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Income Inquality and Growth: A Simulation Study

by

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Abstract

The purpose of this paper is to investigate inequality changes during the course of economic growth of a dynamic optimization problem. One-sector and two-sector growth models that involve distribution in endowments are numerically solved to simulate inequality change starting at low capital accumulation. The simulation result of the one-sector model demonstrates that the "inverse-U" curve may or may not be observed depending on production technology and intertemporal preferences, and that a rapidly growing economy with high saving rates is likely to experience a decline in inequality without "inverse-U". Two-sector models with agriculture and industry demonstrate "inverse-U", "U", and a monotonic decline in inequality.
Keyword: Dynamic optimization problem, Inequality, Inverse-U, Numerical solution
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1. Introduction

Income inequality has been a classical issue in the study of economic growth since Kuznets (1955) addressed his celebrated "inverse-U" hypothesis by observing that inequality rises at first and then turns to decline during the course of economic growth. Kuznets encouraged subsequent studies in this topic, because his discussion was made by observing data for currently developed countries (the United States, England, and Germany) and by simple simulation, restricted to "perhaps 5 per cent empirical information and 95 per cent speculation". Provoked by the study, considerable number of empirical and theoretical researches have contributed to study the relationship between inequality and economic growth.

Theoretical contributions have been made to explain "inverse-U" in the context of structural change from the traditional sector to the modern sector. Robinson (1976) formulated a simple two-sector model to prove the "inverse-U" hypothesis, assuming that mean income of the traditional sector is lower than that of the modern sector given distributions of per capita income in each sector, and that labor migrates from the traditional sector to the modern sector. Recently, Anand and Kanbur (1993) generalized Robinson's result by employing general form of distribution and various inequality measures, allowing a change in relative sectoral mean incomes over time.

Empirical results seem to be in favor of the "inverse-U" hypothesis, but have not been conclusive. Ahluwalia et al. (1979), for example, found the "inverse-U" relationship, but Saith (1983) criticized analyses using cross-country data in testing the Kuznets' hypothesis on time series change. Unfortunately, most empirical studies have used cross-country data, because time series analysis for current developing countries is extremely difficult due to data limitation.

Table 1.1 presents some figures on inequality change in income share of the bottom 60% for selected developing countries. What does the table suggest? First, inequality levels are diversified across countries; in particular, Asian countries seem to be equalized more than Latin American countries. Second, there are various types in inequality change in time series; some countries are

Table 1.1: Inequality change in selected countries

Country by Type		Share of bottom 60%				
		$t=1$	$t=2$	$t=3$	$t=4$	$t=5$
(1)	Equalized					
	Peru	0.179 ('61)	0.180 ('72)	0.266 ('85-6)		
	Philippines	0.247 ('61)	0.248 ('71)	0.300 ('85)	0.310 ('88)	
	Venezuela	0.232 ('70)	0.279 ('87)	0.287 ('89)		
(2)	"Inverse-U"					
	Costa Rica	0.253 ('71)	0.248 ('86)	0.274 ('89)		
	India	0.310 ('54)	0.292 ('64)	0.301 ('75-6)	0.367 ('83)	0.375 ('89-90)
	Korea	0.349 ('65)	0.323 ('76)	0.360 ('88)		
	Mexico	0.217 ('63)	0.197 ('75)	0.219 ('77)	0.242 ('84)	
(3)	"U"					
	Bangladesh	0.326 ('81-2)	0.409 ('85-6)	0.399 ('88-9)		
	Colombia	0.190 ('64)	0.212 ('74)	0.262 ('88)	0.238 ('91)	
	Indonesia	0.270 ('76)	0.372 ('87)	0.367 ('90)		
	Malaysia	0.236 ('73)	0.278 ('87)	0.259 ('89)		
(4)	Others					
	Brazil	0.248 ('60)	0.206 ('70)	0.164 ('72)	0.188 ('83)	0.159 ('89)
	Sri Lanka	0.274 ('63)	0.354 ('73)	0.300 ('80-1)	0.254 ('85-6)	0.389 ('90)

Sources: Ahluwalia et al. (1979) Table 4, and World Bank (various years)

monotonically equalized (although it is possible for these countries to experience a turning point before, during or after the periods in the table), some experience "inverse-U", some encounter "U" conversely, and others are even difficult to tell. It seems that the process of inequality is varied across countries during the course of economic growth.

The relationship between inequality and growth rates has been another concern in economic development. Ahluwalia (1976) tested if faster growth induces higher inequality, and denies positive relationship between growth rates and inequality. Recently, World Bank (1993) observes that high performing Asian countries share common characteristics to successfully achieve rapid economic growth and equality at once.

The purpose of this paper is to investigate the relationship between inequality change and economic growth, without a priori conditions that generate "inverse-U" such as income inequal-

ity between sectors and invariant sectoral income distributions. Section 2 introduces a dynamic optimization problem in which a heterogeneously endowed agent behaves so as to maximize her life-time utility. The model is numerically solved and simulated to demonstrate possible inequality processes during the course of economic growth. Section 3 presents the simulation result of the one-sector model. The simulation demonstrates that whether the "inverse-U" phenomenon is observed depends on production technology and intertemporal preferences, and that a rapidly growing economy with high saving rates is more likely to experience equalization without "inverse-U". Section 4 considers two types of two-sector models; one is a model in which two sectors coexist in the neighborhood, and another is a model in which two sectors are entirely separated into rural area and urban area. The simulation result of the first model is similar to the result of the one-sector model, but the second type demonstrates both the "inverse-U" curve and the "U" curve depending on distribution of initial endowments. Finally, Section 5 gives concluding remarks.

2. One-Sector Growth Model

2.1. One-Sector Economy

Suppose that an agent maximizes her life-time utility given initial wealth in a growing economy starting at low capital accumulation. Agent i solves

$$\begin{aligned} \max \quad & \sum_{t=0}^{\infty} \beta^t u(c_t(i)) \\ \text{s.t.} \quad c_t(i) + s_{t+1}(i) &= w_t + (1 + r_t) \cdot s_t(i) \\ s_0(i) &: \text{given} \end{aligned} \tag{2.1}$$

where β is the discount factor $(0 < \beta < 1)$, $u(\cdot)$ is an instantaneous utility function, $c_t(i)$ is consumption for i, $s_t(i)$ is savings for i, w_t is wages, r_t is return on capital, and subscript t denotes period. Initial endowment $s_0(i)$ is assumed to follow a distribution. Each agent inelastically supplies one unit of labor, and receives wages w_t determined in the labor market. Given the path $\{w_t, r_t\}_{t=0}^{\infty}$

of the aggregate economy, the stream of life-time budget constraint from time t is

$$c_t(i) + \sum_{h=1}^{\infty}\left[(\prod_{j=1}^{h}\frac{1}{(1+r_{t+j})})\cdot c_{t+h}(i)\right] = w_t + \sum_{h=1}^{\infty}\left[(\prod_{j=1}^{h}\frac{1}{(1+r_{t+j})})\cdot w_{t+h}\right] + (1+r_t)\cdot s_t(i)$$

Assuming the CRRA utility function: $u(c) = (c^{1-\mu}-1)/(1-\mu)$ where $\mu > 0$, and using the first order condition $u'(c_t(i)) = \beta \cdot u'(c_{t+1}(i))(1+r_{t+1})$ where $u'(c) = \partial u(c)/\partial c$, consumption policy for agent i is

$$c_t(i) = \frac{w_t + \sum_{h=1}^{\infty}\left[(\prod_{j=1}^{h}\frac{1}{(1+r_{t+j})})\cdot w_{t+h}\right] + (1+r_t)\cdot s_t(i)}{1+\sum_{h=1}^{\infty}\beta^{h/\mu}\left[\prod_{j=1}^{h}(1+r_{t+j})\right]^{1/\mu-1}} \tag{2.2}$$

Production technology $F(K_t, L_t)$ is assumed to be constant returns to scale with respect to labor supply L_t and capital stock K_t. Assuming labor supply is fixed over time, i.e., $L_t = L$, $f(k_t) = F(K_t, L)/L$ ($f' > 0, f'' < 0$) and $k_t = K_t/L$. From the profit maximization condition given capital stock at time t, return on capital is

$$r_t = f'(k_t) - \delta \tag{2.3}$$

where δ is the depreciation rate of capital, and wages are

$$w_t = f(k_t) - (\delta + r_t)\cdot k_t \tag{2.4}$$

which is equivalent to the marginal product of labor. The path $\{w_t, r_t\}_{t=0}^{\infty}$ of the aggregate economy is obtained by solving the following representative agent problem

$$\begin{aligned} \max \quad & \sum_{t=0}^{\infty}\beta^t u(c_t) \\ \text{s.t.} \quad c_t + k_{t+1} - k_t &= f(k_t) - \delta\cdot k_t \\ k_0 &: \text{given} \end{aligned} \tag{2.5}$$

where c_t is the average consumption.

2.2. Choice of Numerical Technique and Simulation Method

To investigate the process of inequality change, the problem is solved by two steps. First, the optimal path of the aggregate economy (2.5) is solved to obtain the growth path $\{k_t\}_{t=0}^{T}$ in order to calculate the path $\{w_t, r_t\}_{t=0}^{T}$ using equations (2.3) and (2.4). Second,consumption policies for agent $i = 1, ..., N$ is approximated from equation (2.2) by choosing sufficiently large T[1] for ∞. Then, inequality change is simulated for sufficiently large N given a distribution of initial endowments.

Numerical Method: Since the analytical solution is not known for the problem (2.5) except special cases, numerical technique is applied to approximate the solution far from the steady state. In selecting numerical technique, three algorithms recently developed are considered; one is the projection method developed by Judd (1992) and others are improved PEA (Parameterized Expectation Approach, developed by den Haan and Marcet (1990)) methods by Christiano and Fisher (1994). Both methods are well developed[2], and show high performance in their own experiments.

Accuracy Check as a Selection Criterion: Selection is made by accuracy tests of the following three methods: the projection method by Judd, the PEA collocation method and the PEA Galerkin method by Christiano and Fisher. In addition, the state variable is defined as $\ln k$ in addition to k to improve accuracy far from the steady state. For the purpose of simulating the course of economic growth, I check accuracy of these methods over a wide range of the state space (overall, accuracy deteriorates as the lower bound becomes small). Accuracy is checked by the test developed by Judd (1992)[3] that checks residual ratio of the Euler equation to consumption policy at 1,000 equidistant grid points over the state space.

The test result supports that all of the three methods show sufficiently high performance using

[1] T is chosen to satisfy that the relative difference between $c_t(i)$ calculated up to T and $c_t(i)$ up to $T+1$ becomes smaller than 1.0×10^{-6}.

[2] Rust (1994a, 1994b) provides a broad and detailed review of numerical methods in Economics.

[3] den Haan and Marcet (1994) proposed another accuracy test method. However, the method is developed for stochastic models focusing on stationary process. Therefore, I employ the test method by Judd that is applied for both deterministic and stochastic models by checking accuracy all over the state space.

$\ln k$. The choice in this paper is the PEA Galerkin[4], because of its good accuracy with relatively low degrees of polynomial over the wide range of the state space .

Distribution: The choice of the distribution is the uniform distribution in which the poor and the rich spread over the society, and the exponential distribution in which distribution is skew toward the poor[5]. Initial endowment $s_0(i)$ is randomly drawn from either the uniform distribution $d(x; \underline{z}, \overline{z}) = 1/(\overline{z} - \underline{z})$ or the exponential distribution $d(x; \lambda) = \lambda e^{-\lambda x}(x > 0)$ for $i = 1, ..., N$. Means of the initial endowments take the same value for the distributions. In the simulation, $x \in [0, 0.1]$ for the uniform case, $\lambda = 20$ for the exponential case, and N equals 1000.

Production Technology and Parameters: Production technology is assumed to take the Cobb-Douglas form: $f(k) = \iota k^{\gamma}$, where ι is a constant such as the steady state $k = 1.0$ and $0 < \gamma < 1$. Parameter values are as follows: β =0.95, δ =0.05, $\gamma = 0.33$ (or 0.5, 0.67), μ =0.5, 1.1, 2.0, 5.0, and 8.0, and initial capital stock $k_0 = 0.05$.

3. Inequality Change of the One-Sector Model

Economic Growth: Before jumping to inequality analysis, it is useful to look at the process of the aggregate economy. Figure 1 presents output growth and saving rates with $\gamma = 0.33$. With higher intertemporal elasticity of substitution (μ^{-1}), an economy saves more at the early stages and grows faster. One interesting characteristics is that initial saving rates are higher than the steady state level with $\mu = 0.5$, 1.1, and 2.0, but lower with $\mu = 5.0$ and 8.0. These two groups also show different result in inequality change.

[4]The Judd method also shows as good performance as the PEA Galerkin with relatively high degrees of polynomials and a moderate range of the state space. In fact, in the deterministic version, only one difference between the Judd method and the PEA Galerkin is the choice of the term to be numerically solved. Judd chooses consumption policy, and PEA methods choose log of the right handside of the Euler equation without the discount factor.

[5]Any distribution such as the pareto distribution can be chosen for the one-sector model; these two distributions are picked up in order to simplify the algorithm of the two-sector model described later.

Inequality Change: Figure 2 presents inequality change by the Gini coefficient change relative to the initial level (actual Gini coefficient levels are varied between the distributions; a horizontal line without symbols is a zero line). Figure 2 (A) contains cases in decreasing inequality with $\mu = 0.5$, 1.1, and 2.0 (U and X in parentheses indicate the uniform distribution and the exponential distribution, respectively), while Figure 2 (B) demonstrates "inverse-U" cases with $\mu = 5.0$, and 8.0[6]. Simulation experiments indicate that a change in parameter values of the initial distribution does not affect this result.

Production technology is another factor for changing the inequality process. Figures 3 compares inequality changes with $\gamma = 0.33, 0.5$, and 0.67. As technology becomes capital-intensive, "inverse-U" tends to disappear. However, this has pros and cons; with $\gamma = 0.67$, an increase in inequality seems to be avoided but a decline seems to be also small comparing to the case with $\gamma = 0.33$.

Lorenz Curve: The Gini coefficient alone may not be enough to represent inequality change in some cases. Particularly, development economists often regard the situation of the poorest class as important; a decrease in the Gini coefficient does not necessarily indicate an improvement of the poorest class. To make up this point, income distributions at $t = 1, 5, 10, 20$, and 50 are also checked in the form of the Lorenz curve or distribution by 10 percent.

Figure 4 presents the Lorenz curves at time $t = 1, 5$, and 50 that shifts without crossing one another. With $\mu = 1.1$, the Lorenz curve quickly shifts toward the 45 degree line, while the change seems to be very small with $\mu = 8.0$. This is true not only by time series change but also by capital accumulation level; at a some capital stock level during the growth process, the Gini coefficient with $\mu = 8.0$ is bigger than the one with smaller μ.

In summary, the simulation result suggests (1) that "inverse-U" can be generated without assuming structural change, (2) that "inverse-U" may or may not be observed depending on in-

[6]The possibility of "inverse-U" is not deniable for the decreasing-inequality cases when capital stock is less than the initial value. However, it was not fruitful to investigate this point, because $k_0 = 0.001$ only extends a few periods before $k_t = 0.05$ at the cost of accuracy of the numerical solution.

tertemporal preferences and production technology, and (3) that a rapidly growing economy with high saving rates is likely to be equalized rapidly without "inverse-U".

4. An Extension: Two-Sector Growth Model

The one-sector model can be easily modified to a two-sector model to include structural change with capital accumulation. Now, assume that the production technology of agriculture is

$$F(A, L_t^A) = \kappa A^\alpha (L_t^A)^{1-\alpha}$$

and that of industry is

$$G(K_t, L_t^I) = \iota (K_t)^\gamma (L_t^I)^{1-\gamma}$$

where A is arable land constant over time, L_t^A is labor in agriculture, L_t^I is labor in industry where $L_t^A + L_t^I = L$ (constant over time), $0 < \alpha < 1$, $0 < \gamma < 1$, and κ and ι are some constants.

Then, who owns which factor? Does an entrepreneur possess land in rural area? Does a farmer hold stocks? Can an agent allocate labor between both sectors? The answer may vary across countries due to their own specific institutional or natural conditions. In this section, I provide two models as extreme cases. One model assumes that an agent allocates labor between both sectors and receives both rent on land and return on capital. The other model assumes that an agent in agriculture provides arable land and labor for agriculture, and an agent in industry provides capital stock and labor for industry.

4.1. Two-Sector Model (A)

The model is very similar to the one-sector model. Suppose that an agent solves the problem

$$\begin{aligned} \max \quad & \sum_{t=0}^{\infty} \beta^t u(c_t(i)) \\ \text{s.t} \quad c_t(i) + s_{t+1}(i) &= w_t + (1 + r_t) \cdot s_t(i) + q_t \cdot A(i) \\ s_0(i), A(i) &: \text{given} \end{aligned} \tag{4.1}$$

where q_t is rent on land, and $A(i)$ is heterogeneous land endowment for agent i fixed over time.

This type of industrialization may be observed in a relatively small country where an agent is able to allocate labor both in his fields and in industry by season, or some members of household work in their fields while other members work at factory or office If an agent is interpreted as a household[7].

Again, the aggregate economy is characterized by the representative agent problem that solves

$$\begin{aligned} \max \quad & \sum_{t=0}^{\infty} \beta^t u(c_t) \\ \text{s.t} \quad c_t + k_{t+1} - k_t &= f(\overline{a}, l_t^*) + g(k_t, 1 - l_t^*) - \delta \cdot k_t \\ k_0 &: \text{given} \end{aligned}$$

where $f(\overline{a}, l_t^*) = F(A, L_t^A)/L$, $g(k_t, 1 - l_t^*) = G(K_t, L - L_t^A)/L$, $\overline{a} = A/L$, $l_t = L_t^A/L$, and l_t^* is the optimal labor allocation to equate wages between sectors given k_t. In the simulation, land distribution is assumed to follow the uniform (exponential) distribution when initial endowment follows the uniform (exponential) distribution.

4.2. Two-Sector Model (B)

Suppose that two sectors are completely separated. An agent (or household) either stays in rural area to work in agriculture or moves to urban area to work in industry, but cannot allocate labor into both sectors. That is, all members of household move together when migrating to urban area, leaving their land behind. Also, agents in urban area alone invest in industry. This type of industrialization may be observed in a relatively big country where industry concentrates in a few big cities.

Suppose that an agent *in industry* solves the problem,

[7]Experiences in Japan could be an example of this type. Many male farmers in north-east part of Japan where heavy snow fall keeps farmers away from fields, moved to big cities as construction workers in the winter season. Also, wives and elderly family members took care of rice fields, while the head of a household and the young work in town. Fei, Ranis and Kuo (1979) discuss experiences of this type in the case of Taiwan.

$$\max \quad \sum_{t=0}^{\infty} \beta^t u(c_t(i)) \tag{4.2}$$

$$\text{s.t} \quad c_t(i) + s_{t+1}(i) = w_t^I + (1 + r_t) \cdot s_t(i)$$

$$s_0(i) \quad : \quad \text{given} \tag{4.3}$$

where w_t^I is wages in industry, $s_0(i)$ follows a distribution for agents in industry at the initial period, and $s_t(i)$ is zero for new comers at time t from rural area at period 1 or after. Suppose that an agent moves from rural area to urban area if income in rural area is less than wage income in urban area. Then, after the migration at time t,

$$w_t^A + q_t \cdot A_t(i) \geq w_t^I \tag{4.4}$$

for all i in rural area where w_t^A is wages in rural area, $A_t(i)$ is land endowment at time t for agent i. Rent on land and return on capital are given by the marginal product of each factor. Land is assumed to be abandoned after small land holders who do not satisfy the condition (4.4) leave rural area. For convenience, abandoned land is assumed to be proportionally allocated to remainders in rural area; that is, when land area B lose owners, remainder's land endowment becomes

$$A_t(i) = \frac{A}{A - B} A_{t-1}(i)$$

Land distribution keeps the same distributional form over time, but the average land holding rises as small farmers leave rural area.

The aggregate economy may not, unfortunately, coincide with the representative agent problem. The following two steps are repeated until getting convergence; (1) calculating the individual problem given the path of the aggregate economy, and (2) adjusting the aggregate economy by summing up individual activities.

4.3. Simulation Result of Two-Sector Models

Two-Sector Model (A): Parameters are as follows: $\gamma = 0.33$ for capital, $\alpha = 0.5$ for land as technology, initial capital stock $k_0(i) \in [0, 0.02]$ for the uniform distribution, $\lambda = 100$ for the

Table 4.1: Income distribution of Two-Sector Model (A)

%	t=1	Δ	t=5	Δ	t=10	Δ	t=20	Δ	t=50
0-10	0.049	−	0.047	+	0.048	+	0.050	+	0.054
10-20	0.056	−	0.054	0	0.054	+	0.056	+	0.060
20-30	0.062	−	0.060	+	0.061	+	0.062	+	0.066
30-40	0.070	−	0.068	+	0.069	+	0.070	+	0.073
40-50	0.079	−	0.078	0	0.078	+	0.079	+	0.081
50-60	0.090	−	0.089	0	0.089	+	0.090	+	0.091
60-70	0.102	0	0.102	0	0.102	0	0.102	0	0.102
70-80	0.120	+	0.121	0	0.121	−	0.120	−	0.118
80-90	0.147	+	0.149	−	0.148	−	0.146	−	0.142
90-100	0.226	+	0.232	−	0.231	−	0.225	−	0.214
GINI	0.267	+	0.279	−	0.277	−	0.265	−	0.241

exponential distribution, initial land distribution $A_0(i) \in [0, 0.2]$ for the uniform distribution, $\lambda = 10$ for the exponential distribution, and $\kappa = \iota$ for simplicity.

Figure 5 presents the simulation result of the structural change and inequality change. As the intertemporal elasticity of substitution (μ^{-1}) becomes higher, the structural change becomes faster. Initial saving rates are low with $\mu = 5.0$ and 8.0 as before, but "inverse-U" is not observed with $\mu = 5.0$ (initial capital stock takes a smaller value than the one-sector model to check "inverse-U" with $\mu = 5.0$ and 8.0). Also, additional experiments indicate that an increase of land endowments contributes to avoid "inverse-U".

Technology of agriculture including natural environment such as climates may also affect inequality change. Figure 5 (C) presents the simulation result with $\mu = 8.0$ and $\alpha = 0.33, 0.5$, and 0.67. As technology of agriculture becomes more labor intensive (i.e., α becomes smaller), "inverse-U" is less likely to be observed. Therefore, it is possible that some countries encounter "inverse-U" and others do not depending on natural environment, even with the same industrial technology, initial capital stock and preferences.

Table 4.1 presents the distribution of income at time $t = 1$, 5, 10, 20 and 50, and its change between these check points from the poorest to the richest by 10%; "+" indicates an increase of

income share for each group, "-" indicates a decrease, and "0" indicates that an absolute change is less than 0.001. The table is an example of the "inverse-U" case with $\mu = 8.0$ and $\alpha = 0.67$. At initial stages ($t = 1 \sim 5$) when inequality rises, the bottom 60% loses its share, and the top 30% gains. Later when inequality declines, the bottom 60% gains, and the top 30% loses.

Two-Sector Model (B): The uniform distribution and the exponential distribution show different movements in inequality changes in Figures 6[8] (A) and (C). Simulation with the uniform distribution demonstrates the "inverse-U" curve, while simulation with the exponential distribution demonstrates the "U" curve. In both cases, higher μ^{-1} which leads higher saving rates and faster growth contributes a decline in inequality.

To investigate why this difference occurs in more detail, Figures 6 (B) and (D) present the ratio of the average income of agriculture to that of industry. With the uniform distribution, the average income of agriculture is lower than that of industry over time, and time series change is relatively small. With the exponential distribution, the average income of agriculture is higher than that of industry at first, but falls quickly and becomes lower with the structural change.

In terms of income distribution by sector, inequality in agriculture falls over time with either distribution. However, inequality change in industry demonstrates "inverse-U" with the uniform distribution, while inequality rapidly falls with the exponential distribution. Thus, "inverse-U" in industry seems to be a major factor to cause "inverse-U" with the uniform distribution, while a fall of relative income of agriculture against industry seems to cause the "U" change in inequality with the exponential distribution.

Table 4.2 presents changes of income distribution with $\mu = 2.0$. With the uniform distribution, (1) the poorest group keeps or gains their income share over time; (2) groups over a wide middle range lose at first stages but the range of the losers becomes smaller (groups in 10-70% lose between $t = 1 \sim 5$, but groups only in 40-60% lose between $t = 5 \sim 10$); and (3) increasing range of the

[8]The simulation with $\mu = 0.5$ failed to obtain convergence.

Table 4.2: Income distribution of Two-Sector Model (B)

Uniform,μ=2.0					Exponential,μ=2.0				
%	t=1-5	t=5-10	t=10-20	t=20-50	%	t=1-5	t=5-10	t=10-20	t=20-50
0-10	+	+	+	+	0-10	+	−	−	−
10-20	−	0	+	+	10-20	+	−	−	−
20-30	−	+	+	+	20-30	+	+	−	−
30-40	−	+	+	+	30-40	+	+	+	+
40-50	−	−	+	+	40-50	+	+	+	+
50-60	−	−	+	+	50-60	−	+	+	+
60-70	−	+	+	0	60-70	−	+	+	+
70-80	+	+	−	−	70-80	−	+	+	+
80-90	+	0	−	−	80-90	−	0	0	−
90-100	+	−	−	−	90-100	−	−	−	−
GINI	+	−	−	−	GINI	−	+	+	+

rich, instead of middle groups, loses as time passes. With the exponential distribution, the bottom 50% is the initial winners, but the bottom 30% loses afterwards. Instead, groups in middle ranges (but slightly biased upwards) are winners when inequality declines.

5. Concluding Remarks

The purpose of this paper has been to investigate inequality changes during the course of economic growth of a dynamic optimization problem, without "stylized facts" such as exogenous income inequality between sectors. The simulation of the one-sector model demonstrates that "inverse-U" can be observed without the structural change, and that the process of inequality may be varied depending on technology and preferences. The simulation result has been consistent with the "growth with equity" experience in East Asia in the sense that a rapidly growing economy with high saving rates seems to have a good chance to avoid initial unequalization. Simulation results of the two-sector models demonstrate more diversified processes in inequality than the one-sector model, depending on technology, preferences, distribution of initial endowments, and factor ownerships.

Since the model was pretty simple, other factors to influence income distribution need to be

considered to interpret the complicated real world. First, the model does not include distribution in wages due to educational level, job experiences, personal ability, social and cultural restrictions, etc. If wages are diversified during the course of development, "inverse-U" may be observed more frequently than the simulation. Second, different technological advancement by sector could alter mean income inequality between sectors and as a result, income distribution in the aggregate economy. Third, the model assumes that return on capital is determined in the financial market in which every agent (at least in industry) has an access, but limited access to the financial market in underdeveloped countries may adversely affect inequality. Also, the simulation result of the two-sector models are restricted by specific assumptions on factor ownerships.

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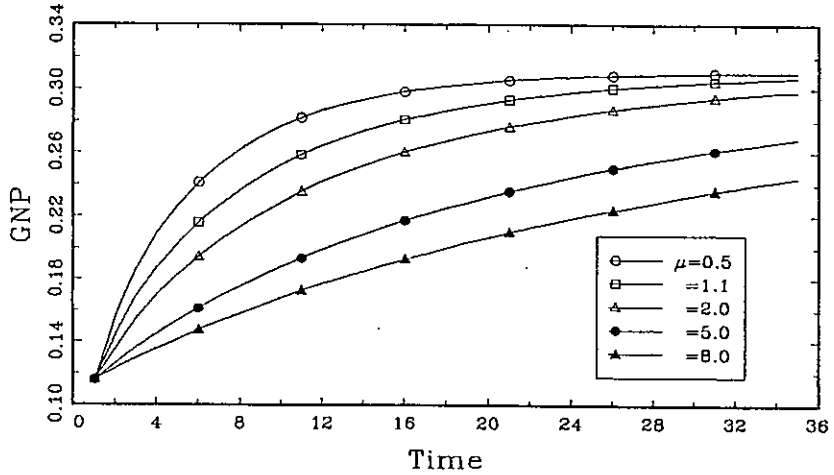
Figure 1: One-Sector Model

(A)GNP Growth



(B) Saving Rate

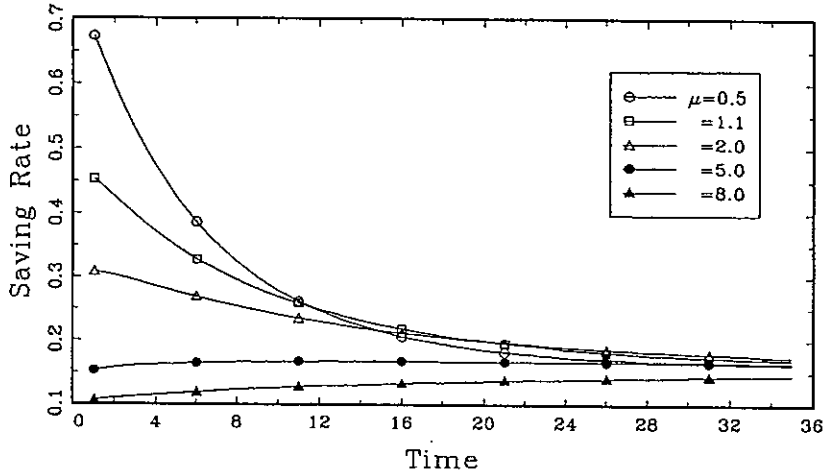
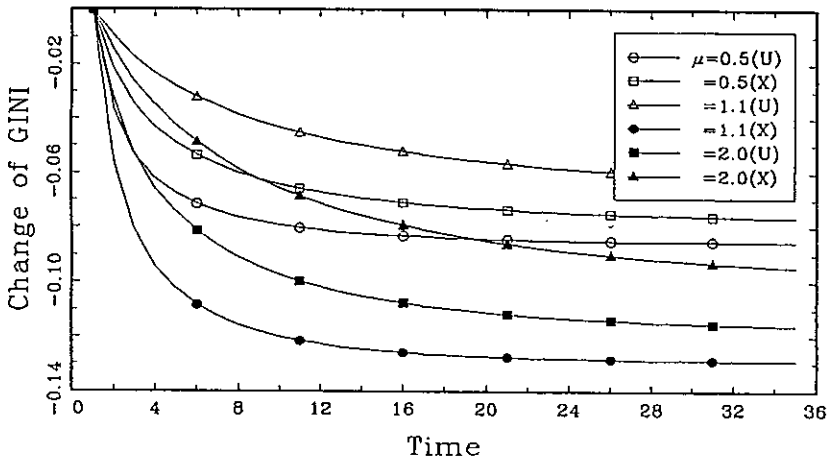


Figure 2: Inequality Change

(A) Monotonic Case



(B) Inverse-U case

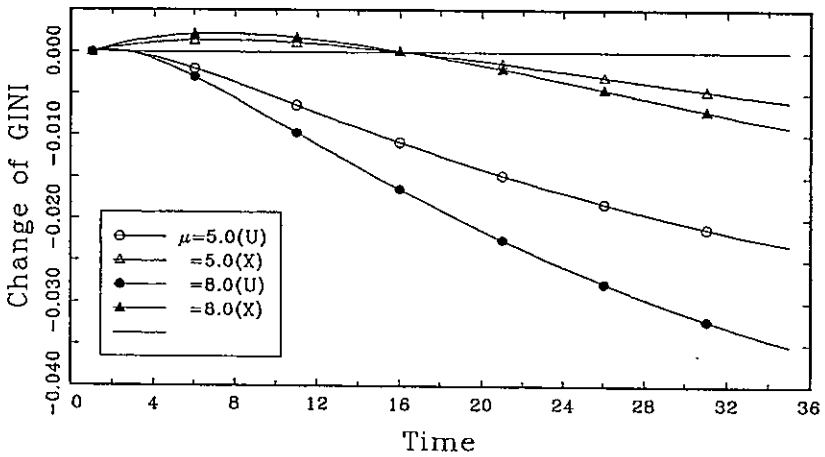


Figure 3: Inequality Change on Technology

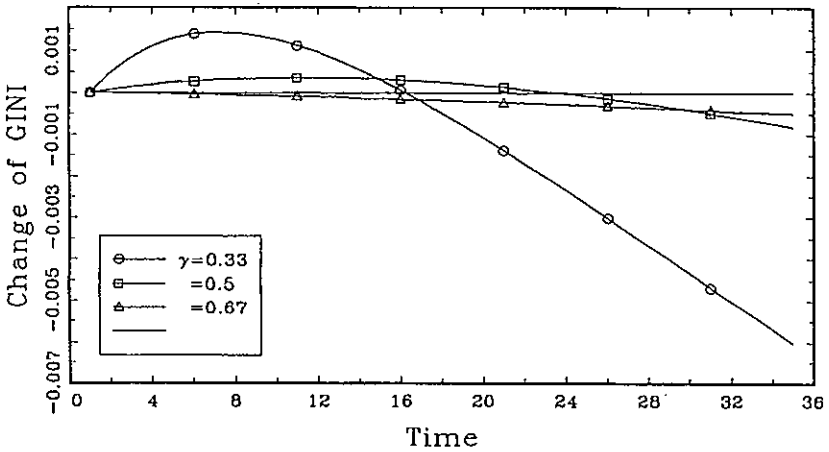
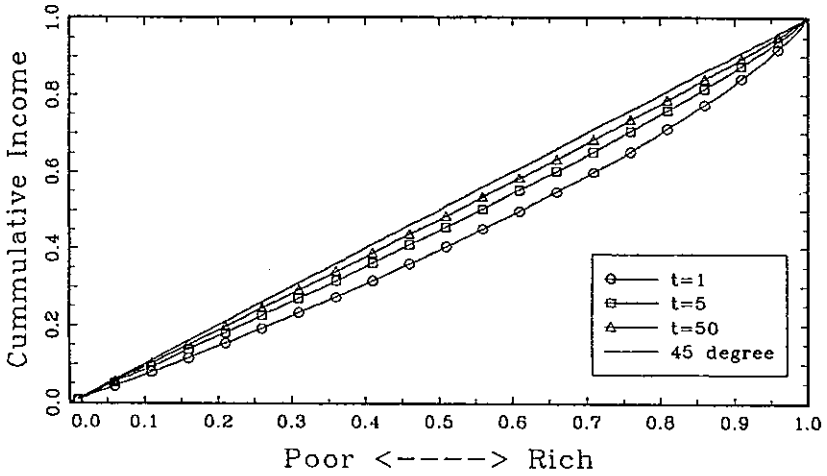


Figure 4: Lorenz Curve

(A) μ=1.1 (Exponential)



(B) μ=8.0 (Exponential)

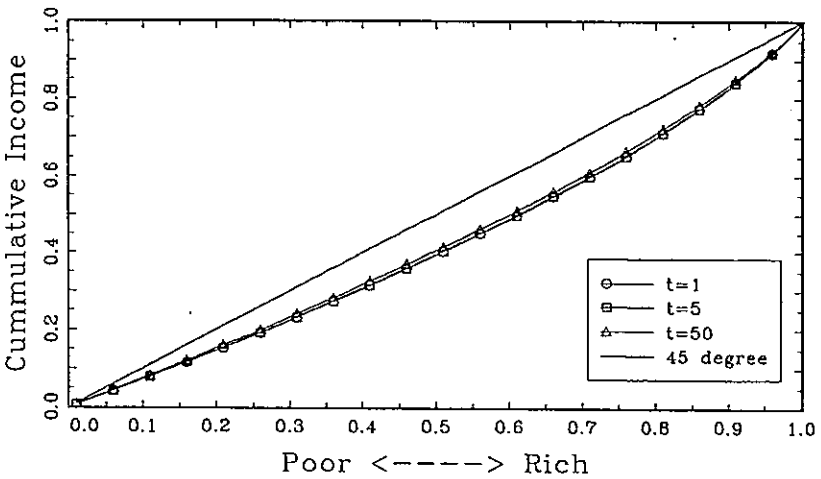
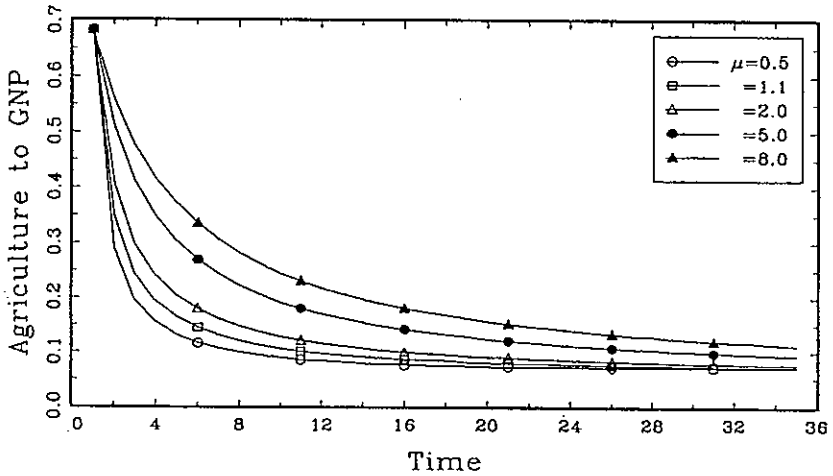
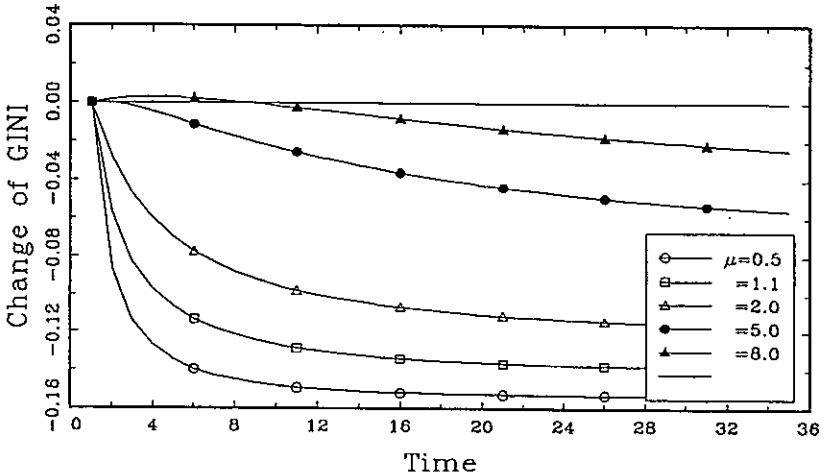


Figure 5: Two-Sector Model (A)

(A) Structural Change



(B) Inequality Change



(C) On Agricultural Technology

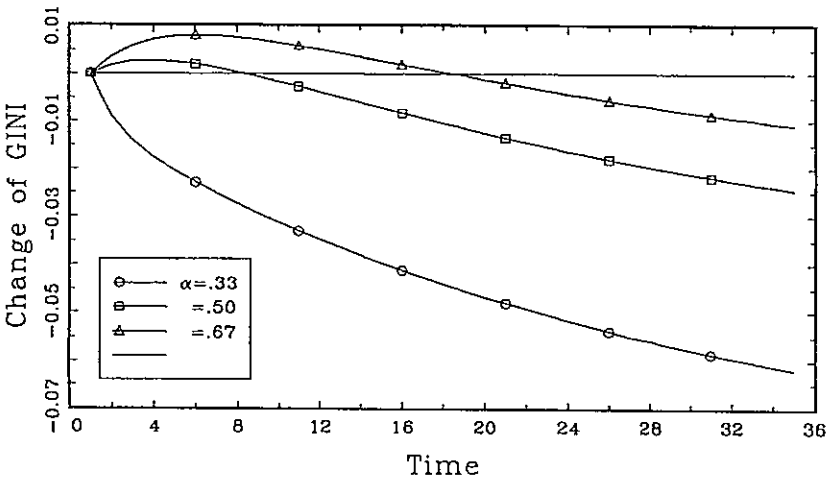
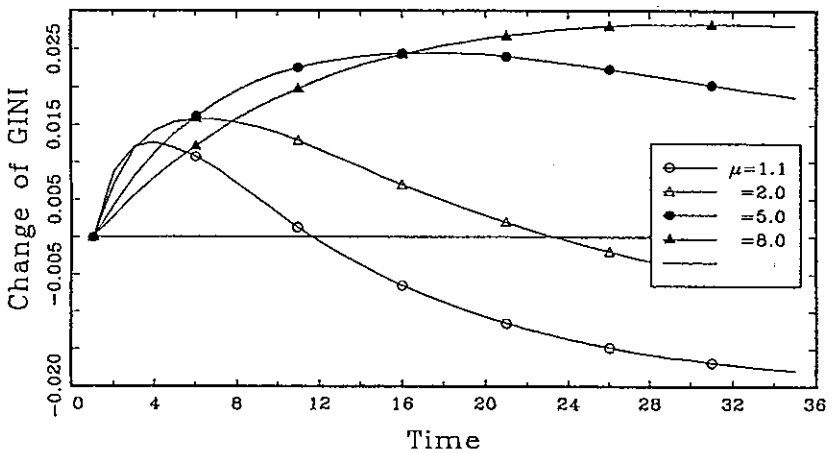
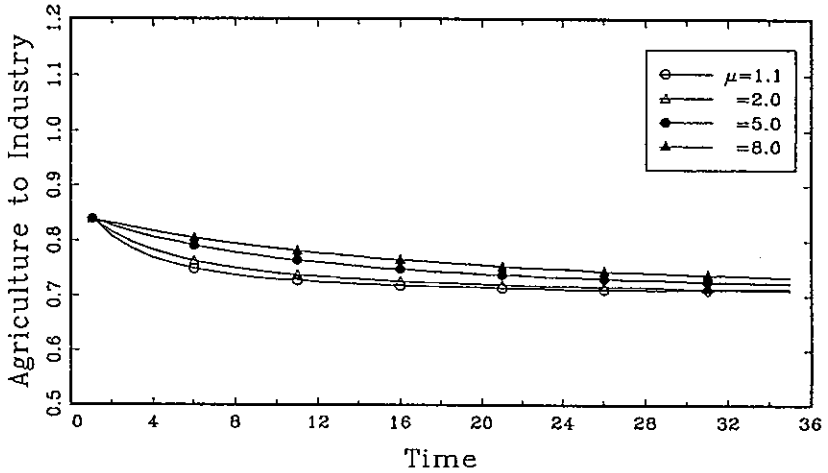


Figure 6: Two-Sector Model (B)

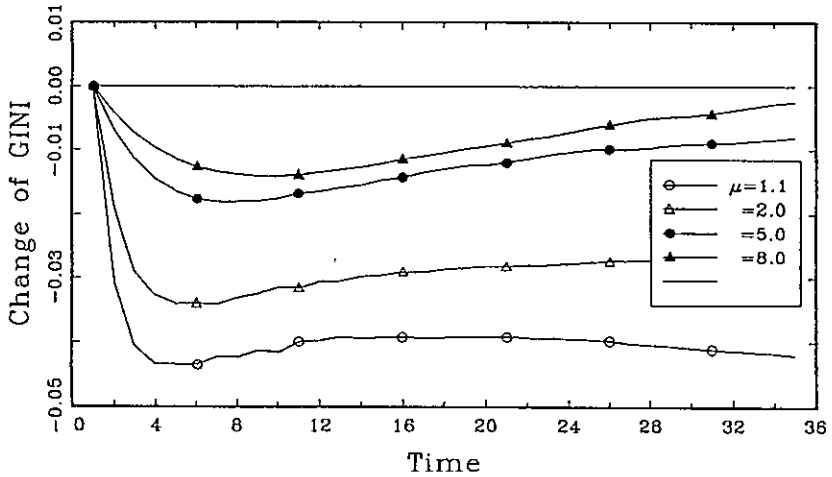
(A) Inequality Change (Uniform)



(B) Income Disparity (Uniform)



(C) Inequality Change (Exponential)



(D) Income Disparity (Exponential)