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Growth and Threshold Effect of Environment in an Overlapping Generations Model

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Abstract

In this paper, in an overlapping generations model where decisions of short-lived individuals have long-lasting intergenerational externalities on both factor productivity and the environment, we analyse the potential conflict between eonomic growth and environmental conservation. Environmental quality causes health damages to individuals and has a *threshold effect* on the probability of health: as soon as it decreases below the threshold level, most people fall ill. We show that if society's preferences shift towards a larger concern for a clean environment and if the threshold effect arises far from the natural state equilibrium of the environment, then economic growth is associated first with declines, then improvements, in environmental quality in production technologies with "constant" returns to scale. However, under the same conditions, growth and conservation become incompatible in production technologies with "increasing" returns to scale. It is also shown that if the threshold level of environmental quality is near the natural state, then there exists a trade-off between growth and conservation under positive environmental investment, which is different from another trade-off under zero investment, that is, the cleaner environment can be attained at the sacrifice of economic growth.

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I. Introduction

Recently, in an overlapping generations model where decision-makings of two-period-lived homogeneous individuals between saving for consumption and investment in the environment have long-lasting *intergenerational* externalities on both factor productivity and the environment, John and Pecchenino (1994) have analysed the potential conflict between economic growth and the maintenance of environmental quality and have showed that a growing economy is initially associated with declines in environmental quality but is later characterised by improvements in it. They have treated environmental quality which gives individuals *amenity values* which are public goods. Indeed, since the amenity property of the environment is essential for landscape, natural environment, biodiversity and so on, their model may well apply to these environmental issues. However, *health damages* to individuals are more important than amenity values in other environmental problems such as ozone layer depletion, radioactive pollution, lead or mercury poisoning and so on. In this paper, we attack these health-environmental problems in a version of the John-Pecchenino model modified as follows.

First, the utility function of each individual depends on consumption and his *health state* in his old age. The latter is a binomial random variable which takes on only two values corresponding to "ill" and "well". He knows the probability of health but can not control it independently. Second, this probability is a monotone increasing function of environmental quality since people become more healthy if the environment is cleaner. Therefore, although environmental quality does not directly enter into the utility function of the individual as a public good, his *expected* utility depends on environmental quality. Consumption of the present generarion decreases health probabilities of future generations because it degrades the environment bequeathed to these generations, but investment by the present generation can improve the environment *collectively*. Hence, consumption and investment have *intergenerational externalities* through their consequences on health probabilities of future generations. Third, we incorporate the *threshold effect* of the environment emphasized by Dasgupta (1982) into the model. That is, as soon as environmental quality decreases less than the threshold level, health probabilities decline quickly so that most of the old fall ill. This effect is formulated such that the health probability function of the individual is strictly *convex* for environmental quality smaller than the threshold level but is strictly *concave* for it larger than this level. Finally, although environmental quality in the John-Pecchenino model can take on positive or negative values as an index of the amenity value, it can take on *non-positive* values only in our model. The reason is that it is defined by a divergence of the *pollution* stock measured in terms of non-negative physical units from the level of zero in the natural state equilibrium without any human activity. Hence, the best and worst levels of environmental quality are zero and negative infinity respectively.

We re-examine possible trade-offs between growth and environmental quality, concentrating our attentions on the comparative dynamic analysis with respect to parameters: the utility rate lossed by illness, the rate of investment in environmental improvement, the environmental degradation rate due to consumption, and the natural depreciation rate and threshold level of environmental quality. In particular, we are interested in the comparative dynamic analysis of the utility loss rate of illness since increases in it imply that society's preferences shift towards a larger concern for a clean environment.[1] We obtain the following main results. In production technologies with *constant* returns to scale, if the following conditions with respect to parameters hold, that is, (i) individuals attach more importance to *health* than to consumption, (ii) environmental quality is greatly improved by investment, (iii) it is not so much degraded by consumption, (iv) the natural depreciation rate of environmental quality is small, and (v) the threshold level of environmental quality is far from the natural state, then a growing economy is

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[1] In recent years, concern for the problem that how increased environmental care affects the long-run growth rate and the optimal public policy has been increasing. For example, Gradus and Smulders (1993) have studied effects on long-term growth in three representative growth models from the literature, and Van der Ploeg and Bovenberg (1994) have investigated impacts on the labour tax rate, public goods and public abatement in the optimal taxation framework.

initially associated with declines in environmental quality, while is later characterised by improvements in it and finally converges to the long-run equilibrium where capital per capita is maintained at the high level and where the best level of environmental quality is attained so that the old are all healthy. If the condition (v) does not hold, then there exists a trade-off between growth and environmental conservation under positive environmental investment in the second stage of economic development, which is different from another trade-off under zero investment in the first. That is, the cleaner environment can be attained at the sacrifice of economic growth. The conditions (ii)-(v) are consistent with our intuition because they imply amelioration of the environmental situation. It is interesting that the condition (i) is not so. This condition seems to be rather *paradoxical* because it is contrary to the popular perception that it will be impossible for the economy to grow as the society becomes more interested in environmental care. It can be explained as follows. Since individuals attach more importance to health than to consumption, they desire *little* capital for the given environmental quality. In production technologies with constant returns to scale, the less capital is, the larger the feasible improvement in environmental quality is. Therefore, although the absolute level of capital is low, it is possible to increase capital and environmental quality simultaneously. It goes without saying that the conditions (ii)-(v) are required for the compatibility between growth and environmental conservation in production technologies with *increasing* returns to scale also. However, since the feasible improvement in environmental quality is large for high levels of capital in these technologies, the reverse of the condition (i) is required. That is, it is necessary that individuals attach more importance to *consumption* than to health. A growing economy is initially associated with declines in environmental quality, but is later characterised by improvements in it and finally the economy can continue to grow infinitely in the best environment.

The rest of the paper is organized as follows. Section II introduces the basic model. Section III characterises the interior and boundary solutions without external increasing returns and explores the problem of the compatibility between economic growth and environmental conservation. Section IV considers equilibrium dynamics with external increasing returns. Finally, section V contains some concluding remarks.

II. The Model

The closed economy considered here has an infinite future of discrete time and consists of overlapping generations of two-period-lived homogeneous individuals and perfectly competitive firms. Each individual works in the young period and retires in the old. We assume no population growth and normalize the size of each generation to unity. The individual born at period t earns wage, w_t, inelastically supplying one unit of labour endowed to firms. We will assume that each individual does not consume in the youth so as to focus attention on the choice between investment in the environment and investment in physical capital. Then, his wage is divided between saving, s_t, for consumption in the old period, c_{t+1}, and investment in environmental maintenance and improvement, m_t. He supplies his saving inelastically to firms and when old he gains the gross return, $(1 + r_{t+1})$. Hence, the young- and old-period budget constraints of the individual are respectively

(1) $$w_t = s_t + m_t,$$

(2) $$c_{t+1} = (1 + r_{t+1}) s_t.$$

We assume that each individual does not fall ill in his youth, so his preference can be defined by a utility function whose arguments are his level of consumption, c, and his state of *health*, h, in the old period. The latter is the binomial random variable which takes on only two values, corresponding to "ill": $h = 0$ and "well": $h = 1$.[2] The probability of good health, which is known by the individual, is denoted by p. We also assume that c and h are independent and that the marginal utility of h is constant. Thus, the utility function of the individual born at period t, which is twice continuously differentiable, can be expressed by:

$$U(c_{t+1}, h_{t+1}) = u(c_{t+1}) + \lambda h_{t+1} \quad \text{where} \quad \lambda > 0, \; u_c > 0, \; u_{cc} < 0.$$

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[2] To avoid the problems of death insurance and bequest disposal, it is assumed that individuals do not die even if they fall ill.

The parameter λ represents the utility lost when he falls ill. The larger the value of λ is, the more he attaches importance to his state of health than his consumption. Note that this parameter does not depend on environmental quality, because we do not consider its amenity value in this paper.

The probability of health depends on the level of environmental quality E, i.e., $p = p(E)$. The cleaner the environment is, the more healthy people are. Thus, it is proper to assume that p is a monotone increasing function of E. Though it is impossible for each individual to control this probability independently, the government can increase it collectively through investment in the environment. The environment has a *threshold effect* on the probability of health. That is, as soon as environmental quality decreases less than the threshold level, most of the old fall ill. The probability function of health is continuous and twice differentiable, while its shapes are quite different at the boundary of the threshold level, E_T. It is strictly *concave* for $E \geq E_T$ but is strictly *convex* for $E \leq E_T$ (see Figure 1), so that

$$\begin{aligned} p(E) &= p^+(E),\ p_E^+ > 0,\ p_{EE}^+ < 0 \quad \text{for all } E \in E^+ \\ &= p^-(E),\ p_E^- > 0,\ p_{EE}^- > 0 \quad \text{for all } E \in E^-, \end{aligned}$$

where $E^+ = \{E : 0 \geq E \geq E_T\}$ and $E^- = \{E : E_T \geq E\}$. We also assume that

(a) $p_E^+ = p_E^- = \omega$ (finite) and $p_{EE}^+ = p_{EE}^- = 0$ at $E = E_T$,

(b) $p^-(E) = 0$ and $p_{EE}^- = 0$ at $E = E_W$,

(c) $p^+(E) = 1$ and $p_{EE}^+ = 0$ at $E = E_M$,

where E_M and E_W are the best and worst levels of environmental quality, respectively (see Figures 1 and 2).

Denoting the "pollution" stock by P, which is measured in terms of *non-negative* physical units, then it evolves according to the following equation:

$$P_{t+1} = (1 - b)P_t + \beta c_t - \gamma m_t \quad \text{for } P_t \geq 0,$$

where $0 < b < 1$, $\beta > 0$ and $\gamma > 0$. In the absence of any human activity, the pollution stock has the autonomous level of zero: $P_N = 0$; the parameter b expresses the natural depreciation rate of the stock. The term βc_t is the degradation of the environment as a result of the consumption of the old at period t, but γm_t measures improvement resulting from investment in the environment by the young. Letting define environmental quality by $E_t = P_N - P_t = - P_t$, then its accumulation equation can be represented as follows:

$$(3) \qquad E_{t+1} = (1 - b) E_t - \beta c_t + \gamma m_t \quad \text{for } E_t \leq 0.$$

Note that environmental quality can take on *non-positive* values only in this model. The best and worst levels of environmental quality are $E_M = 0$ and $E_W = - \infty$, respectively. Since its equilibrium level at the natural state is zero, it holds that $E_N = E_M = 0$.

Those alive at period t are represented by a one-period lived "government" whose responsibility is the improvement of environmental quality for the benefit of agents alive during its period of office. It levies lump-sum taxes on the young to achieve their desired level of environmental maintenance, while leaving the welfare of the old unchanged. Thus, the government is organized by each generation so that it does not have intergenerational altruism. Since individuals are homogeneous and the population is normalized to one, if we assume that the "generational" government at period t has the Bentham social welfare function, then its objective is equivalent to maximizing the *expected* utility of the representative individual, i.e.,

$$p(E_{t+1})\left[u(c_{t+1}) + \lambda\right] + \left[1 - p(E_{t+1})\right] u(c_{t+1})$$

subject to his budget constraints in the young and old periods, (1) and (2), and the accumulation equation (3) of environmental quality with respect to his saving, s_t (≥ 0), his consumption of the old period, c_{t+1} (≥ 0), investment in the environment financed by lump-sum taxes, m_t (≥ 0), and environmental quality, E_{t+1} (≤ 0). If we now assume $\lim_{c \to 0} u_c(c) = \infty$, then since c_{t+1} and s_t are positive, we need not consider their non-negativity conditions. Eliminating c_{t+1}, s_t, and E_{t+1} from (1)-(3), the maximization problem of the government can be finally formulated as follows:

$$\text{Maximize } u[(1 + r_{t+1})(w_t - m_t)] + \lambda p[(1 - b)E_t - \beta c_t + \gamma m_t]$$

subject to

$$0 \le m_t \le \frac{\beta c_t - (1 - b)E_t}{\gamma},$$

with respect to m_t. In this problem, the wage, w_t, the return on saving, r_{t+1}, the consumption of the previous generation, c_t, and environmental quality determined in the preceeding period, E_t, are all given. The first-order conditions for the interior and boundary maxima are respectively given by:[3]

$$(4.1) \qquad \frac{u_c(c_{t+1})}{\lambda p_E(E_{t+1})} = \frac{\gamma}{1 + r_{t+1}} \quad \text{for } 0 < m_t < \frac{\beta c_t - (1 - b)E_t}{\gamma},$$

$$(4.2) \qquad \frac{u_c(c_{t+1})}{\lambda p_E(E_{t+1})} \le \frac{\gamma}{1 + r_{t+1}} \quad \text{for } E_{t+1} = 0 \text{ and } m_t = \frac{\beta c_t - (1 - b)E_t}{\gamma},$$

$$(4.3) \qquad \frac{u_c(c_{t+1})}{\lambda p_E(E_{t+1})} \ge \frac{\gamma}{1 + r_{t+1}} \quad \text{for } m_t = 0,$$

where the left-hand side in each condition is the marginal rate of substitution between environmental quality and consumption (i.e., the marginal "cost" of m_t), and the right-hand side is the marginal rate of transformation (i.e., the marginal "benefit"). Note that $p_E(0) > 0$ in (4.2).

We follow John and Pecchenino (1994) for the formulation of the production technology. Production of output at period t is represented by the production function which relates the total amount of output produced, Y_t, to the total amount of capital stock, K_t, and aggregate labour, N_t: $Y_t = \psi(K_{t-1}) F(K_t, N_t)$, where the function $\psi(K_{t-1})$ is a technological externality that captures enhancements to productivity from

[3] The second-order condition for the interior maximum is $(1 + r)^2 u_{cc} + \gamma^2 \lambda p_{EE} < 0$. This condition is satisfied for $E \in E^+$. Although it is not always satisfied for $E \in E^-$, we assume that it holds.

last period's capital: $\psi'(\cdot) \geq 0$, which is a constant from the perspective of current producers. Thus, although the model exhibits *increasing* returns from an intertemporal social perspective, production at any period is a *constant* returns activity.[4] Since the population is normalized to one, $\psi(K_{t-1})$ can be written as $\psi(k_{t-1})$, where k is the capital-labour ratio. Since the function $F(\cdot)$ exhibits constant returns to scale, output per worker, y_t, can be written as

$$y_t = \psi(k_{t-1})f(k_t) \quad \text{where} \quad f(0) = 0, f'(\cdot) > 0, f''(\cdot) < 0.$$

Markets are competitive and each firm producing output maximizes its profit, so that each factor is paid its marginal product. Hence, the wage rate, w_t, and the interest rate, r_t, at period t are respectively given by

(5) $$r_t = \psi(k_{t-1})f'(k_t) - \delta,$$

(6) $$w_t = \psi(k_{t-1})[f(k_t) - k_t f'(k_t)],$$

where δ is the depreciation rate of capital.

Finally, since the savings of the working generation in the present period should equal the capital stock in the next, the equilibrium condition in the capital market is:

(7) $$k_{t+1} = s_t.$$

A competitive equilibrium for the overlapping generations economy is defined by a sequence $\{k_t, c_t, w_t, r_t, m_t, s_t, E_t\}_1^\infty$, such that satisfies (1)-(3), one of (4.1)-(4.3), and (5)-(7) at each period $t = 1, 2, ...$*ad infinitum*, given $\{k_0, k_1, E_1\}$.[5]

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[4] For recent literature on external increasing returns in growth models, for example, see Romer (1986). He postulated increasing returns to scale at the economy wide level and found that capital spillovers in the aggregate production function can generate steady growth.

[5] The capital stock in the first period, k_1, is held by an initial generation of old agents who supply their capital to firms and consume the proceeds. The firms in this period are endowed with the technology $\psi(k_0)f(k_1)$, where $\psi(k_0)$ is given.

III. Equilibrium Dynamics in Constant Returns to Scale Technologies

In this section, we study equilibrium dynamics in the absence of external increasing returns: $\psi(k) = 1$ for all k. We first analyse the dynamic behaviour in the interior and boundary solutions and then explore the problem of the compatibility between economic growth and environmental conservation.

A. Interior Solution: Positive Environmental Investment Equilibrium

The dynamic system in this case is given by (1)-(3), (4.1), (5) led one period, i.e., $r_{t+1} = f'(k_{t+1}) - \delta$, and (6)-(7). These equations determine endogenous variables $(k_{t+1}, c_{t+1}, w_t, r_{t+1}, m_t, s_t, E_{t+1})$, given predetermined variables (k_t, E_t, c_t). Eliminating (w_t, r_{t+1}, m_t, s_t) from the system, then we can obtain

(8) $$c_{t+1} = (1 - \delta)k_{t+1} + \nu(k_{t+1})f(k_{t+1}) = c(k_{t+1}),$$

(9) $$u_c(c_{t+1})[1 + f'(k_{t+1}) - \delta] = \gamma\lambda p_E(E_{t+1}),$$

(10) $$E_{t+1} = (1 - b)E_t - \beta c_t + \gamma\{[1 - \nu(k_t)]f(k_t) - k_{t+1}\},$$

where $\nu(k) = kf'(k)/f(k)$ is capital's share of output. Now, substituting (8) and (8) lagged one period into (9) and (10), the dynamic system can be finally described in terms of stock variables, k and E:

(11) $$[1 + r(k_{t+1})]\, u_c[c(k_{t+1})] = \lambda\gamma p_E(E_{t+1}),$$

(12) $$E_{t+1} = (1 - b)E_t - \beta(1 - \delta)k_t + \rho(k_t)f(k_t) - \gamma k_{t+1},$$

where $\rho(k) = \gamma[1 - \nu(k)] - \beta\nu(k)$.[6] Given (k_t, E_t) predetermined at period t, equations (11) and (12) determine the *short-run* equilibrium (k_{t+1}, E_{t+1}) at period $t + 1$. We now characterize the dynamic equilibrium path of the economy in $k-E$ space by considering (11) and (12) separately.

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[6] Note that the term $\rho(k)f(k)$ in the right-hand side of (12) expresses the *maximum* improvement in environmental quality feasible by the society, since the term $\gamma[1 - \nu(k)]f(k)$ is the change in environmental quality that would result if the young devoted their *entire* wage income to maintenance and the term $\beta\nu(k)f(k)$ is the change resulting from the consumption of the interest income by the old.

At first, since equation (11) implicitly defines E_{t+1} as a function of k_{t+1} only, we can rewrite it as

(13) $$E_{t+1} = \phi(k_{t+1}; \lambda, \gamma),$$

which represents environmental quality *desired* by the generational government for the given capital. In stationary state, equation (13) becomes

$$\bar{E} = \phi(\bar{k}; \lambda, \gamma),$$

where $\bar{k}$ and $\bar{E}$ denote stationary values. Since this is the stationary first-order condition for welfare maximization, we call it the FOC curve in $k-E$ space. The function $\phi(\bar{k})$ has the partial derivative with respect to $\bar{k}$:

(14) $$\phi_k = \frac{r_k u_c + (1+r) u_{cc} c_k}{\lambda \gamma p_{EE}}.$$

From $u_c > 0, u_{cc} < 0, r_k < 0$ and $c_k > 0$, we can derive sign relations:[7]

$$\text{sign } \phi_k = -\text{ sign } p_{EE}^{+} > 0 \text{ for all } E \in E^{+}$$
$$= -\text{ sign } p_{EE}^{-} < 0 \text{ for all } E \in E^{-},$$

so that the FOC curve can be described by the FOC^{+} curve with a *positive* slope in the E^{+} region and by the FOC^{-} curve with a *negative* slope in the E^{-} region. Hence, these curves have the "symmetrical" relation at the boundary of the threshold level of environmental quality. These results can be explained as follows. Since an increase in capital causes increases in consumption of the old and decreases in the interest rate, the marginal "benefit" of capital [i.e., the numerator in the right-hand side of (14)] is negative. On the other hand, the marginal opportunity "cost" of it (i.e., the denominator), equivalently

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[7] We derive $r_k < 0$ from $r = f'(k) - \delta$ and $f''(k) < 0$ and assume $f''(k)k + f'(k) > 0$ which is sufficient for $c_k > 0$.

the marginal benefit of environmental quality, is negative at environmental quality beyond the threshold level but is positive under this level. Thus, when capital increases, environmental quality must increase in the former but decrease in the latter in order that the first-order condition (11) for the welfare maximization continues to be maintained.

Next, we can rewrite equation (12) as follows:

$$(15) \qquad E_{t+1} = \tau(k_{t+1}; k_t, E_t) = b[\varphi(k_t) - E_t] - \gamma(k_{t+1} - k_t) + E_t,$$

where

$$\varphi(k;\gamma,\beta,b) = \left(\frac{1}{b}\right)\{\rho(k)f(k) - [\beta(1-\delta) + \gamma]k\}.$$

Equation (15) determines (k_{t+1}, E_{t+1}) feasible from the environmental side for (k_t, E_t) predetermined. Since this is a *temporary* environmental condition, we call it the TEC line. Geometrically, given a point $A\ (k_t, E_t)$, a point $B\ (k_{t+1}, E_{t+1})$ lies on the *downward* sloping line through the two points:

$$C\{k_t, E_t + b[\varphi(k_t) - E_t]\} \text{ and } D\{k_t + b[\varphi(k_t) - E_t]/\gamma, E_t\}.$$

In stationary state, the law of motion for the environment is given by

$$\bar{E} = \varphi(\bar{k};\gamma,b,\beta) = \left(\frac{1}{b}\right)\{\rho(\bar{k})f(\bar{k}) - [\beta(1-\delta) + \gamma]\bar{k}\}.$$

Since this is a stationary environmental condition, we call it the SEC curve. It should be now noted that $\rho(\bar{k})f(\bar{k}) - [\beta(1-\delta) + \gamma]\bar{k}$ in the right-hand side is *net* environmental improvement in stationary state because the first term is the maximum improvement in environmental quality feasible by the society and the second term reflects the fact that agents do not devote their entire wage to maintenance: each unit of saving by young agents implies both $(1-\delta)$ extra units of consumption and one less unit of environmental investment. As John and Pecchenino (1994) pointed out, this curve may be less well behaved since $\rho(k)$ may change sign as k varies. In this paper, for the sake of simplicity we assume that the technology is Cobb-Douglas so that $\nu(k)$ and hence $\rho(k)$ is *constant* for all k and further assume that ρ is *positive*.

The FOC and SEC curves and the TEC line are illustrated in Figure 3.[8] The intersection point of the SEC line with the FOC curve represents the *short-run* equilibrium at each period. Such an equilibrium is realized through the adjustment process of *capital* as follows. For the given k_{t+1}, the environmental quality which the generational government desires is given by E^{ϕ}_{t+1} on the FOC curve, while the maximum disposable quality by the environment is E^{τ}_{t+1} on the TEC line. If $E^{\tau}_{t+1} > E^{\phi}_{t+1}$ ($E^{\tau}_{t+1} < E^{\phi}_{t+1}$), then k_{t+1} increases (decreases). Thus, capital is adjusted within the period so as to satisfy $E^{\tau}_{t+1} = E^{\phi}_{t+1}$. The TEC line must cut the FOC curve from *above* in order that the short-run equilibrium is stable. Since the FOC$^+$ curve has a right upward slope, this stability condition is usually satisfied. On the other hand, although this condition is not always satisfied for the FOC$^-$ curve since it has a right downward slope, we will assume that it holds. Hence, the short-run equilibrium point usually lies on the FOC curve.

The *long-run* stationary equilibrium is determined by the intersection point of the SEC curve with the FOC. There is no (non-autarkic) stationary equilibrium if the latter lies everywhere above the former, and there are two stationary equilibria if the two curves intersect twice. It follows from (13) and (15) that in the E^+ region of $k-E$ space, environmental quality and capital both increase (decrease) along the FOC$^+$ curve below (above) the SEC.[9] On the other hand, in the E^- region, capital increases (decreases) while environmental quality decreases (increases) along the FOC$^-$ curve above (below) the SEC. Hence, the stationary equilibrium determined by the intersection

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[8] Firms associate with individuals in the labour and capital markets but have no relation to the environment because business activities are not factors of environmental contamination. Therefore, eliminating the market equilibrium conditions and the profit maximization conditions of firms from the dynamic system, the working of the model can be finally determined by the interrelation between the FOC curve of the generational government and the SEC or TEC curve of the environmnet side.

[9] We assume that parameters, b and γ, in equation (15) both take on sufficiently small values to exclude the so-called "overshooting" phenomenon of short-run eqililibrium points around the long-run equilibrium. Thus, as the TEC line is flat and passes at the neighbourhood of a short-run eqililibrium, such an equilibrium in each period moves gradually along the FOC curve.

point of the SEC curve with the FOC$^+$ is *stable*, but that of the SEC curve with the FOC$^-$ is *unstable*. Only when a sequence of the short-run equilibrium converges into the stable long-run equilibrium point along the FOC$^+$ curve below the the SEC, the economy is growing and environmental quality is improving. This characteristic of the dynamic equilibrium path is the same as that in John and Pecchenino (1994). However, there exists a *trade-off* between growth and environmental conservation in the region of environmental quality less than the threshold level. Such a trade-off is an important property peculiar to our model. If the threshold effect arises in the neighborhood of the natural state equilibrium of the environment, i.e., $E_T \cong E_N = 0$, then growth is incompatible with environmental quality almost everywhere in the interior solution.

It will be useful for the subsequent analyses to study comparative dynamic effects of parameters $(\lambda, \gamma, \beta, b)$ on the dynamic equilibrium path. First, the utility loss coefficient on illness, λ, does not affect the SEC curve, but it follows from $\phi_\lambda = -p_E/\gamma p_{EE}$ that as this parameter decreases, the FOC$^+$ and FOC$^-$ curves shift downward and upward respectively. These results can be explained as follows. The smaller the value of λ is, the more agents attach importance to consumption rather than health. Then, the marginal effect of environmental quality on the health probability, p_E, must increase so as to maintain the first-order condition (11). This can be attained by decreases (increases) of environmental quality in the E^+ (E^-) region. Second, when the environmental improvement parameter of investment, γ, increases, the FOC$^+$ and FOC$^-$ curves both shift in the same way as λ since sign ϕ_γ = sign ϕ_λ. On the other hand, because it can be easily shown that $\varphi_\gamma > 0$, the SEC curve shifts upward, implying that the environmental condition is more improved. Third, the environmental degradation coefficient on consumption, β, and the natural depreciation coefficient on environmental quality, b, do not both shift the FOC curve, but it follows from $\varphi_\beta < 0$ and $\varphi_b < 0$ that the SEC shifts downward with increases in these parameters. These results are consistent with our intuition because the environmental situation is then aggravated. Hence, the FOC$^+$ and FOC$^-$ curves lie *above* the SEC for sufficiently *small* values of (λ, γ) and/or for sufficiently *large* values of (β, b), so that the long-run stationary equilibrium may not exist (see Figure 4).

Finally, note that the expected utility of the representative individual at period t in equilibrium is a function of k_{t+1} and E_{t+1}, i.e., $u[c(k_{t+1})] + \lambda p(E_{t+1})$, so that the marginal rate of substitution between these variables can be given by $u_c c_k / \lambda p_E$. Thus, as the utility loss parameter of illness, λ, increases, the social contour of the expected utility in $k-E$ space becomes *flatter*.

B. Boundary Solution: Zero Environmental Quality Equilibrium

The dynamic system in this case is given by (1)-(3), (4.2), (5) led one period, and (6)-(7). Eliminating $(w_t, r_{t+1}, m_t, s_t, c_t, c_{t+1})$ from the system, it can be written as

(16) $$E_{t+1} = \tau(k_{t+1}; k_t, E_t) = b[\varphi(k_t) - E_t] - \gamma(k_{t+1} - k_t) + E_t,$$

(17) $$E_{t+1} = 0,$$

(18) $$[1 + r(k_{t+1})]\, u_c[c(k_{t+1})] \leq \lambda \gamma p_E(E_{t+1}).$$

Since equation (16) is the same as (15) in the case of interior solution, the TEC line and the SEC curve in $k-E$ space are effective also in this case. Equations (17) and (18) are the first-order conditions for welfare maximization. Substituting (17) into (18), then we obtain the following inequality:

(19) $$[1 + r(k_{t+1})]\, u_c[c(k_{t+1})] \leq \lambda \gamma p_E(0) \quad \text{where } p_E(0) > 0.$$

Since the left-hand side denoting the benefit of capital is a decreasing function of k_{t+1}, this inequality is equivalent to the following inequality,

$$k_{t+1} \geq k_M,$$

where k_M is the value of k_{t+1} at which inequality (19) holds with equality. Thus, the FOC^0 line can be described as the part of the k axis greater than k_M (see Figures 3 and 4). The short-run equilibrium is given by the intersection point of the TEC line with the horizontal FOC^0. When the SEC curve lies above (below) the FOC^0 line, such a equilibrium moves right (left) along this line, so that capital increases (decreases). The long-run equilibrium, if it exists, is determined by the intersection point of the SEC curve with the FOC^0 line. It is stable if the former cuts the latter from above.

C. Boundary Solution: Zero Environmental Investment Equilibrium

It follows from (1), (7) and $m_t = 0$ in (4.3) that $k_{t+1} = s_t = w_t$. Using these relations, we can finally write the dynamic system in this case as follows:

(20) $$k_{t+1} = w(k_t) = [1 - \upsilon(k_t)]f(k_t),$$

(21) $$E_{t+1} = (1 - b)E_t - \beta c\,(k_t),$$

(22) $$[1 + r(k_{t+1})]\,u_c[c(k_{t+1})] \geq \lambda \gamma p_E(E_{t+1}).$$

Equations (20) and (21) determine the short-run equilibrium (k_{t+1}, E_{t+1}) at period $t + 1$, given (k_t, E_t) predetermined at period t. In stationary state, these equations can be represented respectively as follows:

$$k = \bar{k}_z \text{ where } \bar{k}_z = w\,(\bar{k}_z) \text{ and } \bar{E} = \eta(\bar{k}) = -\frac{\beta c\,(\bar{k})}{b}.$$

Substituting (20) and (21) into (22), we obtain the following inequality:

(23) $$\{1 + r[w(k_t)]\}\,u_c\{c[w(k_t)]\} \geq \lambda \gamma p_E\{(1 - b)E_t - \beta c[w(k_t)]\}.$$

If a point (k_t, E_t) is involved in the region constrained by this inequality, then the government at period t should not choose to engage in investment of the environment, that is, $m_t = 0$. Thus, the simultaneous difference equations, (20) and (21), are effective only in this region, and if there exists a long-run stationary equilibrium, then it is known to be *stable* [see Figure 3 in John and Pecchenino (1994)]. It is of course true that $m_t > 0$ in the other region of $k - E$ space.

We denote a set of (k_t, E_t) at which inequality (23) holds with equality by $E_t = \theta(k_t)$. Since it defines the zero maintenance manifold as the set of points where the government is just different between zero and positive environmental maintenance (investment), we call this the ZMM curve (see Figures 3 and 4). The function $\theta(\cdot)$ has the derivative as:

$$\theta_k = \frac{u_c r_w w_k + (1 + r) u_{cc} c_w w_k}{(1 - b)\lambda\gamma p_{EE}} + \frac{\beta c_w w_k}{1 - b}, \tag{24}$$

where $r_w < 0$, $w_k > 0$ and $c_w > 0$. Since the second term in the right-hand side is positive, the ZMM$^+$ curve has a positive slope in the E^+ region. Comparing (24) with (14), we find that this curve has the larger slope than the FOC$^+$. It follows from $p^-_{EE} > 0$ in the E^- region that a slope of the ZMM$^-$ curve is ambiguous. However, this has a negative slope in the neighbourhood of the threshold and worst levels of environmental quality since $p^-_{EE} = 0$ at these levels, so that the ZMM$^-$ curve there has the smaller slope than the FOC$^-$. Differentiating the right-hand side of (23) with respect to environmental quality, then we obtain $(1 - b)\lambda\gamma p_{EE}$. Therefore, investment in the environment is zero in the shaded region above (below) the ZMM$^+$ (ZMM$^-$) curve, which implies that the government should not engage in investment when environmental quality is exceedingly good or bad.

D. Compatibility between Economic Growth and Environmental Conservation

We now synthesize equilibrium dynamics in the interior and boundary solutions analyzed so far separately in order to study the problem of the compatibility between economic growth and environmental conservation. Let us consider an economy that initially possesses little capital per capita and has environmental quality in the neighbourhood of the natural state equilibrium. Namely, it is supposed that the activity level of economy is low while the environment is clean in the initial period. In such an initial state, which is shown by the point I in Figures 3 and 4, it is the best choice for the government to make environmental investment to zero. The dynamic adjustment of the economy entails increases in capital and decreases in environmnetal quality, and it continues untill the shor-run equilibrium point, which would converge to the long-run stable point LRE^2 of the difference equations, (20) and (21), hits the ZMM curve at the switching point S^1. Hence, there exists a trade-off between growth and conservation under *zero* environmnetal investment in the first stage of economic developement.

The economy then enters into the region of *positive* investment, moving from the point S^1 to the point G on the FOC curve. The dynamic adjustment of the economy in this region crucially depends on whether or not this curve lies below the SEC curve. As illustrated in Figure 3, if the FOC curve lies below the SEC due to sufficiently *large* values of (λ, γ) and/or sufficiently *small* values of (β, b), then the economy moves along the FOC^- curve from the point G to the point A where environmental quality is at the threshold level. Thus, there exists *another* trade-off in the second stage, that is, capital decreases but environmental quality increases. However, if the threshold level of environmental quality is very *far* from the natural state, then such a trade-off does not arise because the point G lies on the FOC^+ curve. In the third stage, the economy moves along the FOC^+ curve, holding a *positive* correlation between growth and conservation. As soon as the equilibrium path hits the horizontal FOC^0 line at the point k_M, the economy enters into the region of *zero* environmental quality, and in the final stage it converges to the long-run equilibrium point LRE^1 along this line.[10] At such an equilibrium point, capital and consumption per capita are maintained at the relatively high levels and the best level of environmental quality is attained so that the old are all healthy. Hence, economic growth is compatible with environmental conservation in the third and final stages but is not so in the first and second ones. Environmental quality does never decrease as long as the government invests in the environment and is always beyond the threshold level in the long-run equilibrium. Capital continues to increase untill the equilibrium level except for the second stage. Since the social contour of the expected utility is flat when λ is very large, it decreases in the first stage but increases in the second. These results are interesting because the utilities of successive generations are decreasing (increasing) along a zero (positive) investment path. It goes without saying that they are increasing in the third and final stages.

On the other hand, as illustrated in Figure 4, the FOC curve lies *above* the SEC for sufficiently *small* values of (λ, γ) and/or for sufficiently *large* values of (β, b). In the second stage, the economy moves from the point G to the point A along the FOC^+ curve,

[10] It is also possible that the equilibrium path does not hit the FOC^0 line and converges to the long-run equilibrium point LRE^3 which is the intersection of the SEC curve with the FOC^+.

capital and environmental quality both decreasing.[11] In the third stage, it moves along the FOC^- curve, so that capital increases but environmental quality decreases. It is interesting that in spite of positive investment this trade-off is the same type as that in the first stage. Since there does not exist the long-run stationary equilibrium in the third stage, it seems that the trade-off continues forever. However, such a situation will finish at a finite time because the FOC^- curve intersects the ZMM^-. At the switching point S^2, the economy again enters into the region of *zero* investment. In the final stage, since the economy is governed by the dynamics of the difference equations, (20) and (21), it converges to the long-run stable point LRE^2. Environmental quality decreases in all stages except for the final stage and is always below the threshold level in the long-run equilibrium. Capital increases in the first and third stages but decreases in the other. Hence, economic growth is incompatible with environmental conservation in all periods. Since λ is very small, the social contour of the expected utility is steep, so that it increases in the first and third stages but decreases in the other.

From the above analyses, we find out that the following conditions with respect to parameters $(\lambda, \gamma, \beta, b, E_T)$ must hold in order that economic growth be compatible with environmental conservation.

(i) Agents attach more importance to health than to consumption.

(ii) Environmental quality is greatly improved by investment.

(iii) It is not so much polluted by consumption.

(iv) The natural depreciation rate of environmental quality is small.

(v) The threshold level of environmental quality is far from the natural state.

The conditions (ii)-(v) are consistent with our intuition because they imply amelioration of the environmental situation. It is interesting that the condition (i) is not so. This condition seems to be rather *paradoxical* because it is contrary to the popular perception that it will be impossible for the economy to grow as the society becomes more interested in environmental care.[12] However, it can be explained without any contradiction as follows. As individuals attach more importance to health than to consumption, they

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[11] If the threshold level of environmental quality is very *near* the natural state, then the movement of the economy along the FOC^+ curve may not arise.

desire *little* capital for a given environmental quality. Noting that the production technology of output is subject to *constant* returns to scale, it follows that the less capital is, the larger the feasible environmental quality is. Therefore, although the absolute level of capital is low, it is possible to increase capital and environmental quality simultaneously. On the other hand, if consumption is preferred, then the generational government desires the *high* level of capital but it is infeasible due to the environmental condition.

IV. Increasing Returns to Scale Technologies

In this section, we modify our model to allow for *sustained* output growth because economic growth is usually defined in terms of increased output of goods and services. In production techologies with increasing returns to scale, the interest and wage rates and the level of consumption are functions of current and lagged capital per capita, so that we obtain:

(25) $$r_{t+1} = \psi(k_t) f'(k_{t+1}) - \delta = r(k_{t+1}, k_t),$$

(26) $$w_t = \psi(k_{t-1}) f(k_t) - k_t \psi(k_{t-1}) f'(k_t) = w(k_t, k_{t-1}),$$

(27) $$c_{t+1} = (1 + r_{t+1}) k_{t+1} = c(k_{t+1}, k_t).$$

The dynamic system at interior equilibrium in this case can be finally described in terms of stock variables as follows:

(28) $$E_{t+1} = \phi^*(k_{t+1}; k_t) \Leftrightarrow [1 + r(k_{t+1}, k_t)] u_c [c(k_{t+1}, k_t)] = \lambda \gamma p_E(E_{t+1}),$$

(29) $$E_{t+1} = \pi^*(k_{t+1}; k_t, E_t, k_{t-1}) = b[\varphi^*(k_t, k_{t-1}) - E_t] - \gamma(k_{t+1} - k_t) + E_t,$$

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[12] In an economy where pollution arises from the use of *physical capital*, Gradus and Smulders (1993) examined whether the *long-run* rate of growth is affected by increased environmental care. They showed that (i) it is not affected in the standard neoclassical growth model, (ii) it is decreased if there are constant returns to physical capital in an endogenous growth model developed by Romer (1986), and (iii) it is unaffected or is even higher, depending on whether or not pollution influences agents' ability to learn, if human capital accumulation is the engine of endogenous growth as discussed by Lucas (1988). Since our model without external increasing returns is a version of the standard neoclassical growth model, the long-run growth rate is not affected by increased environmental care.

where

$$\varphi^*(k_t; k_{t-1}) = \left(\frac{1}{b}\right)\{\rho(k_t)\,\psi(k_{t-1})f(k_t) - [\beta(1-\delta) + \gamma]\,k_t\}.$$

Note that capital's share ρ is independent of k_{t-1}. Given predetermined variables (k_{t-1}, k_t, E_t), equations (28) and (29) determine the short-run equilibrium (k_{t+1}, E_{t+1}) at period $t+1$. In stationary state, these equations become

$$\bar{E} = \phi^*(\bar{k}, \bar{k}) \text{ and } \bar{E} = \varphi^*(\bar{k}, \bar{k}),$$

which are the FOC and SEC curves respectively. The long-run stationary equilibrium is given by the intersection point of these curves. It is difficult to describe completely the dynamic equilibrium path of the economy in $k - E$ space because the functions in (28) and (29), $\phi^*(\cdot)$ and $\varphi^*(\cdot)$, depend on predetermined variables, k_t and k_{t-1}, respectively. However, it will be possible to do so *approximately* by equalizing these variables to k_{t+1} and k_t respectively if we assume that the short-run equilibrium at each period moves *gradually* along the FOC curve.[13] Thus, we can give a geometric illustration of the dynamic equilibrium path, as shown in Figures 5 and 6.

The short-run equilibrium at each period can be determined by the intersection point of the FOC curve and the TEC line [i.e., equation (29)] depending on the short-run equilibrium at the preceding period. It follows from equations, (25) and (27), that partial derivatives of r_{t+1} and c_{t+1} with respect to k_t are both positive. Therefore, the FOC curve has the same property as that in constant returns to scale (CRS) technologies, so that it has a right upward slope in the E^+ region but has a right downward slope in the E^-. On the other hand, the SEC curve is not always so. As John and Pecchenino (1994) pointed out, if external increasing returns are sufficiently *strong* that the per capita production function $\psi(k)f(k)$ is *convex*, then the SEC curve is *symmetrical* to that in CRS technologies (see Figures 3 and 5). That is, high environmental quality can be realized by *large* capital in production technologies with increasing returns to scale.

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[13] As noticed in footnote 8, this assumption is satisfied if parameters (b, γ) in equation (29) both take on sufficiently *small* values

There is no (non-autarkic) long-run equilibrium if the FOC curve lies everywhere below the SEC, and there are two stationary equilibria if the two curves intersect twice. It follows from (28) and (29) that in the E^+ region, environmental quality and capital both increases (decreases) along the FOC$^+$ curve below (above) the SEC. On the other hand, in the E^- region, capital increases (decreases) while environmental quality decreases (increases) along the FOC$^-$curve above (below) the SEC. Hence, the long-run stationary equilibrium determined by the intersection point of the SEC curve with the FOC$^+$ is *unstable*, but that of the SEC curve with the FOC$^-$ is *stable*. There exists a trade-off between growth and conservation in environmental quality smaller than the threshold one. The economy is growing and environmental quality is improving, only when a sequence of the short-run equilibrium converges to the natural state level of environmental quality along the FOC$^+$ curve below the the SEC. Comparative dynamic effects of parameters (λ, γ, β, b) on the equilibrium path are the same as those in CRS technologies. We can make the similar analyses for two boundary solutions also and can easily show that the basic results in CRS technologies continue to hold in increasing returns to scale technologies except that the long-run stationary equilibrium under zero investment is a *saddle* point.

Now, let us consider the problem of the compatibility between economic growth and environmental conservation. The dynamic adjustment in the first stage of the economy is the same as that in CRS technologies: capital increases but environmnetal quality decreases. The equilibrium path afterward depends on whether or not the FOC curve intersects the SEC. At first, let us analyse the case illustrated in Figure 5 where the former lies below the latter due to sufficiently *small* values of (λ, β, b) and/or sufficiently *large* value of γ. If the threshold level of environmental quality is near the natural state, then the economy moves along the FOC$^-$curve in the second stage so that capital decreases but environmental quality increases. As soon as environmental quality arrives at the threshold level, it moves along the FOC$^+$ curve in the third stage, holding the positive correlation between growth and conservation. In the final stage, it enters into the region of zero environmental quality and grows infinitely at the best environment. Hence, these characteristics of the equilibrium path are similar to those in CRS technologies illustrated in Figure 3, except that the economy can *ultimately* continue to grow, being compatible with conservation. Next, as illustrated in Figure 6, if the FOC curve intersects the SEC and if the threshold level is far from the natural state, then the economy moves along the FOC^+ curve in the second stage, so that capital and environmental quality both decreases. In the final stage, it moves along the FOC^- curve and converges to the long-run equilibrium LRE^3 where environmental quality is below the threshold level so that most of the old are ill. Growth is incompatible with conservation in all stages. Thus, these characteristics of the equilibrium path are similar to those in CRS technologies illustrated in Figure 4, except that the economy finally arrives at the long-run equilibrium under positive investment.

It follows from the above analyses that the conditions of parameters (γ, β, b, E_T) necessary for the compatibility between growth and conservation are the same as the conditions (ii)-(v) in CRS technologies. However, since the feasible improvement in environmental quality is large for high levels of capital in increasing returns to scale technologies, the reverse of the condition (i) is required. That is, it is necessary that individuals attach more importance to *consumption* than to health. A growing economy is initially associated with declines in environmental quality, but is later characterised by improvements in it and finally the economy can continue to grow infinitely in the best environment.[14]

V. Concluding Remarks

In this paper, in an overlapping generations model where decisions of two-period-lived individuals between saving for consumption and investment in the environment have intergenerational externalities on both factor productivity and the environment, we have analysed the potential conflict between eonomic growth and environmental conservation. Environmental quality gives not amenity values like public goods but causes health damages to individuals and has the threshold effect on the probability of health: as soon as it decreases less than the threshold level, most people fall ill. We have been particularly

[14] Note that the growth rate of the economy, g_t, on the FOC^0 line with the best environment does *not* depend on the utility loss parameter λ because it becomes $g_t = b\,\varphi^*(k_t, k_{t-1})/\gamma k_t$ from (29).

concerned with comparative dynamic analyses with respect to various parameters on the equilibrium path of the economy. The main conclusions can be summarized as follows.

If society's preferences shift towards a larger concern for a clean environment and if the threshold effect arises far from the natural state equilibrium of the environment, then growth is associated first with declines, then improvements, in environmental quality in constant returns to scale technologies. Under the same conditions, economic growth and environmental conservation become incompatible in increasing returns to scale technologies. If the threshold level is near the natural state, then there exists a trade-off between growth and conservation under positive environmental investment. This is different from a trade-off under zero investment, namely, the cleaner environment can be attained at the sacrifice of economic growth. If the natural depreciation rate of environmental quality is small, the environment is greatly improved by investment, and/or it is not so much degraded by consumption, then a growing economy exhibits environmental quality that deteriorates initially and later improves in both technologies.

Finally, it will be useful to make some remarks on our model. First, we have discussed the model in terms of environmental quality, while the analysis has more general applicability to social overhead capital, for example, medical facilities, a sewer system, a garbage burning plant and so on. Second, although we did not refer to the welfare analysis of Pareto-improving policies, overmaintenance of the environment, analogus to overaccumulation of capital, may emerge due to environmental externalities of consumption and investment and externality of capital on factor productivity. Third, it has been assumed that the individual continues to supply one unit of labour endowed to firms even if he falls ill. However, many ill workers work less effectively. This is an important point in the environmental problems such as ozone layer depletion, radioactive pollution and lead or mercury poisoning. Gradus and Smulders (1993) introduced depreciation of human capital by pollution into the Lucas (1988) model. Following them, it may be valuable to re-formulate our model such that labour supplied by the young is an increasing function of environmental quality.

References

Dasguputa, P. (1982). *The Control of Resources*. Basil Blackwell, Oxford.

Gradus, R. and Smulders, S. (1993). "The Trade-off Between Environmental Care and Long-term Growth -- Pollution in Three Prototype Growth Models." *Journal of Economics*, Vol. 58, pp. 25-51.

John, A. and Pecchenino, P. (1994). "An Overlapping Generations Model of Growth and the Environment." *Economic Journal*, Vol. 104. pp. 1393-1410.

Lucas, R. (1988). "On the Mechanics of Economic Development." *Journal of Monetary Economics*, Vol. 21, pp. 3-42.

Romer, P. (1986). "Increasing Returns and Long-run Growth." *Journal of Political Economy*, Vol. 94, pp. 1002-1037.

Van der Ploeg, F. and Bovenberg, A.L. (1994). "Environmental Policy, Public Goods and the Marginal Cost of Public Funds." *Economic Journal*, Vol. 104, pp. 444-454.

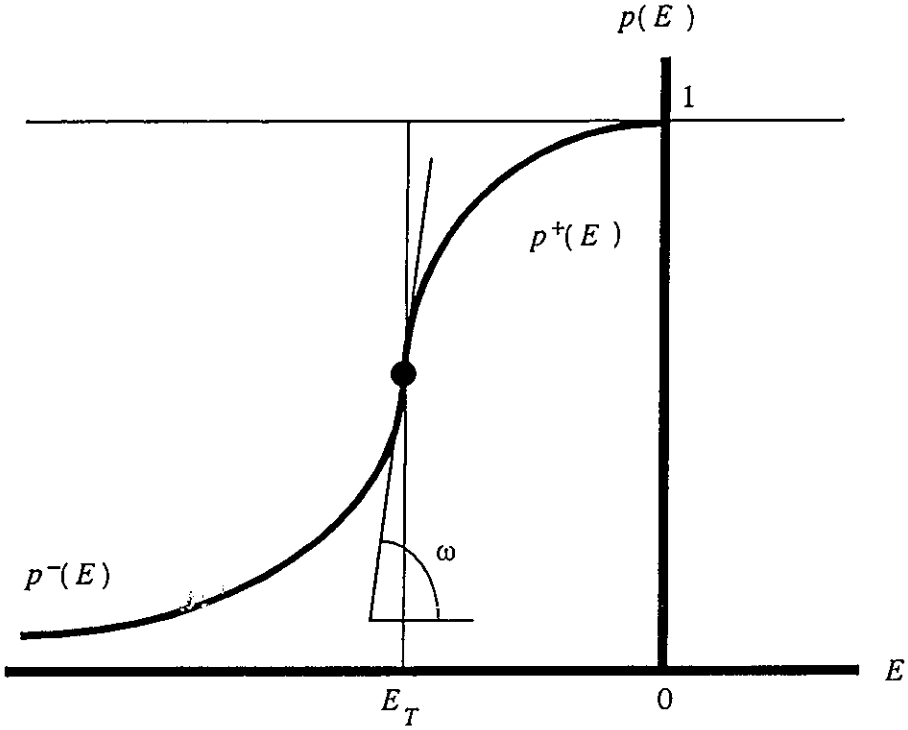

Fig. 1. Threshold effect of environment on the health probability

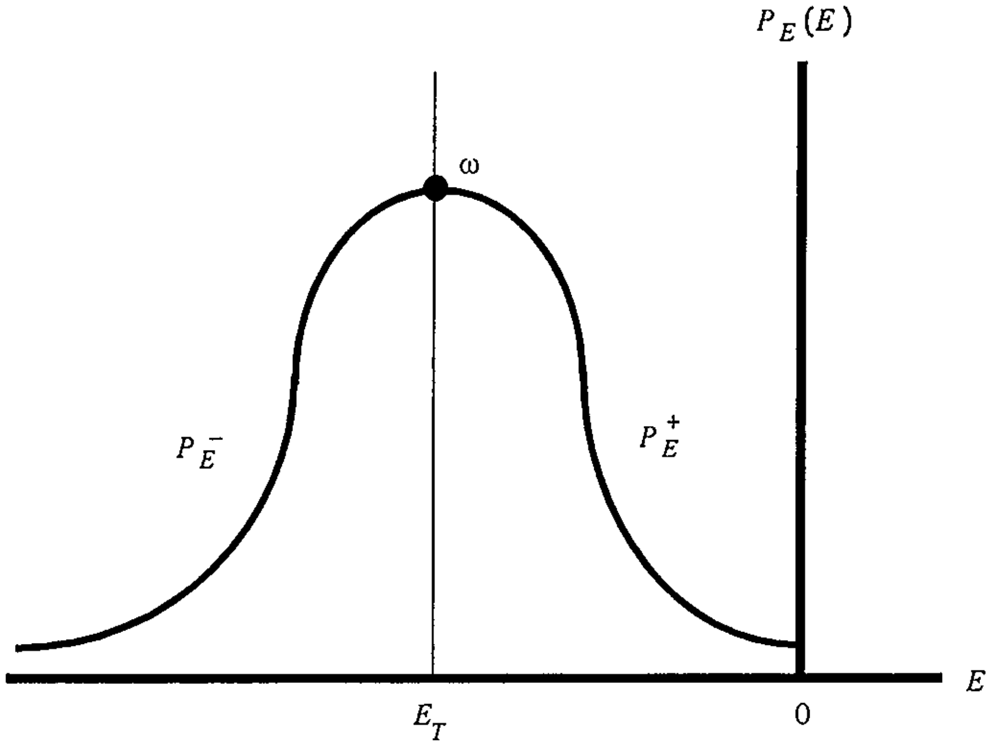

Fig. 2. Marginal effect of environment on the health probability

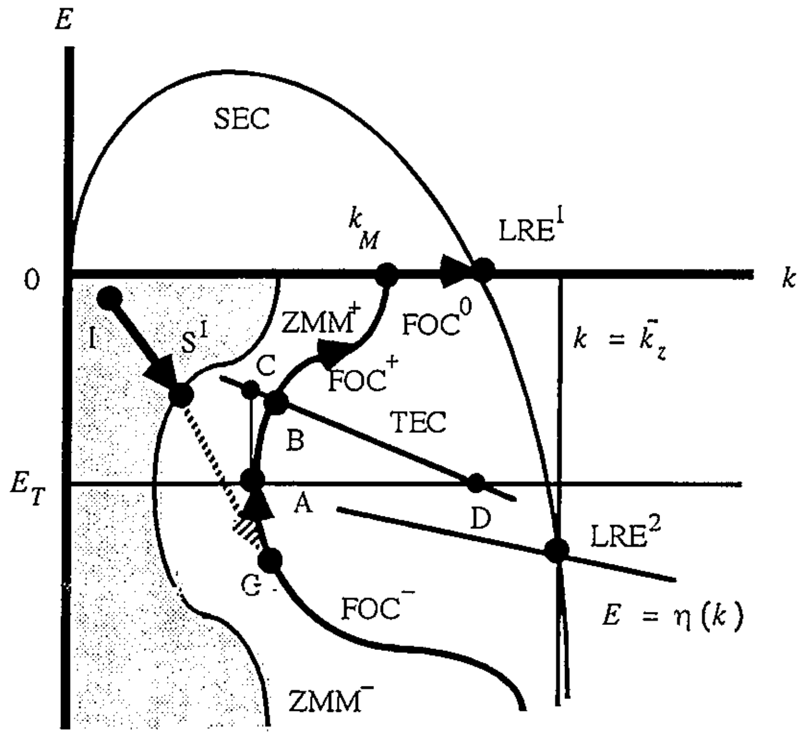

Fig. 3. Equilibrium path in constant returns to scale technologies: the case of the large (λ,γ) and/or the small (β,b)

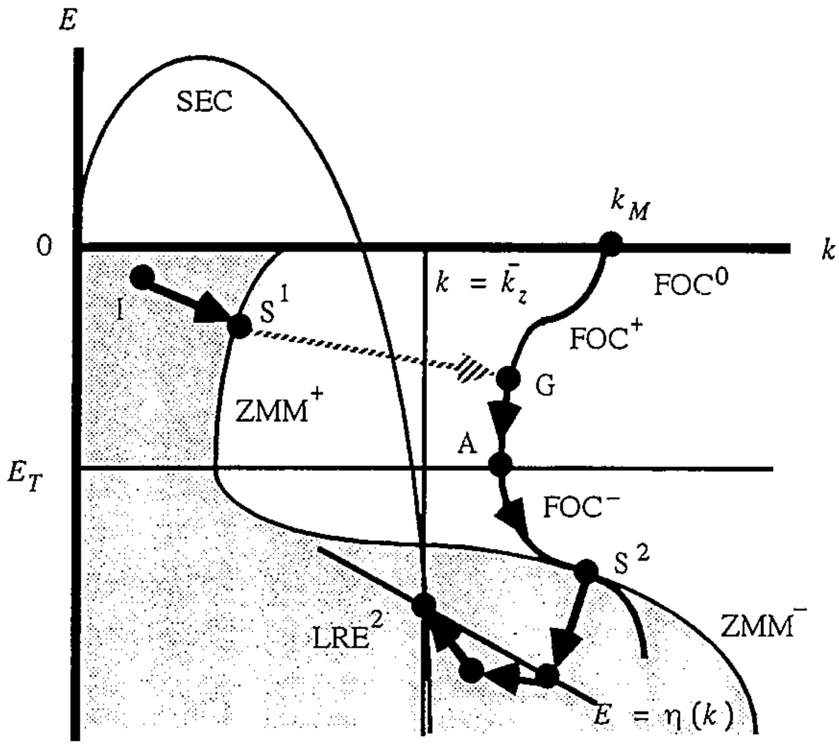

Fig. 4. Equilibrium path in constant returns to scale technologies: the case of the small (λ,γ) and/or the large (β,b)

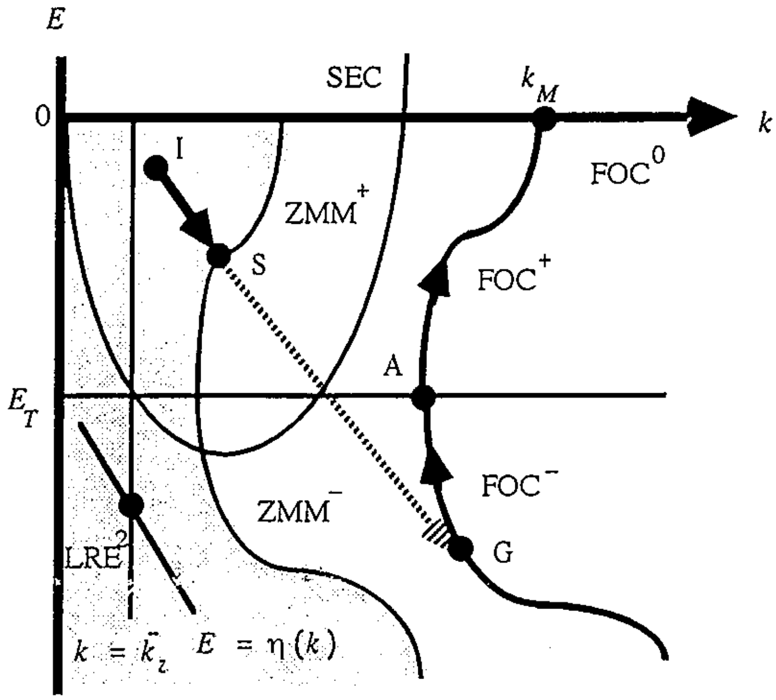

Fig. 5. Equilibrium path in increasing returns to scale technologies: the case of the small (λ, β, b) and/or the large γ

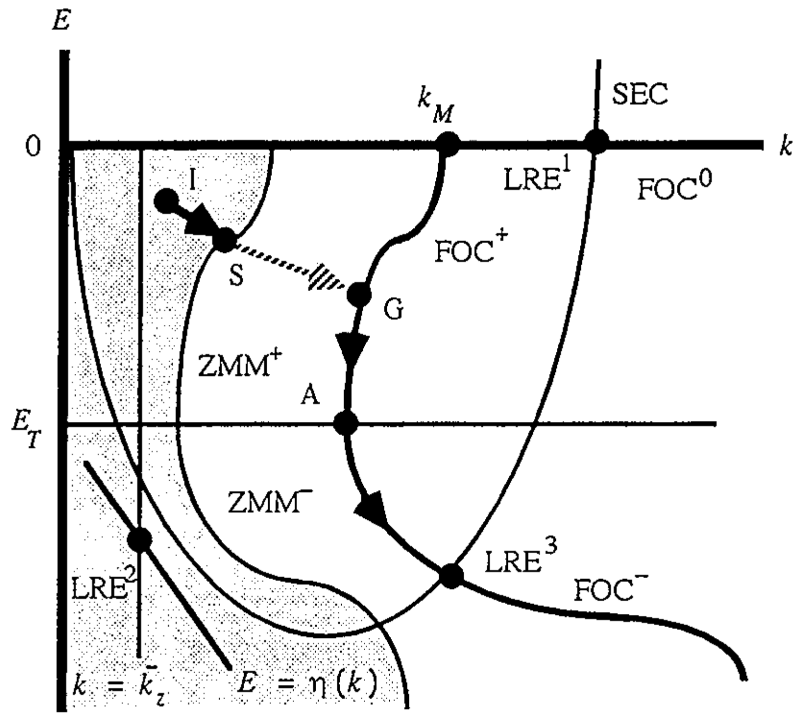

Fig. 6. Equilibrium path in increasing returns to scale technologies: the case of the large (λ, β, b) and/or the small γ