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An Application of Gibbs Sampler to the Married Women Labor Supply

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Abstract

Chib (1992) shows that the Gibbs sampler combined with data augmentation yields a simplified solution to the difficulty of analyzing the standard Tobit model. In this paper the method is applied to the more complicated two equation Tobit model, which is often used in labor economics. Using the actual micro data on female labor supply, the results of this method are compared with those of the modified Heckman's two step method, which is widely used in the empirical analysis. However, the Heckman's two step estimator is known to perform poorly compared with the maximum likelihood estimator (MLE). The Gibbs sampler with data augmentation appears to be an effective alternative procedure when the computational difficulty in MLE exists.

1 Introduction

The Gibbs sampler is a Markov chain Monte Carlo method designed to obtain marginal distributions from nonnormalized joint distributions. Geman and Geman (1984) introduced this in the context of image restoration.[1] More recently, Gelfand and Smith (1990) revealed its potential in a wide variety of the statistical problems. They also generalized the idea of data augmentation proposed by Tanner and Wong (1987) and showed that the data augmentation is closely related to the Gibbs sampler. Since then quite a few papers dealing with the Gibbs sampler have emerged in the statistical literature. Its potential for fitting a wide range of models is still being appreciated.[2] Several papers have also appeared in econometrics (Chib 1992, 1993; Albert and Chib 1993a and 1993b; Shephard 1993; Geweke 1993; Zangari and Tsurumi 1994; Chib and Greenberg 1994).

The Gibbs sampler generates random variables from a marginal distribution indirectly, without having to calculate the density. The difficulty in obtaining marginal distribution from a nonnormalized joint density is integration. The Gibbs sampler replaces such a difficult calculation by a sequence of much easier calculations. Although most applications of the Gibbs sampler have been in Bayesian framework, it is very useful in many situations of the likelihood.[3]

In this paper the Gibbs sampler is applied to the analysis of the generalized Tobit censored regression model. Chib (1992) shows that the Gibbs sampler combined with data augmentation yields a simplified solution to the difficulty of analyzing the standard Tobit model. His simulated results sup-

[1] However, the roots of the method are traced back to Metropolis *et al.* (1953). Hastings (1970) developed this further in the statistical literature.

[2] For a partial list of the various models to which the Gibbs sampler is applied, see Casella and George(1992) and Gelfand (1994).

[3] see Tanner (1993).

port the efficacy of the method even when the degree of censoring is large. We adapt this idea and extend the techniques to the more complicated two-equation Tobit model as is often in labor economics. The estimation problems are especially serious in the analysis of married women labor supply since a large proportion of married women do not work. In addition, the standard theory implies hours of work and wage rate are simultaneously determined. But typically offered wage rate are not available for nonworking women. These lead up to computational difficulty in obtaining the maximum likelihood estimator even based on normality assumption.[4] Instead, Heckman's two step estimator is widely used in the empirical analysis. Monte Carlo experiments such as Wales and Woodland (1980), however, indicate that the Heckman's two step estimator perform rather poorly than the maximum likelihood estimator. Using the actual micro data, we demonstrate that the Gibbs sampler effectively produces samples from the marginal and joint distributions. Using the entire sample, posterior inference is straightforward. The results are compared with those of Heckman's method.

The next section presents the model. In section 3 the conventional Heckman's method and the Gibbs sampler with data augmentation are provided. Section 4 presents the results of these estimation methods using the actual micro data. Our conclusions are in section 5.

2 The Model

We assume that an individual married woman maximizes a well behaved twice differentiable utility function subject to the budget and time constraints.[5] Under the assumption that working women are free to choose their hours of work, both offered wage and reservation wage functions fully characterize the

[4] see Nawata (1993a).

[5] We basically consider Nakamura, Nakamura and Cullen's model (1979), which is an extension of Heckman's model (1974, 1976 and 1979).

decision on female labor supply.[6] The natural logarithm of the reservation wage derived from the utility maximization is expressed as

$$\begin{aligned} l(w_{Ri}) &= \beta_0 + \beta_1 h_i + \beta_2 l(w_i) + \beta_3 A_i + \beta_4' Z_i + \epsilon_{R_i} \quad \text{for } h_i > 0 \\ &= \beta_0 + \beta_3 A_i + \beta_4' Z_i + \epsilon_{R_i} \qquad\qquad \text{for } h_i = 0 \end{aligned} \tag{1}$$

where $l(w_{R_i})$ and $l(w_i)$ are natural logarithm of the reservation wage and market offered wage, respectively, A_i represents the nonwife income including husband's earned income, Z_i denotes a vector of exogenous variables arising from the previous economic choices or chance events such as education and the number of the children, h_i denotes hours of work, ϵ_{R_i} is a disturbance, and β_0, β_1, β_2, β_3 and β_4 are the parameters of the reservation wage function. The market offered wage is assumed to be given by

$$l(w_i) = b_0 + b_1' Z_i^* + b_2' D_i + \epsilon_{w_i} \tag{2}$$

where Z_i^* and D_i are vectors of personal and regional characteristic variables, respectively, ϵ_{w_i} is a disturbance, and b_0, b_1 and b_2 are the parameters of the offered wage function.

If $w_i > w_{R_i}$ at $h_i = 0$, the woman works and her working hours adjust continuously to $w_i = w_{R_i}$ in equilibrium. If $w_i \leq w_{R_i}$ at $h_i = 0$, she doesn't work and her hours of work are always zero. Thus the resulting forms for hours of work and offered wage equation are derived from (1) and (2),

$$l(w_i) = b_0 + b_1' Z_i^* + b_2' D_i + \epsilon_{w_i} \tag{3}$$

$$\begin{aligned} h_i &= \frac{1}{\beta_1}[(1-\beta_2)l(w_i) - \beta_0 - \beta_3 A_i - \beta_4' Z_i - \epsilon_{R_i}] \quad \text{if } w_i > w_{R_i} \\ &= 0 \qquad\qquad \text{if } w_i \leq w_{R_i} \end{aligned} \tag{4}$$

Note that these are the structural forms of equations.

[6] See details in Appendix 1.

3 Estimation

3.1 Conventional Method

There are two crucial problems in estimating hours of work and offered wage equation such as equations (3) and (4); sample selection bias and simultaneous bias.[7] Heckman's two step estimator (1976, 1979) is most widely used as a method solving the above problems. It can be simply calculated by the probit maximum likelihood and then the ordinary least squares (hereafter OLS).[8] Heckman (1976) originally proposed this method to the reduced form equations. However, we apply this method to the structural equations such as (4) and (5) along with two-stage least squares.[9]

Assume a joint normal distribution of ϵ_{R_i} and ϵ_{w_i} in reservation wage and offered wage equations with means zero and the covariance,

$$E(\epsilon_{w_i}^2) = \sigma_w^2, \quad E(\epsilon_{R_i}^2) = \sigma_R^2,$$

$$E(\epsilon_{R_i}\epsilon_{w_i}) = \sigma_{wR},$$

$$E(\epsilon_{R_i}\epsilon_{w_j}) = 0 \quad \text{for } i \neq j.$$

Then the probability that ith woman works is given by

$$Pr(\text{i works}) = Pr[\{l(w_i) - l(w_{R_i})\}|_{h=0} > 0]$$

$$= Pr[(\epsilon_{w_i} - \epsilon_{R_i}) > \{(\beta_0 - b_0 + \beta_3 A_i + \beta_4 Z_i - b_1 Z_i^* - b_2 D_i\}]$$

[7]The importance of sample selectivity was first recognized in labor economics by the work of Gronau (1974). Heckman (1974, 1976,1978b and 1979), Hanoch (1980) and so on considered the simultaneous problem as well as sample selectivity.

[8]At the second step, weighted least squares was used in Nakamura *et al* (1979) and Nakamura and Nakamura (1981) and so on since they took account of the heterogeneous variances of disturbances over a cross section.

[9]This procedure is sometimes called Heckit generalized tobit estimators (see Berndt (1991) and Nakamura and Nakamura (1992)). In this procedure three steps are necessary. The advantage of this procedure is that the effect of offered wage on hours of work can be seen explicitly.

$$= 1 - Pr[(\epsilon_{w_i} - \epsilon_{R_i})/\sigma < I_i]$$

$$= 1 - \Phi(I_i) \tag{5}$$

where $\Phi(\cdot)$ is the cumulative density of standard normal distribution and

$$I_i = \frac{1}{\sigma}[(\beta_0 - b_0) + \beta_3 A_i + \beta_4 Z_i - b_1 Z_i^* - b_2 D_i] \tag{6}$$

$$\sigma^2 = Var(\epsilon_{w_i} - \epsilon_{R_i}) = \sigma_w^2 + \sigma_R^2 - 2\sigma_{wR} \tag{7}$$

The likelihood function of the probit model for a labor force participation is

$$L = \prod_{i=1}^{k} (1 - \Phi(I_i)) \prod_{i=k+1}^{n} \Phi(I_i) \tag{8}$$

where k is the number of the workers and n indicates the total number of the sample. Thus parameter estimates of the probit analysis are for the coefficients $(\beta_0 - b_0)/\sigma$, β_3/σ, β_4/σ, $-(b_1/\sigma)$ and $-(b_2/\sigma)$

Using the data restricted to the working women, the regression functions of hours of work and offered wage equations are, respectively,[10]

$$\begin{aligned} E(l(w_i)|h_i > 0) &= b_0 + b_1 Z_i^* + b_2 D_i + E(\epsilon_{w_i}|h_i > 0) \qquad (9) \\ &= b_0 + b_1 Z_i^* + b_2 D_i + \left(\frac{\sigma_{hw}}{\sigma_h}\right)\lambda_i \\ E(h_i|h_i > 0) &= \frac{1}{\beta_1}[(1 - \beta_2)l(w_i) + \beta_0 + \beta_3 A_i - \beta_4 Z_i] + E(\epsilon_{h_i}|h_i > 0) \\ &= \frac{1}{\beta_1}[(1 - \beta_2)l(w_i) + \beta_0 + \beta_3 A_i - \beta_4 Z_i] + \sigma_h \lambda_i \end{aligned}$$

$$\tag{10}$$

where $\lambda_i = \phi(I_i)/\Phi(I_i)$ is the inverse of Mill's ratio, $\phi(I_i)$ is the density of standard normal distribution, σ_h^2 is the variance of error in (5) and σ_{hw} is the covariance of errors in (4) and (5). Heckman (1974) suggested the conditions for identification in this model.[11]

Heckit generalized tobit estimators are as follows.

(1) Estimate parameters of probit model and then calculate the inverse of

[10] see Johnson and Kotz (1972).

[11] There should be at least one variable in Z^* and D excluded from A and Z in this case.

Mill's ratio by using them.

(2) Append the predicted inverse of Mill's ratio to offered wage equation and estimate it by ordinary least squares using data for working women.[12]

(3) Use fitted values from (2) as instrument variable and append the predicted inverse of Mill's ratio to hours of work equation. Then estimate it by two-stage least squares.

The asymptotic variance-covariance matrix of the estimator is obtained by the method of White (1980).

3.2 Gibbs sampling with data augmentation

We rewrite offered wage and hours of work equation, respectively,

$$w_i = x'_{1i}\beta_1 + u_1 \tag{11}$$

$$h_i = x'_{2i}\beta_2 + u_2 \tag{12}$$

where we redefine the natural logarithm of wage as w_i for brevity. This w_i is also included in x'_{i2}, u_1 and u_2 are assumed to be a joint normal distribution with variance σ_1^2 and σ_2^2, respectively, and covariance σ_{12}.

Chib (1992) shows that Gibbs sampling algorithm is enormously simplified when the latent data corresponding to censored data is added to the sampler. The Gibbs sampling requires the full conditional distribution; $f(h_c|h_u, \beta_2, \sigma_2^2)$, $f(w_c|w_u, \beta_1, \sigma_1^2)$, $g(\beta_2|h_c, h_u, \sigma_2^2)$, $g(\beta_1|w_c, w_u, \sigma_1^2)$, $g(\sigma_2^2|h_c, h_u, \beta_2)$ and $g(\sigma_1^2|w_c, w_u, \beta_1)$, where suffices "c" and "u" correspond to censored and uncensored data, respectively.

From the normality assumption in section 3.3, the conditional distribution of h_c is a truncated normal with h_c ranging from $-\infty$ to zero, i.e.

$$f(h_{c_i}|h_u, \beta_2, \sigma_2^2) = f_N(h_{c_i}|x'_{2i}\beta_2, \sigma_2^2)/(1 - \Phi(x'_{2i}\beta_2/\sigma_2^2)) \tag{13}$$

[12] Since offered wage equation is structural form as well as reduced form equation, these estimates are consistent.

where $-\infty < h_{c_i} < 0$, i is from censored data, f_N denotes normal density and Φ is cumulative standard normal distribution. As for offered wage, a woman cannot choose her wage rate. Thus the truncated nature of offered wage diminishes.

$$f(w_{c_i}|w_u, \beta_1, \sigma_1^2) = f_N(w_{c_i}|x'_{1i}\beta_1, \sigma_1^2) \tag{14}$$

Let $w' = (w'_c, w'_u)$ and $h' = (h'_c, h'_u)$ be $1 \times n$ vectors and define $\hat{\beta}_1$ and $\hat{\beta}_2$ as the corresponding OLS estimates,

$$\hat{\beta}_1 = (X'_1X_1)^{-1}X'_1w \tag{15}$$

$$\hat{\beta}_2 = (X'_2X_2)^{-1}X'_2h \tag{16}$$

where X_1 and X_2 are $n \times k_1$ and $n \times k_2$ matrices corresponding to the vectors x_{1i} and x_{2i}. If we assume diffuse priors $\pi(\beta_1, \sigma_1^2) \propto \sigma_1^{-2}$ and $\pi(\beta_2, \sigma_2^2) \propto \sigma_2^{-2}$, then the conditional distributions of β_1 and β_2 are given by,

$$g(\beta_2|h_c, h_u, \sigma_2^2) = f_N(\beta_2|\hat{\beta}_2, \sigma_2^2(X'_2X_2)^{-1}) \tag{17}$$

$$g(\beta_1|w_c, w_u, \sigma_1^2) = f_N(\beta_1|\hat{\beta}_1, \sigma_1^2(X'_1X_1)^{-1}) \tag{18}$$

The conditional distributions of σ_1^2 and σ_2^2 follow immediately from,

$$g(\sigma_2^2|h_c, h_u, \beta_2) = f_{IG}\left(\sigma_2^2|\frac{n}{2}, \frac{2}{(h - X_2\beta_2)'(h - X_2\beta_2)}\right) \tag{19}$$

$$g(\sigma_1^2|w_c, w_u, \beta_1) = f_{IG}\left(\sigma_1^2|\frac{n}{2}, \frac{2}{(h - X_1\beta_1)'(h - X_1\beta_1)}\right) \tag{20}$$

where f_{IG} denotes inverted gamma distribution.

Since offered wage equation is the structural form as well as the reduced form, hours of work doesn't affect it. However, note that offered wage affects the hours of work. If starting values $(\beta_1^{(0)}, \sigma_1^{2(0)})$ and $(\beta_2^{(0)}, \sigma_2^{2(0)})$ are given, the resulting Gibbs sampling iterates the following;[13]

[13]Devroye (1986) shows an efficient method to sample from the truncated normal distribution. In this case, we draw from $x'_i\beta + \sigma^2\Phi^{-1}(U\Phi(-x'_i\beta/\sigma^2))$, where U is uniform distribution.

$$1 \quad w_{c_i} \sim N(x'_{1i}\beta_1, \sigma_1^2)$$

$$1' \quad h_{c_i} \sim \text{truncated } N(x'_{2i}\beta_2, \sigma_2^2)$$

$$2 \quad \beta_1 \sim N(\hat{\beta}_1, \sigma_1^2(X_1'X_1)^{-1})$$

$$2' \quad \beta_2 \sim N(\hat{\beta}_2, \sigma_2^2(X_2'X_2)^{-1})$$

$$3 \quad \sigma_1^2 \sim IG\left(\frac{n}{2}, \frac{2}{(w - X_1\beta_1)'(w - X_1\beta_1)}\right)$$

$$3' \quad \sigma_2^2 \sim IG\left(\frac{n}{2}, \frac{2}{(h - X_2\beta_2)'(h - X_2\beta_2)}\right)$$

$$4 \quad \text{go to } 1$$

Note that at steps 1 and 1', we impute the latent data. After iterations of sufficiently large t, it can be shown that the obtained $(w_c^{(t)}, \beta_1^{(t)}, \sigma_1^{2(t)})$, $(h_c^{(t)}, \beta_2^{(t)}, \sigma_2^{2(t)})$ simulates the draw from the joint distribution.[14] Repeating the above t iterations M times, M iid draws $(w_{cj}^{(t)}, \beta_{1j}^{(t)}, \sigma_{1j}^{2(t)})$, $(h_{cj}^{(t)}, \beta_{2j}^{(t)}, \sigma_{2j}^{2(t)})$ from the joint distribution are obtained. Note also that each sampled values such as $w_{cj}^{(t)}$, $\beta_{1j}^{(t)}$, $\sigma_{1j}^{2(t)}$ and so on converge in distribution to the relevant marginal distribution.[15] Inference is straightforward based on the entire sample.

4 The empirical results

4.1 The Data Set

The data used are the 1975 Panel Study of income Dynamics (hereafter PSID) conducted by the University of Michigan. This data set has been used often for the study of married women's labor supply. The sample consists of 753 white married women aged 30 - 60, of whom 428 worked (56.8 per cent) during 1975. The measure of labor supply is annual hours at work, which

[14] For proof, see Tierney (1991).

[15] See Gelfand and Smith (1990).

is the product of the number of weeks for 1975 and the average number of hours of work per week. The measure of wage is the average hourly earnings, derived by dividing the woman's total labor income for 1975 by the annual hours at work defined by the above. Altogether there are 19 variables such as the age, the education, the number of the children and a dummy variable for large cities. The arithmetic means and standard deviations of 11 variables to compare the samples between working women and nonworking women are provided in Appendix 2. The vectors Z, Z^* and D of equations in the model are also presented in Appendix 2.

4.2 Preliminary Analysis

As a preliminary analysis, the Gibbs sampling with data augmentation is applied to the following model by Mroz (1987).[16]

$$h_i = a_0 + a_1 l(w_i) + a_2 Y_i + a_3' Z_i + e_i \tag{21}$$

where h_i is the woman's hours of work during a given year, w_i presents her offered wage rate, Y_i denotes nonwife income, Z_i is a set of control variables including the number of children less than six years old and the number of children between the ages of six and eighteen, e_i is a stochastic disturbance and a_0, a_1, a_2 and a_3 are the parameters of the hours of work function. Mroz reports the results when parameters in equation (21) are estimated by Tobit maximum likelihood and the modified Heckman's two step method.[17] As the starting values of the Gibbs sampling, the OLS estimates restricted to the working women are given. Table 1 shows the results reported by Mroz with the one derived from the Gibbs sampling explained in section 3.2.

[16] Mroz (1987) uses this simple model for a systematic analysis of several theoretic and statistical assumptions used in many empirical models of female labor supply.

[17] see 782 page in Mroz's paper (1987).

TABLE 1

	Log of offered wage	Nonwife income /1000	# of children under 6	# of children between 6 and 18	Standard error of regression
Heckman	64	-1.0	-183	-106	
	(227)	(9.2)	(408)	(34)	
Tobit	261	-22.9	-1035	-97	1.6×10^6
	(357)	(5.1)	(140)	(48)	
Gibbs sampler	4.1	-22.3	-1066	-106	1.6×10^6
with data	(92.5)	(5.3)	(135)	(42)	
augmentation					

a. Numbers in parentheses are standard deviations.

The Gibbs sampling estimates are very close to those of Tobit maximum likelihood. Although the coefficient estimates for log of offered wage rate are different between these methods, the large standard error of the estimate appears to yield the difference. The coefficient estimates for the number of children and nonwife income are larger negative than those of Heckman's method. In other words, Heckman's method has small effect of these two variables on women's hours of work. Altogether, this preliminary analysis supports that the Gibbs sampler with data augmentation effectively produce samples from marginal distribution even when applied to the two equation Tobit model.

4.3 The empirical results

The Gibbs sampling with data augmentation and the modified Heckman's method are used to estimate the simultaneous equations in (3) and (4). The results are presented in Table 2 and Table 3.

TABLE 2

Comparison of estimates for log of offered wage equation

Explanatory variables	Heckman's method	Gibbs sampling with data augmentation
Constant	-0.529*	-0.484*
	(0.306)	(0.285)
Age	-0.00064	-0.00015
	(0.00705)	(0.00485)
# of years of schooling	0.107**	0.106**
	(0.015)	(0.014)
Previous labor market experience in years	0.043**	0.040**
	(0.018)	(0.014)
Square of the above	-0.00084*	-0.00076*
	(0.00043)	(0.00041)
Dummy 1 if living in a large city; 0 otherwise	0.057	0.063
	(0.065)	(0.068)
Unemployment rate in the region	-0.0046	-0.0041
	(0.0094)	(0.0117)
Selection bias (λ)	0.029	
	(0.211)	
Standard error of regression	0.6689	0.6698

a. Numbers in parentheses are standard deviations. White's heteroscedastic-consistent estimates (1980) are used in Heckman's model.
b. Coefficients with two asterisks are significant at a 5 percent level. Coefficients with one asterisk are significant at a 10 percent.

TABLE 3

Comparison of estimates for hours of work equation

Explanatory variables	Heckman's method	Gibbs sampling with data augmentation
Constant	2654.8** (340.40)	1157.364** (517.935)
Log of offered wage	1750.6** (340.01)	136.881* (102.170)
Nonwife income including husband's earned income	-0.0032 (0.0036)	-0.0226** (0.0045)
Age	-18.702** (6.0)	-36.882** (8.207)
# of years of schooling	-198.27** (36.994)	113.495** (25.673)
# of children under age 6	-389.21** (113.62)	-1065.89** (130.182)
# of children between 6 and 18	-71.334** (29.945)	-104.283** (42.655)
Selection bias (λ)	138.66 (126.09)	
Standard error of regression	725.95	1276.11

a. Numbers in parentheses are standard deviations. White's heteroscedastic-consistent estimates (1980) are used in Heckman's model.

b. Coefficients with two asterisks are significant at a 5 percent level. Coefficients with one asterisk are significant at an 10 percent.

The sample selectivity bias in offered wage equation is not significant at all, as is the case for Heckman's (1976, 1977, 1980) results. The estimation results from these two methods are very similar for the offered wage. The number of years of the woman's schooling is significant at a 95 per cent and positive on the hours of work as expected. The previous labor market experience is also significantly positive.[18]

As for the hours of work, there is some evidence in the existence of sample selectivity bias in estimating it only for working women. But it is not significant at a 10 per cent. All coefficient estimates from the Gibbs sampling are significant with at least a 10 per cent level and are in agreements with the Heckman's (1980) empirical results. The offered wage has a positive effect on the hours of work. This implies no backward bending in the relationship between them and supports the Heckman's (1974) suggestion. Heckman's conventional method underestimates the coefficients of nonwife income, her age and number of children. In addition, the number of years of schooling is significantly negative in marked contrast to the Gibbs sampling results.

5 Conclusion

Chib(1992) shows that the Gibbs sampler with data augmentation yields a solution to the difficulty of analyzing the standard Tobit model. In this paper the Gibbs sampler with data augmentation is applied to the two equation Tobit model, which is most often used in labor economics. In the empirical analysis on the married women labor supply, this model is typically used since a large proportion of the married women do not work and the standard theory implies female labor supply and offered wage rate are simultaneously determined. Although MLE is known to be a better estimator, the Heckman's

[18] Previous analysis indicates the endogeneity of labor market experience (see Heckman (1978b) for example). But Mroz (1987) shows that it is unnecessary to treat labor market experience as endogenous when controlling for self selection.

two step estimator is widely used because of its computational simplicity.

As a preliminary analysis, the Gibbs sampling with data augmentation is applied to the simple model of female labor supply given by Mroz (1987) using the actual micro data. The result from the Gibbs sampling is very close to the MLE, which is different from Heckman's two step estimator. This supports the efficacy of the Gibbs sampling when applied to the generalized Tobit model. Then the Gibbs sampling is applied to the hours of work and the offered wage function derived from the utility maximization problem. All coefficients estimates are significant and the signs are in agreement with Heckman's (1980) empirical results. Compared with the results of the Gibbs sampling, Heckman's conventional method yields small coefficients estimates for the nonwife income, the woman's wage and so on. The Gibbs sampling with data augmentation appears to be an effective procedure when the computational difficulty in MLE exists.

Appendix 1

An individual married woman is assumed to maximize a twice-differentiable quasi-concave conditional utility function $U(G, l; Z)$ subject to the budget and time constraints,

$$pG = A + wh \tag{22}$$

$$T = h + l \tag{23}$$

where G denotes a Hicksian composite good with the unit price p, l represents the time of the nonmarket activity ("leisure" for simplicity), $h(= T - l)$ is the time of market work with the market offered wage w, T is the total time available, A represents the nonwife income including the husband's earned income, and Z denotes a vector of exogenous variables arising from the previous choices or chance events such as education and the number of the children. Assuming $0 \leq h < T$, maximizing the Lagrangian for this nonlinear problem leads to the Kuhn-Tucker conditions [19] such as (22), (23), $\mu \geq 0$,

$$U_G - \lambda p = 0 \tag{24}$$

$$U_l - \lambda w - \mu = 0 \tag{25}$$

$$\mu h = 0 \tag{26}$$

where $U_G = \partial U / \partial G$, $U_l = \partial U / \partial l$, and λ and μ are an unconstrained and a nonnegative Lagrangian multiplier, respectively .

The marginal utilities of G and l are assumed to be positive, $U_G > 0$ and $U_l > 0$. Thus condition (24) yields $\lambda = U_G/p > 0$. The quasi-concavity of the utility function implies that $\partial^2 U/\partial G^2 < 0$, $\partial^2 U/\partial l^2 < 0$ and $\partial^2 U/\partial G \partial l > 0$. In addition, from (24) and (22) , we get $\lambda = U_G/p = f(h, p, w, A, Z)$ in equilibrium. Rearranging condition (25) and letting $w_R \equiv \frac{U_l}{\lambda}$,

$$w = \frac{U_l}{\lambda} - \frac{\mu}{\lambda} \equiv w_R - \frac{\mu}{\lambda} \tag{27}$$

[19] see Nakamura *et al.* (1979, 1983, 1992)

where U_l/λ is defined as the reservation wage, w_R, which is the shadow price of a woman's time, i.e. asking wage. Since in equilibrium $w_R = (pU_l)/U_G$, the shadow price is the value of her marginal unit of time in the money value. From conditions (27) and $\mu \geq 0$, if $h > 0$, $\mu = 0$ yields $w = w_R$, whereas if $h = 0$, $\mu > 0$ yields $w < w_R$. Manipulating (24) and (22) with the definition of w_R, we get $w_R = g(h, p, w, A, Z)$ for $h > 0$ and $w_R = h(p, A, Z)$ for $h = 0$.

Appendix 2

TABLE 4

Sample Means

Explanatory variables	Workers	Nonworkers	All
Women's hours of work	1302.93 (776.27)	0 (0)	740.58 (871.31)
# of children under age 6	0.14 (0.39)	0.36 (0.64)	0.24 (0.52)
# of children between 6 and 18	1.35 (1.36)	1.36 (1.33)	1.35 (1.32)
# of years of schooling	12.66 (2.29)	11.80 (2.18)	12.29 (2.28)
Women's wage	4.18 (3.3)	0 (0)	2.37 (3.24)
Nonwife income including husband's earned income	24130.42 (11671.26)	21698.05 (12728.15)	23080.59 (12190.20)
Her mother's years of schooling	9.52 (3.30)	8.90 (3.42)	9.25 (3.37)
Her father's years of schooling	8.99 (3.52)	8.90 (3.42)	8.81 (3.57)
Unemployment rate in the region	8.55 (3.03)	8.72 (3.22)	8.62 (3.11)
Dummy 1 if living in a large city; 0 otherwise	0.64 (0.48)	0.65 (0.48)	0.64 (0.48)
Previous labor market experience in years	13.04 (8.06)	7.46 (6.92)	10.63 (8.07)
Numbers of observations	428	325	753

a. Numbers in parentheses are standard deviations.

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