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Realization theory of general discrete event systems and the uniqueness problem of DEVS formalism

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abstract

One way to construct state space representations for the class of general discrete event systems is presented in this paper. A general discrete event system is an input-output system that is past-determined and stationary, and has the discrete event input space and discrete event-determinacy. The constructed state space representation is minimal in a class of dynamical system representations. The realization theory provides an answer to the uniqueness problem of representations for discrete event systems. As an application of the theory it is shown that a reduced and reachable DEVS, which is originated by Zeigler[1], is unique up to isomorphism in the class of discrete event dynamical system representations.

1 Introduction

Discrete event systems are firstly formulated by [1] from the systems theoretic view. The formalism is called discrete event system specification (DEVS). State variables were considered as one of the descriptive variables of discrete event models. That is, they are part of the definition of the systems itself.

Another understanding for state variable has been given in [2, 3]. A general system is given as a set of input-output data (in the form of time functions) and then the properties of a specific system, such as stationarity and linearity, are used to construct its state space representation (i.e., state transition mechanism). From their point of view, state is not part of the definition of the system, but a derived, artificial variable. Until now, no realization theory for general discrete event systems has been available. The realization problem we consider is the following. "*For a given discrete event system* $S \subseteq X \times Y$*, construct its state space representation.*"

In general many possible variables can be used as state variables to recognize the behavior of a system. The construction of a state space representation presented in this paper is minimal in a class of dynamical system representations. It is applied to the uniqueness problem of representation for discrete event systems. From the result it is shown that the DEVS state space representation is the universal state space representation if it is reachable and reduced.

2 Basic definition and notation

The time set is denoted by T, which is either the set of non-negative integers or the set of non-negative reals. That is, the result in this paper holds true for both time sets.

Definition 1. System

A system is a relation of an input set and output set. If those two sets are sets of time functions defined on the same time set, then the system is called a time system.

The input set is denoted by X and the output set by Y. A system is denoted by $S \subseteq X \times Y$. It is always assumed that for any $x \in X$ there is $y \in Y$ such that $(x, y) \in S$ and that for any $y \in Y$ there is $x \in X$ such that $(x, y) \in S$. The value set of inputs of a time system is called an input alphabet and the value set of output is called an outputs alphabet. Let x be a function from the time set T to an input alphabet. For any $t, t' \in T^\infty$, $t \le t'$, the restriction $x|[t, t')$ is written by $x_{tt'}$, where $[t, t') = \{t'' \mid t'' \in T, t \le t'' < t'\}$ as usual. That is, $x_{tt'}(\tau) = x(\tau)$ for any τ, $t \le \tau < t'$. The notation x^t is used interchangeably for x_{0t}. Also $\bar{x}_{tt'}$ denotes $x|[t, t']$, where $[t, t'] = [t, t') \cup \{t'\}$, and x_t denotes $x_{t\infty}$. The set of all restrictions of all members of X is defined by $\underline{X} = \{x_{tt'} \mid x \in X$ and $t, t' \in T^\infty\}$. We think of a shift operator σ^t and a shift-left operator λ^t for a time function. That is, for any r and t, $r \ge t$, $(\sigma^t(x))(r) = x(r-t)$ holds. If $r < t$ then $(\sigma^t(x))(r)$ is undefined. And $(\lambda^s(x))(t) = x(s+t)$ for any s and $t \in T$. A time system S is called stationary if $S|_{[t,\infty)} \subseteq \sigma^t(S)$ holds, or equivalently $\lambda^t(S) \subseteq S$ holds, for any $t \in T$, where $\sigma^t(S) = \{(\sigma^t(x), \sigma^t(y)) \mid (x, y) \in S\}$ and $S|_{[t,\infty)} = \{(x_t, y_t) \mid (x, y) \in S\}$. For a stationary system S, it is always assumed that $\lambda^t(X) = X$ holds for any t. S is called strongly stationary if $S|_{[t,\infty)} = \sigma^t(S)$ holds for any t. Let x and x' be arbitrary functions from T to the same alphabet A. For any $t \in T$, we can define another element $x'': T \to A$ by

$$x''(t'') = \begin{cases} x(t''), \text{ if } t'' < t, \\ x'(t''), \text{ if } t'' \ge t. \end{cases}$$

x'' is called the concatenation of x^t and x'_t and is denoted by $x'' = x^t \bullet x'_t$.

Definition 2: Past-determinacy [2]

Let $S \subseteq X \times Y$ be a time system. S is called past-determined from $k \in T$ if and only if the following conditions hold.

(i) $\forall (x, y) \in S, \forall (x', y') \in S, \forall t > k,$

$$\text{if } (x^k, y^k) = (x'^k, y'^k) \text{ and } \bar{x}_{kt} = \bar{x}'_{kt}, \text{ then } \bar{y}_{kt} = \bar{y}'_{kt} \text{ holds.}$$

(ii) $\forall (x^k, y^k), \forall x_k, \exists y'_k,$

$$\text{if } (x^k, y^k) \in S^k \text{ then } (x^k \cdot x_k, y^k \cdot y'_k) \in S.$$

Lemma 1

Let $S \subseteq X \times Y$ be a time system that is past-determined from k. Let $(x, y) \in X \times Y$ be arbitrary. Then, $(x, y) \in S$ if and only if for any t, $t \geq k$, $(x^t, y^t) \in S^t$ holds.

We need the following property to cope with the past-determinacy.

Definition 3: Finite observability [3]

A time system $S \subseteq X \times Y$ is said to be finitely observable from $k \in T$, if

$$(\forall x \in X)\ (\forall\ y, y' \in S(x))\ (\, y^k = y'^k \rightarrow y = y').$$

Proposition 1 [3]

A time system $S \subseteq X \times Y$ is past-determined from $k \in T$ if and only if the following two conditions hold:

(i) S is pre-causal: that is,

$$(\forall x, x' \in X)\ (\forall \tau \geq k)\ (\bar{x}^{\tau} = \bar{x}'^{\tau} \rightarrow S(x)|_{[0,\tau]} = S(x')|_{[0,\tau]}\,)$$

(ii) S is finitely observable from k.

The state space is a key concept by which we can grasp the behavior of dynamic systems.

Definition 4: Initial response function [2]

Let $S \subseteq X \times Y$ be a time system. A function ρ_0: $C \times X \rightarrow Y$ is called an initial response function of S if it satisfies the condition such that $(x, y) \in S$ if and only if $\rho_0(c, x) = y$ holds for some $c \in C$.

An initial response function gives global correspondence between inputs and outputs through states. If ρ_0 satisfies the following condition then it is said to be causal: if $\bar{x}_{0t} = \bar{x}'_{0t}$ holds then $\rho_0(c, x)|_{[0,t]} = \rho_0(c, x')|_{[0,t]}$ holds for each $c \in C$ and $x, x' \in X$. If a time system S has a causal initial response function then S is called causal. If ρ_0 satisfies the following condition then ρ_0 is said to be reduced: If $\rho_0(c, x) = \rho_0(c', x)$ holds for each $x \in X$ then $c = c'$.

In order to recognize the internal change of a system we need the state transition mechanism which is defined as follows.

Definition 5: Dynamical system representation [2]

Let $S \subseteq X \times Y$ be a stationary system. A pair of families of mappings $<\rho, \phi>$ where $\rho = \{\rho_t \mid \rho_t : C \times X_t \to Y_t \text{ and } t \in T\}$ and $\phi = \{\phi_{tt'} \mid \phi_{tt'} : C \times X_{tt'} \to C \text{ and } t, t' \in T, t \leq t'\}$ is a (time invariant) dynamical system representation of S if the following conditions are satisfied:

(i) ρ is a response family of S.
That is, for each $t > 0$ and $(x_t, y_t) \in S_t$ there exists $c \in C$ such that $\rho_t(c, x_t) = y_t$ holds.

(ii) ρ is time invariant.
That is, $\rho_t(c, x_t) = \sigma^t(\rho_0(c, \sigma^{-t}(x_t)))$ holds for each t, c and x_t.

(iii) Each function $\phi_{tt'}$ satisfies the following conditions:
(α) $\rho_t(c, x_t)|T_{t'} = \rho_{t'}(\phi_{tt'}(c, x_{tt'}), x_{t'})$ where $x_t = x_{tt'} \cdot x_{t'}$
(β) $\phi_{tt''}(c, x_{tt''}) = \phi_{t't''}(\phi_{tt'}(c, x_{tt'}), x_{t't''})$ where $x_{tt''} = x_{tt'} \cdot x_{t't''}$
(γ) $\phi_{tt}(c, x_{tt}) = c$

(iv) Each functions $\phi_{tt'}$ is time invariant. That is, $\phi_{tt'}(c, x_{tt'}) = \phi_{0\tau}(c, \sigma^{-t}(x_{tt'}))$ where $\tau = t' - t$.

In the above definition the set C is called the state space of the dynamical system representation $<\rho, \phi>$. The family ϕ which satisfies the conditions (iii)-(β), (γ) and (iv) is called a family of time invariant state transition functions. The condition (iii)-(β) is called the semigroup property.

It has been proved that every stationary system has a time invariant dynamical system representation even if it is not causal [2, 3]. For a causal system we can decompose its dynamical system representation into a state space representation where the response family is decomposed.

Definition 6: State space representation [3]

Let $S \subseteq X \times Y$ be a stationary system with the input alphabet A and the output alphabet B. Let C be an arbitrary set. C is a state space for S if there exist a family of functions $\phi = \{\phi_{tt'} \mid \phi_{tt'} : C \times X_{tt'} \to C \text{ and } t, t' \in T, t \leq t'\}$ and a function $\mu : C \times A \to B$ such that

(i) $S = \{(x, y) \mid$ there exists some $c \in C$ such that $y(t) = \mu(\phi_{0t}(c, x_{0t}), x(t))$ for any $t \in T\}$

(ii) ϕ is a family of time invariant state transition functions.

The pair $<\phi, \mu>$ is called a (time invariant) state space representation of S.

If S has a state space representation then a causal initial response function ρ_0: $C \times X \to Y$ is defined by $\rho_0(c, x)(t) = \mu(\phi_{0t}(c, x_{0t}), x(t))$ for each $t \in T$. Also, defining $\underline{\rho} = \{\rho_t \mid \rho_t : C \times X_t \to Y_t$, where $\rho_t(c, x_t) = \sigma^t(\rho_0(c, \sigma^{-t}(x_t)))\}$, we have a causal dynamical system representation $<\underline{\rho}, \underline{\phi}>$ of S from a state space representation $<\underline{\phi}, \mu>$. The $<\underline{\rho}, \underline{\phi}>$ is called the dynamical system representation defined by $<\underline{\phi}, \mu>$.

A state space representation is a wide-spread framework to recognize dynamics of a time system in a causal way.

A discrete event system is a special time system defined as follows:

Definition 7: Discrete event system

A time system $S \subseteq X \times Y$ is a discrete event system if satisfies the following conditions:

(i) S is strongly stationary.

(ii) S is past-determined from $k \in T$.

(iii) The input space X is a discrete event input space. That is,

(iii-1) The input alphabet A for X is P(A') which is the power set of A'.

(iii-2) The constant valued function $\tilde{\Lambda}$ is in X, where $\tilde{\Lambda}(t) = \Lambda$ for any $t \in T$ and Λ is the empty set.

(iv) S has the discrete event-determinacy: For any $(\hat{x}^k, \hat{y}^k) \in S^k$, $(x^k, y^k) \in S^k$, $c \in S(\Lambda)$, $x' \in X$ and $y' \in Y$, if $(\hat{x}^k \cdot \Lambda_k, \hat{y}^k \cdot \sigma^k(c)) \in S$, $(x^k \cdot \Lambda_k, y^k \cdot \sigma^k(c)) \in S$ and $(\hat{x}^k \cdot \sigma^k(x'), \hat{y}^k \cdot \sigma^k(y')) \in S$ hold then $(x^k \cdot \sigma^k(x'), y^k \cdot \sigma^k(y')) \in S$.

Taking the set A' as the tasks to be executed or processed in the system, the condition (iii) shows parallel execution of tasks, while Λ represents nonevent. The discrete event-determinacy (iv) can be interpreted that each element of $S(\bar{X})$ virtually decides the "internal state" of S, where $S(\bar{X}) = \{y \mid \exists \hat{x}^k, \exists \hat{y}^k, (\hat{x}^k \cdot \Lambda_k, \hat{y}^k \cdot \sigma^k(y)) \in S\}$. In the rest of this paper $\tilde{\Lambda}$ is simply denoted by Λ. The empty set Λ is called nonevent.

3 Realization of discrete event system

In the case of a linear system S the linear space S(0) of state responses is used as the canonical state space [3] , where $S(0) = \{y \mid (0, y) \in S$ and 0 is the constant function whose value is always $0\}$. Similar to this, for a discrete event system we will use the space $S(\Lambda)$ as a state space, where $S(\Lambda)$ is the set of all the response to the constant nonevent Λ. That is, $S(\Lambda)$ is the all possible behavior of the system without external event. Intuitively speaking, $S(\Lambda)$ holds all internal behavioral variety of the system.

In this section let $S \subseteq X \times Y$ be an arbitrarily fixed discrete event system that is past-determined from k.

Proposition 2

$S(\bar{X}) = S(\Lambda)$. If S is assumed to be stationary instead of strongly stationary, we have $S(\bar{X}) \subseteq S(\Lambda)$.

Based on this proposition and the discrete event-determinacy, the following function can be defined for S. Define the function $\rho_0 : S(\Lambda) \times X \to Y$ by the correspondence: $\rho_0(c, x) = y$ if and only if there exists $(\hat{x}^k, \hat{y}^k) \in S^k$ such that $(\hat{x}^k \cdot \Lambda_k, \hat{y}^k \cdot \sigma^k(c)) \in S$ and $(\hat{x}^k \cdot \sigma^k(x), \hat{y}^k \cdot \sigma^k(y)) \in S$ hold.

The function ρ_0 is well-defined. In fact, for any $c \in S(\Lambda)$ there exists $(\hat{x}^k, \hat{y}^k) \in S^k$ such that $(\hat{x}^k \cdot \Lambda_k, \hat{y}^k \cdot \sigma^k(c)) \in S$ holds. And there also exists $\sigma^k(y)$ with $(\hat{x}^k \cdot \sigma^k(x), \hat{y}^k \cdot \sigma^k(y)) \in S$ because of the past-determinacy of S. Assume also that there also exists $(x'^k, y'^k) \in S^k$ such that $(x'^k \cdot \Lambda_k, y'^k \cdot \sigma^k(c)) \in S$ holds. In this case the discrete event-determinacy assures that $(x'^k \cdot \sigma^k(x), y'^k \cdot \sigma^k(y)) \in S$ holds. Thus ρ_0 is well-defined.

Proposition 3

The function ρ_0 defined above is an initial response function of S. That is, $(x, y) \in S$ if and only if there exists some $c \in S(\Lambda)$ such that $\rho_0(c, x) = y$.

Proposition 4

The function ρ_0 has the following properties.

(1) ρ_0 is causal;

(2) ρ_0 is reduced;

(3) For any $c \in S(\Lambda)$, $\rho_0(c, \Lambda) = c$ holds;

(4) For any $c \in S(\Lambda)$, $x \in X$ and $t \in T$, $\rho_0(c, x)|_{[0,t)} = \rho_0(c, x^t \cdot \Lambda_t)|_{[0,t)}$ holds.

Now we define the state space representation for a discrete event system. Let $t \in T$ be arbitrary. Define $\phi_{0t} : S(\Lambda) \times X_{0t} \to S(\Lambda)$ by $\phi_{0t}(c, x^t) = \lambda^t(\rho_0(c, x^t \cdot \Lambda_t))$. Let $\underline{\phi} = \{\phi_{tt'} \mid \phi_{tt'} : S(\Lambda) \times X_{tt'} \to S(\Lambda)$ and $t, t' \in T, t \le t'\}$, where $\phi_{tt'}(c, x_{tt'})$ is defined as $\phi_{0\tau}(c, \sigma^{-t}(x_{tt'}))$ and $\tau = t' - t$. Define a function $\mu : S(\Lambda) \times A \to B$ as $\mu(c, a) = \rho_0(c, x)(0)$, where x is an arbitrary but $x(0) = a$. Since ρ_0 is causal μ is well-defined.

Proposition 5

The pair $\langle \underline{\phi}, \mu \rangle$, defined above, is a time invariant state space representation of S.

The state space representation $<\underline{\phi}, \underline{\mu}>$ is called the S(Λ)-realization of S. The following result is a property of S(Λ)-realization.

Proposition 6

The S(Λ)-realization is reachable. That is, for any $c \in S(\Lambda)$ there exist $c' \in S(\Lambda)$ and $x^k \in X^k$ such that $c = \phi_{0k}(c', x^k)$.

4 The uniqueness problem of representation for discrete event systems

The definition of the uniqueness problem of representation for a system S and its importance are stated in [3] as follows.

The uniqueness problem of representation:

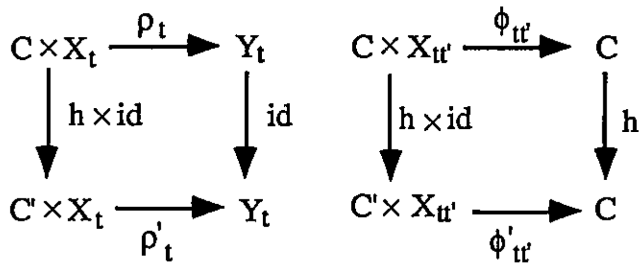
Given a time system S, find conditions on S under which S has a unique state space representation up to isomorphism.

They point out that "the state space approach is meaningless for the analysis of a system unless the uniqueness problem is solved in a positive way; otherwise the whole family of representations of S which may be differ in various degree must be used simultaneously whenever the system is being investigated." In order to provide the answer to the problem of representation for discrete event systems, we first have to construct a state space representation and then show its uniqueness in a class of state space representations or more strongly in a class of dynamical system representations. In the previous section the S(Λ)-realization has been constructed for a discrete event system S. The uniqueness of S(Λ)-realization up to isomorphism is shown in this section.

In the following of this section $S \subseteq X \times Y$ is an arbitrarily fixed discrete event system that is past-determined from k. We first need a way to compare a dynamical system representation with others.

Definition 8: Morphism [3]

Let $<\underline{\rho}, \underline{\phi}>$ and $<\underline{\rho}', \underline{\phi}'>$ be time invariant dynamical system representations of S. Then a mapping $h: C \rightarrow C'$ is called a morphism from $<\underline{\rho}, \underline{\phi}>$ to $<\underline{\rho}', \underline{\phi}'>$, if the diagrams in Fig. 1 are commute. That is, for any $t \in T$, $c \in C$ and $x \in X$, it holds that $\rho_t(c, x_t) = \rho'_t(h(c), x_t)$ and $h(\phi_{0t}(c, x^t)) = \phi'_{0t}(h(c), x^t)$. If h is bijective then $<\underline{\rho}, \underline{\phi}>$ is called isomorphic to $<\underline{\rho}', \underline{\phi}'>$, which is denoted by $<\underline{\rho}, \underline{\phi}> \cong <\underline{\rho}', \underline{\phi}'>$.



(Fig. 1 Commutative diagram)

A morphism h from $<\underline{\rho}, \underline{\phi}>$ to $<\underline{\rho}', \underline{\phi}'>$ is denoted by $h : <\underline{\rho}, \underline{\phi}> \rightarrow <\underline{\rho}', \underline{\phi}'>$. If there is a morphism from $<\underline{\rho}, \underline{\phi}>$ to $<\underline{\rho}', \underline{\phi}'>$ that is not surjective, then $<\underline{\rho}, \underline{\phi}>$ can be "embedded" into $<\underline{\rho}', \underline{\phi}'>$. Thus we can intrinsically think of that $<\underline{\rho}', \underline{\phi}'>$ is possibly bigger and then redundant than $<\underline{\rho}, \underline{\phi}>$. If a morphism is surjective, then $<\underline{\rho}', \underline{\phi}'>$ is smaller than $<\underline{\rho}, \underline{\phi}>$.

We can have the dynamical system representation defined by $S(\Lambda)$-realization. In the following the dynamical system representation is also called $S(\Lambda)$-realization and is denoted by $<\underline{\rho}^*, \underline{\phi}^*>$. For discrete event systems the following property of dynamical system representations is imposed.

Definition 9: Discrete event-response function

Let a function ρ_0: $C \times X \rightarrow Y$ be an initial response function of S. It is called a discrete event-response function of S if it satisfies the following condition: For any $c \in C$ and $x \in X$ there exist $c', c'' \in C$ and $\hat{x}^k \in X^k$ such that $\rho_0(c, \Lambda) = \lambda^k \rho_0(c', \hat{x}^k \bullet \Lambda_k)$, $\rho_0(c, x) = \lambda^k \rho_0(c'', \hat{x}^k \bullet \sigma^k(x))$ and $\rho_0(c', \hat{x}^k \bullet \Lambda_k)|[0, k) = \rho_0(c'', \hat{x}^k \bullet \sigma^k(x))|[0, k)$.

If ρ_0 of a dynamical system representation $<\underline{\rho}, \underline{\phi}>$ of S is a discrete event-response function then $<\underline{\rho}, \underline{\phi}>$ is called a discrete event dynamical system representation of S. It is easy to see for a dynamical system representation $<\underline{\rho}, \underline{\phi}>$ that ρ_0 is a discrete event-response function if and only if for any $c \in C$ and $x \in X$ there exists $(\hat{x}^k, \hat{y}^k) \in S^k$ such that $(\hat{x}^k \bullet \Lambda_k, \hat{y}^k \bullet \sigma^k[\rho_0(c, \Lambda)]) \in S$ and $(\hat{x}^k \bullet \sigma^k(x), \hat{y}^k \bullet \sigma^k[\rho_0(c, x)]) \in S$ hold.

The following is the main theorem on the uniqueness problem of representation for discrete event systems.

Theorem 1

Let $<\underline{\rho}, \underline{\phi}>$ be a discrete event dynamical system representation of S and C its state space. Then there always exists a surjective morphism $h : <\underline{\rho}, \underline{\phi}> \rightarrow <\underline{\rho}^*, \underline{\phi}^*>$, which is defined by $h(c) = \rho_0(c, \Lambda)$.

Corollary

Let $\langle \underline{\rho}, \underline{\phi} \rangle$ be a discrete event dynamical system representation of S and C its state space. Assume $\langle \underline{\rho}, \underline{\phi} \rangle$ is reduced. Then $\langle \underline{\rho}, \underline{\phi} \rangle$ and S(Λ)-realization are isomorphic.

The above theorem and its corollary shows that the S(Λ)-realization is minimal in the class of discrete event dynamical system representations of S.

In the following of this section we consider which property of a dynamical system representation of S implies its initial response function to be a discrete event-responce.

First we examine reachability.

Proposition 7

If a causal dynamical system representation $\langle \underline{\rho}, \underline{\phi} \rangle$ of S is reachable, then it is a discrete event dynamical system representation.

Corollary

Let $\langle \underline{\rho}, \underline{\phi} \rangle$ be a causal dynamical system representation of S. Assume $\langle \underline{\rho}, \underline{\phi} \rangle$ is reachable. Then there exists a surjective morphism from $\langle \underline{\rho}, \underline{\phi} \rangle$ to the S(Λ)-realization.

Definition 10: Reduced on nonevent

Let $\langle \underline{\rho}, \underline{\phi} \rangle$ be a dynamical system representation of S. Both ρ_0 and $\langle \underline{\rho}, \underline{\phi} \rangle$ are said to be reduced on nonevent if the following holds: If $\rho_0(c, \Lambda) = \rho_0(c', \Lambda)$ then $c = c'$.

Proposition 4-(3) shows that $\langle \underline{\rho}^*, \underline{\phi}^* \rangle$ is reduced on nonevent. The reducedness on nonevent is a stronger property than reducedness.

Proposition 8

If a dynamical system representation of a discrete event system is reduced on nonevent then it is reduced.

The reducedness of $\langle \underline{\rho}, \underline{\phi} \rangle$ on nonevent is so strong that we can provide an isomorphism between $\langle \underline{\rho}, \underline{\phi} \rangle$ and S(Λ)-realization.

Proposition 9

Let $\langle \underline{\rho}, \underline{\phi} \rangle$ be a causal dynamical system representation of S. Assume $\langle \underline{\rho}, \underline{\phi} \rangle$ is reduced on nonevent. Then there always exists an isomorphic morphism $h : \langle \underline{\rho}, \underline{\phi} \rangle \to \langle \underline{\rho}^*, \underline{\phi}^* \rangle$ defined by $h(c) = \rho_0(c, \Lambda)$.

The following property is interesting.

Definition 11: Response-determinacy on nonevent

If a dynamical system representation $\langle \underline{\rho}, \underline{\phi} \rangle$ of S satisfies the following property then both ρ_0 and $\langle \underline{\rho}, \underline{\phi} \rangle$ are said to have response-determinacy on nonevent.

$$(\forall c \in C)\ (\forall c' \in C)\ (\forall x \in C)[\rho_0(c, \Lambda) = \rho_0(c', \Lambda) \rightarrow \rho_0(c, x) = \rho_0(c', x)].$$

This property is rather strong as we can see below, though it is satisfied by $S(\Lambda)$-realization.

Proposition 10

Let $\langle \underline{\rho}, \underline{\phi} \rangle$ be a dynamical system representation of S. If $\langle \underline{\rho}, \underline{\phi} \rangle$ is reduced on nonevent then it has response-determinacy on nonevent.

Proposition 11

Let $\langle \underline{\rho}, \underline{\phi} \rangle$ be a dynamical system representation of S. If $\langle \underline{\rho}, \underline{\phi} \rangle$ has response-determinacy on nonevent then it is a discrete event dynamical system representation.

Corollary

Let $\langle \underline{\rho}, \underline{\phi} \rangle$ be a causal dynamical system representation of S. Assume $\langle \underline{\rho}, \underline{\phi} \rangle$ has response-determinacy on nonevent. Then there always exists a surjective morphism $h : \langle \underline{\rho}, \underline{\phi} \rangle \rightarrow \langle \underline{\rho}^*, \underline{\phi}^* \rangle$ defined by $h(c) = \rho_0(c, \Lambda)$.

5 DEVS state space representation

In this section the state space representation for a legitimate DEVS is defined and then the uniqueness of DEVS state space representation is provided. In general there can be many possible state space representations for a system specified by DEVS formalism. Some of such possible representations might not be specified by DEVS. What we will show, as an application of the realization theory in the previous sections, is that a reduced and reachable DEVS state space representation is isomorphic to $S(\Lambda)$-realization. This means that such DEVS state space representation is unique up to isomorphism in a class of representations.

A discrete event system specification DEVS is defined below.

Definition 12: Discrete event system specification :DEVS [1]

A discrete event system specification (DEVS) is a sextuple

$M = <A_M, S_M, B_M, \delta_M, \lambda_M, ta>$,

where A_M is a set called the external event set, S_M is a set called the sequential states set, B_M is a set called the output value set, δ_M is a function called the quasitransition function, λ_M is a set called the output function, and ta is a function called the time advance function with the following properties:

(i) $ta:S_M \to T^\infty$

(ii) $\delta_M:Q_M \times (A_M \cup \{\Lambda\}) \to S_M$,

$\delta_M(s, e, \Lambda) = \delta_\Lambda(s)$ for all $(s, e) \in Q_M$,

where $Q_M = \{(s, e) \mid s \in S_M \text{ and } 0 \le e \le ta(s)\}$, Λ is a special symbol not in A_M and $\delta_\Lambda:S_M \to S_M$.

(iii) $\lambda_M : Q_M \to B_M$.

A DEVS itself is not a state space representation nor does not show state transition mechanism. We need the following property to construct a state space representation from a DEVS.

Definition 13: Legitimacy of DEVS [1]

Let $M = <A_M, S_M, B_M, \delta_M, \lambda_M, ta>$ be a DEVS. Define $\delta:S \times I^+ \to S$ as $\delta(s, 0) = s$, and $\delta(s, k+1) = \delta_\Lambda(\delta(s, k))$ for each integer $k > 0$. Also define $\Sigma:S \times I^+ \to R$ as $\Sigma(s, 0) = 0$, and $\Sigma(s, k) = \sum_{p=0}^{k-1} ta(\delta(s, p))$ for each $k > 0$. M is said to be legitimate, if $\lim_{k \to \infty} \Sigma(s, k) = \infty$ holds.

In order to construct a state space representation for a legitimate DEVS M , we define $m_{s,e,\tau}$ under the same notation in the above definition according to [1].

$m_{s,e,\tau} = \max \{n \mid \Sigma(s, n) < e + \tau \}$

Thus $m_{s,e,\tau}$ is the maximum possible number of times of internal state transitions from the initial state (s, e), while $\Sigma(s, n)$ means the total elapsed time for (n -1) times internal transitions for the internal state s.

Define the input segments as follows. $\Omega_\Lambda = \{ \Lambda_\tau \mid [0, \tau) \to \{\Lambda\} \text{ for each } \tau \in T\}$. $\Omega_A = \{a_\tau \mid [0, \tau) \to A_M \cup \{\Lambda\}$, where $a_\tau(0) = a$ and $a_\tau(t) = \Lambda$ for each $t, 0 < t < \tau\}$. $\Omega_G = \Omega_A \cup \Omega_\Lambda$. $\Omega_G{}^1 = \Omega_G$, $\Omega_G{}^{i+1} = \Omega_G{}^i \bullet \Omega_G = \{\omega\omega' \mid \omega \in \Omega_G{}^i \text{ and } \omega' \in \Omega_G \}$. Here $\omega\omega'$ means that $\omega \bullet \sigma^t(\omega')$, if ω is a segment such as $\omega:[0, t) \to A_M \cup \{\Lambda\}$. $\Omega_G{}^+ = \cup_{i \in I^+} \Omega_G{}^i$. Each element of $\Omega_G{}^+$ has its unique mls (maximal length segment) decomposition which is defined below [1].

It is assumed that Ω_G is closed under shift-left operators, that is, $\lambda^t(\omega) \in \Omega_G{}^+$ for any $\omega:[t_0, t_1) \to A_M \cup \{\Lambda\}$ and any t, $t_0 \le t < t_1$. For a segment $\omega \in \Omega_G{}^+$, $\omega:[t_0, t_1) \to A_M \cup \{\Lambda\}$, the left segment of ω for t, $t_0 \le t < t_1$, is $\omega|_{[t_0, t)}$. We say that $\omega_1, \omega_2, \ldots$,

$\omega_n \in \Omega_G$ is an mls decomposition by Ω_G of $\omega \in \Omega_G^+$, if $\omega = \omega_1 \cdots \omega_n$ and for each i = 1, ..., n, ω_i is the longest element in Ω_G which is a left segment of $\omega_i \cdots \omega_n$.

Let X be a discrete event input space with the alphabet $P(A_M)$. X is the input space of the resultant state system S_D for the legitimate DEVS M. S_D is defined later.

The quasitransition function can be extended to the function $\delta_G: Q_M \times \Omega_G \to Q_M$ by $\delta_G(s, e, \Lambda_\tau) = (s, e+\tau)$, if $e+\tau \le ta(s)$ and $\delta_G(s, e, \Lambda_\tau) = \delta_G(\delta_\Lambda(s), 0, \Lambda_{e+\tau - ta(s)})$, otherwise. And $\delta_G(s, e, a_\tau) = \delta_G(\delta_M(s, e, a), 0, \Lambda_\tau)$. This definition is well-defined if M is legitimate. Define the output function $\mu: Q_M \times (A_M \cup \{\Lambda\}) \to Q_M$ by $\mu(s, e, z) = (s, e)$, if $z = \Lambda$ and $e < ta(s)$; $(\delta_\Lambda(s), 0)$, if $z = \Lambda$ and $e = ta(s)$; $(\delta_M(s, e, z), 0)$, if $z \neq \Lambda$.

For any $t \in T$ and $x^t \in X^t$, there exists the mls decomposition of x^t in Ω_G^+[1]. For each $\omega \in \Omega_G^+$ denote $D(\omega) = [t_0, t_1)$ if ω is a segment that is defined on $[t_0, t_1)$. Let $a^1 \bullet b^2 \bullet \ldots \bullet c^n$ be the mls decomposition of x_{0t}, where $a^1(0) = a$ and $a^1(t) = \Lambda$ for each $t \in D(a^1) = [0, t_1)$, $b^2(t_1) = b$ and $b^2(t) = \Lambda$ for each $t \in D(b^2)$, and so on. Define a state transition function $\phi = \{\phi_{tt'} \mid \phi_{tt'} : Q_M \times X_{tt'} \to Q_M$ and $t, t' \in T, t \le t'\}$ as follows. $\phi_{0t}(s, e, x_{0t}) = \delta_G(\ldots(\delta_G(\delta_G(\mu(s, e, a^1(0)), \Lambda^1), b^2), \ldots, c^n)$, and $\phi_{tt'}(s, e, x_{tt'}) = \phi_{0\tau}(s, e, \sigma^{-t}(x_{tt'}))$, where $\tau = t' - t$.

We define the system S_D for a legitimate DEVS M, which shows the state transition of M.

$(x, y) \in S_D \subseteq X \times (Q_M)^T$ iff there exists $(s, e) \in Q_M$ such that $y(t) = \mu(\phi_{0t}(s, e, x_{0t}), x(t))$ for any $t \in T$.

Since S_D represents the state transition mechanism of a legitimate DEVS M, it is called the state system of M. The pair $<\phi, \mu>$ for S_D defined from a legitimate DEVS is called the DEVS state space representation (of M).

The importance of the legitimacy is that the DEVS state space representation is always possible. This fact can be stated as follows.

Theorem 2 [1]

Let M be a legitimate DEVS. Then M can be extended to a time invariant state space representation.

Now the system defined by the transition function is actually a discrete event system.

Proposition 12

The state system S_D of a legitimate DEVS is stationary and past-determined from arbitrary positive time, and has a discrete event input space and the discrete event-determinacy.

Proposition 13

If the DEVS state space representation $<\phi, \mu>$ for a legitimate DEVS M is reachable then S_D is strongly stationary.

Since the DEVS formalism has been used so widely as a theoretical framework for modeling and discrete event simulation, it is believed that the definition of DEVS specification is natural and universal in some sense. Now we can provide its uniqueness in the following theorem, which shows the minimality of the DEVS state space representation in the class of dynamical system representations which are completely past-determined with its initial response function.

Theorem 3

If the dynamical system representation that is defined by the DEVS state space representation for a legitimate DEVS is reduced and reachable then it is isomorphic to $S_D(\Lambda)$-realization, where S_D is the state system of the legitimate DEVS.

The above theorem shows that a reduced and reachable DEVS representation is unique up to isomorphism in the class of discrete event dynamical system representations.

6 Conclusion

The construction shown here is the first attempt how to construct a state space representation for discrete event systems defined as input-output data. Also the state space representation, called $S(\Lambda)$-realization, has provides a solution to the uniqueness problem for discrete event systems. Through the minimality of $S(\Lambda)$-realization we have shown a universal property of DEVS formalism. In this sense DEVS formalism is shown to be a good framwork in modeling deiscrete event systems.

Two kinds of discreteness are usually considered for discrete event models. They are the discreteness of the the value set of inputs and that of input events. Usual computer programs for discrete event models assume the finiteness of input events that can be stated below.

event(x)$\cap$[t, t') is finite for any $x \in X$ and $t, t' \in T$, $t < t'$,
where event(x) = $\{t \mid x(t) \neq \Lambda\}$.

It is belived that this discreteness is essential for a discrete event system. And the DEVS formalism for discrete event models also assumes the condition. In the realization of this paper does not require this finiteness. The discreteness of the value space of the input of a system has been needed. Surprisingly enough, Theorem 3 shows that a reduced and reachable DEVS state space representation is unique up to isomorphism for the resultant discrete event system without the finiteness condition for the input events. That fact shows the ability of DEVS formalism in compactly describing state transition mechanism of systems. That is, if the DEVS formalism is used to describe a system whose input events are not restricted to be finite, and if it is legitimate, reduced and reachable, then it is a unique state space representation.

There are many other ways to describe "discrete event models." Examples are Petri-nets [4] and discrete event processes [5]. It is valuable to provide their mutual relationship in precise and exact manner. As a future topic the realization theory of this paper could play the role of a meta-theory for it.

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