.000
1	1981	0.656	0	0.344	0	0.009	0	0.313	0.678
0.9938	1982	0.307	0	0.693	0	0.932	0	0.062	0
0.9605	1983	0	0	1.000	0	0.016	0	0	0.945
0.9852	1984	0	0	1.000	0	0.016	0	0	0.969
0.9943	1985	0	0	1.000	0	0	0.014	0	0.980
0.9952	1986	0	0	1.000	0	0	0.014	0	0.981
1	1987	0.173	0.025	0.802	0	0.791	0.209	0	0
0.9880	1988	0	0	0.798	0.202	0	0.626	0	0.362
0.9857	1989	0	0	0.707	0.293	0.986	0	0	0
1	1990	1.000	0	0	0	0	1.000	0	0

Bold: the greatest virtual values of negative and positive indicators for each year

TABLE II

DEA unlivable years and their reference sets

		DEA measure															
		DEA unlivable year and combination coefficients of its reference set															
Reference set	1958	1959	1960	1961	1963	1966	1972	1975	1982	1983	1984	1985	1986	1988	1989		
1956	0.40	0.62					0.01										
1957	0.49	0.06	0.30														
1962			0.46	0.62	0.34												
1964				0.14		0.26											
1965		0.08		0.27	0.66	0.11											
1967	0.04	0.27	0.28			0.64	0.05										
1968																	
1969																	
1970																	
1971																	
1973								0.30									
1974								0.60	0.02								
1976								0.69									
1977																	
1978																	
1979																	
1980								0.29								0.05	
1981									0.81	0.81	0.62	0.45	0.21				
1987									0.13	0.19	0.38	0.55	0.79	0.51	0.21		
1990									0.06					0.44	0.79		

Bold: the greatest combination coefficient for each DEA unlivable year

TABLE III
DEA and DEA/AR measures

Year	DEA/AR1 measure	DEA measure	DEA/AR2 measure
1956	1	1	0.7764
1957	1	1	0.8141
1958	0.9872	0.9943	0.7963
1959	0.9442	0.9442	0.7451
1960	0.9390	0.9531	0.7862
1961	0.9366	0.9609	0.8667
1962	1	1	0.9215
1963	0.9905	0.9923	0.9321
1964	0.9797	1	0.9893
1965	1	1	0.9863
1966	0.9639	0.9762	0.9462
1967	1	1	0.9944
1968 [†]	1	1	1
1969 [†]	1	1	1
1970	0.9736	1	1
1971 [†]	1	1	1
1972	0.9770	0.9770	0.9604
1973 [†]	1	1	1
1974 [†]	1	1	1
1975	0.9907	0.9907	0.9504
1976	1	1	0.9719
1977	1	1	0.9835
1978	1	1	0.9675
1979 [†]	1	1	1
1980 [†]	1	1	1
1981	1	1	0.9755
1982	0.9839	0.9938	0.9381
1983	0.9603	0.9605	0.8924
1984	0.9849	0.9852	0.8922
1985	0.9940	0.9943	0.9173
1986	0.9951	0.9952	0.9004
1987	1	1	0.9157
1988	0.9552	0.9880	0.9355
1989	0.9691	0.9857	0.9607
1990 [†]	1	1	1
The number of DEA(/AR) livable years			
	18	20	9

[†] Year of both DEA/AR1 and DEA/AR2 livable