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Abstract

This paper aims to examine investment behavior in post-war Korea, motivated by argument on the role of the government in driving economic growth in East Asia by encouraging investment for technological advancement. This paper quantitatively evaluates whether or not more investment was made than would have been made by market forces in Korea. A stochastic growth model with externalities is numerically solved by using a projection method to simulate investment behavior in Korea for the period 1960-1990. The estimation result suggests that market forces alone are not enough to explain the rise in investment rates over time in Korea.
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1. Introduction

Strong economic performance in East Asia in the last decades is deserved to be called "miracle". Sources of the success in East Asia has been sought in outward oriented policy, high rates of investment, high standard of education, rich and low-waged labor supply, cultural background, and so forth. In addition, the growth experience in East Asia attracts attention in the context of the role of the government in driving economic growth. Following an early contribution by Jones and Sakong (1980), Amsden (1989), Wade (1990), and the World Bank (1993) investigate importance of the role of the government in these countries in detail by giving rich examples.

Pack and Westphal (1986) argue that, in order to achieve international competitiveness, *more investment is required than would have been allocated by market forces in a laissez-faire regime.* They seek for a reason in market imperfections associated with technological change; technology is not acquired automatically, but markets do not fully reflect the cost of acquisition of new technology. Wade (1990) argues, more strongly, that the static conditions of market failure are irrelevant for growth and innovation; that is, efficient allocation of resources in a current market may not be efficient when considering future growth. However, this literature has neither offered theoretical models nor estimation results to justify the above claim.

This paper aims to *quantitatively* investigate whether or not more investment was made than would have been made by market forces, focusing on the Republic of Korea (henceforth, Korea) whose government is considered the strongest interventionist in East Asia. This paper applies a growth model with externalities to analyze investment behavior in Korea during the period 1960-1990.

The recent resurgence of neoclassical growth theory has contributed to a better understanding of the mechanism of economic growth (e.g. Romer 1986, Lucas 1988). The analysis on a model with externalities has found that (human or physical) capital could be underinvested in the competitive equilibrium compared to a social optimum, because each agent behaves without considering externalities of her investment. This result supports the view that seeks the role of government in an effort to encourage investment in the acquisition of new technology. This paper applies this class of models to investigate whether or not more investment was encouraged than in a laissez-faire regime in Korea.

Before proceeding with the analysis, I give an outline of the growth experience of Korea and the role of the government to encourage investment in corporation with the private sector.

In 1960, the real per capita GDP of Korea was $923, which was almost the same level as Taiwan ($964), Thailand ($985), and the Philippines ($1,183) [1] . By 1985, it was $3,858: slightly lower than Taiwan, about 50% higher than Thailand, and more than twice that of the Philippines. The annual average growth rate of Korea was 5.7% during 1960-1985, while the world average was 2.1%. Also, Korea improved remarkably in investment to GDP ratio from 8.6% in 1960 to 29.2% in 1988, while the investment ratio in the U.S. ranged from 14.9% to 20.1% during the same period.

After the Korean War (1950-53) separated from relatively resource-rich and industrialized North Korea, post-war reconstruction was financed by the U.S. [2] In the early 60s, Korea suffered from political instability and economic stagnation. After the collapse of President Rhee's government in 1960, Park Chung-Hee took power by military coup in 1961, and became President in 1963. President Park implemented various economic policies, *giving the highest priority to economic growth.* To promote industrialization, Five-year Economic Development Plans have been implemented since 1962. Investment was encouraged by low-interest policy (commercial banks were under the control of the government up to the 70s) and by strong "advice" from bureaucrats. In addition to the well-known Monthly Export Promotion Meeting which involved President Park, government officials and business executives all in collaboration, Park had his own economic control room for checking the progress of economic targets so that he could quickly take steps to solve problems that interfered with economic growth. The government nursed *Chaebol* (conglomerates in the manufacturing sector) and kept a close relationship so that the government could urge them to drive industrial growth. Direct foreign investments were strictly regulated, although joint venture was encouraged so that advanced technology would be acquired. In the interim that followed Park's assassination in 1979 until President Young-Sam Kim was elected in 1992, economic liberalization took the first step under the military regime.

Selective industrial policy is another significant economic policy to promote industrialization. President Park launched the Heavy and Chemical Industries (HCI) Program in 1973, borrowing heavily from

[1]The figures in this paragraph are from Summers and Heston (1991), real GDP per capita in 1985 international prices.
[2]The rest of this section is based on World Bank (1992), Amsden (1989), Mason et. al (1980), Jones and Sakong (1980), and conversation with Joon Kyung Kim (Korean Development Institute).

abroad. Export industries had a priority for low interest finance as well as heavy industries. In the early 80s, heavy industry surpassed light industry in export; in the mid 80s, Korea successfully returned foreign debt.

The plan of this paper is as follows: Section 2 describes a stochastic growth model with externalities, and develops the method for analysis. Section 3 gives preparatory analysis for simulation and estimation. Section 4 presents the result of the analysis, and Section 5 gives concluding remarks.

2. A Stochastic Growth Model with Externalities

2.1. The Model

Consider the following aggregate production technology

$$Y_t = \theta_t \cdot A_t \cdot F(K_t, N_t)$$

where Y_t is the aggregate production, θ_t is a stochastic shock on the production, A_t is productivity level, $F(K_t, N_t)$ is a constant returns to scale (CRS) technology, K_t is capital stock, N_t is labor input (population) with a growth rate $n_t = \frac{N_t}{N_{t-1}}$, and subscript t denotes period. The output per capita is

$$y_t = \theta_t \cdot A_t \cdot f(k_t)$$

where y_t is per capita production, k_t is per capita capital stock, and $f(k_t) = F(K_t, N_t)/N_t$.

A representative agent maximizes the expected intertemporal utility

$$\begin{aligned} \max \quad & E_0 \sum_{t=0}^{\infty} \beta^t \, u(c_t) \qquad (2.1) \\ s.t. \quad & c_t + n_{t+1} \cdot k_{t+1} - k_t = y_t - \delta \cdot k_t \\ & y_t = \theta_t \cdot A_t \cdot f(k_t) \\ & k_0 : given \\ & \ln \theta_t = \rho \cdot \ln \theta_{t-1} + \epsilon_t, \qquad \epsilon_t \sim i.i.d.N(0, \sigma^2) \end{aligned}$$

where β is the discount factor ($0 < \beta < 1$), $u(c_t)$ is the instantaneous utility function: $\frac{c_t^{1-\mu}-1}{1-\mu}$, μ^{-1} is the intertemporal elasticity of substitution ($\mu > 0$), c_t is per capita consumption, δ is the disposal rate of capital, and θ_t is an AR(1) stochastic shock on productivity in the log term.

I assume that improvement in productivity is a function of average capital stock: $A_t = A(K_t/N_t)$ [3]. While there are many possible sources of increase in productivity (such as education), increasing returns to scale of capital could be one of the major sources of the growth of productivity [4]. Other exogenous changes in productivity are separated into stochastic shock θ_t.

In a competitive economy, an agent makes an investment decision without considering externalities of her investment on productivity growth. This is in contrast with an investment behavior in which coordination effort is made in investment decision with considered externalities. In the first case, the Euler equation is

$$u'(c_t) = \beta \cdot E_t[\ u'(c_{t+1}) \cdot \frac{1}{n_{t+1}} \cdot (\theta_{t+1} A(k_{t+1}) f'(k_{t+1}) + 1 - \delta)\ ] \tag{2.2}$$

while the Euler equation in the latter case is

$$u'(c_t) = \beta \cdot E_t[\ u'(c_{t+1}) \cdot \frac{1}{n_{t+1}} \cdot (\theta_{t+1}(A(k_{t+1}) f'(k_{t+1}) + A'(k_{t+1}) f(k_{t+1})) + 1 - \delta)\ ] \tag{2.3}$$

if externalities are perfectly considered. Conventionally, I call the first case the competitive equilibrium (CE) economy, and the second case is the social optimum (SO) economy. In a SO economy, investment rates could be higher than in a CE economy in this model.

2.2. The Simulation and Estimation Method

To analyze investment behavior in the Korean economy, I simulate both the CE case with the Euler equation (2.2) and the SO case with the Euler equation (2.3), using aggregate data in Korea during 1960- 1990. Because no exact solution is known for Problem (2.1), a numerical technique is applied to obtain solutions. The solution method must obtain non Pareto-optimum solution to analyze a CE economy, and a transitional process starting from low capital stock must be analyzed in the context of

[3] I assume $A_t = A(k_t)$ discounted by population, rather than $A_t = A(K_t)$. This implies that the productivity is also stationary when $\{k_t, y_t\}$ is stationary, while aggregate capital may increase with population growth.

[4] In a developing country, new technology is often introduced by new capital from advanced countries, which leads human capital accumulation through on-the-job learning (e.g. Lucas, 1993). Also, improvement of infrastructure that raises the productivity could accompany with capital accumulation. Previous studies show that there exists total factor productivity (TFP) growth in Korea. Chen (1979) reports 4.99% annual TFP growth rate during 1955-1970 at the aggregate level; Nishimizu and Robinson (1984) report 3.71%, and Dollar and Sokoloff (1990) report 6.1% in the manufacturing sector. Kwon (1986) estimates that 38.1% of 2.95% TFP growth is attributed to increasing returns to scale during 1961-1980, and 61.5% out of 4.80% during 1972-1978. Also, Summers (1990) points out the relationship between technical progress and capital accumulation in cross country studies.

a growing economy. In choosing a numerical method, the Euler-equation approach is preferable to the value function iteration; also, linearization around the steady state is not a good choice for an analysis of a non-stationary process.

Considering these conditions, I choose the algorithm developed by Judd (1992) [5], which applies projection methods to approximate the solution using the Euler equations. A projection method finds a set of coefficients $\{a_1, a_2, ..., a_n\}$ to approximate the solution V in an entire space by orthogonal polynomials $\hat{V} = a_1 x_1 + a_2 x_2 + ... + a_n x_n$ in a subspace spanned by a set of bases $\{x_1, x_2, ..., x_n\}$. In the model, the consumption policy $c = h(k, \theta)$, where k and θ are state variables, is approximated by $\hat{h}(k, \theta; \mathrm{a})$, where a is a set of coefficients of the polynomial.

Step 1: Estimate parameters of the production function

First, estimate a $(l \times 1)$ parameter vector $\mathbf{z}$ which is a set of parameters of a parameterized production function and an AR(1) process of stochastic shock. The discount factor β and the inverse of intertemporal elasticities of substitution μ are estimated by the following steps.

Step 2: Solve the problem numerically

Next, obtain a numerical solution using estimated parameters in Step 1, with an initial guess of (β, μ). The numerical method computes a in consumption policy $c = \hat{h}(k, \theta; \mathrm{a})$ given estimates of the parameter set $\mathbf{z}$ and (β, μ).

Step 3: Calculate $\{ c_t \}$

Calculate a simulated sequence $\{ \hat{c}_t(\beta, \mu) \}$, where $\hat{c}_t(\beta, \mu) = \hat{h}(k_t, \hat{\theta}_t; \mathrm{a})$ for the period 1960-1990, with the actual capital stock data $\{k_t\}$ and estimated stochastic shocks $\{\hat{\theta}_t\}$ given estimates of the parameter set $\mathbf{z}$.

Step 4: Repeat Step 2 and Step 3 to estimate (β, μ) to fit the numerical solution to the actual data

[5] Numerical methods to solve stochastic dynamic models are competed in Taylor and Uhlig (1990) and associated short papers by 10 researcher groups. Other contributions to solve this class of models are, for example, Baxter (1991), Coleman (1991), Marcet and Marimon (1992), and Christiano and Fisher (1994).

Assuming errors are distributed normally in the log term

$$\ln c_t = \ln \hat{c}_t(\beta, \mu) + \eta_t$$

where $\eta \sim$ i.i.d., $N(0, \sigma_\eta)$. Find $(\hat{\beta}, \hat{\mu})$ by nonlinear regression.

Step 5: Construct the asymptotic 95% confidence interval of $\{ \hat{c}_t(\hat{\beta}, \hat{\mu}) \}$

Since the numerical solution and an estimated sequence of stochastic shocks are functions of the parameter set estimated in Step 1, I construct the asymptotic 95% confidence interval of the numerical solution $\{ \hat{c}_t \}$ by the Delta Method as follows. The simulated consumption policy is

$$\hat{c}_t \equiv \tilde{h}(k_t, \theta_t(\mathbf{z}), \mathbf{z})$$

By the Delta theorem, when $\sqrt{n}(\hat{\mathbf{z}} - \mathbf{z}) \xrightarrow{D} N(0, \Sigma_{\mathbf{z}})$,

$$\sqrt{n} \; (\; \tilde{h}(k_t, \theta_t(\hat{\mathbf{z}}), \hat{\mathbf{z}}) - \tilde{h}(k_t, \theta_t(\mathbf{z}), \mathbf{z}) \;) \xrightarrow{D} N(0, D\Sigma_{\mathbf{z}}D')$$

where D is the $(1 \times l)$ gradient vector

$$D = [\; \frac{\partial \tilde{h}(k_t, \theta_t(\mathbf{z}), \mathbf{z})}{\partial \mathbf{z}}|_{\mathbf{z}=\hat{\mathbf{z}}} \;]$$

Since the gradient vector D cannot be calculated analytically, D is numerically calculated. The asymptotic confidence interval is constructed by

$$[\; \hat{c}_t - c_\alpha \cdot [\frac{1}{n} D\Sigma_{\mathbf{z}}D']^{1/2}, \; \hat{c}_t + c_\alpha \cdot [\frac{1}{n} D\Sigma_{\mathbf{z}}D']^{1/2} \;]$$

where c_α is a critical value.

3. Preparatory Analysis

3.1. Estimation of a Parameter Set

This subsection provides the estimation results of the production function (Step 1 in Section 2-2). The production function is estimated using annual data as follows.

Step 1: Calculate TFP

First, calculate TFP that consists of externalities of capital (A_t) and a stochastic shock (θ_t). TFP growth

Table 3-1: Average Annual Growth Rate (%)

GDP	Capital(K)	Population(N)	TFP
8.34%	8.04%	1.42%	3.86%

(TFPG) is calculated by

$$TFPG = \frac{dY}{Y} - w \cdot \frac{dN}{N} - (1-w) \cdot \frac{dK}{K}$$

where w is the ratio of compensation of employees to GDP. Set TFP = 1.0 in the initial year, and calculate the sequence of TFP and y/TFP as an estimate of f.

Step 2: Estimate the CES production function

The CRS technology F(K,N) is parameterized as the CES production function. I avoid using the Cobb-Douglas production function that fixes the ratio of labor compensation to output. The CES production function is parameterized

$$f(k) = (ak^{\tau} + b)^{1/\tau}$$

Estimate (a, b, τ) by nonlinear regression in the log term.

Step 3: Estimate externalities and stochastic shocks

In handling the data, all economic terms other than production are assumed to be exogenous including trade deficit, government consumption, and an increase of stocks as a transaction cost, in order to focus on the optimal allocation policy between consumption and investment [6]. Thus,

$$\begin{aligned} C_t + I_t &= Y_t - (G_t + M_t - X_t + \Delta S_t) \\ &\equiv g_t \cdot Y_t \end{aligned}$$

[6] In terms of financial account, the deficit of total spending from income must be financed by external borrowing. In this paper, I implicitly assume that the government controlled international capital flow, including direct foreign investment (DFI). This seems to be a reasonable assumption for Korea, because foreign loans required governmental approval (Jones and Sakong, 1980), and direct foreign investment has been strictly regulated; DFI was consistently less than 1% in Korea, while other Newly Industrialized Countries in East Asia accepted a DFI of more than 5% of GDP.

where G_t is government consumption, M_t is import, X_t is export, ΔS_t is a change of stocks, and g_t is the fraction of GDP that is available for private consumption and investment. Since the process $\{g_t\}$ is quite stable around 91%, I treat this process with the exogenous stochastic shocks together. Assume

$$g_t \cdot y_t \equiv \bar{g} \cdot \theta_t^g \cdot A(k_t) \cdot f(k_t)$$

where $\bar{g}$ is the average of $\{g_t\}$, and $\theta_t^g = \frac{g_t}{\bar{g}}\theta_t$. Parameterizing $A(k) = \bar{A}k^\gamma$ [7], estimate $(\bar{A}, \gamma)$ and an AR(1) process of $\{ \theta_t^g \}$, using the sequence $\{\widehat{\theta_t A_t}\}$, where $\widehat{\theta_t A_t} = y_t/\hat{f}_t$, and $\hat{f}_t = f(k_t; \hat{a}, \hat{b}, \hat{\tau})$

$$\begin{aligned} \ln \frac{g_t}{\bar{g}} \cdot \theta_t A_t &= \ln \bar{A} + \gamma \cdot \ln k + \ln \theta_t^g \\ \ln \theta_t^g &= \rho \ln \theta_{t-1}^g + \epsilon_t \end{aligned}$$

by nonlinear regression. An estimated sequence $\{\hat{\theta}_t^g\}$ is used to simulate the consumption policy $\{\hat{c}_t\}$, where $\hat{c}_t = \hat{h}(k_t, \hat{\theta}_t^g; \mathrm{a})$. A best guess is made for the disposal rate δ with capital stock and investment data (see Appendix).

Table 3-2 reports the results of estimation of the production function [8] . The nonlinear estimation for the CRS part of the production function f(k) converged at two different points. One estimation for the CES function reports negative τ which is reasonable if considering an increase of the labor share to value added. The other estimation result is almost equivalent to the Cobb-Douglas function, i.e., $\tau = 0$; the asymptotic 95% confidence interval of τ is [-0.0101, 0.0103]. Therefore, Table 3-2 reports both the CES case and the Cobb-Douglas case. To see the fitness of nonlinear estimation on the CES production function, ranges of residuals to the dependent variable are compared in the table in percentage. Both the CES case and the Cobb-Douglas case seem to fit the data well; the maximum absolute error is less than 0.5% in the CES case, while the maximum error is 0.827% in the Cobb-Douglas case with $R^2 = 0.9995$.

In the externalities part, estimates of γ seem to be quite high [9] . One possible reason is that the Korean government intentionally created increasing returns to scale by focusing on targeted industries in its industrial policy. Export promotion may contribute to form increasing returns to scale by concentrating special goods for export by learning by doing. However, another possible reason may be an

[7] Sheshinski (1967) estimates the "experiences" coefficients γ using cumulative gross investment. In cross country estimates, estimated values are positive for all 11 industries, and the average γ is 0.2.

[8] Data between 1970-1990 (n=21) are used for calculation of TFP and estimation of the CRS production functions. Other estimation for Korea uses 1960-1990 data (n=31).

[9] Estimation results with negative externalities of capital (i.e., $\gamma < 0$) are not considered in the analysis.

Table 3-2: Estimated Parameters

standard deviation in parentheses

		CES		Cobb-Douglas	
CES Production function $f=(ak^{\tau}+b)^{1/\tau}$	a	0.6155	(0.0378)	0.4620	(0.0039)
	b	0.5332	(0.0019)	0.5379	(0.0025)
	τ	-0.1804	(0.0387)	8.55×10^{-5}	(0.0052)
Cobb-Douglas production function $f=a\cdot k^{\tau}$	$\ln a$			-0.7125	(0.0041)
	τ			0.4621	(0.0024)
					$R^2=0.9995$
	error(%)	[-0.474, 0.409]		[-0.733, 0.827]	
Externalities $\bar{A}k^{\gamma}$	$\ln\bar{A}$	-0.3408	(0.2765)	-0.3486	(0.2791)
	γ	0.4450	(0.1149)	0.4481	(0.1163)
AR(1) process of stochastic shocks	ρ	0.8736	(0.0856)	0.8731	(0.0847)
	σ	0.0401		0.0396	
		$R^2=0.984$	D-W=0.987	$R^2=0.985$	D-W=1.001

overestimation of spillover effects of capital accumulation on productivity. A correlation between capital accumulation and TFP growth does not necessarily imply causality; TFP growth may be caused by R&D efforts, education, and other exogenous technological progress even without capital accumulation. Thus, an estimate of $A(k)$ is considered to evaluate the effects of capital accumulation at maximum.

With the estimates of parameters, the production function $A(k)f(k)$ is globally concave in k with the Cobb-Douglas production function. With the CES function, the function is convex-concave in k. However, the value of k at the (inflection) point is as low as 0.254, while the lowest value of k during 1960-1990 was 2.105. Hence, the estimated production function is concave over the range solved numerically.

In estimating externalities, there still remains autocorrelation in residuals, although the Durbin-Watson statistic is much better than assuming i.i.d. However, it becomes quite difficult to obtain a numerical solution by using a model with higher-order serial correlation including $\{\theta_{t-1},\theta_{t-2},\ldots\}$ in addition to (k_t,θ_t) as state variables, because of the "curse of dimensionality". Moreover, an introduction of higher-order models may not be meaningful, because the annual time series data are limited to 31 samples (1960-1990). Bearing these limitations in mind, I chose an AR(1) process instead of seeking

Table 3-3: Accuracy Check

errors in %

			1960-1990		whole grid	
		(# of coeff& grid)	average	max	average	max
CES	CE	(2,2,2,2)	1.256	2.788	1.083	7.376
		(4,3,4,3)	8.73×10^{-2}	1.61×10^{-1}	1.055	3.062
		(7,5,7,5)	4.27×10^{-3}	1.15×10^{-2}	1.072	2.659
		(10,6,10,6)	9.13×10^{-4}	3.24×10^{-3}	1.073	2.655
	SP	(10,6,10,6)	1.400	3.142	4.33×10^{-1}	7.071
		(14,7,14,7)	5.24×10^{-1}	1.294	4.17×10^{-1}	3.744
		(20,5,20,5)	1.79×10^{-1}	4.87×10^{-1}	4.09×10^{-1}	2.020
Cobb-Douglas	CE	(10,6,10,6)	1.679	3.295	1.041	8.649
		(12,6,12,6)	9.40×10^{-1}	2.083	1.029	6.566
		(15,6,15,6)	5.60×10^{-1}	1.318	1.053	5.140
		(20,5,20,5)	(fail to converge)			
	SP	(10,6,10,6)	97.824	98.009	1.304	98.348
		(x,6, x,6)	(fail to converge for x=12,15)			

higher-order process.

3.2. Accuracy Check

The accuracy of each numerical solution is tested using the accuracy test developed by Judd (1992) to choose an appropriate degree of polynomials for the numerical solution, and to ensure appropriation of each numerical solution. The accuracy test evaluates "the one-period optimization error in consumption terms" (the Euler equation error) at the actual (k_t, θ_t) for all sample years, in addition to (100×80) equidistant grids over state spaces as in Judd (1992). The range of capital stock $[k_m, k_M]$ is set to (1) 2.105 (the minimum k during 1960-1990) $\times 0.95$, and (2) $k_M = p \times k^d_{ss}$, where k^d_{ss} is the deterministic steady state and $p = 1.5 \sim 2.0$. Accuracy at the actual data point is tested as well as the accuracy of equidistant grids over the range $[2.105, (k^d_{ss} + k_M)/2]$.

Table 3-3 reports both average and maximum (absolute) errors of sample experiments in the CE case with $(\beta, \mu) = (0.95, 1.0)$ and in the SO case with $(\beta, \mu) = (0.95, 2.5)$. The degree of polynomials and collocation grids are indicated in parentheses. For example, (4,3,4,3) indicates the use of a polynomial of degree (4,3) for (k, θ), (i.e., the number of coefficients a to be solved is 12), and the latter (4,3) indicates that the projection method uses (4,3) collocation grids of (k, θ).

Using the CES production function, the accuracy in the CE case is improved rapidly at our interest points even with polynomials of low degree; the (10,6,10,6) case detects only 0.00324% error at the maximum. This is because k_m is fairly large compared to $k^d_{ss} = 31.432$. (In the algorithm developed by Judd, the accuracy becomes worse as k_m is set smaller.) On the other hand, the SO case suffers from large errors with the polynomial of the same degree as the CE case, because k_m is at a much smaller level relative to $k^d_{ss}(= 493.7)$ than in the CE case. With (20,5,20,5) sacrificing the polynomial degree of θ, the numerical solution barely obtains less than 1% error at the maximum at our interest points . Figure 3-1 shows the simulated investment rate in the CE case calculated by the numerical solution of consumption policy. Except for the (2,2,2,2) case, the simulation results seem to be fairly uniform. From this result, I deem that the (20,5,20,5) case in the SO case, which is almost as accurate as the (4,3,4,3) case in the CE case, could be acceptable for the analysis. In the analysis, (10,6,10,6) is employed for the CE case, and (20,5,20,5) is employed for the SO case. The accuracy of each numerical solution is tested to keep the same or better accuracy.

I check the accuracy with another set of estimates: the Cobb-Douglas CRS function case. Beeause k^d_{ss} is disastrously large (1.45×10^3 for the CE case, 2.76×10^6 for the SO case), the numerical method fails to obtain satisfactorily small errors even with a high order polynomial, or even ends up with a convergence failure in solving the problem. For this reason, I decide to drop the Cobb-Douglas case from the analysis. Another reason is that the CES function with negative τ seems more compelling in observing a rise in the compensation to employees to GDP ratio over time in Korea. In the CE case, simulation results with the CES function and the Cobb-Douglas function look very similar at our interest points during 1960-1990.

Table 4-1: Estimation of μ

	$\hat{\mu}$	(asymptotic standard error)	the asymptotic 95% confidence interval	
			Lower Bound	Upper Bound
CE	0.82689	(0.11461)	0.60226	1.05153
SP	3.51408	(0.36740)	2.79398	4.23419

4. Results

4.1. Investment Behavior

Throughout this section, an optimal consumption policy is converted into an investment rate ($\equiv \frac{I}{I+C}$, i.e., government consumption is excluded) in figures to illustrate the results clearly.

As explained in Section 2, it is technically possible to estimate both β and μ at once. Unfortunately, however, because of the difficulty in obtaining convergence of nonlinear regression, the discount factor β is temporarily fixed to estimate the inverse of the intertemporal elasticity of utility μ. A change of β causes a change of the deterministic steady state; consequently, a change of the range of state space of capital stock is inevitable (a change of μ does not cause this problem). Nonlinear regression may bid up β while hunting for the best fit for β in the iteration, which may disastrously expand the state space. Hence, β is fixed to 0.95 from 10% interest rate of 1-year time deposit between 1984 November and 1988 December when the average annual inflation rate of CPI was 4.54% in Korea [10].

With $\beta = 0.95$, Table 4-1 reports estimates of μ, its asymptotic standard error, and the asymptotic 95% confidence interval both in the CE case and in the SO case. In comparing the CE case and the SO case, people must be more indifferent to postponing consumption to the future in the CE case than in the SO case to obtain the same level of investment as the SO case. With the same (β, μ), a CE economy invests much less than a SO economy. Furthermore, investment rates in the CE case seems to be more volatile in response to a stochastic shock than in the SO case.

Given $\hat{\mu}$, Figure 4-1 and Figure 4-2 compares the actual path of investment behavior and the simulated path with a numerical solution in the CE case and in the SO case, respectively. The 95% asymptotic

[10]Source: Bank of Korea, *Economic Statistic Yearbook, 1992*

confidence intervals in figures are calculated by estimation of the production function which includes the CRS part, externalities and a stochastic shock; because of the good fit of the CES production function, most of the 95% asymptotic confidence interval comes from the externalities part.

In both Figure 4-1 in the CE case and Figure 4-2 in the SO case, the actual paths lie outside the 95% asymptotic confidence intervals of the simulated paths for most of the years. Numerical solutions generate much flatter investment paths over time than the actual path in which investment rates dramatically rose over time. By changing μ, a simulated path shifts parallel rather than rotating. Higher intertemporal elasticity of substitution (μ^{-1}) results in a higher investment rate in any of the years. The numerical solution tends to overpredict investment for the 60s and for the early 70s, and underpredict for the late 70s and for the 80s.

A change of β does not help to rotate the simulated path to match the actual path, either. Figure 4-3 shows the result of a trial with the best fit $\mu = 0.82689$ in the CE case with $\beta = 0.94$, 0.95, and 0.96. Again, an adjustment of β ends up to shift the simulated path to near parallel.

According to the above estimation results, the dynamic model fails to explain the change in investment behavior over time during 1960-1990. Either by assuming a competitive economy or a social optimum, the model does not predict the steep rise in investment rates occurred in Korea.

What is the reason why actual investment and the simulation differ so much? One possibility is that the Korean economy in the 60s and 70s was still on the transitional process from an inefficient economy that suppresses investment toward a modern economy that provides efficient market system that can be explained by the dynamic model.

In the early and mid 60s, low interest rate policy could make a saving rate even lower than in a competitive economy. In the second half of the 60s, the government raised the (deposit) bank interest rate to encourage bank savings (in the 70s, the real interest rate was lowered again), which could contribute to increasing investments. In addition, the prevailed curb markets may be another reason to lower investment rates statistically. Also, contribution of export promotion policy to modernization of Korean economy through competition in the world market cannot be neglected.

For this explanation, however, Korean people must have a high discount factor and/or high intertemporal elasticity of utility, such as $(\beta, \mu) = (0.96, 0.827)$. Moreover, considering investment rates in the

U.S. (14.9-20.1% during 1960-1988) [11], investment rates in Korea in the 80s seem to be pretty high as a competitive economy.

Another explanation may not be deniable following the view that evaluates the role of government in encouraging investment to advance technology; the investment behavior in the Korean economy could change beyond a competitive economy toward a social optimum, thanks to the coordination effort by the government to promote investment. Assuming that government interventions to encourage investment functions but does not perfectly capture externalities to be a social optimum, actual consumption must be equal to or greater than the simulated consumption policy in a SO economy, and is likely to be lower than the consumption in a CE economy. By nonlinear regression, μ can be estimated to minimize the sum of squared residuals *outside* the CE-SO bound, instead of estimating the CE case and the SO case separately.

With an estimate $\hat{\mu}$ using this method, Figure 4-4 gives a good intuition of the dynamics of the Korean economy for this explanation. The simulated CE-SO bound with an estimate $\hat{\mu} = 3.49766$ nearly covers the actual path, although it is still not wide enough. In the first half of the 60s, investment rates are fairly low; in the second half of the 60s, investment rates overtake the CE case. From 1972 to 1978, the economy shifts from CE to SO. Finally in the 80s, the actual path almost coincides with the SO economy. In the 70s, the Korean economy seems to achieve a major change in investment behavior. As introduced in the Introduction, the period 1973-79 is the time when intensive investments were made to encourage the Heavy and Chemical Industries. Although an investment in these industries alone may not be enough to explain the change in economic behavior in Korea, it could be one of the major drives to pull up investment rates.

According to the above explanation, what is the reason why high investment rates persisted in the 80s when economic liberalization started? One possibility is investment in infrastructure in the early 80s. Korean bureaucrats quickly took action in response to the negative growth between 1979-1980 to recover economic growth; as one of the actions, more investment in infrastructure was made in the early 80s than before. Annual average growth rate of capital stock of electricity, gas and water rose from 13.2% during 1971-79 to 16.8% during 1979-1985, while the rate in the manufacturing dropped from

[11] Summers and Heston (1991).

17.6% to 14.2% (calculated from Pyo 1992).

Question still remains in the above explanation. Why does the actual path range beyond the CE-SO bound? It is possible that the "true" bound is wider than the simulated bound if the 95% confidence interval is considered. However, the CE-SO bound shifts its position rather than changing the width, by adjusting a possible parameter set $\hat{z}$. Moreover, this bound is constructed by evaluating externalities of capital at maximum in estimating the externalities. With an overestimation of externalities of capital, the actual path must be well inside the bound.

There could be many other interpretations of the estimation result. It is also possible that the government pursued economic growth so much as to reduce social welfare in the late 70s and 80s. As mentioned in the Introduction, President Park gave the highest priority to economic growth; there are no grounds to believe that President Park and following military governments were social welfare maximizers. Also, it is not impossible that (β, μ) has been changed over time such that people become more patient from the mid 60s to the 80s.

4.2. Cumulative Effects on GDP growth

In Section 4-1, we examined point-wise optimal policy given the actual capital stock and estimated stochastic shock. This subsection briefly presents a cumulative effect on economic growth starting from the initial capital stock in 1960 and given the sequence of stochastic shocks.

Figure 4-5 shows the growth path of per capita GDP under CE with $\hat{\mu} = 3.49766$ (by the CE-SO bound estimation), and with $\hat{\mu} = 0.82689$ (by the estimation in the CE case). The difference of the GDP 30 years after is impressive; with $\hat{\mu} = 0.82689$, per capita GDP reaches only 39.4% of the actual level in 1990; with $\hat{\mu} = 3.49766$, it is as low as 23.3 % [12]. In addition, simulated paths take more than half decade to recover from the big (negative) shock in 1979, while the actual economy took only one year to restore the GDP level before the shock.

As for growth, the average annual growth rate of per capita GDP would have been 3.2% with the best fit $\hat{\mu} = 0.82689$, and 1.2% with $\hat{\mu} = 3.49766$ in the CE case, instead of the actual growth rate 6.2%.

[12] I leave a "perfect" social planner's economy out of consideration in this subsection, since it may not actually be the social optimum due to the partial contribution of capital on productivity growth (recall that SP scenario takes the upper bound of spillover effects).

If the Korean economy took the CE path of the estimated CE-SO bound, the average annual growth rate could have been less than the world average (1.6% during 1965-1989 [13]), instead of achieving the hyper growth. These simulated growth rates are obtained as stochastic shocks exogenous, that is, by fixing government consumption, foreign borrowing, and even political stability as exogenous in the simulation. Also, efforts to create increasing returns to scale are assumed to be given. The difference in growth rates simply accrues to the difference in allocation policy between consumption and investment.

5. Concluding Remarks

The purpose of this paper has been to quantitatively investigate whether or not more investment was made in Korea to promote technological advancement than would have been observed by market forces. The estimation result suggests that market forces alone are not enough to explain the investment behavior in Korea. Investment rates in Korea have dramatically increased, but the simulation of a growth model with externalities fails to reproduce such investment behavior either with or without considering externalities in the investment decision. The simulation illustrates an increase in investment rates in the 60s and in the 80s, but not in the 70s by market forces; particularly, the simulation does not predict the increase in investment during 1972-1978.

It is not deniable that the high investment rates accelerated the hyper-growth of Korea, and the dynamic model does not predict such a rapid improvement of investment rates. The simulation on growth given the capital stock in 1960 illustrates the possibility that the Korean economy might have been left in stagnation if the Korean economy had taken a similar investment behavior as to the simulation.Therefore, the result is not at least against the positive view of the role of government in East Asia. Unfortunately, however, there are many possibilities why the rise in investment rates occurred. As mentioned before, it is possible that investment rates rose thanks to the governmental effort to encourage investment to promote technological advancement, while it is also possible that impediments that bother competitive economic system quickly disappeared in the case of Korea. In any case, however, the role of the government could be important to promote economic development. As previously studied, there are rich circumstantial evidences that governments could play an important role in driving economic growth. It is quite possible

[13] World Bank, World Development Report 1991

that government intervention was one of the sources of change in investment behavior, although further analysis is required to prove the relationship between government interventions and investment behavior.

The technical interest of this paper is the use of simulation with estimated parameters and a numerical method to analyze a stochastic growth model with externalities. Open parameters may be estimated by nonlinear regression with relatively small samples. In addition, by constructing the asymptotic confidence interval, simulation result can be more convincing than a simple "calibration" using estimates of parameters. Unfortunately, the simulated sequence with a numerical solution did not fit to the actual data series well in this paper. In spite of rejection by the statistical test, however, the simulation could be helpful in illustrating how and why the model and the actual economy are different.

Appendix : Data

G (government consumption), C (private consumption), Y (GDP), I (Gross capital formation)
Sources: (1970-88) Bank of Korea, *National Accounts 1990*
(1961-69, 89-90) Bank of Korea, *Economic Statistics Yearbook, various years*

N (population), n: growth rate of population
Source: National Statistical Office, *Korean Statistical Yearbook*
The growth rate of the population was nearly 3% in 1960, and fell to almost 1% in the late 80s. Therefore, the growth rate is assumed to be a function of (per capita) capital stock and be deterministic. The function is nonparametrically smoothed by cubic spline before 1985 and fixed at 1.0% after 1986.

w (compensation of employees)
Sources: (1970-88) Bank of Korea, *National Accounts 1990*, (89-90) Bank of Korea, *Economic Statistics Yearbook, various years*
The primary sector is excluded from calculation, because of the unreliably low rate of w. The production function is estimated using 1970-1990, because of the limitation of the data.

K (capital stock)
Source: the capital stock estimated by Pyo (1992), using the National Wealth Survey every ten years (1968, 1977, 1987) and gross capital formation data from National Income Account every year. Pyo offers both real and nominal, and both gross and net capital stock, summarizing 9 industries, or 5 asset types. This paper uses real gross capital stock (gross fixed reproducible assets) with the total of 9 industries as capital stock.
A problem of this estimation is that estimated capital retirement is negative. Pyo (1992) explains "that the capital formation data in national income accounts are underestimated and that the capital gains accrued are reflected only in latter national wealth survey". There seems to be an inconsistency between the 1977 and 1987 National Wealth Survey, because the capital stock data from the late 70s to the early 80s could be distorted by high inflation. Therefore, a further adjustment on (real) gross capital stock $\tilde{K}_t$ is made. An adjusted real gross capital stock is calculated by $(\tilde{K}_t)$/(K gain deflator), with K gain deflator at 1985 = 1.0, where capital gain for each year by $\frac{\tilde{K}_t}{(1-\delta)\tilde{K}_{t-1}+I_t}$, with $\delta = 0.02$. The retirement rate of capital (δ) is calculated from the disposal rate of tangible fixed assets in the manufacturing sector between 1984-1990 (Sources: Korea Statistical Association, *Report on Mining and Manufacturing Survey, various years*; Economic Planning Board, *Report on Industrial Census, 1988*)

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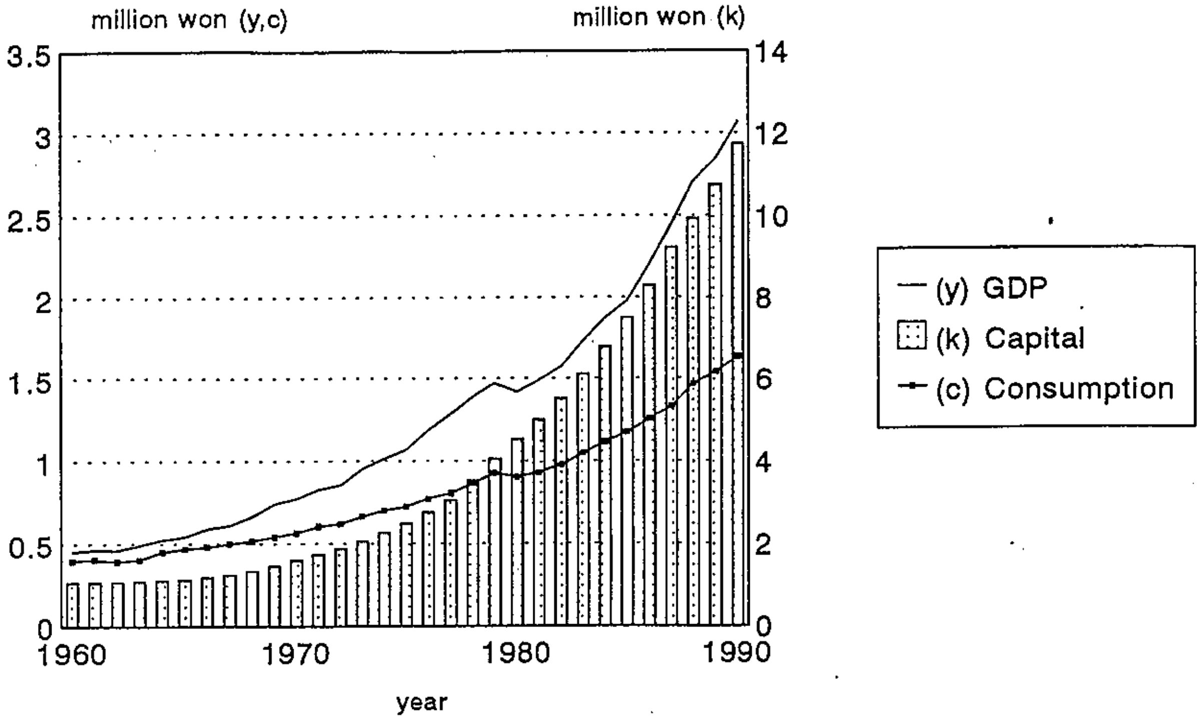
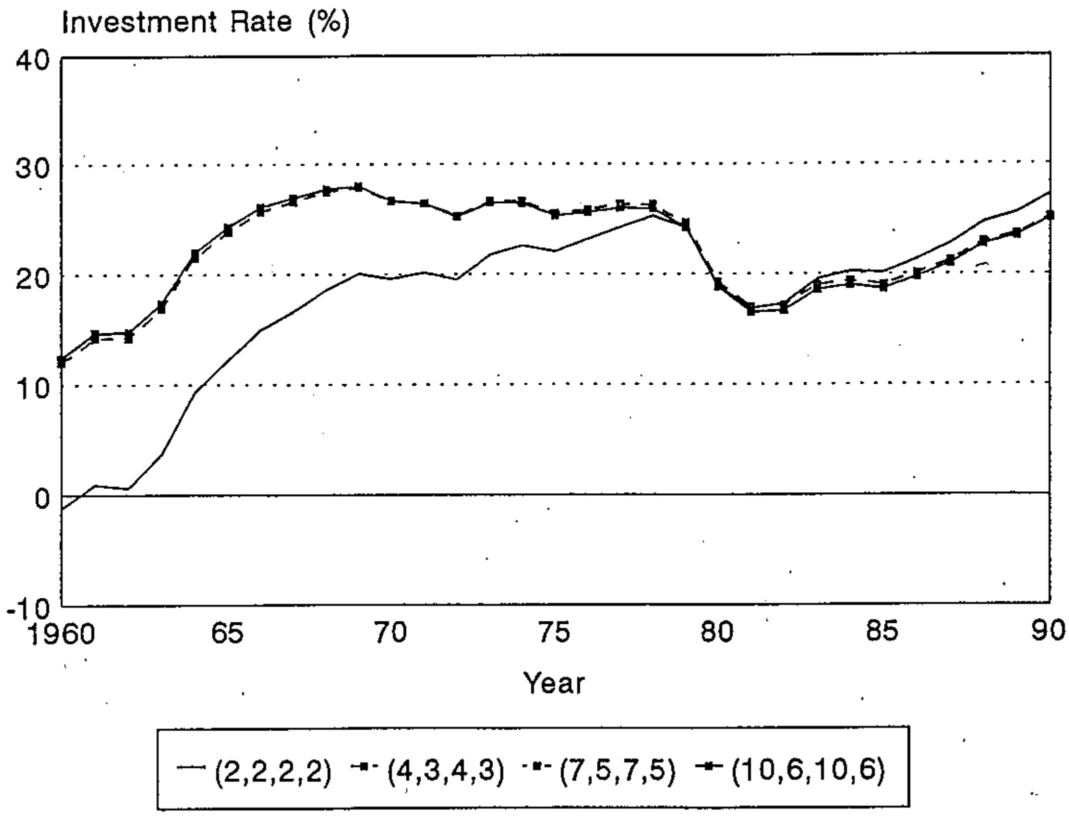
Figure 1-1: per capita GDP, Consumption, and Capital Stock
- Korea (1960 - 1990) -



million won at 1985 constant prices
Sources: in Appendix

Figure 3-1: Accuracy (CE case)



mu=1.0

Figure 4-1: Best fit mu and 95% Confidence Interval
- CE case -

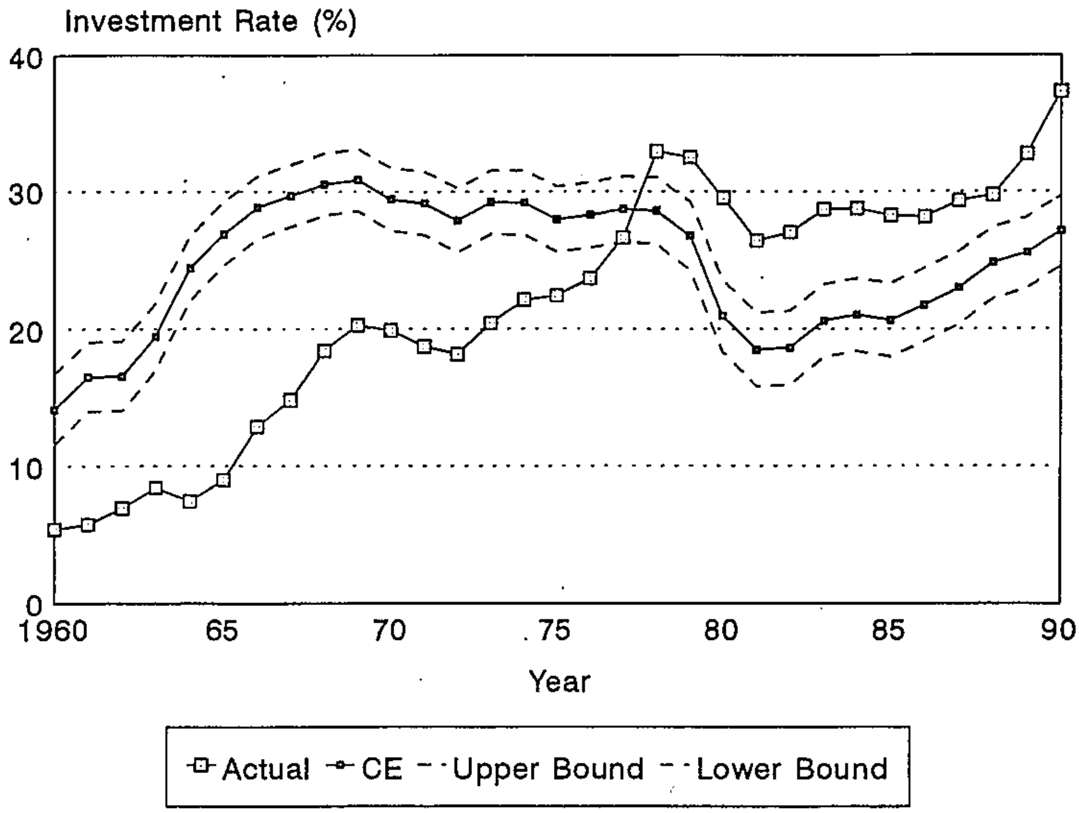


Figure 4-2: Best fit mu and 95% Confidence Interval
- SO case -

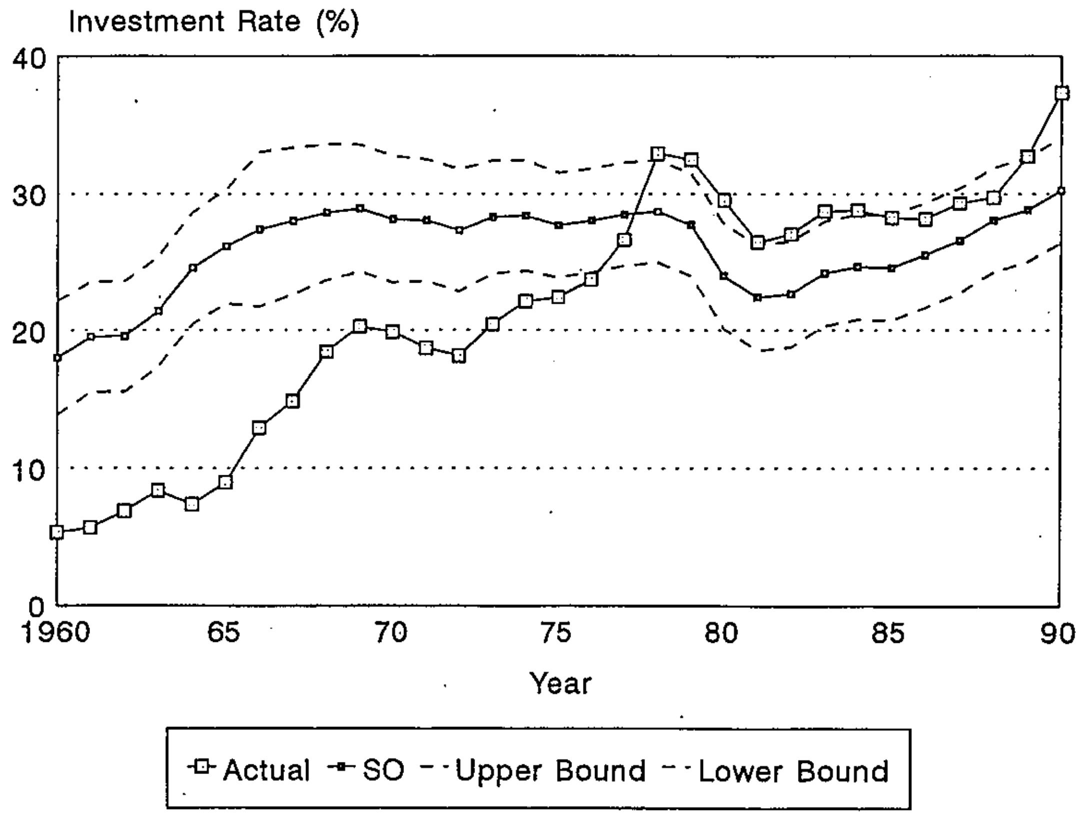


Figure 4-3: Discount Factor (CE)

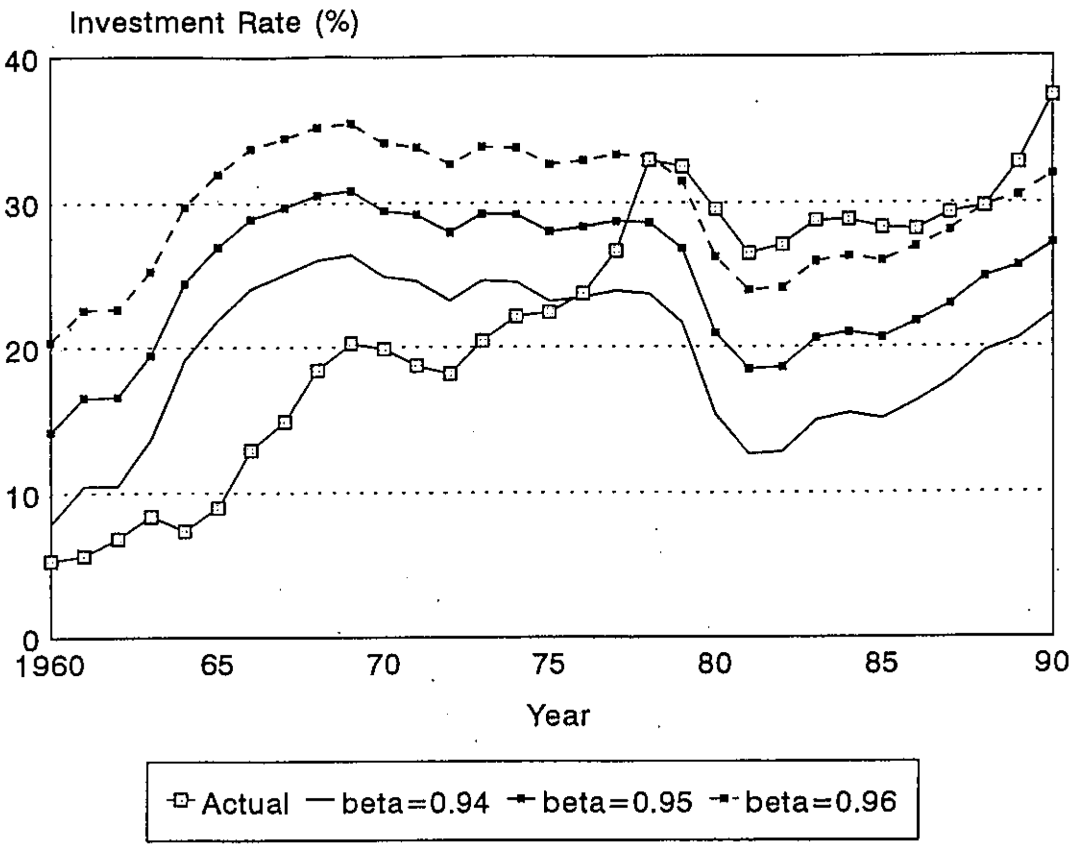


Figure 4-4: Transition from CE to SO
(mu = 3.498)

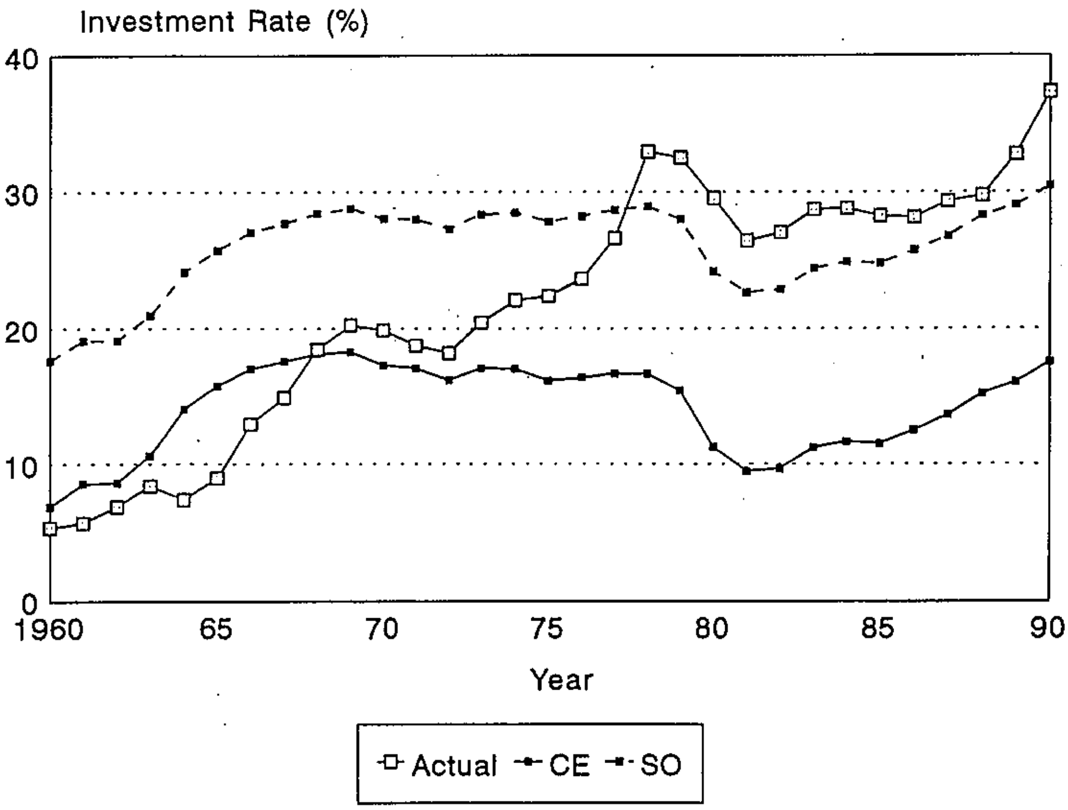


Figure 4-5: Cumulative Effect on GDP
- Actual vs. Simulation (CE) -