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ABSTRACT. In this paper, I present a general equilibrium model of hierarchal inter-urban systems that consist of more than two ranks of cities. By supposing that the business firms at any city rank need to transport their intermediary inputs from the firms at a higher rank and to communicate with the households, a spatial equilibrium of such hierarchical inter-urban systems is obtained, in which the city size and city number are shown to be dependent upon the transportation and communication costs. By comparing the market equilibrium and socially optimal solutions, it is found that the equilibrium city size would be larger while the equilibrium city number be less than the optimal ones throughout all the city ranks.

A GENERAL MODEL OF HIERARCHICAL INTER-URBAN SYSTEMS: LAND USE, CITY SIZE AND CITY NUMBER

1. INTRODUCTION

Concerning the determination of city size and city number in a hierarchical urban system, one may trace back to the central place theory of Christaller (1933), Lösch (1954) and Beckmann (1958), which, however, has been criticized for lacking a solid economic foundation (Henderson, 1972; Eaton and Lipsey, 1982). A rigorous analysis of the city size in view of modern microeconomic theory was carried out by Henderson (1974, 1987) and Kanemoto (1980, Chapter 2). Unfortunately, in their works the inter-urban (or inter-city) interactions (transportation and transaction of goods and services) have not been explicitly revealed, and the inter-urban spatial structure has not been analyzed. In fact, if we introduce some transportation relationship between the cities into their models, only a land-use pattern of agglomerated cities would be obtained because it can save the transportation cost most efficiently. So, from these models it seems difficult to show the dispersion of central places which is widely observed from the real world. This also means that the existing economic theory of city size should incorporate some and-use theoretic developments into consideration.

On the other hand, in the field of land-use theory, although there have been a large number of literature using the microeconomic principles developed since Alonso's seminal work (1964), the endogenous non-monocentric land-use model presented firstly by Fujita and Ogawa (1982) seems to be the most applicable one to the vast inter-urban space. In their work, the possibility of multicentric urban configuration has been demonstrated, but the hierarchy of urban system, which is one of the most important characters in the real urban world, has not been considered. From view of this point, Zheng (1990) presents a model of inter-urban system in which the urban hierarchy has been explicitly taken into account in studying the inter-urban spatial structure. The focus of his work is on the derivation of various land-use patterns from a simple tow-rank city system, but the determination of city size and city number in a more general hierarchical urban system still remains unclear. Most recently, there appear a number of studies on the hierarchical urban system by using the microeconomic theory, e.g., Fujita, Ogawa and Thisse (1988), Fujita (1993), and Fujita, Krugman and Mori (1994). However, their works seem to emphasize the explanation of the formation of such urban systems, having paid little attention to the law governing the city size and city number in the urban hierarchy that remains to be studied.

The purpose of this paper is to investigate the principle governing city size and city number in a general hierarchical inter-urban system. The system considered here is an urban hierarchy in a one-dimensional region, which has a tree-shaped structure, i.e., cities at any rank are influenced exclusively by the city at the next higher rank (not the cities at the all higher ranks). In this urban system, every city contains a business firm and a class of households employed by the firm. By supposing that the business firm at any city rank has to transport its intermediary inputs from the firm at the next higher rank and that it needs to communicate with the households, we shall show a spatial equilibrium of the urban system that depends on the magnitude of transportation cost. So, the resultant city size and city number will be shown to be dependent upon the transportation and communication costs. Furthermore, by comparing to the socially optimal solution of city size and city number, we also find that the equilibrium city size would be too large while the equilibrium city number be too less throughout all the city ranks.

The paper is organized as follows. In Section 2, a general model of hierarchical inter-urban systems will be presented, whose equilibrium and optimal solutions are analyzed in Sections 3 and 4, respectively. Finally, the paper will be concluded in Section 5.

2. THE MODEL

The Hierarchical Inter-urban System

Let us consider a long strip of homogeneous land upon which a system of cities are to develop.The width of the the strip is one unit of distance while its length is supposed to be sufficiently long. (See Figure 1) For simplicity, the structure of the urban hierarchy considered here is thought to be tree-shaped in that a city at any rank would dominate a number of cities at the next lower rank by providing intermediary inputs for firms. Note that such a domination only occurs between any two neighbor ranks of cities but not between other kinds of pairs of city ranks. Every city consists of a business firm and a class of households employed by the firm. In the cities of the same ranks, business firms and households are considered to be identical in terms of production technology and job suitableness. But they are different from those in the cities of other ranks.

Figure 1 is about here

Households

There are the same number of types of households as that of city ranks in the urban system. Each type of households are supposed to be distributed at a constant density, which may differ from that of other types. Such a uniform distribution of households is often assumed in the literature of location theory.

It is assumed that households at any city rank are to be employed by the business firms at the same rank. They commute to the places where the firms are located and gain the wage as their only source of income. Using this income, they pay the commuting cost that is supposed to be proportional to the distance they commute, and pay for the consumption of goods and services that are assumed to be imported from the outside markets. For simplicity, we assume that the households do not consume land at all.

Business Firms

Like the households, there are the same number of types of business firms as that of city ranks in the linear region. It is assumed that a business firm at any city rank produces some export goods by using the intermediary inputs transported from the firms at the next higher city rank and hiring the households in the same city rank.

Concerning the behavior of business firms, we shall propose a few additional assumptions as follows. First of all, in buying the intermediary goods from firms of the higher rank, besides the price the firm has to pay the transportation cost that is assumed to be proportional to the distance between the two transacting firms. So, for a firm at x , the cost of transporting intermediary inputs from a firm of higher rank at z can be given by

(2.1) $$T(k,x) = k|x - z|S$$

where k is the transportation cost per unit distance, and S the amount of goods purchased.

Secondly, as Zheng (1990) has assumed, in employing households the firms need to communicate with them in order to find capable men and women for the work, and such a communication in general bears a cost that has to be deducted from the firm's profit. More specifically, the cost of communication between firms and households is supposed to be proportional to the distance between them. Thus, for a firm at x, the total cost of communicating with the households living in segment $[a, b]$ can be expressed by

$$C(c,x) = c\int_a^b h(y)|y-x|dy \tag{2.2}$$

where c is the communication cost per unit distance, and $h(y)$ the density of households at y.

In doing so, for such a firm at x, if N and S denote the amounts of labor (households) and intermediary goods inputted, respectively, its profit can be written as

$$\pi = pQ(N,S) - wN - qS - T(k,x) - C(c,x) \tag{2.3}$$

where $Q(N, S)$ is the firm's production function, p the price of export products, w the wage, and q the price of intermediary goods.

3. MARKET EQUILIBRIUM

Additional Assumptions

First of all, without loss of generality we assume that the hierarchical inter-urban system considered here is formed in such a process that the cities at the first (highest) rank will appear at first, then cities of the second and other lower ranks will emerge in due order. Here, we shall show the formation of cities at the first and second ranks, and leave the growth of cities at other lower ranks for the reader to imagine in a similar fashion.

Since the structure of the urban system is tree-shaped, we can suppose there is only one city to be formed at the first rank, for simplicity. As has been assumed in the last section, the business firm of this city will buy intermediary inputs from the firms at higher city ranks (in this case, the firms outside the linear region) and communicate with households of the type to be employed for its production who are uniformly distributed in the long strip. To save the communication cost, the firm will choose to locate in the center of the region.

The question left is where the firms at the second rank will be located, how many households they will employ, and how many such firms will enter this linear region. For simplicity, we suppose that there will appear two identical series of such firms symmetrical about the regional center, and the firms will be located in such an order that starts from the place nearest to the center. Furthermore, it is assumed that the length of the strip is sufficiently long compared to the number of firms to be located. So, if the length is let be $2F$, it can be thought to be divided just by $2n$ firms of this city rank. Here, in the half strip on the right side of the center, there will be n firms (or cities) of the second rank. In the following, we shall only show the location of firms, the city size and city number on the right-side half strip. The situation on the left side will follow by a similar argument. (See Figure 2)

Figure 2 is about here

Next, more specifically we assume that the firms in question have fixed-coefficient production technology, i.e.,

(3.1) $$Q(N,S) = \min(\alpha N, \beta S) + \gamma$$

where α , β and γ are positive parameters. Denote the variables of the i th firm of this rank by subscript i , according to (2.3) the profit of the firm can be expressed by

(3.2) $$\pi_i = p(\alpha N_i + \gamma) - w_i N_i - (q + kx_i)\frac{\alpha}{\beta} N_i - c\int_{f_{i-1}}^{f_i} h|y - x_i| dy, \qquad i = 1,2,\cdots,n$$

where x_i is the location of the firm, h the fixed density of the households to be employed by the firms of this rank, and f_{i-1} and f_i are boundaries of the area where these households are living. Notice that f_{i-1} and f_i are also boundaries of the i th city of the second rank. For the i th firm, f_{i-1} could be considered as given while f_i is to be determined by

(3.3) $$f_i = f_{i-1} + \frac{N_i}{h}$$

Finally, concerning the prices of goods appeared in this model, we can suppose that since the products of firms are all exported to the markets outside the region, the price of one of them, say p , is exogenously given. As for the price of intermediary goods, q, if we assume the goods are purchased through the outside market, it can also be treated as a constant.

Equilibrium Conditions

Under the situation described in the last subsection, let us think about the equilibrium conditions for the hierarchical inter-urban system considered here.

Suppose that the firm is to maximize its profit by determining the amount of labor (households) and the place of its location. Here, the firm is not allowed to change the wage for competing with other firms for households, since it will make the problem too complicated to solve. In doing so, we have two equilibrium conditions as follows

(3.4) $$\frac{\partial \pi_i}{\partial N_i} = 0, \qquad i = 1,2,\cdots,n$$

(3.5) $$\frac{\partial \pi_i}{\partial x_i} = 0, \qquad i = 1,2,\cdots,n$$

In addition, if we consider that the entry of firms into the linear region will continue until the firm's profit becomes zero, there will be one more condition as

(3.6) $$\pi_i = 0, \qquad i = 1,2,\cdots,n$$

Now let us manipulate these conditions. Firstly, from (3.2) and (3.4) we get

(3.7) $$\frac{\partial \pi_i}{\partial N_i} = p\alpha - w_i - (q + kx_i)\frac{\alpha}{\beta} - c(f_{i-1} + \frac{N_i}{h} - x_i) = 0$$

which yields

(3.8) $$w_i = p\alpha - (q + kx_i)\frac{\alpha}{\beta} - c(f_{i-1} + \frac{N_i}{h} - x_i)$$

By using (3.2) and (3.5), we obtain

(3.9) $$\frac{\partial \pi_i}{\partial x_i} = -k\frac{\alpha}{\beta}N_i - ch\frac{\partial}{\partial x_i}\int_{f_{i-1}}^{f_i} |y - x_i| dy$$

$$= -k\frac{\alpha}{\beta}N_i - ch\frac{\partial}{\partial x_i}\left[\int_{f_{i-1}}^{x_i}(x_i - y)dy + \int_{x_i}^{f_i}(y - x_i)dy\right]$$

$$= -k\frac{\alpha}{\beta}N_i - ch(2x_i - 2f_{i-1} - \frac{N_i}{h}) = 0$$

which gives the following expression

(3.10) $$x_i = f_{i-1} + (1 - \frac{\alpha k}{\beta c})\frac{N_i}{2h}$$

Second, by substituting (3.8) into (3.2) we have

(3.11) $$\pi_i = p\gamma - c(f_{i-1} + \frac{N_i}{h} - x_i)N_i - ch\int_{f_{i-1}}^{f_i}|y - x_i|dy$$

$$= p\gamma - c(f_{i-1} + \frac{N_i}{h} - x_i)N_i - ch\left[\int_{f_{i-1}}^{x_i}(x_i - y)dy + \int_{x_i}^{f_i}(y - x_i)dy\right]$$

$$= p\gamma - c(f_{i-1} + \frac{N_i}{h} - x_i)N_i - \frac{ch}{2}\left[(f_{i-1} + \frac{N_i}{h} - x_i)^2 + (x_i - f_{i-1})^2\right]$$

which, by using (3.10), will give

(3.12) $$\pi_i = p\gamma - \frac{c}{4h}\left[(\frac{\alpha k}{\beta c} + 1)^2 + 2\right]N_i^2$$

So, from (3.6) we get

(3.13) $$N_i = 2\sqrt{\frac{p\gamma h}{c\left[(\frac{\alpha k}{\beta c} + 1)^2 + 2\right]}}$$

Here, if we use (3.3) there turns out to be

(3.14) $$f_i - f_{i-1} = \frac{N_i}{h} = 2\sqrt{\frac{p\gamma}{ch\left[(\frac{\alpha k}{\beta c} + 1)^2 + 2\right]}}$$

Since in the last subsection we assumed the length of the whole strip is $2F$ and its center could be considered as the origin, we have

(3.15) $$f_0 = 0$$

(3.16) $$f_n = F$$

The solution for the difference equation system composed by (3.14)-(3.16) will yield

$$(3.17)\qquad f_i = 2i\sqrt{\frac{p\gamma}{ch\left[(\frac{\alpha k}{\beta c}+1)^2+2\right]}}, \qquad i=1,2,\cdots,n$$

$$(3.18)\qquad n = \frac{F}{2}\sqrt{\frac{ch\left[(\frac{\alpha k}{\beta c}+1)^2+2\right]}{p\gamma}}$$

In summary, from the equilibrium conditions we finally derived a system of equations, i.e., (3.8), (3.10), (3.13), (3.17) and (3.18), for the following five unknowns, w_i , x_i , N_i , f_i and n ($i = 1, 2, ..., n$).

Equilibrium Properties

In this subsection, we shall show some important equilibrium properties of the hierarchical inter-urban system. In the first place, from the equation system described we can obtain the equilibrium wage as follows

$$(3.19)\qquad w_i = p\alpha - q\frac{\alpha}{\beta} - c\left[2i\frac{\alpha k}{\beta c} - \left(\frac{\alpha k}{\beta c}\right)^2 + 1\right]\sqrt{\frac{p\gamma}{ch\left[(\frac{\alpha k}{\beta c}+1)^2+2\right]}}, \qquad i=1,2,\cdots,n$$

which yields

$$(3.20)\qquad \frac{dw_i}{di} = -2\frac{\alpha k}{\beta}\sqrt{\frac{p\gamma}{ch\left[(\frac{\alpha k}{\beta c}+1)^2+2\right]}} < 0$$

This means that in equilibrium, the wage paid by firms to households will depend on the location of firms. The more distant the firms are located away from the regional center, the lower the wage they pay will be.

Concerning the location of firms, substitution of (3.13) and (3.17) into (3.10) gives

$$(3.21)\qquad x_i = (2i - \frac{\alpha k}{\beta c} - 1)\sqrt{\frac{p\gamma}{ch\left[(\frac{\alpha k}{\beta c}+1)^2+2\right]}}, \qquad i=1,2,\cdots,n$$

whose derivatives with respect to k and i are respectively as follows

(3.22) $$\frac{dx_i}{dk} = -\frac{2\alpha}{\beta c}\sqrt{\frac{p\gamma}{ch\left[(\frac{\alpha k}{\beta c}+1)^2+2\right]}}\left[\left(\frac{\alpha k}{\beta c}+1\right)i+1\right] < 0$$

(3.23) $$\frac{dx_i}{di} = 2\sqrt{\frac{p\gamma}{ch\left[(\frac{\alpha k}{\beta c}+1)^2+2\right]}} > 0$$

(3.22) implies that when the cost of transporting intermediary goods (that is, some high-grade goods or services for the production activities, such as the information through face-to-face contacting) from the regional center increases, the firms tend to locate nearer to the center. Based on the result shown by (3.22) and (3.23), it is easy to draw a group of x_i curves for different values of i on the plain of x and k, which is shown in Figure 3. From this figure, we can see that the spatial structure of the hierarchical inter-urban system is heavily influenced by the cost of transporting goods or services from the regional center to the periphery. When the transportation cost is very high, most of the firms would locate within a limited area surrounding the center, and the spatial structure would become very concentrated. If the transportation cost is lowered due to the technological progress (say the development of new technology for contacting), the firms would tend to locate in a dispersed manner in order to save the cost of communicating with the uniformly distributed households.

Figure 3 is about here

As for the city size (i.e., the household number employed by the firm) and the city number (the number of firms entered the region), (3.13) and (3.18) have shown their determinants, respectively. Obviously, both of the city size and city number are also dependent upon the magnitudes of transportation and communication costs (k/c). This result, however, has not been pointed out by the existing economic theoriy of city size (say Henderson, 1987), because it has neglected the interactions between firms of different cities.

Finally, let us check over the second-order conditions for the firm's profit-maximizing problem. The second derivatives of (3.7) and (3.9) yield the following expressions

(3.24) $$\frac{\partial^2 \pi_i}{\partial N_i^2} = -\frac{c}{h}$$

(3.25)
$$\frac{\partial^2 \pi_i}{\partial N_i \partial x_i} = \frac{\partial^2 \pi_i}{\partial x_i \partial N_i} = -k\frac{\alpha}{\beta} + c$$

(3.26)
$$\frac{\partial^2 \pi_i}{\partial x_i^2} = -2ch$$

(3.27)
$$\begin{vmatrix} \frac{\partial^2 \pi_i}{\partial N_i^2} & \frac{\partial^2 \pi_i}{\partial N_i \partial x_i} \\ \frac{\partial^2 \pi_i}{\partial x_i \partial N_i} & \frac{\partial^2 \pi_i}{\partial x_i^2} \end{vmatrix} = \left[(\sqrt{2}+1)c - k\frac{\alpha}{\beta}\right]\left[(\sqrt{2}-1)c + k\frac{\alpha}{\beta}\right]$$

For the profit-maximizing problem to have a maximum, the second-order matrix should be negative definite. Since the right hand side of (3.24) is negative, that of (3.27) should be positive. By calculation, this second-order condition is equivalent to

(3.28)
$$1-\sqrt{2} < \frac{\alpha k}{\beta c} < 1+\sqrt{2}$$

Since α , β , k are c are positive parameters, (3.28) becomes

(3.29)
$$0 < \frac{\alpha k}{\beta c} < 1+\sqrt{2}$$

which implies that for the defined hierarchical inter-urban system to reach a stable equilibrium, the parameters representing the firm's marginal productivities, and the costs of transportation and communication should satisfy some requirements.

4. SOCIAL OPTIMUM

The market equilibrium discussed so far may not be necessarily optimal from the perspective of social welfare. In this section, we shall show a socially optimal solution for the hierarchical inter-urban system defined, and compare it with the equilibrium solution.

Optimal Conditions

The social optimality here is defined as such an allocation of regional resources that maximizes the total of social net profits from the whole urban system. The word "net" means that the cost of households' commuting should be deducted from the business firms' profits, because it is one kind of social costs. take the cities at the second rank in the right-side half strip of the region as an illustration, as we did in the last section. The optimality can be expressed as the following maximization problem

$$
(4.1)\quad \begin{cases} \max \phi = \sum_{i=1}^{n} \lambda_i \Big[p(\alpha N_i + \gamma) - w_i N_i - (q + kx_i)\dfrac{\alpha}{\beta} N_i - c\int_{f_{i-1}}^{f_{i-1}+N_i/h} h|y - x_i|dy \\ \qquad\qquad - t\int_{f_{i-1}}^{f_{i-1}+N_i/h_i} h|y - x_i|dy \Big] \\ \text{with respect to} \quad N_i,\ x_i,\ \lambda_i\ (i = 1,2,\cdots,\text{n}) \end{cases}
$$

where ϕ is the total of social net profits, λ_i the shadow price of the i th city, and t the commuting cost per unit distance.

The first-order conditions of the above maximizing problem give

$$
(4.2)\quad \frac{\partial \phi}{\partial N_i} = \lambda_i \left[p\alpha - w_i - (q + kx_i)\frac{\alpha}{\beta} - (c+t)(f_{i-1} + \frac{N_i}{h} - x_i) \right] = 0
$$

$$
(4.3)\quad \frac{\partial \phi}{\partial x_i} = \lambda_i \left[-k\frac{\alpha}{\beta} N_i - (c+t)h\frac{\partial}{\partial x_i}\int_{f_{i-1}}^{f_{i-1}+N_i/h} |y - x_i|dy \right] = 0
$$

$$
(4.4)\quad \frac{\partial \phi}{\partial \lambda_i} = p(\alpha N_i + \gamma) - w_i N_i - (q + kx_i)\frac{\alpha}{\beta} N_i - (c+t)\int_{f_{i-1}}^{f_{i-1}+N_i/h} h|y - x_i|dy = 0
$$

From (4.2), we have

$$
(4.5)\quad w_i = p\alpha - (q + kx_i)\frac{\alpha}{\beta} - (c+t)(f_{i-1} + \frac{N_i}{h} - x_i) \qquad i = 1,2,\cdots,n
$$

which is a condition concerning the wage level that correspond to (3.8) in the equilibrium. By calculating (4.3), we obtain the following expressions

$$
(4.6)\quad -k\frac{\alpha}{\beta} N_i - (c+t)h(2x_i - 2f_{i-1} - \frac{N_i}{h}) = 0
$$

$$
(4.7)\quad x_i = f_{i-1} + \left[1 - \frac{\alpha k}{\beta(c+t)} \right] \frac{N_i}{2h}, \qquad i = 1,2,\cdots,n
$$

(4.7) is the counterpart of the equilibrium condition (3.10).

Substitution of (4.5) and (4.7) into (4.4) yields

$$
(4.8)\quad p\gamma - \frac{c+t}{4h}\left\{ \left[\frac{\alpha k}{\beta(c+t)} + 1 \right]^2 + 2 \right\} N_i^2 = 0
$$

$$
(4.9)\quad N_i = 2\sqrt{\frac{p\gamma h}{(c+t)\left\{ \left[\dfrac{\alpha k}{\beta(c+t)} + 1 \right]^2 + 2 \right\}}}
$$

Here, by using the equations of (3.3), (3.15) and (3.16) we get

$$f_i = 2i\sqrt{\frac{p\gamma}{(c+t)h\left\{\left[\frac{\alpha k}{\beta(c+t)}+1\right]^2+2\right\}}}, \qquad i = 1,2,\cdots,n \tag{4.10}$$

$$n = \frac{F}{2}\sqrt{\frac{(c+t)h\left\{\left[\frac{\alpha k}{\beta(c+t)}+1\right]^2+2\right\}}{p\gamma}} \tag{4.11}$$

In this way, from the first-order conditions we obtained a system of equations, (4.5), (4.7), (4.9), (4.10) and (4.11), for solving the following five variables, w_i, x_i, N_i, f_i and n ($i = 1, 2, \ldots, n$).

Comparison between Equilibrium and Optimum

Let us compare the solutions of city size and city number between the market equilibrium and social optimum. Denote N_i^{eq} and N_i^{opt} as the equilibrium and optimal solutions of the city size, respectively. Using (3.13) and (4.9), we have

$$N_i^{eq} - N_i^{opt} = 2\sqrt{p\gamma h}\left\langle \frac{1}{\sqrt{c\left[(\frac{\alpha k}{\beta c}+1)^2+2\right]}} - \frac{1}{\sqrt{(c+t)\left\{\left[\frac{\alpha k}{\beta(c+t)}+1\right]^2+2\right\}}}\right\rangle \tag{4.12}$$

from which comparison of the denominators in the angle bracket yields

$$\begin{aligned}\Delta &\equiv (c+t)\left\{\left[\frac{\alpha k}{\beta(c+t)}+1\right]^2+2\right\} - c\left[(\frac{\alpha k}{\beta c}+1)^2+2\right] \\ &= t\left[3-\frac{\alpha^2k^2}{\beta^2c(c+t)}\right]\end{aligned} \tag{4.13}$$

By using the second-order condition (3.29), we get

$$\Delta = t\left[3-\frac{\alpha^2k^2}{\beta^2c(c+t)}\right] > t\left[1-\left(\frac{\alpha k}{\beta c}\right)^2\right] > 0 \tag{4.14}$$

Thus, (4.12) becomes

$$N_i^{eq} - N_i^{opt} > 0 \tag{4.15}$$

which means that the equilibrium city size is larger than the optimal one.

Next, by letting n^{eq} and n^{opt} be the equilibrium and optimal solutions of the city number, respectively, from (3.18), (4.11) and the inequality of (4.14) we obtain the comparing result as follows

$$(4.16)\quad n^{eq}-n^{opt}=\frac{F}{2}\sqrt{\frac{h}{p\gamma}}\left\langle\sqrt{c\left[(\frac{\alpha k}{\beta c})^{2}+1\right]}-\sqrt{(c+t)\left\{\left[\frac{\alpha k}{\beta(c+t)}\right]^{2}+1\right\}}\right\rangle<0$$

That is, in view of social optimality the equilibrium city number is too less.

The inconsistency between equilibrium and optimal solutions of the city size and city number seems to result from the commuting cost of households. In the market equilibrium, unlike in the social optimum, the households' commuting cost in general does not need to be considered in the firms' profit-maximizing behavior, so by this saving the firms can employ a few more households that causes the equilibrium city size to be larger than the optimal one. At the same time, due to the limited length of the region, larger city sizes means that less number of cities could enter the space. Thus, the equilibrium city number will be less than the social optimum.

This result can be applied to all ranks of the urban system. That is, throughout all the city ranks, by the market mechanism the city size would be larger and the city number be less compared to the social optimum. The policy implication from this conclusion is that in the real urban world, to realize the socially optimal urban system, we need to control the possibly excessive city sizes and meanwhile to increase the number of cities in all over the urban system.

5. CONCLUDING REMARKS

In this paper, we have presented a general model of hierarchical inter-urban systems which are supposed to be tree-shaped, containing a number of ranks of cities constituted by firms and households of different types. By assuming that a firm at any city rank has to pay the cost of transporting its intermediary inputs from the firm at the next higher rank and the cost of communicating with its employed households at the same rank, we have shown that there seems to exist an equilibrium of spatial structure in which the location of firms may be concentrated or dispersed, depending on the transportation cost. In the equilibrium, the resulted city size and city number of each rank are shown to be dependent upon the magnitude of transportation and communication costs. However, these equilibrium solution is not necessarily socially optimal. It has also been shown that by the market principle, the city size would be too large while the city number too less compared to the social optimum. Although the possibility of excessive city size has also been pointed out before by a few urban economists, say Henderson (1987), we want to emphasize that the framework of theorizing used here differs considerably in such aspects that the transaction and transportation between cities have been explicitly taken into account.

This work might be considered to have generalized the model of Zheng's (1990) in the sense that it can be applied to a hierarchical inter-urban system with more than two ranks of cities. The next step we should take is to make the present work more general and reasonable. As the another direction of further works, like Zheng (1991), we shall try to carry out an empirical study on some metropolitan areas in the real world, since for the present rigorous empirical studies of the city size seem much less than the related theoretical works.

REFERENCES

Alonso, William. 1964. *Location and Land Use*. Cambridge, Massachusetts: Harvard university Press.

Beckmann, Martin J. 1958. "City Hierarchies and the Distribution of City Size," *Economic Development and Cultural Change*, 6, 243-248.

Christaller, Walter. 1933. *Die Zentralen Orte in Süddeutschland* [translated by C. W. Baskin (1966) as *Central Places in Southern Germany*]. Englewood Cliffs, New Jersey: Prentice-Hall.

Eaton, B. Curtis. and Richard G. Lipsey. 1982. "An Economic Theory of Central Places," *Economic Journal*, 92, 56-72.

Fujita, Masahisa. 1993. "Monopolistic Competition and Urban Systems," *European Economic review* , 37, 308-315.

Fujita, Masahisa and Hideaki Ogawa. 1982. "Multiple Equilibria and Structural Transition of Non-monocentric Urban Configurations," *Regional Science and Urban Economics* , 12, 161-196.

Fujita, Masahisa, Paul Krugman, and Tomiya Mori. 1994. "On the Evolution of Hierarchical Urban System," Paper presented at the 41st North American Meetings of Regional Science Association International, Niagara Falls, Ontario, Canada.

Fujita, Masahisa, Hideaki Ogawa, and Jacques-Francois Thisse. 1988. "A Spatial Competition Approach to Central Place Theory: Some Basic Principles," *Journal of Regional Science* , 28, 477-494.

Henderson, J. Vernon. 1972. "Hierarchical Models of City Size: An Economic Evaluation," *Journal of Regional Science* , 12, 435-441.

Henderson, J. Vernon. 1974. "The Sizes and Types of Cities," *American Economic Review* , 64, 640-657.

Henderson, J. Vernon. 1987. "System of C15ities and Inter-City Trade," in Richard Arnott (ed.) *Systems of Cities and Facility Location* . New York: Harwood Academic Publishers, pp.1-71.

Kanemoto, Yoshitsugu. 1980. *Theories of Urban Externalities* . Amsterdam: North-Holland.

Lösch, August. 1954. The Economics of Location (W.H. Woglom and W. F. Stolper, Trans.) New Haven: Yale University Press.

Zheng, Xiao-Ping. 1990. "The Spatial Structure of Hierarchical Inter-urban Systems: Equilibrium and Optimum," *Journal of Regional science* , 30, 375-392.

Zheng, Xiao-Ping. 1991. "Metropolitan Spatial Structure and Its Determinants: A Case-study of Tokyo," *Urban Studies* , 28, 87-104.

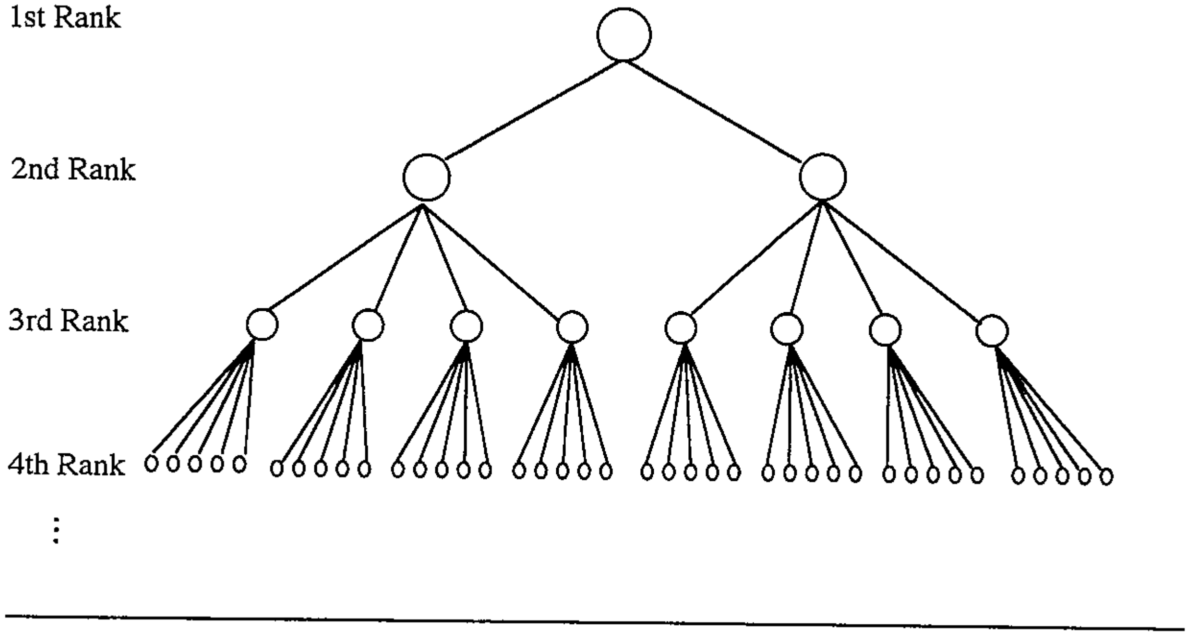

FIGURE 1: The Hierarchical Inter-urban System

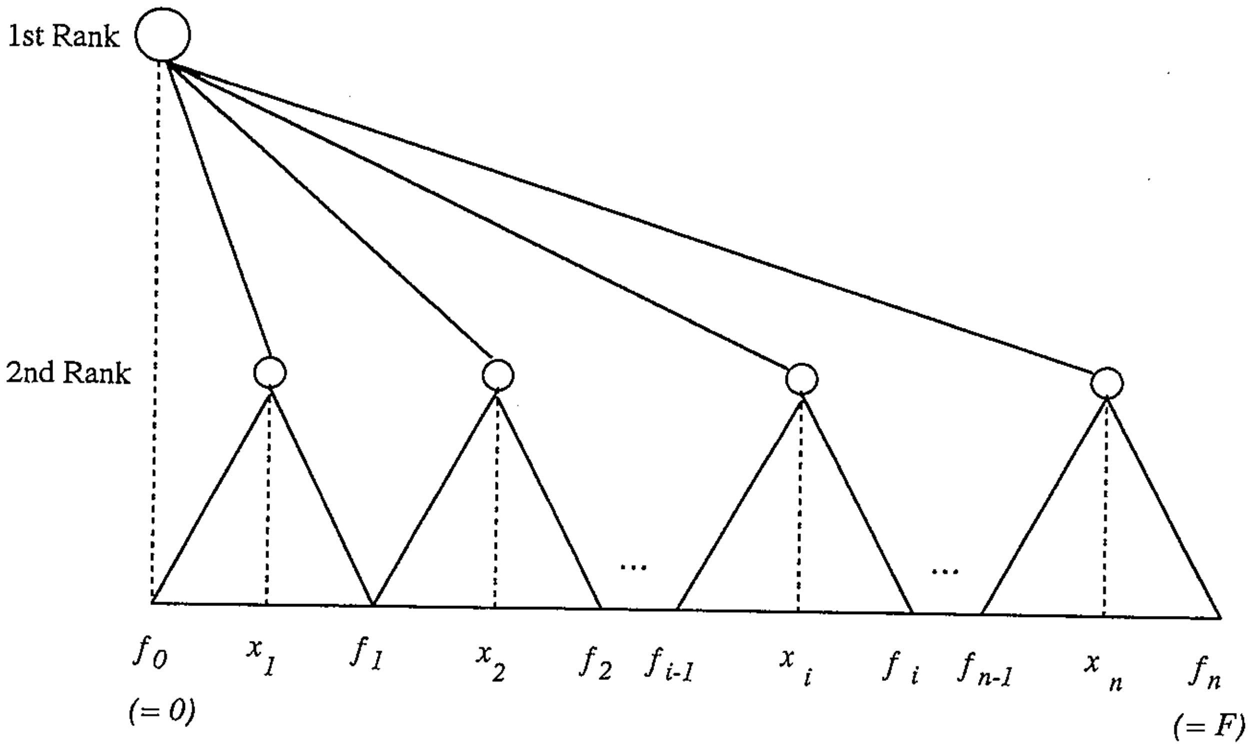

FIGURE 2: The First and Second Ranks of Cities (Only the Right-side Half Strip)

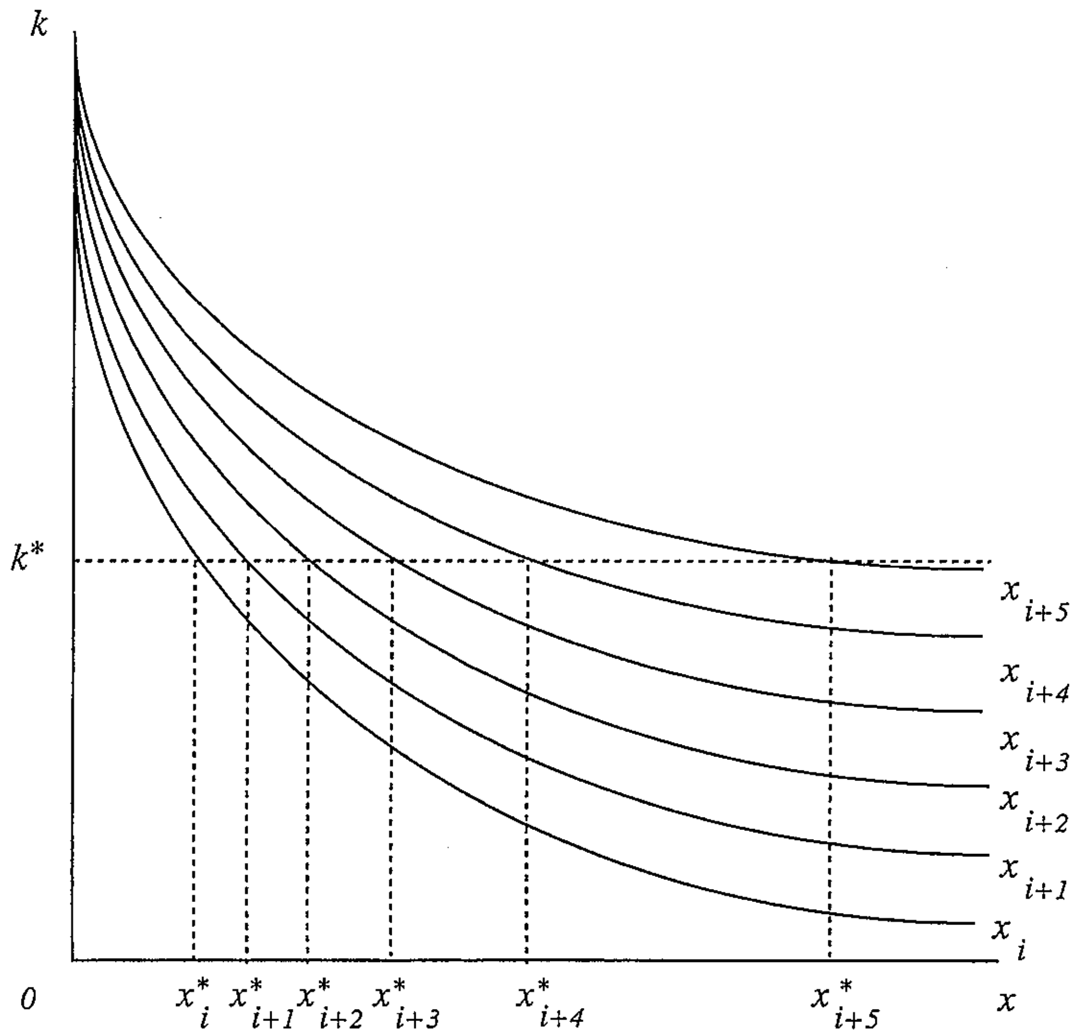

FIGURE 3: Location of Firms (x) and Transportation Cost (k)