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Regional Allocation of Population with Hometown Preferences : Normative Analysis

by

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Abstract

Introduction of hometown preferences into the utility function of the individual changes the socially optimal allocation of population among different regions. In addition, it makes a difference between the Rawlsian scheme and the Benthamite scheme concerning the social objective. This paper attempts to generalize the Wildasin-type normative model of regional economies to the case in which the emotional part is included in the utility function.

1. Introduction

Regional allocation of population has been one of the most important issues in normative and positive regional economics. (See Hartwick [1980], Boadway and Flatters [1982], Wildasin [1986], Wildasin [1987].) In most of the literature, the utility function of each resident in a region is assumed to be a rational one whose arguments consist of consumption of a private good and the supply of a local public good and nothing else. However, the utility function of a resident could contain some emotional or psychological part that infuences a preference to live in his or her hometown even though the region does not offer a high level of consumption. Such factor was explicitly dealt with by Mansoorian and Myers ([1993], hereafter MM). However, MM also introduced strategic behaviour of local authority in their model, a factor that is still a sort of novelty in regional economic analysis. This made interpretation of their results rather complicated (or obscure).

Since many local authorities adhere to traditional community preference-type behaviour (i.e. they are less rational than MM assumed), it is worthwhile to analyze the effect of hometown preference on the socially optimal allocation of population among regions as well as on the market equilibrium allocation of the same with free mobility of people.

This paper attempts to introduce hometown preferences into the Wildasin-type model of a regional economic system. Each local government typically maximizes the utility of a representative resident taking the current population of the region as a fixed amount (myopic behaviour) with an equal utility constraint between different regions. This equal utility constraint becomes equality between marginal residents in different regions when there are hometown preferences. As expected, the hometown preference may make the response of population movement to interregional income transfer less predictable. If this preference were more or less symmetric among regions, the regional allocation of population would become more equal than the same without such preferences. The purpose of the paper is to analyze these points with a formal model.

2. The Model and the social objectives

An economy consists of two regions, $i=1, 2$. The total population is N, and individuals are idiosyncratic in the sense that they have the same systematic part of the utility function, but differ in the form of an emotional part, which embodies their hometown preferences. Individuals are parameterized by n, $n \in [0, N]$, and it is assumed that n has a uniform distribution of height 1/N.

The utility of individual n is, therefore, expressed as

$$U^n = \begin{cases} U(C_1, G_1) + \gamma(N-n), & \text{if he/she lives in region 1} \\ U(C_2, G_2) + \gamma n, & \text{if he/she lives in region 2} \end{cases} \qquad (1)$$

in which C_i is the per capita consumption of the private good in region i, and G_i is the supply of the local public good in region i ($i=1, 2$). (1) implies the interregionally symmetric character of the hometown preferences. If the regional allocation of population is determined only by the emotional part of the utility function, it will undoubtedly be 50:50.

Region i has a productive capacity expressed by production function $F_i(n_i)$ ($i=1, 2$), but the total production of the economy is distributed by an integrated supply and demand condition. For simplicity, I assume that G_1 and G_2 are exogenously fixed (say, by a national standard), and that n_1, the population of region 1, is the only controllable variable[1].

For a social planner of the economy, there are two objectives. One is the maximization of the sum of individual utilities and can be termed the Benthamite criterion. The other is the maximization of the marginal resident's utility in each region and can be termed the Rawlsian criterion[2]. In this paper, these two criteria are not equivalent to each other, as explained below.

In Figure 1, lines AU_1 and BU_2 are distributions of residents' utility in each region, including potential parts (EU_1 and EU_2). Owing to the equal utility condition of marginal residents, the total population is allocated to the two regions as indicated by point F. The (equalized) utility of marginal residents, W^R, is then equal to the height of point E, i.e.

$$W^R = \frac{U_1 + U_2}{2} + \frac{\gamma}{2} N \tag{2}$$

By (2), it is clear that the Rawlsian criterion is equivalent to the maximization of the simple average of systematic parts of residents' utilities in the two regions.

On the other hand, the sum of individual utilities for the whole economy, W^B, is measured by the area surrounded by the heavy line segments in Figure 1. After some calculation, it is given as

$$W^B = \frac{(U_1 - U_2)^2}{4\gamma} + N(U_1 + U_2) + \frac{3}{2}\gamma N^2 \ , \ \text{for } |U_1 - U_2| \leqq \gamma N \tag{3}$$

Given (3), it is obvious that the Benthamite criterion is not equivalent to the Rawlsian criterion<3>.

These two criteria will be analyzed in turn.

3. Rawlsian scheme

The social planner's problem in this case is as follows:

$$\underset{\langle C_1, C_2, n_1 \rangle}{\text{Maximize}} \ U(C_1, G_1) + \gamma(N - n_1) \tag{4}$$

$$\text{subject to } U(C_2, G_2) + \gamma n_1 = U(C_1, G_1) + \gamma(N - n_1) \tag{5}$$

$$F_1(n_1) + F_2(N - n_1) - C_1 n_1 - C_2(N - n_1) - G_1 - G_2 = 0 \tag{6}$$

The equal utility of the marginal residents condition,(5), is a conscious constraint for the social planner and it is not a natural outcome of free mobility. See Wildasin [1987] 1136.

Optimizing conditions will be

$$(1-\lambda)\frac{\partial U}{\partial C_1} - \mu n_1 = 0 \ , \quad \lambda \frac{\partial U}{\partial C_2} - \mu(N - n_1) = 0 \tag{7}$$

$$-\gamma + 2\lambda\gamma + \mu\left\{\frac{dF_1}{dn_1} - \frac{dF_2}{d(N-n_1)} - C_1 + C_2\right\} = 0 \tag{8}$$

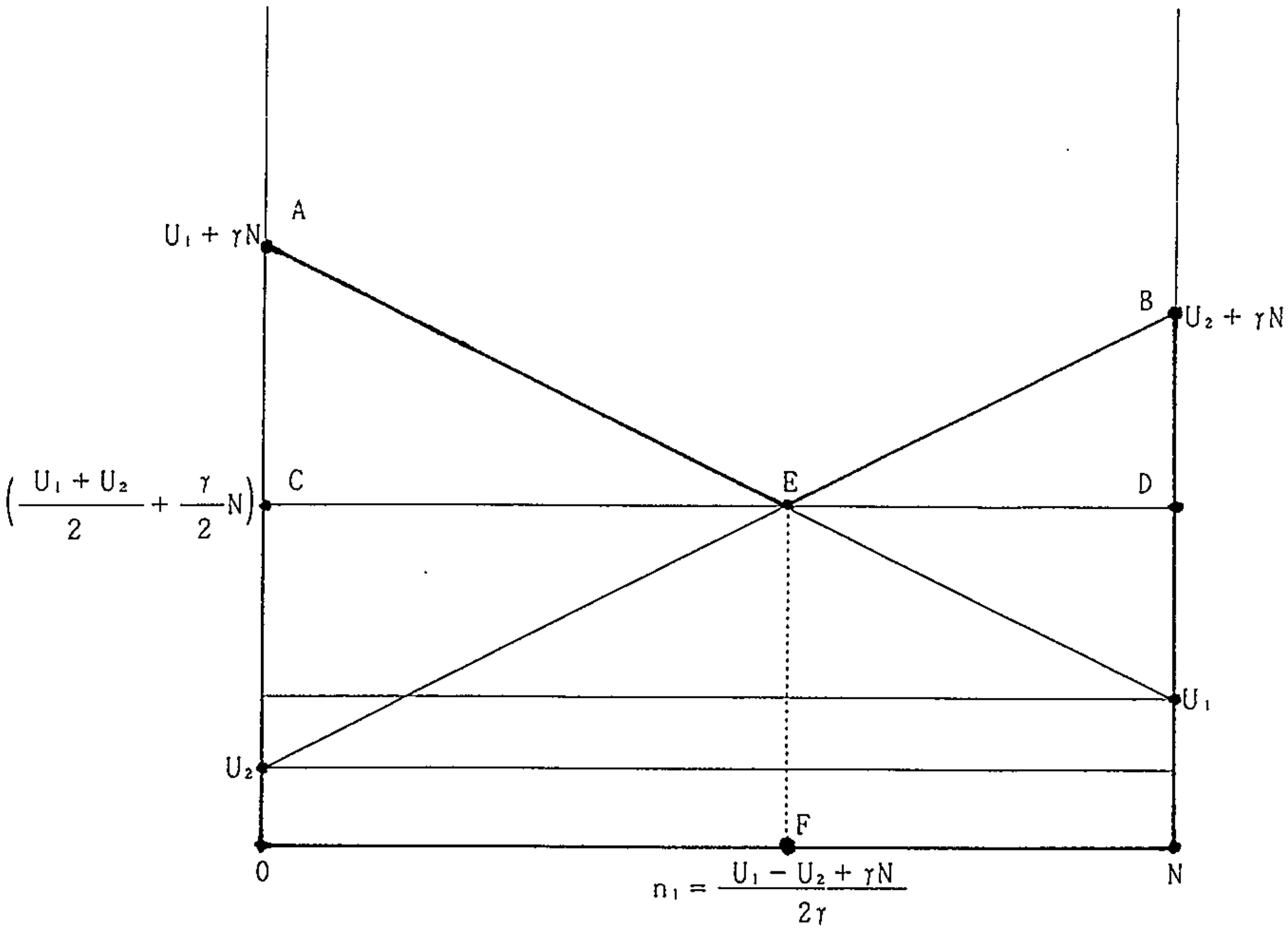

Figure 1. Allocation of Population

in which λ and μ are Lagrangean multipliers corresponding to (5) and (6). From (7) and (8), the following equation is derived:

$$F_1' - C_1 - n_1 \gamma / U_{c1} = F_2' - C_2 - (N - n_1) \gamma / U_{c2} \tag{9}$$

in which $F_1' = \frac{d F_1}{d n_1}$, $F_2' = \frac{d F_2}{d(N - n_1)}$, $U_{c1} = \frac{\partial U}{\partial C_1}$, and $U_{c2} = \frac{\partial U}{\partial C_2}$.

(9) can be interpreted intuitively as follows: both sides of (9) imply the net contribution of an additional entrant to each region, and should be equalized at the optimum. (9) is usually called the net social benefit condition of migration (nsbcm).

Three unknowns of the problem, C_1, C_2, and n_1 are solved by (5), (6), and (9). It is useful to make a comparative static analysis with regard to a change in γ concerning this solution. Total differentiation of (5), (6), and (9) gives the following result:

$$\begin{bmatrix} U_{c1} & -U_{c2} & -2\gamma \\ -n_1 & -(N-n_1) & \{(F_1' - C_1) - (F_2' - C_2)\} \\ -\left(1 + n_1 \gamma \frac{U_{c1c1}}{U_{c1}^2}\right) & \left\{1 - (N - n_1) \gamma \frac{U_{c2c2}}{U_{c2}^2}\right\} & \left(F_1'' - \frac{\gamma}{U_{c1}} + F_2'' - \frac{\gamma}{U_{c2}}\right) \end{bmatrix} \begin{pmatrix} \frac{d C_1}{d \gamma} \\ \frac{d C_2}{d \gamma} \\ \frac{d n_1}{d \gamma} \end{pmatrix}$$

$$= \begin{pmatrix} 2 n_1 - N \\ 0 \\ \frac{n_1}{U_{c1}} - \frac{N - n_1}{U_{c2}} \end{pmatrix} \tag{10}$$

in which $U_{c1c1} = \frac{\partial^2 U}{\partial C_1^2}$, $U_{c2c2} = \frac{\partial^2 U}{\partial C_2^2}$, $F_1' = \frac{d F_1}{d n_1}$, $F_2' = \frac{d F_2}{d (N - n_1)}$,

$F_1'' = \frac{d^2 F}{d n_1^2}$, and $F_2'' = \frac{d^2 F}{d (N - n_1)^2}$.

Calculating (10) for a general case would be tedious, so that let us concentrate on the case when γ is zero at the initial stage. Then, the determinant of LHS matrix in (10), Q, becomes

$$Q = -\{U_{c1}(N - n_1) + U_{c2} n_1\} (F_1'' + F_2'') \tag{11}$$

If $F_i'' < 0$, $i = 1, 2$, as usually assumed, Q is definitely positive. In this case, $\frac{d n_1}{d \gamma}$ is calculated as follows:

$$\frac{d n_1}{d \gamma} = \frac{1}{Q}\left[(N - 2 n_1) N + \left\{\frac{U_{c1}}{U_{c2}}(N - n_1)^2 - \frac{U_{c2}}{U_{c1}} n_1^2\right\} \right] \tag{12}$$

Let us evaluate (12) for some special cases.

(i) $F_1(\cdot) \equiv F_2(\cdot)$ and $G_1 = G_2$: Perfectly symmetric regions.

In this case, obviously $C_1 = C_2$, $U_{c1} = U_{c2}$, and $n_1 = \frac{N}{2}$ at the initial stage. Then it is easily shown that

$$\frac{d\, n_1}{d\, \gamma} = 0 \tag{13}$$

Introduction of the emotional part into the utility function does not change the initial 50:50 allocation of population.

(ii) $G_1 = G_2$:

In this case again, $C_1 = C_2$, $U_{c1} = U_{c2}$. Therefore

$$\frac{d\, n_1}{d\, \gamma} = \frac{1}{Q}\{2\ N\ (N - 2\ n_1)\} \tag{14}$$

If $n_1 < \frac{N}{2}$ initially, then $\frac{d\, n_1}{d\, \gamma} > 0$, and $\frac{d\, n_1}{d\, \gamma} < 0$ *vice versa.*

Introduction of the emotional part moves the initial allocation of population to a more even direction.

By the envelope theorem, it is shown that

$$\frac{d\, \phi}{d\, \gamma} = \frac{\partial\, \phi}{\partial\, \gamma} = (N - n_1) + \lambda\ (2\ n_1 - N) \tag{15}$$

in which ϕ is the Lagrangean form associated with the maximization problem (4) - (6). Starting from $\gamma = 0$, $(N - n_1)$ in (15) indicates an increase of utility brought about by the introduction of the emotional part itself. λ is the change of the value of the objective function when the equal utility constraint is marginally relaxed: let us assume that it is positive<4>. $\lambda\,(2\ n_1 - N)$ in (15) implies that the systematic part of the utility in region 1 is increased when $n_1 > \frac{N}{2}$ initially and is decreased when $n_1 < \frac{N}{2}$.

4. Benthamite Scheme

The social planner's problem in this case is as follows:

$$\underset{\langle C_1, C_2, n_1 \rangle}{\text{Maximize}}\ U(C_1, G_1)\ n_1 + U(C_2, G_2)\ (N - n_1) + \gamma\left(\frac{N^2}{2} + n_1\ N - n_1{}^2\ \right) \tag{16}$$

$$\text{subject to } U(C_2, G_2) + \gamma\ n_1 = U(C_1, G_1) + \gamma(N - n_1) \tag{17}$$

$$F_1(n_1) + F_2(N - n_1) - C_1\ n_1 - C_2(N - n_1) - G_1 - G_2 = 0 \tag{18}$$

Optimizing conditions are

$$n_1 U_{c1} - \lambda U_{c1} - \mu n_1 = 0 \tag{19}$$

$$(N - n_1) U_{c2} + \lambda U_{c2} - \mu (N - n_1) = 0 \tag{20}$$

$$U(C_1, G_1) - U(C_2, G_2) + \gamma (N - 2 n_1) + 2 \lambda \gamma + \mu (F_1' - F_2' - C_1 + C_2) = 0 \tag{21}$$

Through (19) and (20), the Lagrangean multipliers for this problem are solved as

$$\lambda = \frac{1}{R} \{ (N - n_1) n_1 (U_{c1} - U_{c2}) \} \tag{22}$$

$$\mu = \frac{1}{R} N U_{c1} U_{c2} \tag{23}$$

in which $R = U_{c1}(N - n_1) + U_{c2} n_1 > 0.$ (24)

Again I start from the initial situation of $\gamma = 0$, and analyze a special case in which $G_1 = G_2$.

In this case $C_1 = C_2$ and $U_{c1} = U_{c2}$ by (17) and $\gamma = 0$. Therefore $\lambda = 0$, $\mu = U_{c1} = U_{c2}$, and (21) becomes

$$U(C_1, G_1) + U_{c1}(F_1' - C_1) - \gamma n_1 = U(C_2, G_2) + U_{c2}(F_2' - C_2) - \gamma (N - n_1) \tag{25}$$

Since (25) is exactly the same as (9) when $\gamma = 0$ and then $U(C_1, G_1) = U(C_2, G_2)$, the equivalence between the Rawlsian case and the Benthamite case is reaffirmed when there is no emotional part.

Ignoring the changes in λ and μ, a comparative static corresponding to the change in γ is shown as:

$$\begin{bmatrix} U_{c1} & -U_{c2} & -2\gamma \\ -n_1 & -(N - n_1) & \{(F_1' - C_1) - (F_2' - C_2)\} \\ U_{c1c1}(F_1' - C_1) & -U_{c2c2}(F_2' - C_2) & U_{c1}F_1'' + U_{c2}F_1'' - 2\gamma \end{bmatrix} \begin{pmatrix} \frac{dC_1}{d\gamma} \\ \frac{dC_2}{d\gamma} \\ \frac{dn_1}{d\gamma} \end{pmatrix} = \begin{pmatrix} 2n_1 - N \\ 0 \\ 2n_1 - N \end{pmatrix} \tag{26}$$

The determinant of LHS matrix in (26), S, becomes

$$S = -\{U_{c1}(N - n_1) + U_{c2} n_1\} (U_{c1}F_1'' + U_{c2}F_2'') > 0 \tag{27}$$

Then

$$\frac{dn_1}{d\gamma} = \frac{1}{S}(2n_1 - N)[n_1 U_{c2c2}(F_2' - C_2) - (N - n_1) U_{c1c1}(F_1' - C_1) - \{U_{c1}(N - n_1) + U_{c2} n_1\}] \tag{28}$$

In the special case of $G_1 = G_2$, (28) becomes

$$\frac{d\, n_1}{d\, \gamma} = \frac{1}{S}[(2\, n_1 - N)^2\, U_{cc}(F' - C) - (2\, n_1 - N)\, U_c\, N] \tag{29}$$

Although a bit obscured by the existence of the first term in [ ] of (29), there will again be a tendency that the introduction of the emotional part moves the initial allocation of population in a more even direction. I can say very little, however, about the qualitative nature of the model via the analytical method; therefore some numerical simulations are very important to explore the characteristics of the model. These will be undertaken in the next section.

By the envelope theorem, it is shown that

$$\frac{d\, \phi^*}{d\, \gamma} > (\frac{N^2}{2} + n_1\, N - n_1{}^2) + \lambda(2\, n_1 - N) \tag{30}$$

in which ϕ^* is the Lagrangean form associated with (16) - (17). A similar discussion concerning (15) can be made about (30).

5. Numerical Simulations

(i) Rawlsian Scheme

For numerical simulations, functions and exogenous variables are specified as follows:

$$\left.\begin{array}{lll} \text{Systematic part of utility function} & & U = 0.8 \log C + 0.2 \log G \\ \text{Production functions} & F_1 = 1.2\, N_1{}^{0.6}\ , & F_2 = N_2{}^{0.6} \\ \text{Total population} & N = 100 & \\ \text{Parameter of emotional part} & \gamma\ : \text{to be specified} & \end{array}\right\} \tag{31}$$

In order to reduce the arbitrariness of the simulations, G_1 and G_2 are also optimized and result in two Samuelsonian conditions for these simulations.

From Samuelsonian conditions and the supply-demand equation, (6), the following equation is derived:

$$(1.25\, n_1)\, C_1 + (1.25\, n_2)\, C_2 = 1.2\, n_1{}^{0.6} + n_2{}^{0.6} \tag{32}$$

in which $n_2 = 100 - n_1$. From the nsbcm condition, (9), another equation is derived:

$$(\frac{\gamma\, n_1}{0.8} + 1)\, C_1 - (\frac{\gamma\, n_2}{0.8} + 1)\, C_2 = 0.72\, n_1{}^{-0.4} - 0.6\, n_2{}^{-0.4} \tag{33}$$

(32) and (33) form simultaneous equations for C_1 and C_2, and the latter two unknowns are solved for any value of n_1 starting from an initial value. The

utility levels of residents in two regions are then compared and n_1 is adjusted incrementally to increase the population of the region with higher utility until the equal utility condition is reached.

(See Appendix A for Basic program.)

The results of the simulations are summarized in Table 1. As expected, the greater the relative importance of the emotional part, the more even the allocation of population. Consumption patterns of two regions become more similar when the population allocation approaches 50:50. The systematic parts of the utilities in two regions show complicated behaviour. There are two opposite forces operating here: one is a relaxing of the equal utility constraint brought about by the introduction of the emotional part; the other is a loss of productive efficiency caused by the departure from the original allocation without the emotional part.

γ	0	0.001	0.01	0.1	1
U1	-1.17001	-1.12834	-0.68614	3.81259	48.81260
U2	-1.17001	-1.12834	-0.68614	3.81259	48.81240
U1S	-1.17001	-1.16179	-1.16205	-1.17920	-1.18644
U2S	-1.17001	-1.19499	-1.21032	-1.19570	-1.18855
C1	0.17099	0.17833	0.18701	0.18551	0.184227
C2	0.22230	0.19795	0.18166	0.18259	0.18384
G1	3.36837	2.96689	2.75022	2.32261	2.30288
G2	1.17853	1.65543	2.16140	2.27861	2.29794
n1	78.79	66.55	52.41	50.08	50.00
n2	21.21	33.45	47.59	49.92	50.00

notes: UIS = systematic part of the I-th region's utility, I = 1, 2.

Table 1. Simulations for Rawlsian Scheme

(ii) Benthamite Scheme

The same specification used in the previous subsection is employed here. Samuelsonian conditions are again taken into consideration in this case.

First an initial value of n_1 is selected; then C_1 and C_2 are solved by (17) and (18). The net contribution by the marginal entrant in each region is then calculated with (21):

$$\left.\begin{aligned} V_1 &= 0.8\log C_1 + 0.2\log G_1 + \gamma n_2 + \lambda\gamma + \mu(F_1' - C_1) \\ V_2 &= 0.8\log C_2 + 0.2\log G_2 + \gamma n_1 - \lambda\gamma + \mu(F_2' - C_2) \end{aligned}\right\} \quad (34)$$

λ and μ in (34) are calculated by (22) and (23). If $V_1 > V_2$, the value of n_1 is slightly increased and *vice versa*. This process is repeated until V_1 becomes equal to V_2. (See Appendix B for Basic Program)

The results of the simulations are summarized in Table 2. When the value of γ is increased, the allocation of population first moves to a more even pattern but soon returns to an uneven pattern again and approaches the extreme pattern of $n_1 = 100$. This is rather unexpected behaviour for a regional economy. A possible explanation is that the equal utility constraint of the systematic part of the utility is de facto relaxed by the introduction of the emotional part so that the social planner can allocate more population to a more productive region, i.e. region 1. The first region's utility dominates the social welfare in this case and approaches the level of the Benthamite solution without the emotional part as well as without the equal utility constraint. (See Appendix C)

This behaviour of the Benthamite scheme is indeed intriguing and thus deserves further investigation.

γ	0	0.0001	0.001	0.005	0.01
U1	-1.17004	-1.16645	-1.12956	-0.959194	-1.05616
U2	-1.17003	-1.16645	-1.12956	-0.959192	-1.05615
U1S	-1.17004	-1.16891	-1.16888	-1.10906	-1.11981
U2S	-1.17003	-1.17399	-1.19024	-1.30932	-1.99249
W	-117.004	-116.33	-110.342	-81.4142	-61.5762
C1	0.170988	0.1727	0.180371	0.18608	0.173698
C2	0.222336	0.214959	0.192564	0.180481	0.124255
G1	3.36851	3.25522	2.73621	3.2576	4.06602
G2	1.17831	1.32221	1.89292	1.35244	0.197749
n1	78.8013	75.396	60.6796	70.0259	93.6341
n2	21.1987	24.604	39.3204	29.9741	6.36591
λ	-4.0566	-3.83222	-1.54941	0.645138	2.31329
μ	0.224737	0.226847	0.23122	0.230457	0.211759

notes: UIS = systematic part of the I-th region's utility, I = 1, 2.

Table 2. Simulations for Benthamite Scheme

6. Conclusion

An emotional element has been introduced to the individual utility function of the Wildasin-type normative regional economic system. There emerges a sharp contrast concerning the results of such a change in the model between the Rawlsian and Benthamite schemes of social planning. In the former, a tendency toward more equal allocation of population is observed; but in the latter, after a small change of a similar tendency, a move to more extreme allocation (a corner solution) is observed.

The next target of the analysis should be that of a positive model of regional economies, in which the market itself determines the regional allocation of population.

Notes

* The author wishes to thank Mr. Daniel Baron for his valuable comments to the original version of this paper.

〈1〉 Since the condition of the optimal allocation of population is, so to say, orthogonal to the condition of the optimal supply of the public good (Samuelsonian condition), the inclusion of G_i into controllable variables does not change the main results of the analysis that follows. See Wildasin [1987] 1138.

〈2〉 See Jurion (1983) for an example of such dichotomy.

〈3〉 The condition of $|U_1 - U_2| \leqq \gamma N$ follows from $0 \leqq n_1 \leqq N$. See Figure 1.

〈4〉 A positive λ means that U_1 becomes larger than U_2 when the equal utility constraint is relaxed in the Wildasin-type problem.

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Appendix A. Basic Program for Rawlsian Scheme Simulation

```
10   REM HOMETOWN 1
11   INPUT "A= "; A
12   INPUT "G= "; G
13   INPUT "N1= "; N1
20   N2=100-N1
30   Q=-[1.25*N1)*((G*N2)/0.8+1)-(1.25*N2)*((G*N1)/0.8+1)
40   C1=(1/Q)*(-(1.25*N1^0.6+N2^0.6)*((N2*G/.08+1)
          -1.25*N2*(0.72*N1^(-0.4)-0.6*N2^(-0.4))
50   C2=(1/Q)*(1.25*N1*(0.72*N1^(-0.4)-0.6*N2^(-0.4))
          -(1.2*N1^0.6+N2^0.6)*((N1*G)/0.8+1))
60   G1=0.25*C1*N1
70   G2=0.25*C2*N2
80   U1=0.8*LOG(C1)+0.2*LOG(G1)+G*N2
90   U2=0.8*LOG(C2)+0.2*LOG(G2)+G*N1
91   U1S=U1-G*N2
92   U2S=U2-G*N1
100  D=U1-U2
105  PRINT "N1= "; N1; "C1= "; C1; "C2= "; C2; "G1= "; G1;
          "G2= "; G2; "U1= "; U1; "U2= "; U2; "D= "; D;
          "U1S= "; U1S; "U2S= "; U2S; "G= "; G
110  IF ABS(D)<0.00001      GOTO 1000
130  N1=N1+A*D
140  GOTO 20
1000 STOP
1100 END
```

Appendix B. Basic Program for Benthamite Scheme Simulation

```
10   REM: BENTHAMITE
20   INPUT "G="; G
30   INPUT "A="; A
40   INPUT "N1="; N1
50   GOSUB 1000
51   G1=0.25*C1*N1
52   G2=0.25*C2*N2
60   R=(C1/0.8)*N2+(C2/0.8)*N1
70   L=(1/R)*(N2*N1*(C1/0.8-C2/0.8))
80   M=(1/R)*!00*(C1/0.8)*(C2/0.8)
81   U1S=0.8*LOG(C1)+0.2*LOG(G1)
82   U2S=0.8*LOG(C2)+0.2*LOG(G2)
83   U1=U1S+G*N2
84   U2=U2S+G*N1
85   W=U1S*N1+U2S*N2+G*(5000+100*N1-N1^2)
90   V1=0.8*LOG(C1)+0.2*LOG(G1)+G*N2+L*G+M*(0.72*N1^(-0.4)-C1)
100  V2=0.8*LOG(C2)+0.2*LOG(G2)+G*N1-L*G+M*(0.6*N2^(-0.4)-C2)
104  H=ABS(V1-V2)
105  PRINT "N1="; N1; "N2="; N2; "C1="; C1; "C2="; C2; "G1="; G1;
         "G2="; G2; "W="; W;   "U1="; U1; "U2="; U2; "U1S="; U1S;
         "U2S="; U2S; "L="; L; "M="; M; "G="; G
107  IF H<0.0000001   GOTO 2020
110  N1=N1+A*(V1-V2)
120  GOTO 50
1000 N2=100-N1
1010 LET C=0.1
1020 C2A=(1.2*N1^(0.6)+N2^(0.6)-1.25*C1*N1)/(1.25*N2)
1030 C2B=EXP(LOG(C1)+0.2*LOG(0.25*N1)+G*(N2-N1)-0.2*LOG(0.25*N2))
1040 IF ABS(C2A-C2B)<0.000001   GOTO 2000
1050 C2=(C2A+C2B)/2
1070 C1=(1.2*N1^(0.6)+N2^(0.6)-1.25*C2*N2)/(1.25*N1)
1080 GOTO 1020
2000 RETURN
2020 STOP
2030 END
```

Appendix C. Results of Simulations without the Emotional Part

Numerical Examples

$$\begin{cases} U_i = 0.8 \log C_i + 0.2 \log G_i, \quad i = 1, 2 \\ F_1 = 1.2\, N_1^{0.5}, \quad F_2 = N_2^{0.5} \\ N = N_1 + N_2 = 100 \end{cases}$$

	(i) Social Optimum	(ii) UNDS Market Equilibrium	(iii) Autarky Market Equilibrium	(iv) Producers' Sovereignty ‖ Maximum Output	(v) Benthamite Optimum
U_1	-1.170036	-1.173624	-1.171553	-1.177282	-1.109519
U_2	-1.170036	-1.173624	-1.171553	-1.177282	-1.433375
$\tilde{U}$	——	——	——	——	-1.163053
C_1	0.170985	0.166651	0.174163	0.178556	0.179576
C_2	0.222351	0.248279	0.208995	0.195596	0.179576
G_1	3.368762	3.666690	3.105879	2.731915	3.747287
G_2	1.177985	0.744293	1.497828	1.897280	0.741146
N_1	78.808521	88.008775	71.332709	61.201827	83.4696
N_2	21.191479	11.991225	28.667291	38.798173	16.5304
I	——	0.088220	——	——	——
Y	22.7338	22.0549	23.0185	23.1460	22.447