No. **638**

Effects of capacity constraints and concentration on the pricing behavior in oligopolistic industries with demand fluctuations

by

Toshikazu Kawakami

August 1995

Effects of capacity constraints and concentration on the pricing behavior in oligopolistic industries with demand fluctuations

Toshikazu Kawakami*

Institute of Socio-Economic Planning, University of Tsukuba

Abstract

This paper introduces a capacity in Rotemberg and Saloner's model on firm's collusive pricing behavior so that the capacity cost serves as a cost in adjusting firm's output level. Then, we shall find that countercyclical movements of prices can occur if firms can set up sufficiently large capacities, or if concentration is high.

Correspondence: Toshikazu Kawakami

Institute of Socio-Economic Planning, University of Tsukuba

Tsukuba, Ibaraki 305, Japan

E-mail: kwkm@aries.sk.tsukuba.ac.jp

*I am especially grateful to my advisor, Professor Yoshihiko Otani for his help and encouragement. I would also like to thank Professors Hiroshi Atsumi, Hiroyuki Odagiri and Hitoshi Matsushima for their helpful comments. Any remaining errors are my own.

1 Introduction

Since Green and Porter (1984) and Rotemberg and Saloner (1986), several attempts to explain firms' pricing behavior by using the repeated game framework have been made. Green and Porter have formalized the issue of secret price cutting in which price drops, remains low for some time and then rises. According to traditional interpretations, this phenomenon is explained as a breakdown and rearrangement of a cartel for some reason. In contrast, Green and Porter showed that this can be described as a perfect equilibrium which would be collusive if the demand function were random and firms could observe neither demand shocks nor their rivals' prices. That is, when each firm doesn't know whether the reason why its profit falls is due to a price cut by its rivals or due to low demand, it must charge low price for some time to punish its rivals if it observes a low profit. Therefore, they emphasize that fluctuations of prices can occur even if there is no breakdown of a cartel. Moreover, their analysis may be interpreted as an attempt to explain price wars during recessions because low demand leads to a price cut and such an interpretation can be found in various articles. However we would like to interpret this phenomenon as an application of imperfect monitoring rather than as a description of the relationship between firm's pricing behavior and business cycles.

Rotemberg and Saloner intentionally explored the relationship between collusive prices and business conditions. The reason why Rotemberg and Saloner carried out such an analysis follows from empirical findings in which a positive correlation of labor demand and the wage rate can be found. Labor demand curves are usually thought to be downward-sloping in wages in traditional economics because labor demand will be determined by the marginal product of labor if product markets are perfectly competitive. If product markets are imperfectly competitive, labor demand curves are determined as follows:

$$f'(x) = \mu w$$

where $f'(x)$ is the marginal product of labor, μ is the markup, and w is the real wage. If, for some reason, an increase in demand makes the markup go down, we may explain this positive correlation. Hence, Rotemberg and Saloner need the theory that prices move countercyclically.

In their model it is supposed that firms play an infinite price-setting game under demand uncertainty, such that in each stage firms set the price after observing the state of demand. If the discount factor is sufficiently high, firms can collusively charge the monopoly price regardless of the state of demand. But if the discount factor becomes low to some extent, the monopoly price at high demand cannot be sustained because the incentive to cut price is greater at high demand than at low demand. Thus they have to make their collusive price lower at high demand. Moreover, Rotemberg and Saloner emphasize that the collusive price at high demand can be lower than at low demand when the discount factor stays within some region.

Another point in their work deals with concentration[1]. In industrial organization, around 1930's, it was found that prices move procyclically in

[1]In their article they don't describe this point explicitly. However we realize the followings by studying their article in detail.

some industries and countercyclically in some other industries. Since then, researchers have faced the following two questions.

1. Why do prices move countercyclically?

2. In which industries do prices move countercyclically?

Many researchers in industrial organization have studied the latter question through emprical analyses. By studying the relationship between price movements and concentration, Wachtel and Adelsheim (1977) found one of probable facts. Using the U.S. data, they showed that prices are likely to move countercyclically when concentration is high whereas prices move procyclically when concentration is medium or low[2].

Rotemberg and Saloner provided an explicable answer to the former question. But their explanation would not apply to the latter if the fact that Wachtel and Adelsheim found was correct. In Rotemberg and Saloner's model, prices move procyclically when concentration is either high or low, while prices move countercyclically when concentration is in the middle range. This is because they assume that firms' cost functions obey constant returns to scale. As a result, firms can adjust the output level freely without incurring extra costs and the gain from deviation is greater as the number of firm in the industry increases. Accordingly, under the condition that firms collude, the possibility that prices move countercyclically is greater, the lower is the degree of concentration. But in reality it may not be easy to increase outputs for a short period and firms may incur extra costs in increasing

[2] Cowling (1983) also found that using the U.K. data.

outputs for various reasons. Thus some modification on the difficulty of adjusting the output level in Rotemberg and Saloner is needed to provide some explanation to the former question. This paper considers this modification. We shall introduce a capacity constraint and a capacity cost to measure the cost of adjusting the output level.

In our model firms play an infinitely repeated game. Firms decide at the first stage the size of a capacity to install. The demand curve at each period in the second stage is determined stochastically. At each period in the second stage firms set their prices after they observe the state of demand. Then we shall find that countercyclical movements of prices are likely to occur when capacities installed at the first stage are sufficiently high or costs to maintain capacities are sufficiently low and procyclical movements of prices always occur when capacities installed at the first stage are to some extent small or costs to maintain capacities are sufficiently high. Furthermore we shall find that countercyclical movements of prices are likely to occur when concentration is high.

2 Model

In this paper we consider a situation in which there are n firms in a market playing an infinitely repeated game. In the first stage firms set up their capacity K, a value that is unchanged over time in the second stage. In the second stage, they engage in an infinitely repeated game of price competition given the capacity level chosen in the first stage. Firms can produce up to K units of the product at zero marginal cost but cannot produce more than

K. And they must pay a unit cost of θ for one unit of the installed capacity at every period.

2.1 Demand

The demand curve takes a linear form for simplicity given as follows:

$$D(\alpha; p) = \alpha - p,$$

or $P(\alpha; x) = D^{-1}(\alpha; p) = \alpha - x$ in the inverse demand form. Suppose α is determined stochastically and the state of demand can be either high or low. More concretely, the value of α is $\overline{\alpha}$ with probability β and $\underline{\alpha}$ with probability $1 - \beta$ where $\overline{\alpha} \geq \underline{\alpha} > 0$. At each period in the second stage, firms choose their prices simultaneously after they observe the state of demand.

Now suppose the following efficient-rationing rule[3],that is, if l firms choose prices strictly below p and m firms choose exactly p, then the demand it faces when firm i chooses p is given by:

$$D(p|\alpha; p_1, \cdots, p_{i-1}, p_{i+1}, \cdots, p_n) = max\left\{0, (\alpha - p - lK)/m\right\},$$

where $n \geq l + m$.

2.2 Price competition

In this section we analyze the price setting stage. We suppose that every firm in this industry sets up a capacity collusively and we call this variable

[3]See, e.g, Tirole (1988).

K^c satisfying the condition that $K^c \leq \overline{\alpha}$ [4]. We suppose that at some period t, each firm chooses p^c, a collusive price, as long as all firms have charged p^c in every period preceding t. Otherwise, it sets its price prevailing in a static Nash equilibrium. We have two cases for p^c given K^c.

It is impossible that firms set their price below $P(\alpha; nK^c)$. Because they can produce at most nK^c. Figure 1 shows the range of the collusive price when $\frac{1}{2n}\alpha \geq K^c$. In this case firms can charge their price above $P(\alpha; nK^c)$, but they don't get more profits than charging $P(\alpha; nK^c)$. Hence firms set their collusive price at $P(\alpha; nK^c)$, then there is no excess capacity in the industry and they get the maximal profits $K^c(P(\alpha; nK^c) - \theta)$.

On the other hand figure 2 shows the other case: $\frac{1}{2n}\alpha \leq K^c$. When firms set their collusive price in $[P(\alpha; nK^c), nK^c]$, they get more profits than when they charge $P(\alpha; nK^c)$ because of the definition of the demand function. However firms never charge their price in $[\frac{\alpha}{2}, nK^c]$. This is because there is a price in $[P(\alpha; nK^c), \frac{\alpha}{2}]$ which gives them the same collusive profits as the price in $[\frac{\alpha}{2}, nK^c]$ but with less incentive of cheating. Hence firms choose a collusive price in $[P(\alpha; nK^c), \frac{\alpha}{2}]$ and they get the profits $\frac{1}{n}p^c D(\alpha; p^c) - \theta K^c$.

We summarize the collusive price and profits. The collusive price is as follows:

$$p^c \quad \begin{cases} = P(\alpha; nK^c), & \text{if } \frac{1}{2n}\alpha \geq K^c; \\ \in [P(\alpha; nK^c), \frac{\alpha}{2}], & \text{if } \frac{1}{2n}\alpha \leq K^c. \end{cases}$$

[4]If $K^c > \overline{\alpha}$, $K^c - \overline{\alpha}$ is of no use. Thus we may restrict our collusive capacity to the range $[0, \overline{\alpha}]$.

And the collusive profits is as follows:

$$\pi_i^c(\alpha; p^c, K^c) = \begin{cases} K^c(P(\alpha; nK^c) - \theta), & \text{if } \frac{1}{2n}\alpha \geq K^c; \\ \frac{1}{n}p^c D(\alpha; p^c) - \theta K^c, & \text{if } \frac{1}{2n}\alpha \leq K^c. \end{cases}$$

Next we consider the per period profit earned by firm i, denoted by $\pi^{ch}(\alpha; p^c, K^c)$ when the state of demand is α and firm i optimally deviates from p^c. And let the net gain by deviation be denoted by $NG(\alpha; p^c, K^c)$[5].

Suppose all firms charge p^c. We have three cases on $\pi_i^{ch}(\cdot)$ given K^c. When $\frac{1}{n}D(\alpha; p^c) \geq K^c$, each firm produces at full capacity. Hence his optimal deviation provides no net gain.

The case when $\frac{1}{n}K^c \leq \frac{1}{n}D(\alpha; p^c) \leq K^c$ is shown in figure 3. Suppose any firm will deviate and cut his price by a small amount, then he gets all demand. However he produces at most K^c. Hence his optimal deviation is that he charges a slightly lower price and produces at full capacity level. Then he gets approximately $K^c(p^c - \theta)$. Hence the net gain is $p^c\{K^c - \frac{1}{n}D(\alpha; p^c)\}$.

The case when $D(\alpha; p^c) \leq K^c$ is shown in figure 4. A firm also gets all demand by cutting his price and has enough K^c to fill it. As $p^c \leq \frac{\alpha}{2}$, cutting his price lower yieldes less profits. Hence his optimal deviation is that he charges a slightly lower price and produces $D(\alpha; p^c)$. Then he gets approximately $p^c D(\alpha; p^c) - \theta K^c$. Hence the net gains is $\frac{n-1}{n}p^c D(\alpha; p^c)$.

Therefore we summarize the above net gain $NG(\alpha; p^c, K^c)$ as follows:

$$NG(\alpha; p^c, K^c) = \begin{cases} 0, & \text{if } D(\alpha; p^c) \geq nK^c; \\ p^c\{K^c - \frac{1}{n}D(\alpha; p^c)\}, & \text{if } K^c \leq D(\alpha; p^c) \leq nK^c; \\ p^c\{\frac{n-1}{n}D(\alpha; p^c)\}, & \text{if } D(\alpha; p^c) \leq K^c. \end{cases}$$

[5]That is, $NG(\alpha; p^c, K^c) = \pi^{ch}(\alpha; p^c, K^c) - \pi^c(\alpha; p^c, K^c)$.

Let $\pi^N(\alpha; p^N(\alpha; K^c))$ denote the per period profit earned by firm i in a static Nash equilibrium when the state of demand is α[6]. Let $\overline{p}^c$ and $\underline{p}^c$ denote the cartel prices when the demand is high and low, respectively, and let δ denote the discount factor.

From now on, we investigate $\{\overline{p}^c, \underline{p}^c\}$ so as to maximize expected profit subject to incentive constraints. Noting that the temptation to undercut the cartel price depends on whether the realized state is high or low, such a problem can be described precisely as follows:

$$\max_{\{\overline{p}^c, \underline{p}^c\}} \beta\pi^c(\overline{\alpha}; \overline{p}^c, K^c) + (1-\beta)\pi^c(\underline{\alpha}; \underline{p}^c, K^c)$$

subject to

$$NG(\overline{\alpha}; \overline{p}^c, K^c) \leq \frac{\delta}{1-\delta}V, \tag{1}$$

for the high demand state; and

$$NG(\underline{\alpha}; \underline{p}^c, K^c) \leq \frac{\delta}{1-\delta}V, \tag{2}$$

for the low demand state, where

$$V \equiv \beta\left\{\pi^c(\overline{\alpha}; \overline{p}^c, K^c) - \pi^N(\overline{\alpha}; K^c)\right\}$$
$$+(1-\beta)\left\{\pi^c(\underline{\alpha}; \underline{p}^c, K^c) - \pi^N(\underline{\alpha}; K^c)\right\}.$$

In these inequalities (1) and (2), the left-hand sides are the short-run gains by deviating from the collusion and the right-hand sides are the long-run losses. As the discount factor increases, the payoffs to be sustained as a

[6]The value of $\pi^N(\alpha; p^N(\alpha; K^c))$ is determined as in the theorem by Davidson and Deneckere (1990). See appendix A.

collusive equilibrium become bigger. On the other hand, the bigger were the short-run gains or the smaller the long-run losses, the discount factor to sustain a collusive equilibrium would have to be larger to satisfy the incentive constraints.

If firms set the same price in both state of demand in Rotemberg and Saloner's model, the temptation to deviate is always bigger when demand is high than when demand is low. But this is not necessarily true for our model, because the incentive to deviate can be larger when demand is low.

Our interest is in the relationship between firms' pricing behavior and capacity constraints. But firms' pricing behavior depends not only on K^c but also on δ. Moreover, if K^c changes, V also changes. Therefore, the set of δ satisfying the incentive constraints depends on K^c for any given $(\overline{p}^c, \underline{p}^c)$. It is not very simple to describe the relationship fully. Fortunately it turns out to be sufficient that we analyse $NG_i(\cdot)$ to investigate the relationship between the pricing behavior and capacity constraints.

Suppose that n and K^c are given. Suppose, in addibR tion, that $p^c = \rho$ (a constant value). Then we can draw $NG_i(\cdot)$ as in figure 5 where the horizontal axis shows α. $NG_i(\cdot)$ is linear and increasing when $\alpha \in [2\rho, \rho + K^c]$. (This is shown by the segment AB.) It is also linear but decreasing when $\alpha \in [\rho + K^c, \rho + nK^c]$. (This is shown by the segment BC.) And it is not defined when $\alpha > \rho + nK^c$ because firms cannot produce above nK^c. Over $\rho + K^c$, the profits when a firm deviates is limited by capacity constraints. Hence a kink is at the point B. Next we investigate the effect of the change of K^c on $NG_i(\cdot)$. This is shown in figure 6. we suppose that the values of $\overline{\alpha}$ and $\underline{\alpha}$ are given as in figure 6. The kinked curves ABE, ACF and ADG correspond to $NG(\alpha;\rho,\dot{K}^c)$, $NG(\alpha;\rho,\hat{\hat{K}}^c)$ and $NG(\alpha;\rho,\ddot{K}^c)$ respectively where $\dot{K}^c < \hat{\hat{K}}^c < \ddot{K}^c$ and $NG(\underline{\alpha};\rho,\hat{\hat{K}}^c) = NG(\overline{\alpha};\rho,\hat{\hat{K}}^c)$. That is, the graph shifts rightward as K^c increases. Furthermore $NG(\overline{\alpha};\rho,K^c) \leq NG(\underline{\alpha};\rho,K^c)$ for $K^c \leq \hat{\hat{K}}^c$ and $NG(\overline{\alpha};\rho,K^c) \geq NG(\underline{\alpha};\rho,K^c)$ for $K^c \geq \hat{\hat{K}}^c$. In other words $\hat{\hat{K}}^c$ is the watershed whether the net gain by deviation at high demand is bigger than at low demand under the condition that firms charge the same price at both demand. From these arguments we can obtain the next proposition about the critical value $\hat{K}^c$ whether prices can move countercyclically or not:

Proposition 1

Prices *can move* countercyclically when $K^c \in [\hat{K}^c, \overline{\alpha}]$, while prices *move* procyclically when $K^c \leq \hat{K}^c$, where

$$\hat{K}^c = \frac{1}{n}(\overline{\alpha} + \frac{n-2}{2}\underline{\alpha}) \quad .$$

Proof: To compare $NG(\underline{\alpha};\rho,K^c)$ and $NG(\overline{\alpha};\rho,K^c)$ let us consider $NG(\overline{\alpha};\rho,K^c) - NG(\underline{\alpha};\rho,K^c)$. Note that $\overline{\alpha} \leq \rho + nK^c$. Then there are two cases to be considered. If $D(\underline{\alpha};\rho) \geq \frac{1}{n}D(\overline{\alpha};\rho)$, then

$$NG(\overline{\alpha};\rho,K^c) - NG(\underline{\alpha};\rho,K^c) =$$
$$\begin{cases} \frac{1}{n}\rho(\underline{\alpha} - \overline{\alpha}), & \text{if } D(\underline{\alpha};\rho) \geq K^c \geq \frac{1}{n}D(\overline{\alpha};\rho); \\ \rho\{K^c - \frac{1}{n}(\overline{\alpha} - \rho)\} - \frac{n-1}{n}\rho(\underline{\alpha} - \rho), & \text{if } D(\overline{\alpha};\rho) \geq K^c \geq D(\underline{\alpha};\rho); \\ \frac{n-1}{n}\rho(\overline{\alpha} - \underline{\alpha}), & \text{if } K^c \geq D(\overline{\alpha};\rho). \end{cases}$$

If $D(\underline{\alpha};\rho) \leq \frac{1}{n}D(\overline{\alpha};\rho)$, then

$$NG(\overline{\alpha};\rho,K^c) - NG(\underline{\alpha};\rho,K^c) =$$

$$\begin{cases} \rho\{K^c - \frac{1}{n}(\overline{\alpha} - \rho)\} - \frac{n-1}{n}\rho(\underline{\alpha} - \rho), & \text{if } D(\overline{\alpha};\rho) \geq K^c \geq \frac{1}{n}D(\overline{\alpha};\rho); \\ \frac{n-1}{n}\rho(\overline{\alpha} - \underline{\alpha}), & \text{if } K^c \geq D(\overline{\alpha};\rho) \quad . \end{cases}$$

In both cases $NG(\overline{\alpha};\rho,K^c) - NG(\underline{\alpha};\rho,K^c)$ is increasing on K^c. And it is negative when K^c is small, while it is positive when K^c is large. Therefore there is $\hat{\hat{K}}^c$ such that for every K^c such that $K^c \geq \hat{\hat{K}}^c$

$$NG(\overline{\alpha};\rho,K^c) \geq NG(\underline{\alpha};\rho,K^c)$$

, and for every K^c such that $K^c \leq \hat{\hat{K}}^c$

$$NG(\overline{\alpha};\rho,K^c) \leq NG(\underline{\alpha};\rho,K^c)$$

where

$$\hat{\hat{K}}^c = \frac{1}{n}(\overline{\alpha} + (n-1)\underline{\alpha} - n\rho) \quad .$$

Note that $\hat{\hat{K}}^c$ is minimum when p^c is maximum or $\frac{\underline{\alpha}}{2}$, i.e.,

$$\hat{K}^c = \frac{1}{n}(\overline{\alpha} + \frac{n-2}{2}\underline{\alpha}) \quad .$$

This proposition has the following economic implications.

1. Countercyclical movements of prices can occur if firms set up sufficiently larger capacities. On the other hand, procyclical movements of prices always occur if firms set up sufficiently smaller capacities.

2. When n increases, $n\hat{K}^c$ increases. Hence if nK^c does not change so much regardless of its concentration, then the movements of prices will be procyclically as n increases.

The reason why the first sentence of the economic implication 2 issues is the following. When $\underline{p}^c$ is $\frac{\alpha}{2}$, they must have excess capacities in collusively charging $\frac{\alpha}{2}$ at high demand so that $\overline{p}^c$ can be lower than $\frac{\alpha}{2}$ (in other words, they can charge their collusive price countercyclically). How large excess capacities do they need for charging $\overline{p}^c$ lower than $\frac{\alpha}{2}$ at that time? Excess capacities at high demand are smaller than ones at low demand in charging the same price at both demand. If each firm has less capacity than $\frac{\alpha}{2}$, it is capacity constrained even at low demand when it undercuts its price slightly. As a result, $NG(\underline{\alpha}; \frac{\alpha}{2}, K^c) \geq NG(\overline{\alpha}; \frac{\alpha}{2}, K^c)$ for $K^c \leq \frac{\alpha}{2}$ and any n. Thus all firms have more capacities than $\frac{\alpha}{2}$ so that $\overline{p}^c$ can be lower than $\frac{\alpha}{2}$. Then each firm always gets an extra demand of almost $\frac{n-1}{n}(\frac{\alpha}{2})$ if it deviates at low demand. For the above reasons firms can charge $\overline{p}^c$ lower than $\frac{\alpha}{2}$ if they have excess capacities more than $\frac{n-1}{n}(\frac{\alpha}{2})$ when they all charge $\frac{\alpha}{2}$ at high demand. That is, the minimum capacity level of one firm when they can charge their collusive price countercyclically is as follows:

$$\underbrace{\frac{1}{n}(\overline{\alpha} - \frac{\alpha}{2})}_{\text{active capacity}} \quad + \quad \underbrace{\frac{n-1}{n}(\frac{\alpha}{2})}_{\text{excess capacity}} \quad .$$

Hence the minimum capacity level in the industry is as follows:

$$\underbrace{\overline{\alpha} - \frac{\alpha}{2}}_{\text{active capacity}} \quad + \quad \underbrace{(n-1)\frac{\alpha}{2}}_{\text{excess capacity}} \quad .$$

While the term of the active capacity is constant, the term of the excess capacity increases with increasing number of firms.

One of our major interests is on the relationship between firms' pricing behavior and the degree of concentration in the industry. In order to answer this question it remains to analyze the choice of capacity firms choose. We shall analyze this in the next section.

2.3 Incentives at the first stage

In this section we analyze the capacity choice. In our model, in order for firms to collude in the second stage they must set up their capacities so that the collusive profit in the future may be greater than the noncooperative profit in the second stage. This relation can be expressed as follows:[7]

$$E\Pi_i^c(\overline{\alpha}, \underline{\alpha}; K^c) - \theta K^c \geq E\Pi_i^*(\overline{\alpha}, \underline{\alpha}; K_i^N, K_{-i}^N, \theta), \tag{3}$$

where

$$E\Pi_i^c(\overline{\alpha}, \underline{\alpha}; K^c) \equiv \beta \pi_i^c(\overline{\alpha}; \overline{p}^c(\overline{\alpha}, K^c)) + (1-\beta)\pi_i^c(\underline{\alpha}; \underline{p}^c(\underline{\alpha}, K^c)) \quad .$$

The left-hand side is firm i's *ex-ante* per period profit when all firms behave collusively. And the right-hand side is the one when they play the static Nash equilibrium strategy in the price-setting stage. [8]

2.3.1 Capacity choice when firms don't behave collusively in the price-setting stages

Let us consider K_i^N and the per period profit in choosing K_i^N, that is, $E\Pi_i^*(\overline{\alpha}, \underline{\alpha}; K_i^N, K_{-i}^N, \theta)$. To analyze this profit we shall employ the theorem

[7]We don't consider mixed strategies, but only pure strategies on capacity choice in the first stage.

[8]Let K_i^N denote the capacity in equilibrium which is installed by firm i when it plays the static Nash equilibrium strategy in the price-setting stage.

due to Davidson and Deneckere (1990).[9] As their theorem indicates, firm 1's profit is distinguished in several regions when the state of demand at the period is realized. Here note that α can be either $\overline{\alpha}$ or $\underline{\alpha}$ and all firms charge their prices after observing the state:

$$E\Pi_i^*(\overline{\alpha},\underline{\alpha};K_i,K_{-i},\theta) \equiv \beta\pi_i^N(\overline{\alpha};K_i,K_{-i},\theta) + (1-\beta)\pi_i^N(\underline{\alpha};K_i,K_{-i},\theta) \quad .$$

We only consider a symmetric capacity choice: $K_i = \bar{K}$ for all i. Then there exists a symmetric static Nash equilibrium for the capacity choice given by

$$K^N = \begin{cases} \frac{1}{2n}\{\overline{\alpha} - \frac{\theta}{\beta}\}, & \text{if } \beta(\overline{\alpha}-\underline{\alpha}) \geq \theta; \\ \frac{1}{n+1}\{\beta\overline{\alpha} + (1-\beta)\underline{\alpha} - \theta\}, & \text{if } \beta(\overline{\alpha}-\underline{\alpha}) \leq \theta; \end{cases}$$

as is shown in Appendix B.

In this model all firms are to decide their capacity level cooperatively no matter how they behave in the price setting stage. That is, if the inequality (3) is satisfied, then they are to cooperate one another and if not, they all agree to choose K_i^N. So we don't have to consider punishments in the first stage.

2.3.2 Capacity choice when firms collusively behave in the price-setting stage

By investigating the inequality (3), we can find that there exists $\hat{\theta}$ such that there cannot be countercyclical movements of prices if $\theta \geq \hat{\theta}$ and there can be countercyclical movements of prices if $\hat{\theta} \geq \theta \geq 0$. And $\hat{\theta}$ will be

[9] See appendix A.

determined by the next equation:

$$\beta\pi_i^c(\overline{\alpha};\frac{\alpha}{2}) + (1-\beta)\pi_i^c(\underline{\alpha};\frac{\alpha}{2}) - \hat{\theta}\hat{K}^c = E\Pi_i^*(\overline{\alpha},\underline{\alpha};K_i^N,K_{-i}^N,\hat{\theta}) \quad .$$

When prices can move countercyclically, the value of $E\Pi_i^c(\overline{\alpha},\underline{\alpha};K^c)$ is maximum at $\underline{p}^c = \overline{p}^c = \frac{\alpha}{2}$. And the value of θK^c is minimum when $K^c = \hat{K}^c$. Hence, $\hat{\theta}$ must be determined by the above equation.

Table 1 exhibits the value of $\hat{\theta}$ which we calculate for the case when $\beta = \frac{1}{2}$ and $\overline{\alpha} = 10$. The following features are found in table 1.

1. The value of $\hat{\theta}$ decreases as the difference between $\overline{\alpha}$ and $\underline{\alpha}$ increase.

2. The value of $\hat{\theta}$ is miximum at $n = 3$ or 4. And it decreases with increasing n.

The former means that there is a small possibility of countercyclical movements of prices in the industry where the difference of demand between booms and recessions is larger. This is because countercyclical pricing makes firms lose larger profits at booms as the difference is larger. The latter demonstrates that the possibility of countercyclial movements of prices increase as concentration increases. This result is consistent with the empirical analysis by Wachtel and Adelsheim.

3 Concluding Remarks

This paper considers the effect of capacity cost to capture the cost of adjusting the output level in firms' collusive pricing behavior. We have shown the following results. If the capacity cost is sufficiently low, firms can set up sufficiently large capacities and prices can move countercyclically. While prices move procyclically if the capacity cost is sufficiently high. Furthermore prices are more likely to move countercyclically as concentration is higher.

However there may be a question we should further consider. It is about the capacity choice. We have assumed that firms only compare the collusive profit and the noncooperative profit when they choose their capacity level. But the capacity choice is influenced by various (in particular, exogenous) factors. In addition, we have assumed the existence of a symmetric equilibrium to evaluate the noncooperative profit. Thus our results in this paper should be considered tentative. We would like to carry out further investigation to make our results more appropriate.

Appendix

Appendix A

On the Bertrand-Edgeworth duopoly, Kreps and Scheinkman (1983) derived the profit function given capacities. Davidson and Deneckere (1990) modify it for linear demand case. And we modify their result applicable to the n firms case.

Theorem

Firms except for i are supposed to choose the same capacity level $\bar{K}$. For each pair (K_i,$\bar{\mathcal{K}}$), where $\bar{\mathcal{K}}$ means all firms except i choose $\bar{K}$, and for any α, the static price-setting game with capacity constraints has a unique static Nash equilibrium:

1. If $(n-2)\bar{K}+K_i > \alpha$ or $\bar{K} > \frac{\alpha}{n-1}$, the equilibrium is in pure strategies, all firms charge $p = 0$ and firm i gets $-\theta K_i$ and another firms get $-\theta\bar{K}$.

2. If $K_i \leq \alpha - n\bar{K}$ and $K_i \leq \frac{1}{2}\{\alpha - (n-1)\bar{K}\}$, the equilibrium is in pure strategies, all firms charge $p = \alpha - K_i - (n-1)\bar{K}$ and firm i gets $K_i\{\alpha-\theta-K_i-(n-1)\bar{K}\}$ and another firms get $\bar{K}\{\alpha-K_i-(n-1)\bar{K}\}$.

3. If $K_i \geq \bar{K}$ and $(n-2)\bar{K}+K_i < \alpha$ and $K_i \geq \frac{1}{2}\{\alpha-(n-1)\bar{K}\}$, the equilibrium is in mixed strategies and firm i gets $\frac{1}{4}\{\alpha-(n-1)\bar{K}\}^2-\theta K_i$ and another firms get $\frac{\bar{K}}{4K_i}\{\alpha-(n-1)\bar{K}\}^2-\theta\bar{K}$.

4. If $K_i > \bar{K}$ and $\bar{K} < \frac{\alpha}{n-1}$ and $K_i > \alpha - n\bar{K}$, the equilibrium is in mixed strategies and firm i gets $\frac{K_i}{4\bar{K}}\{\alpha-(n-2)\bar{K}-K_i\}^2-\theta K_i$ and another firms get $\frac{1}{4}\{\alpha-(n-2)\bar{K}-K_i\}^2-\theta\bar{K}$.

Appendix B

As we mentioned in subsection 2.3.1, we only consider a symmetric capacity choice. K^N is not below $\frac{\alpha-\theta}{n+1}$. This is because, if $K^N \leq \frac{1}{2}\{\alpha-\theta-(n-1)\bar{K}\}$, then any firm i can get more profit by increasing its capacity. (Note that $\frac{\partial\pi_i(\alpha;K_i,\bar{\mathcal{K}},\theta)}{\partial K_i}\Big|_{K_i=\bar{K}} = \alpha-(n+1)\bar{K}-\theta \geq 0$ if $K_i < \frac{\alpha-\theta}{n+1}$.) Therefore it doesn't choose in such a region.

K_i is not above $\frac{\overline{\alpha}-\theta}{n+1}$, either. This is because when $\bar{K} = \frac{\alpha}{n+1}$, then $\frac{\partial E\Pi_i^*(\alpha;K_i,\bar{\mathcal{K}},\theta)}{\partial K_i}\Big|_{K_i=\bar{K}} = -\theta$. As a result firms cannot get more profit by increasing their capacity. Therefore it chooses $K^N \in [\frac{\overline{\alpha}}{n+1}, \frac{\alpha-\theta}{n+1}]$.

If $\frac{\alpha-\theta}{n+1} \leq \bar{K} \leq \frac{\alpha}{n-1}$,

$$\frac{\partial E\Pi_i^*(\overline{\alpha},\underline{\alpha};K_i,\bar{\mathcal{K}},\theta)}{\partial K_i}\Big|_{K_i=\bar{K}} = \beta\{\overline{\alpha}-(n+1)\bar{K}-\theta\}+(1-\beta)\{\underline{\alpha}-(n+1)\bar{K}-\theta\}.$$

And if $\frac{\alpha}{n-1} \leq \bar{K} \leq \frac{\overline{\alpha}}{n+1}$,

$$\frac{\partial E\Pi_i^*(\overline{\alpha},\underline{\alpha};K_i,\bar{\mathcal{K}},\theta)}{\partial K_i}\Big|_{K_i=\bar{K},} = \beta\{\overline{\alpha}-(n+1)\bar{K}-\theta\}+(1-\beta)(-\theta\bar{K}).$$

Hence if

$$\frac{\partial E\Pi_i^*(\overline{\alpha},\underline{\alpha};K_i,\frac{\alpha}{n-1},\cdots,\frac{\alpha}{n-1},\theta)}{\partial K_i}\Big|_{K_i=\frac{\alpha}{n-1}} \geq 0,$$

then $\frac{\alpha-\theta}{n+1} \leq K^N \leq \frac{\alpha}{n-1}$ and

$$K^N = \frac{1}{n+1}\{\beta\overline{\alpha}+(1-\beta)\underline{\alpha}-\theta\}.$$

And if

$$\frac{\partial E\Pi_i^*(\overline{\alpha},\underline{\alpha};K_i,\frac{\alpha}{n-1},\cdots,\frac{\alpha}{n-1},\theta)}{\partial K_i}\Big|_{K_i=\frac{\alpha}{n-1}} \leq 0,$$

then $\frac{\alpha}{n-1} \leq K^N \leq \frac{\overline{\alpha}}{n+1}$ and

$$K^N = \frac{1}{2n}(\overline{\alpha}-\frac{\theta}{\beta}).$$

References

[1] Benoit, J.P. and Krishna, V. "Dynamic Duopoly: Prices and Quantities." *Review of Economic Studies*, Vol.54 (1987), pp, 383-396.

[2] Cowling, K. "Excess Capacity and the Degree of Collusion: Oligopoly Behavior in the Slump." *Manchester School of Economics and Social Studies*, Vol.51 (1983), pp,341-359

[3] Machin, S. and Reenen, V. "Profit Margins and The Business Cycle: Evidence from UK Manufacturing Firms." *The Journal of Industrial Economics*, Vol.41 (1993), pp, 29-50.

[4] Brock, W.A. and Scheinkman, J.A. "Price Setting Supergames with Capacity Constraints." *Review of Economic Studies*, Vol. 52 (1985), pp. 371-382.

[5] Davidson, C. and Deneckere, R. "Excess Capacity and Collusion." *International Economic Review*, Vol. 31 (1990), pp. 521-542.

[6] Green, E.J. and Porter, R.H. "Noncooperative Collusion Under Imperfect Price Information." *Econometrica*, Vol. 52 (1984), pp. 87-100.

[7] Haltwanger, J. and Harrington, J.E.,Jr. "The Impact of Cyclical Demand Movements on Collusive Behavior." *RAND Journal of Economics*, Vol. 22 (1991), pp. 89-106.

[8] Kreps, D.M. and Scheinkman, J.A. "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes." *Bell Journal of Economics*, vol. 14 (1983), pp. 326-337.

[9] Kreps, D.M. *Game Theory and Economic Modelling* Oxford University Press (1990).

[10] Odagiri, H and Yamashita, T "Price Mark-ups, Market Structure, and Business Fluctuation in Japanese Manufacturing Industries" *Journal of Industrial Economics*, Vol 35 (1987), pp. 317-331

[11] Osborne, M.J. and Pitchik, C. "Price Competition in Capacity-Constrained Duopoly'." *Journal of Economic Theory*, Vol. 38 (1986), pp. 238-280.

[12] Rotemberg, J.J. and Saloner, G. "A Supergame-Theoretic Model of Price Wars During Booms." it American Economic Review, Vol. 76 (1986), pp. 390-407.

[13] Rotemberg, J.J. and Saloner, G. "The Cyclical Behavior of Strategic Inventories." *Quarterly Journal of Economics*, Vol. 104 (1989), pp. 73-97.

[14] Rotemberg, J.J. and Woodford, M. "Markups and the Business Cycle." *NBER Macroeconomics Annual*,(1991).

[15] Scherer, F. *Industrial Market Structure and Economic Performance*, Second Edition, Chicago,Rand McNally (1980)

[16] Staiger, R.W. and Wolak, F.A. "Collusive Pricing with Capacity Constraints in the presence of demand uncertainty." *RAND Journal of Economics*, Vol.23 (1992), pp. 203-220.

[17] Tirole, J. *The Theory of Industrial Organization*, MIT press (1988).

[18] Wachtel, H.M. and Adelsheim, P.D. "How Recession Feeds Inflation: Price Markups in a Concentrated Economy." *Challenge*, Vol. 20 (1977), pp. 6-13.

Table 1

n \ α	8	7	6	5	4	3
2	0.467	0.431	0.319	0.112	-	-
3	0.561	0.567	0.506	0.347	0.033	-
4	0.562	0.577	0.525	0.364	0.000	-
5	0.538	0.551	0.492	0.309	-	-
6	0.507	0.513	0.441	0.220	-	-
7	0.475	0.472	0.384	0.112	-	-
8	0.443	0.431	0.323	-	-	-
9	0.414	0.392	0.263	-	-	-
10	0.386	0.355	0.204	-	-	-
11	0.361	0.320	0.146	-	-	-
12	0.338	0.287	0.089	-	-	-
13	0.316	0.256	0.034	-	-	-
14	0.300	0.228	-	-	-	-
15	0.278	0.201	-	-	-	-
23	0.171	0.033	-	-	-	-
24	0.161	0.019	-	-	-	-
25	0.151	0.002	-	-	-	-
26	1.142	-	-	-	-	-
55	0.003	-	-	-	-	-
56	0.001	-	-	-	-	-
57	-	-	-	-	-	-

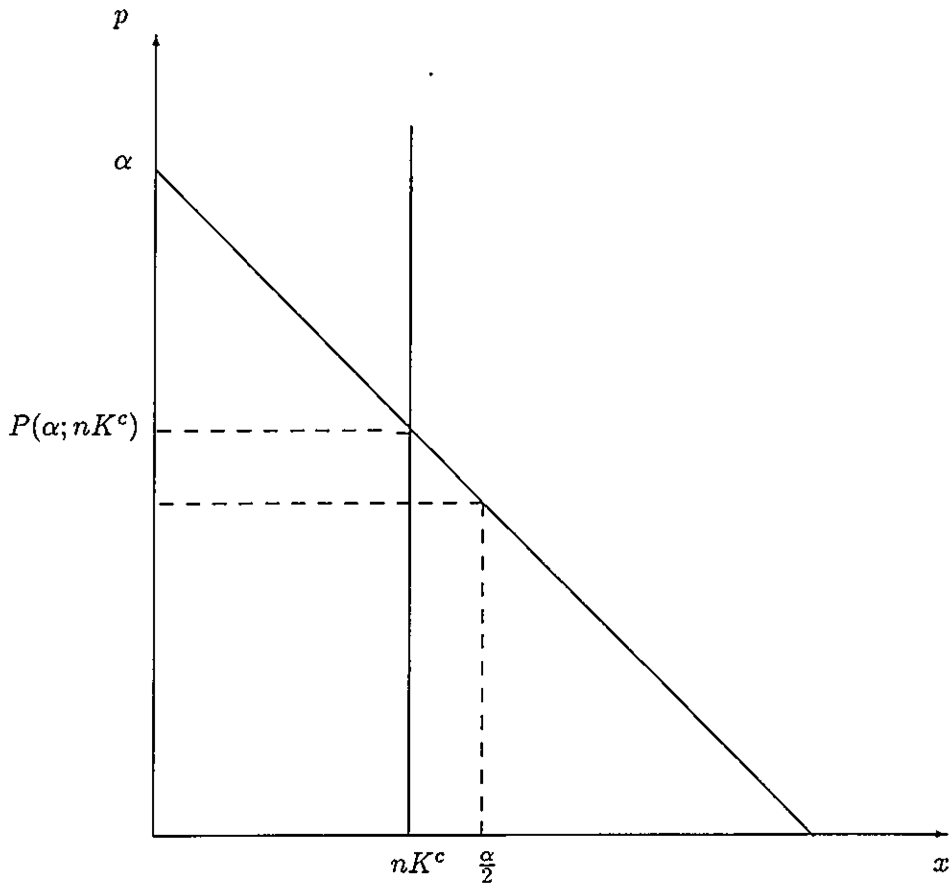

Figure 1: $\frac{1}{2n}\alpha \geq K^c$

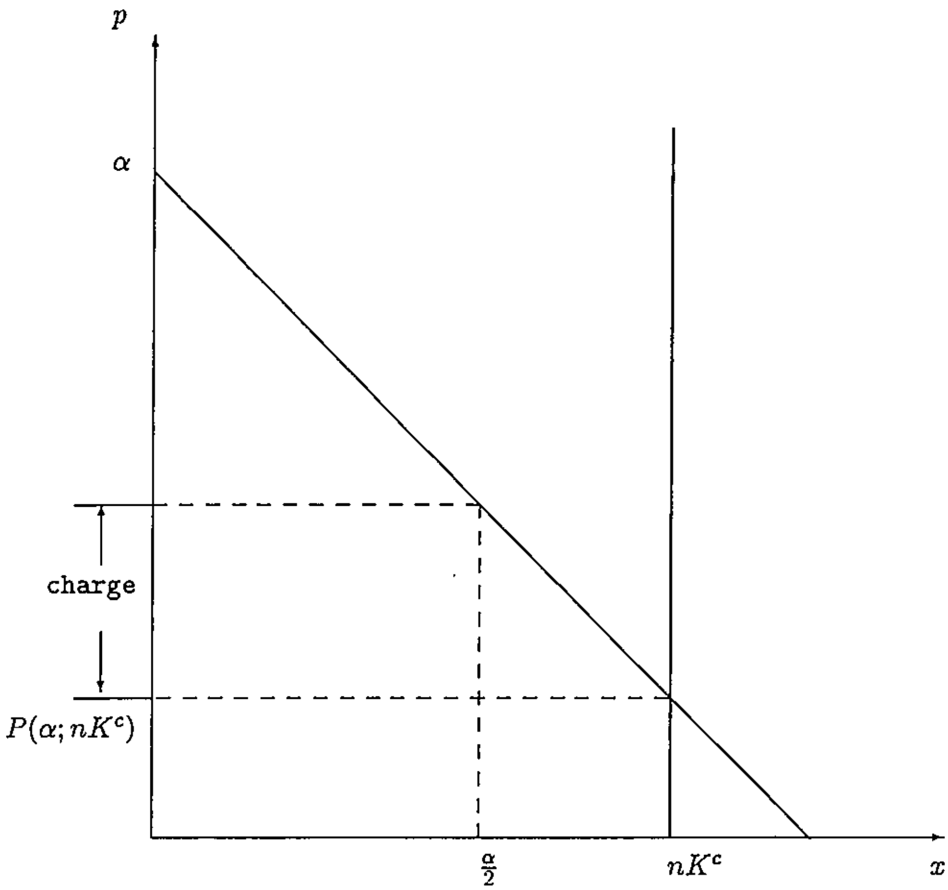

Figure 2: $\frac{1}{2n}\alpha \le K^c$

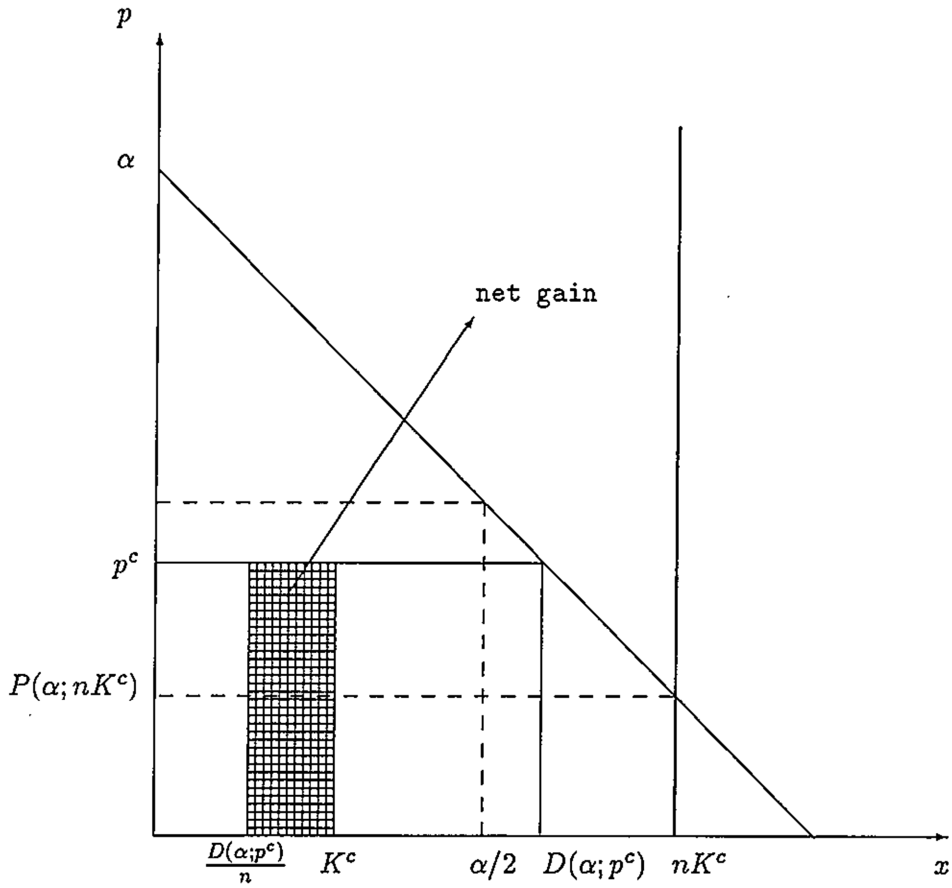

Figure 3: $\frac{1}{n}K^c \leq \frac{1}{n}D(\alpha; p^c) \leq K^c$

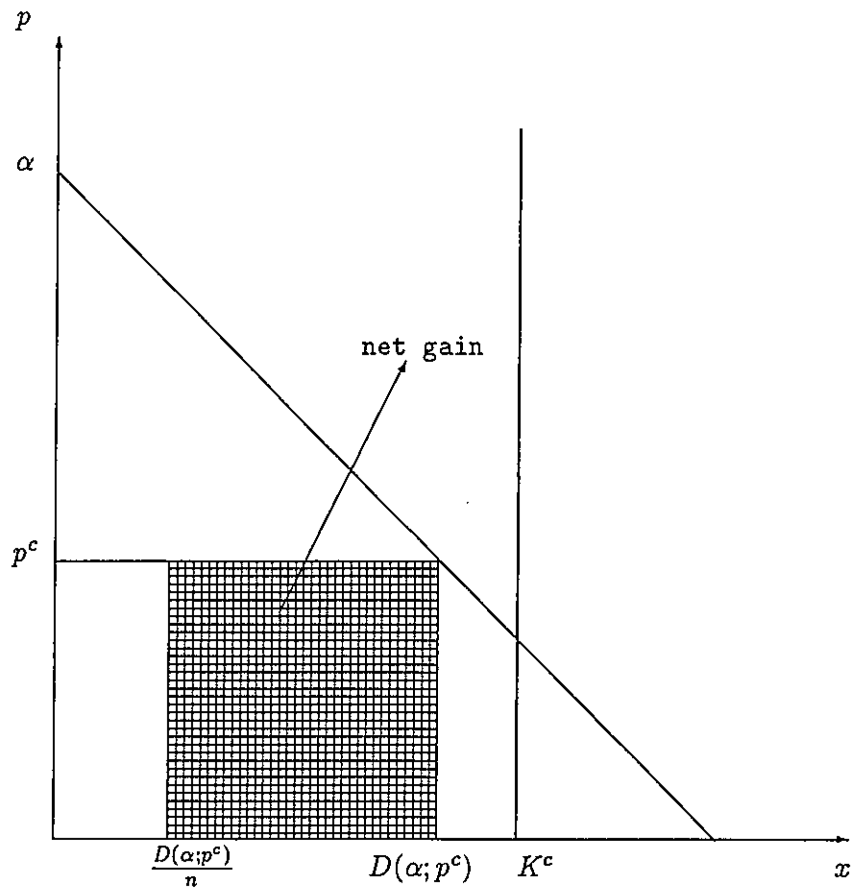

Figure 4: $D(\alpha;p^c) \leq K^c$

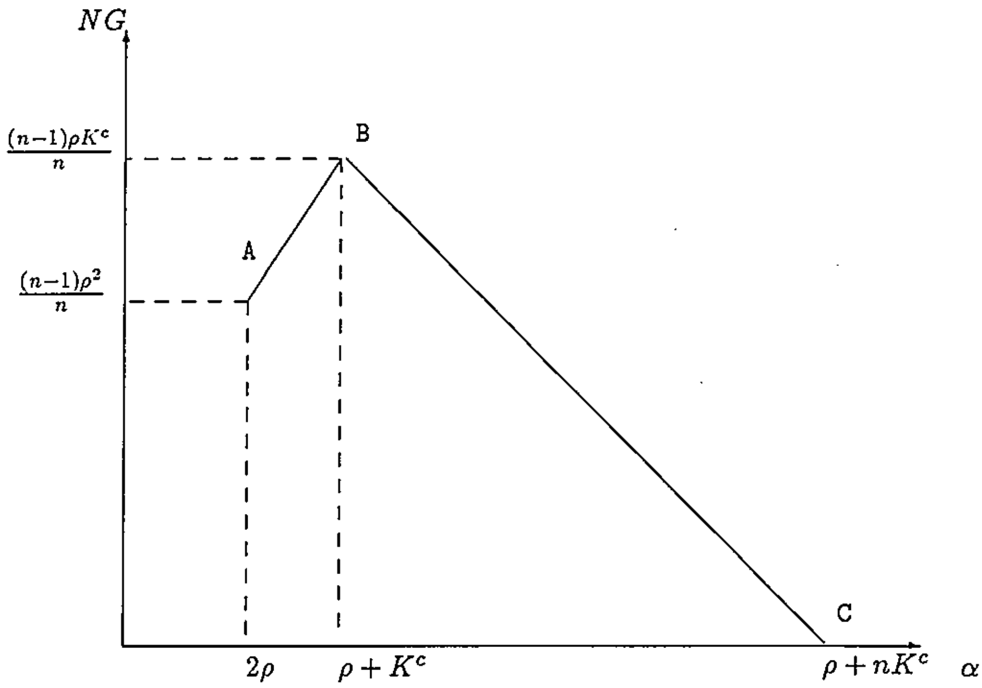

Figure 5: $NG_i(\cdot)$

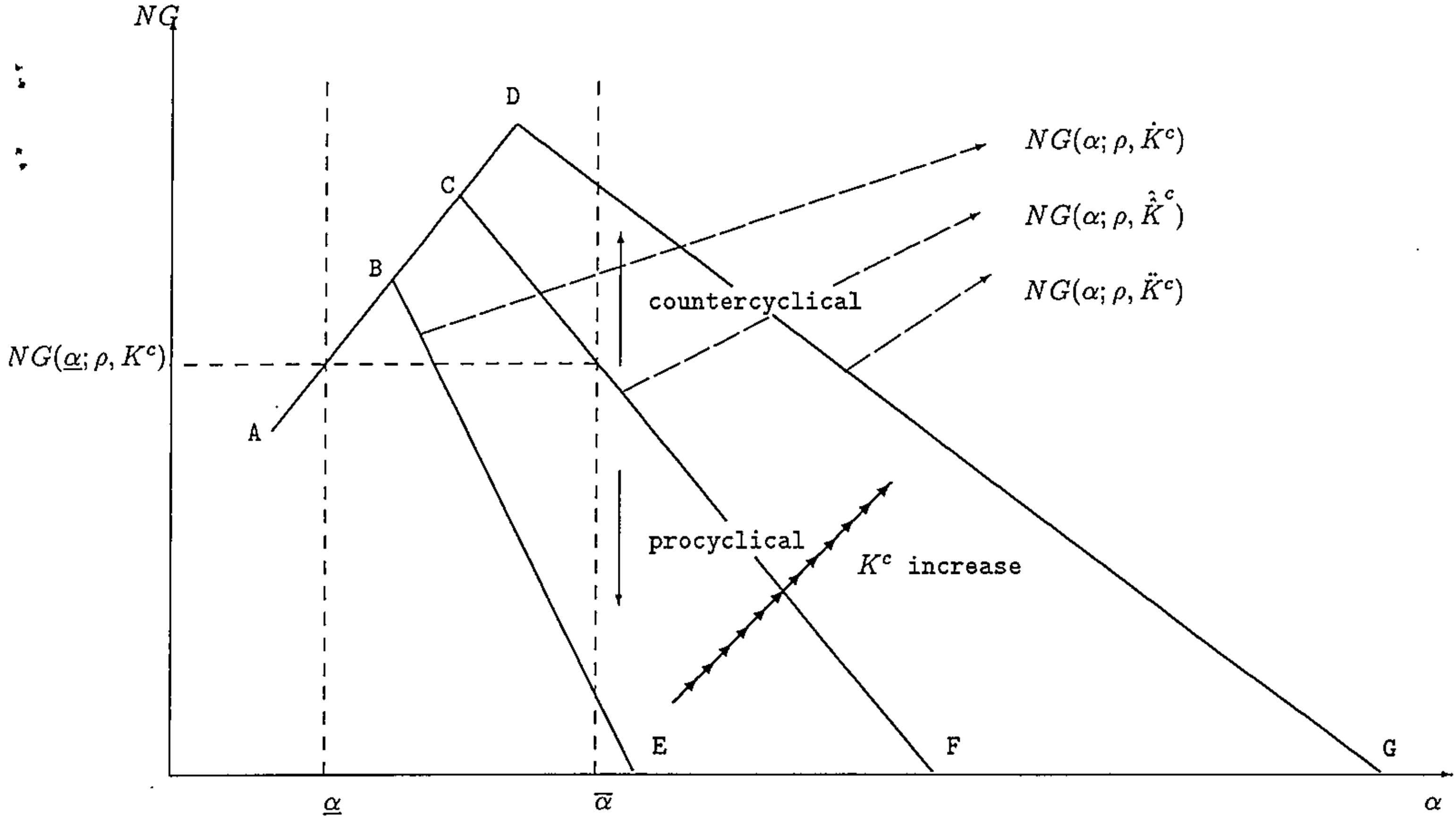

Figure 6: How does $\hat{\hat{K}}^C$ determine?