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A Statistical Reexamination of the Solow Growth Model

by

Hideki Nakamura

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Hideki Nakamura*
Institute of Socio-Economic Planning, University of Tsukuba

Abstract

We reexamine the paper, "A contribution to the empirics of economic growth," by Mankiw, Romer and Weil (1992). Using a battery of specification error tests, it shows that neither the Solow nor an augmented Solow growth model incorporating exogenous human capital accumulation may be empirically appropriate in steady state specification. But, an augmented Solow model taking convergence to the steady state into account provides consistency with the cross-country data.

Correspondence: Hideki Nakamura
Institute of Socio-Economic Planning, University of Tsukuba
Tsukuba, Ibaraki 305, Japan
E-mail: hnakamur@shako.sk.tsukuba.ac.jp

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1. Introduction

There has been a thorough reworking of economic growth theory in past decade. This reworking is motivated by the empirical failure of the Solow (1956) growth model to explain the fact that the growth rate of income per capita of each country is uncorrelated with the initial income per capita. The main reformulation is carried out through endogenous growth theory advocated by Romer (1986,1990) and Lucas (1988). This theory can explain international differences in economic growth by factors such as human capital investment and R&D activities which are endogenous to the model. In contrast, the Solow growth model describes economic growth by exogenous factors such as physical capital investment, population growth and technological progress.

Mankiw, Romer and Weil (1992) investigated whether the Solow growth model is consistent with the cross-country data constructed by Summers and Heston (1988). They showed that an augmented Solow model which considers the accumulation of both physical and human capital provides a reasonably good fit to the data. They also examined whether poor countries tend to grow faster than rich countries. They reported evidence on convergence in contrast to endogenous growth models.

This paper reexamines the empirical study of Mankiw, Romer and Weil (1992) using a battery of specification error tests. We use the data which are shown in the appendix of Mankiw, Romer and Weil (1992). We shall show that the Solow model which assume the steady state may be misspecified even after introducing human capital. An augmented Solow model taking convergence into account will be shown to provide a statistically good description of the cross-country data.

In section 2 we explain the battery of specification error tests employed. We specify the Solow model assuming the steady state and show estimation results in section 3 and add human capital to the Solow model in section 4. In section 5 the Solow model taking convergence to the steady state into account are investigated. Section 6 summarizes the paper.

2. Test Statistics

The Jarque-Bera (1980) [J.-B.] statistic tests whether or not the residuals are distributed as normal. The J.-B. statistic is asymptotically distributed as $\chi^2(2)$ under the null hypothesis that residuals' measure of symmetry and kurtosis is that of the normal distribution. The alternative hypothesis is that residuals' distribution belongs to the Pearson family.

The Goldfeld and Quandt (1965) [G.-Q.], Breusch and Pagan (1979) [B.-P.] and White (1980) statistics test heteroscedasticity of residuals. The G.-Q. statistic is distributed as $F_{(n-p-2k)/2,(n-p-2k)/2}$ under the null hypothesis of homoscedasticity where n is the number of observations, p is the number of excluded observations which are located at the center and k is the number of regressors. The B.-P. statistic is asymptotically distributed as $\chi^2(p)$ under the null hypothesis of homoscedasticity where p is the number of variables in an auxiliary regression. We assume that the variance of disturbances is a linear function of the regressors under the alternative hypothesis of heteroscedasticity, that is, $p = k - 1$ where k is the number of regressors in the original regression and "1" is for a constant. The White statistic is asymptotically distributed as $\chi^2(k(k-1)/2)$ under the null hypothesis of homoscedasticity where k is the number of regressors in the original regression and $k(k-1)/2$ is the number of regressors in an auxiliary regression. The White test does not assume a specific form of heteroscedasticity.

Ramsey's (1969) regression specification error test [RESET] is intended to detect a nonzero mean of the disturbances. The RESET statistic under the null hypothesis of no missing explanatory variables is distributed as $F_{p-1,n-k-p+1}$ where p is the maximum power of the estimated dependent variable in the original regression, raised in an auxiliary regression, n is the number of observations and k is the number of regressors in the original regression. We examine $p = 2$ and 3 respectively.

The Hausman (1978) statistic is regarded as the test of independence. The test of independence examines whether or not the stochastic regressors are independent of the disturbances. Under the null hypothesis of independence, the Hausman statistic is asymptotically distributed as $\chi^2(p)$ where p is the number of stochastic regressors to be tested.

Because there is a possibility of break points in a sample which may give misleading inference, we use Brown, Durbin and Evans' (1975) cusum [Cusum], cusum of squares [CusumSq] tests and a sequence of Chow (1960) tests [SCT].[1] The Cusum and CusumSq tests use the recursive residuals. Each of Cusum, CusumSq test and SCT investigates the stability of coefficients of regressors. The Cusum test is useful when the departure from constancy of parameter vectors is systematic and the CusumSq test is in-

[1]Durbin (1969) provided critical values for significance levels of the cusum of squares test.

tended to catch haphazard rather than systematic types of instability. The SCT is a sequence of Chow tests, that is, the split point of a sample is moved sequentially. The Chow statistic is distributed as $F_{k,n-2k}$ where k is the number of regressors and n is the number of observations.

We assume in estimations of sections 3.2, 4.2 and 5.2 that there is no structural break and the disturbances are independent of regressors and are independently and identically distributed with the normal distribution which has mean zero and a finite variance.[2] We should keep in mind that the equation is usually assumed to be otherwise well-behaved when one property is tested.

3. The Textbook Solow Model

3.1 Specification

We shall follow Mankiw, Romer and Weil (1992) in reviewing the Solow model.[3] We assume a Cobb-Douglas production function whose technological progress is the Harrod neutral type:

$$Y(t) = K(t)^{\alpha}(A(t)L(t))^{1-\alpha}, \tag{1}$$

where Y is output, K is capital, L is labor, A represents the level of technology, and $0 < \alpha < 1$. L and A are assumed to grow exogenously at rates n and g respectively:

$$L(t) = L(0)e^{nt}, \tag{2}$$

[2]The regressors were treated as nonstochastic variables in Goldfeld and Quandt (1965), Breusch and Pagan (1979), Brown, Durbin and Evans (1975) and Chow (1960).

[3]All equations and notations of variables in our study are the same as Mankiw, Romer and Weil (1992).

$$A(t) = A(0)e^{gt}. \quad (3)$$

The evolution of the stock of capital per effective unit of labor is governed by

$$\dot{k}(t) = sy(t) - (n + g + \delta)k(t) = sk(t)^{\alpha} - (n + g + \delta)k(t), \quad (4)$$

where $k \equiv K/AL$, $s \equiv I/Y$, $y \equiv Y/AL$ and δ is the rate of depreciation where I is the investment in capital. k converges to a steady-state value k^* defined by

$$k^* \equiv [s/(n + g + \delta)]^{1/(1-\alpha)}. \quad (5)$$

Substituting k^* into the production function and taking logarithm, a steady-state income per capita is given by

$$\ln(\frac{Y(t)}{L(t)}) = \ln A(0) + gt + \frac{\alpha}{1-\alpha}\ln(s) - \frac{\alpha}{1-\alpha}\ln(n + g + \delta). \quad (6)$$

g and δ are assumed to be constant across countries. In contrast, $A(0)$ is assumed as follows: $\ln A(0) = a + \epsilon$, where a is a constant and ϵ is a country-specific shock and is independently and identically distributed with the normal distribution which has mean zero and a finite variance. Therefore, log income per capita at time 0 becomes

$$\ln(\frac{Y}{L}) = a + \frac{\alpha}{1-\alpha}\ln(s) - \frac{\alpha}{1-\alpha}\ln(n + g + \delta) + \epsilon. \quad (7)$$

3.2 Results

Mankiw, Romer and Weil (1992) used data from the Real National Accounts constructed by Summers and Heston (1988). The annual data cover the period from 1960 to 1985. Y/L is the real GDP in 1985 divided by the working-age population in that year where working age is from 15 to 64. I/Y is the fraction of real investment including government investment in real GDP averaged over the period 1960 to 1985. n is the average growth rate of the working-age population during 1960 to 1985. $g + \delta$ is assumed to be 0.05 across countries.[4]

Mankiw, Romer and Weil (1992) considered three samples of countries. The first consists of all countries in which data are available and oil production is not the key industry. The first sample is called the non-oil sample and the sample size is 98. The second sample excludes small countries from the first sample because idiosyncratic factors may be important in the determination of their real income. The small countries are defined as countries whose real output figures are extremely small in the primary data by Summers and Heston (1988) or whose populations in 1960 are less than one million. This sample is called the intermediate sample and the size is 75. The final sample is subsample of both the first and the second samples and consists of the 22 OECD countries whose populations are greater than one million. This sample is named the OECD sample.[5]

If the sample countries have different economic structure, *that is*, there is international differences in the parameters α and a, we should consider the existence of structural differences in the sample. Because economic structure may be distinguished by the initial level of development, the existence of structural differences in the sample may appear according to the initial

[4] Mankiw, Romer and Weil (1992) indicated that reasonable changes in this assumption influence the estimates little.

[5] Statistics using asymptotic theory may suffer from the small sample size.

income per capita. Therefore, we rearrange the data in ascending order according to the initial income per capita, *that is*, the real GDP in 1960 divided by the working population in that year.[6] We set p to be 32, 23 and 6 in the non-oil, intermediate and OECD samples respectively in the Goldfeld and Quant test where p in the number of excluded observations which are located at the center.[7] We assume that $\ln(I/Y)$ is a stochastic regressor to be tested in the test of independence and a constant, $\ln(n+g+\delta)$ and the lagged variable of $\ln(I/Y)$ are used as the instrumental variables for unrestricted estimation and a constant and the lagged variable of $\ln(I/Y) - \ln(n+g+\delta)$ are used as the instruments for restricted estimation.[8]

Each estimation is conducted by the ordinary least squares method with or without a restriction where the restriction is the equality of the coefficients on $\ln(I/Y)$ and $\ln(n+g+\delta)$. Both unrestricted and restricted estimation results of equation (7) are shown in table 1. The signs of saving and population growth coefficients agree with theory in all samples.[9] These coefficients are highly significant except for the OECD sample. Mankiw, Romer and

[6]The Goldfeld and Quandt test for heteroscedasticity, the test of independence and the stability tests of parameters may be affected by the arrangement of cross-section data.

[7]See section 4.2 in our study.

[8]The sorted data from number 1 to $n-1$ were used as the instruments for estimation of the data from number 2 to n. It may be difficult to find the other instruments in our study as discussed in Mankiw, Romer and Weil (1992). When the variable $ln(n+g+\delta)$ was taken as one of stochastic regressors to be tested, the results did not change so much.

[9]Although the estimates of coefficients in our study did not accurately coincide with those of Mankiw, Romer and Weil (1992), the differences were very small and may not affect our conclusion.

Weil (1992) showed that the restriction that the coefficients on $\ln(s)$ and $\ln(n+g+\delta)$ be equal is not rejected in any sample.

The Jarque-Bera statistic for normality is significant at the 5% significance level for restricted estimation in the intermediate sample. Neither statistics for heteroscedasticity nor RESET for missing explanatory variables is significant in any case. The Hausman statistic for independence is significant at the 5% significance level except for unrestricted estimation in the intermediate sample and restricted estimation in the OECD sample.

These facts may be due to an existence of break points in a sample or a misspecification of the model or a violation of underlying assumptions. Figures 1, 2 and 3 are the results of parameter stability tests for the restricted Solow model.[10] Three figures for each of non-oil, intermediate and OECD sample are for the Cusum, CusumSq tests and the SCT respectively.

The Cusum graph exceeds the upper 5% significance level except for the low initial-income group in all samples.[11] The CusumSq graph stays below the lower 5% significance level for the middle initial-income group in the non-oil sample. The SCT shows that the P-values are almost zero at all points in each sample.[12]

[10]The probability of type I error in the parameter stability tests was considered at each point. Because the results of Cusum, CusumSq tests and SCT for unrestricted estimation were similar to those for restricted estimation, the results for unrestricted estimation were not reported.

[11]The data were sorted in ascending order according to the initial income per capita.

[12]The Chow test which considered nonlinearity of parameters was conducted by using the technique which was discussed in chapter 11 of Davidson and MacKinnon (1993). This SCT graph in each sample was similar to the graph which was reported in this study.

These results seem to be due to a misspecification of the Solow model or a violation of underlying assumptions rather than the existence of break points in a sample.[13]

4. The Augmented Solow Model and Steady State

4.1 Specification

We investigate the effect of adding human-capital accumulation to the Solow model because the importance of human capital to the process of growth has been recently stressed by economists.

We assume the production function as follows:

$$Y(t) = K(t)^{\alpha} H(t)^{\beta} (A(t)L(t))^{1-\alpha-\beta}, \tag{8}$$

where H is the stock of human capital and $\alpha + \beta < 1$.[14]

Human capital is assumed to depreciate at the same rate as physical capital. The evolution of physical and human capital per effective unit of labor are governed by

$$\dot{k}(t) = s_k y(t) - (n + g + \delta)k(t), \tag{9a}$$

$$\dot{h}(t) = s_h y(t) - (n + g + \delta)h(t), \tag{9b}$$

[13] When both the low and the high initial income samples for which each of the non-oil and the intermediate samples was approximately, equally splitted were regressed, the conclusion did not change.

[14] Our model would become an endogenous growth model with the assumption of $\alpha+\beta = 1$.

where $h \equiv H/AL$, s_k and s_h are respectively the fractions of output invested in physical and human capital.

Both k and h converge to steady-state values defined by

$$k^* = (\frac{s_k^{1-\beta} s_h^{\beta}}{n+g+\delta})^{1/(1-\alpha-\beta)} \qquad and \qquad h^* = (\frac{s_k^{\alpha} s_h^{1-\alpha}}{n+g+\delta})^{1/(1-\alpha-\beta)}. \tag{10}$$

Substituting equation (10) into the production function and taking logarithms, income per capita becomes

$$\ln(\frac{Y(t)}{L(t)}) = \ln A(0) + gt - \frac{\alpha+\beta}{1-\alpha-\beta}\ln(n+g+\delta) + \frac{\alpha}{1-\alpha-\beta}\ln(s_k)$$
$$+\frac{\beta}{1-\alpha-\beta}\ln(s_h). \tag{11}$$

There is an alternative way to determine income per capita. Combining equation (11) with equation (10) yields

$$\ln(\frac{Y(t)}{L(t)}) = \ln A(0) + gt + \frac{\alpha}{1-\alpha}\ln(s_k) - \frac{\alpha}{1-\alpha}\ln(n+g+\delta) + \frac{\beta}{1-\alpha}\ln(h^*). \tag{12}$$

Because the level of human capital is included in the disturbances in equation (7), there is a possibility that omitting the human-capital term biases the estimation of equation (7).

4.2 Results

$\ln(s_h)$ is assumed to be proportional to $\ln(school)$ where *school* is the rate of the working-age population in secondary school which is averaged for the period 1960 to 1985. We assume that (i) $\ln(I/Y)$ and $\ln(school)$ are the stochastic regressors to be tested in the test of independence, (ii) a constant, $\ln(n+g+\delta)$ and the lagged variables of $\ln(I/Y)$ and $\ln(school)$ are used as the instrumental variables for unrestricted estimation, and (*iii*) a constant and the lagged variables of $\ln(I/Y)-\ln(n+g+\delta)$ and $\ln(school)-\ln(n+g+\delta)$ are used as the instruments for restricted estimation.

The unrestricted and restricted estimation results of equation (11) are shown in table 2 where the restriction is the equality among the coefficients on $\ln(I/Y)$, $\ln(n+g+\delta)$ and $\ln(school)$. The sign conditions for saving, population growth and human capital measure agree with the theory in all samples. The coefficient of human capital measure is highly significant in all cases and all coefficients of saving and population growth are significant except for the OECD sample. Adding the human capital measure raises the adjusted R^2 in all samples. Mankiw, Romer and Weil (1992) showed that the restriction among the coefficients on $\ln(I/Y)$, $\ln(n+g+\delta)$ and $\ln(school)$ is not rejected and concluded that the augmented Solow model provides a reasonable description of the data.

The White statistic for heteroscedasticity is significant for the restricted estimation in the OECD sample. Each of RESET(2) and RESET(3) for missing explanatory variables is highly significant for both unrestricted and restricted estimations in the non-oil sample. The Hausman statistic for independence is highly significant for both estimations in the non-oil sample and for restricted estimation in the intermediate sample.

The results of parameter stability tests for the restricted augmented Solow model are reported in figures 4, 5 and 6. The Cusum graph exceeds the upper 5% significance level from the middle initial-income group in each of non-oil and intermediate sample. The CusumSq graph approaches the lower 5% significance level for the middle initial-income group in each of non-oil and intermediate samples. The SCT reports that the P-values are almost zero at all points in each sample.

These results may originate from a misspecification of the augmented Solow model or a violation of underlying assumptions.[15]

5. Convergence

5.1 Specification

Endogenous growth models are characterized by the assumption of non-decreasing returns to scale with respect to reproducible inputs. This assumption implies that a country which saves more grows faster indefinitely and income per capita needs not converge. On the other hand, in the Solow model income per capita in a country reaches the steady state which depends on exogenous parameters of the country as explained in sections 3 and 4. In this section we look at the converging path of income per capita in the Solow model. The speed of convergence in the augmented Solow model is given as follows by approximating around the steady state:

$$\frac{d\ln(y(t))}{dt} = \lambda[\ln(y^*) - \ln(y(t))], \tag{13}$$

where $\lambda \equiv (n+g+\delta)(1-\alpha-\beta)$ and y^* is the steady-state level of income per capita.

From equation (13), the following equation can be derived:

$$\ln(y(t)) = (1-e^{-\lambda t})\ln(y^*) + e^{-\lambda t}\ln(y(0)). \tag{14}$$

[15]The conclusion did not change when both the low and the high initial income samples for which each of the non-oil and the intermediate samples was approximately, equally splitted were regressed.

Subtracting $\ln(y(0))$ in equation (14) yields

$$\ln(y(t)) - \ln(y(0)) = (1 - e^{-\lambda t}) \ln(y^*) - (1 - e^{-\lambda t}) \ln(y(0)). \tag{15}$$

This is the equation for unconditional convergence.

Substituting for y^*, equation (15) becomes

$$\ln(y(t)) - \ln(y(0)) = (1 - e^{-\lambda t}) \ln A(0) + (1 - e^{-\lambda t}) \frac{\alpha}{1 - \alpha - \beta} \ln(s_k) +$$

$$(1 - e^{-\lambda t}) \frac{\beta}{1 - \alpha - \beta} \ln(s_h) - (1 - e^{-\lambda t}) \frac{\alpha + \beta}{1 - \alpha - \beta} \ln(n + g + \delta) - (1 - e^{-\lambda t}) \ln(y(0)). \tag{16}$$

This equation represents conditional convergence.

5.2 Results

$y(0)$ and $y(t)$ are respectively the real GDP in 1960 and 1985 divided by the working-age population in that year.

The results of tests for unconditional convergence are shown in table 3. The coefficient of initial income per capita is not significant and the adjusted R^2 seems to be low in each of non-oil and intermediate sample. We can see a trend toward convergence in the OECD sample. Mankiw, Romer and Weil (1992) indicated that the estimate of λ is significant in the nonlinear estimation for the OECD sample.

Only the RESET(3) for missing explanatory variables is significant in the non-oil sample. Figures 7, 8 and 9 report the results of parameter stability tests for unconditional convergence. The SCT shows that there seems to be a structural change in the middle initial-income group for the non-oil sample.

It may be due to the small sample size that the P-values stay below 0.05 in the low initial-income group for the OECD sample.

These results imply that the unconditional convergence may be valid for the OECD sample.

The tests for convergence of the Solow model are reported in table 4. The sign conditions agree with the theory and all coefficients are highly significant for restricted estimation in each sample.

Each of Jarque-Bera statistic for normality and Goldfeld-Quandt statistic for heteroscedasticity is significant for both unrestricted and restricted estimations in the intermediate sample. The Breusch-Pagan statistic for heteroscedasticity is also significant for unrestricted estimation in each of non-oil and intermediate samples. Figures 10, 11 and 12 are the results of parameter stability tests for convergence in the Solow model. The SCT indicates that there may be a structural change in the middle initial-income group for the non-oil sample.

These results imply that the Solow model which incorporates convergence to a steady state may be appropriate for the OECD sample.

The tests for convergence of the augmented Solow model are shown in table 5. The sign conditions agree with the theory and the coefficient of $\ln(school)$ is highly significant in each of non-oil and intermediate samples. *That is*, the human capital measure may be an omitted variable for the convergence tests of the Solow model. Mankiw, Romer and Weil (1992) showed that the restriction among the coefficients on $\ln(I/Y)$, $\ln(n+g+\delta)$ and $\ln(school)$ is not rejected and convergence occurs at about the rate which the augmented Solow model predicts.

Only the Jarque-Bera statistic for normality is significant at the 5% significance level for both unrestricted and restricted estimations in the intermediate sample. When the residuals of intermediate sample are plotted, there seem to exist a few outliers. It may be due to those outliers that the statistic for normality is rejected. Figures 13, 14 and 15 report the results of parameter stability tests for convergence in the augmented Solow model. Though each of Cusum and CusumSq graphs for the non-oil sample and the CusumSq graph for the intermediate sample approaches the upper 5% significance level, it does not exceed considerably. The Cusum, CusumSq tests and the SCT show that there may not be a structural change in any sample.

Overall, the Solow model taking convergence to the steady state into account provides a reasonably good fit to the OECD sample and the model specification of the augmented Solow model which considers convergence to the steady state may be appropriate in the non-oil and intermediate samples. Because the non-oil and intermediate samples include less advanced countries compared to the OECD sample, we can infer that the human capital investment considered in our study may be important for economic growth in an early stage and may become unnecessary in a later stage.[16] We use the percentage of the working-age population in secondary school as the variable of human capital investment in our study. This variable can capture a lower level of educational attainment of countries. Therefore, it may be reasonable that human capital investment used in our study is not significant for eco-

[16]We confirmed this observation by regressing both the low and the high initial income samples for which each of the non-oil and the intermediate samples was approximately, equally splitted.

nomic growth in the OECD sample. If we use a variable which indicates a higher level of educational attainment as human capital investment, such a variable would be significant for economic growth in a later stage.

6. Concluding Remarks

Mankiw, Romer and Weil (1992) examined whether or not the Solow model can explain international differences in income per capita. They showed that international differences in income per capita can be explained by accumulation of physical and human capital and population growth while maintaining the assumption of decreasing returns and convergence at about the rate implying by the Solow model.

This paper casts doubt on the results of Mankiw, Romer and Weil (1992) that the augmented Solow model which assumes the steady state can explain the cross-section data. The assumption that all countries stay the steady state at 1985 may be valid in neither the Solow nor the augmented Solow model and empirical investigation of the Solow model which assume the steady state may be misleading. We reinforced their results that the Solow model which incorporates convergence to the steady state may be valid for the OECD sample and the augmented Solow model taking convergence to the steady state into account can describe the non-oil and intermediate samples reasonably well. Therefore, investment of physical and human capital for which human capital investment used in our study can represent a lower level of educational attainment may be necessary for economic growth at an early stage but investment of human capital may not be important in a later stage.[17]

[17] Azariadis and Drazen (1990) considered the Diamond (1965) model that multiple, locally stable stationary states can exist where the multiplicity comes from increasing social returns to scale in human capital accumulation. Their study can describe threshold externality in human capital accumulation.

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Table 1 Estimation of the Solow model

\ sample	non-oil	interm.	OECD	\ sample	non-oil	interm.	OECD
constant	5.43	5.35	8.02	constant	6.87	7.09	8.62
t-value	3.43**	3.46**	3.19**	t-value	57.00**	48.71**	16.17**
$\ln(I/Y)$	1.42	1.32	0.50	$\ln(I/Y)-$			
t-value	9.95**	7.71**	1.15	$\ln(n+g+\delta)$	1.49	1.49	0.55
$\ln(n+g+\delta)$	-1.99	-2.02	-0.74	t-value	11.93**	10.29**	1.52
t-value	-3.53**	-3.78**	-0.87				
$\bar{R}^2$	0.59	0.59	0.01	$\bar{R}^2$	0.59	0.59	0.06
s.e.e.	0.69	0.61	0.38	s.e.e.	0.69	0.61	0.37
J.-B.	2.63	5.65	2.57	J.-B.	3.72	8.91*	2.82
B.-P.	4.61	3.96	3.38	B.-P.	0.33	0.00	2.91
G.-Q.	0.75	0.46	0.24	G.-Q.	0.74	0.46	0.24
White	5.27	4.08	3.23	White	0.01	0.21	2.57
RESET(2)	3.60	0.47	1.62	RESET(2)	3.18	0.48	1.26
RESET(3)	1.83	0.36	2.49	RESET(3)	1.66	0.26	2.05
Hausman	4.90*	3.03	6.29*	Hausman	10.63**	7.57**	2.88

Note: interm. is the abbreviation of intermediate, $\bar{R}^2$ is the adjusted multiple coefficient of determination, s.e.e. is the standard error of regression, J.-B. is asymptotically distributed as $\chi^2(2)$ under normality, G.-Q. is distributed as $F_{(n-p-2k)/2,(n-p-2k)/2}$ under homoscedasticity where $k=3$ for unrestricted estimation and $k=2$ for restricted estimation, B.-P. is asymptotically distributed as $\chi^2(2)$ under homoscedasticity for unrestricted estimation and as $\chi^2(1)$ for restricted estimation, White is asymptotically distributed as $\chi^2(3)$ under homoscedasticity for unrestricted estimation and as $\chi^2(1)$ for restricted estimation, RESET(p) is distributed as $F_{p,n-k-p+1}$ under no missing explanatory variables, Huasman is asymptotically distributed as $\chi^2(1)$ under independence, the rejection region is set at one side except for the $t-$test to a constant and the G.-Q. test, all values are rounded, n is the sample size, $n = 98$ in the non-oil sample, $n = 75$ in the intermediate sample and $n = 22$ in the OECD sample.

Table 2 Estimation of the augmented Solow model

\ sample	non-oil	interm.	OECD	\ sample	non-oil	interm.	OECD
constant	6.84	7.79	8.64	constant	7.85	7.97	8.72
t-value	5.81**	6.53**	3.90**	t-value	56.08**	51.58**	18.70**
$\ln(I/Y)$	0.70	0.70	0.28	$\ln(I/Y)-$			
t-value	5.25**	4.65**	0.71	$\ln(n+g+\delta)$	0.74	0.71	0.28
$\ln(n+g+\delta)$	−1.75	−1.50	−1.08	t-value	5.97**	5.15**	0.85
t-value	−4.20**	−3.72**	−1.42	$\ln(school)-$			
$\ln(school)$	0.65	0.73	0.77	$\ln(n+g+\delta)$	0.66	0.73	0.77
t-value	9.00**	7.67**	2.62**	t-value	9.06**	7.87**	2.70**
$\bar{R}^2$	0.78	0.77	0.24	$\bar{R}^2$	0.78	0.77	0.28
s.e.e.	0.51	0.45	0.33	s.e.e.	0.51	0.45	0.32
J.-B.	2.01	1.94	0.17	J.-B.	2.58	2.06	0.17
B.-P.	3.69	2.41	5.51	B.-P.	1.48	0.33	5.35
G.-Q.	1.04	0.82	0.30	G.-Q.	1.02	0.72	0.39
White	6.31	5.79	9.87	White	2.13	3.93	8.25*
RESET(2)	16.74**	1.55	1.86	RESET(2)	15.15**	1.25	1.75
RESET(3)	8.65**	0.77	0.90	RESET(3)	8.27**	0.62	0.83
Hausman	11.84**	1.62	0.06	Hausman	12.04**	10.01**	0.01

Note: G.-Q. is distributed as $F_{(n-p-2k)/2,(n-p-2k)/2}$ under homoscedasticity where $k = 4$ for unrestricted estimation and $k = 3$ for restricted estimation, B.-P. is asymptotically distributed as $\chi^2(3)$ under homoscedasticity for unrestricted estimation and as $\chi^2(2)$ for restricted estimation, White is asymptotically distributed as $\chi^2(6)$ under homoscedasticity for unrestricted estimation and as $\chi^2(3)$ for restricted estimation, RESET(p) is distributed as $F_{p,n-k-p+1}$ under no missing explanatory variables, and Huasman is asymptotically distributed as $\chi^2(2)$ under independence.

Table 3 Tests for unconditional convergence

\ sample	non-oil	interm.	OECD
constant	−0.27	0.59	3.69
t-value	−0.70	1.36	5.38**
$\ln(y(60))$	0.094	−0.004	−0.34
t-value	1.90	−0.07	−4.34**
$\bar{R}^2$	0.03	−0.01	0.46
s.e.e.	0.44	0.41	0.18
J.-B.	0.26	0.62	2.26
B.-P.	0.34	0.16	2.50
G.-Q.	0.86	0.58	0.47
White	0.44	0.22	2.49
RESET(2)	1.73	1.96	3.54
RESET(3)	4.50*	2.13	2.50
Hausman	0.52	0.05	0.65

Note: G.-Q. is distributed as $F_{(n-p-2k)/2,(n-p-2k)/2}$ under homoscedasticity where $k = 2$, B.-P. is asymptotically distributed as $\chi^2(1)$ under homoscedasticity, White is asymptotically distributed as $\chi^2(1)$ under homoscedasticity, RESET(p) is distributed as $F_{p,n-k-p+1}$ under no missing explanatory variables, and Hausman is asymptotically distributed as $\chi^2(1)$ under independence.

Table 4 Tests for convergence: the Solow model

\ sample	non-oil	interm.	OECD	\ sample	non-oil	interm.	OECD
constant	1.92	2.25	2.14	constant	1.11	1.82	3.10
t-value	2.30**	2.63**	1.81	t-value	3.13**	4.65**	5.15**
$\ln(y(60))$	−0.14	−0.23	−0.35	$\ln(y(60))$	−0.15	−0.23	−0.35
t-value	−2.71**	−3.98**	−5.32**	t-value	−2.94**	−4.19**	−5.37**
$\ln(I/Y)$	0.65	0.65	0.39	$\ln(I/Y)-$			
t-value	7.47**	6.22**	2.22*	$\ln(n+g+\delta)$	0.62	0.62	0.47
$\ln(n+g+\delta)$	−0.30	−0.46	−0.77	t-value	7.50**	6.59**	3.13**
t-value	−0.99	−1.49	−2.22*				
$\bar{R}^2$	0.38	0.35	0.62	$\bar{R}^2$	0.38	0.36	0.63
s.e.e.	0.35	0.33	0.15	s.e.e.	0.35	0.32	0.15
J.-B.	2.36	14.71**	3.13	J.-B.	1.94	13.29**	1.16
B.-P.	9.15*	8.90*	4.43	B.-P.	4.20	4.59	1.13
G.-Q.	0.68	0.35*	0.72	G.-Q.	0.68	0.38*	0.82
White	11.39	10.34	8.11	White	4.34	4.30	5.07
RESET(2)	0.05	1.37	0.28	RESET(2)	0.10	1.15	0.28
RESET(3)	0.52	0.73	0.74	RESET(3)	0.24	0.62	0.14
Hausman	0.37	0.00	1.30	Hausman	0.00	0.42	1.15

Note: G.-Q. is distributed as $F_{(n-p-2k)/2,(n-p-2k)/2}$ under homoscedasticity where $k = 4$ for unrestricted estimation and $k = 3$ for restricted estimation, B.-P. is asymptotically distributed as $\chi^2(3)$ under homoscedasticity for unrestricted estimation and as $\chi^2(2)$ for restricted estimation, White is asymptotically distributed as $\chi^2(6)$ under homoscedasticity for unrestricted estimation and as $\chi^2(3)$ for restricted estimation, RESET(p) is distributed as $F_{p,n-k-p+1}$ under no missing explanatory variables, and Huasman is asymptotically distributed as $\chi^2(2)$ under independence.

Table 5 Tests for convergence : the augmented Solow model

\ sample	non-oil	interm.	OECD	\ sample	non-oil	interm.	OECD
constant	3.02	3.71	2.76	constant	2.46	3.09	3.55
t-value	3.65**	4.08**	2.29*	t-value	5.19**	5.83**	5.61**
$\ln(y(60))$	−0.29	−0.37	−0.40	$\ln(y(60))$	−0.30	−0.37	−0.40
t-value	−4.68**	−5.43**	−5.67**	t-value	−4.93**	−5.57**	−5.81**
$\ln(I/Y)$	0.52	0.54	0.33	$\ln(I/Y)-$			
t-value	6.03**	5.26**	1.91*	$\ln(n+g+\delta)$	0.50	0.51	0.40
$\ln(n+g+\delta)$	−0.51	−0.54	−0.86	t-value	6.09**	5.33**	2.61**
t-value	−1.75*	−1.89*	−2.56*	$\ln(school)-$			
$\ln(school)$	0.23	0.27	0.23	$\ln(n+g+\delta)$	0.24	0.27	0.24
t-value	3.89**	3.37**	1.57	t-value	3.98**	3.32**	1.69
$\bar{R}^2$	0.46	0.43	0.65	$\bar{R}^2$	0.47	0.44	0.66
s.e.e.	0.33	0.30	0.15	s.e.e.	0.33	0.30	0.15
J.-B.	1.52	9.15*	1.10	J.-B.	1.57	8.92*	0.29
B.-P.	8.82	8.19	9.04	B.-P.	4.48	3.77	5.08
G.-Q.	0.77	0.45	0.60	G.-Q.	0.77	0.43	0.63
White	17.04	16.14	13.90	White	6.85	11.70	7.35
RESET(2)	1.26	0.04	0.17	RESET(2)	0.92	0.04	0.11
RESET(3)	0.95	0.84	0.21	RESET(3)	0.72	0.42	0.07
Hausman	0.26	0.00	0.54	Hausman	0.14	0.71	0.20

Note: G.-Q. is distributed as $F_{(n-p-2k)/2,(n-p-2k)/2}$ under homoscedasticity where $k = 5$ for unrestricted estimation and $k = 4$ for restricted estimation, B.-P. is asymptotically distributed as $\chi^2(4)$ under homoscedasticity for unrestricted estimation and as $\chi^2(3)$ for restricted estimation, White is asymptotically distributed as $\chi^2(9)$ under homoscedasticity for unrestricted estimation and as $\chi^2(6)$ for restricted estimation, RESET(p) is distributed as $F_{p,n-k-p+1}$ under no missing explanatory variables, and Huasman is asymptotically distributed as $\chi^2(3)$ under independence.

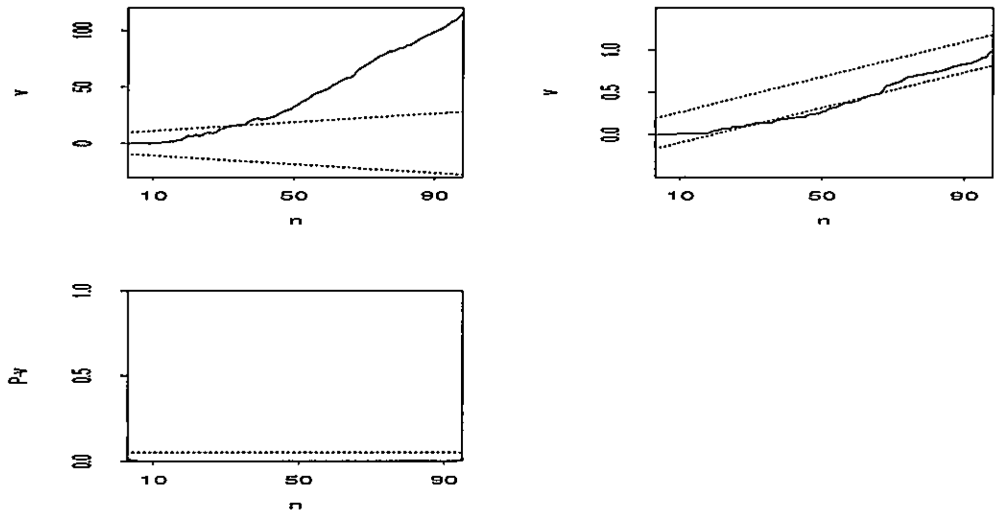

Figure 1 Cusum, CusumSq and SCT (Solow, non-oil)

Note: All figures are those of restricted estimation, the vertical and horizontal axis indicates the values of test statistic and the numbers of data respectively, figures on the upper-left and -right side are those of Cusum and CusumSq tests respectively, the solid line represents the values of test statistic and the dotted lines are the 5% significance lines, figure on the lower-left side is that of SCT, solid line shows the P-values of Chow test statistic, and dotted line is the 5% significance line.

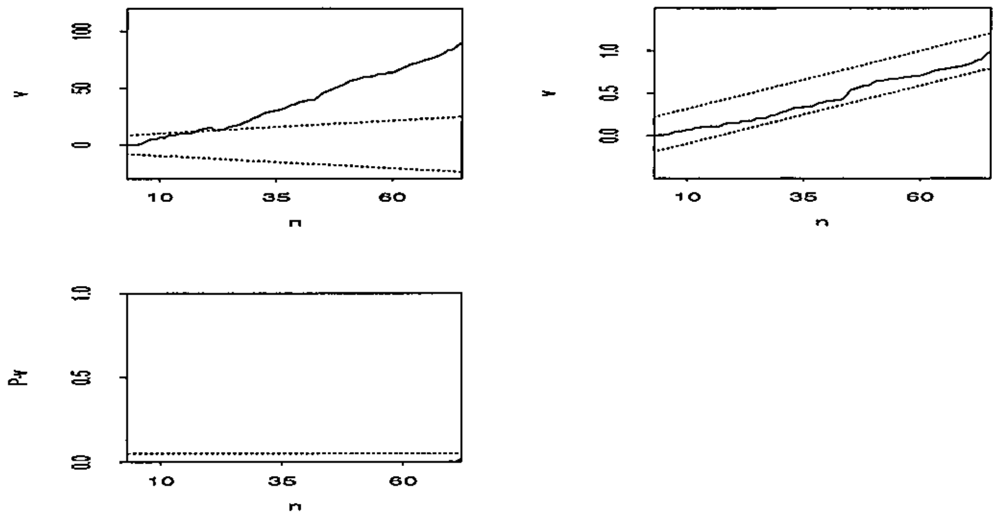

Figure 2 Cusum, CusumSq and SCT (Solow, interm.)

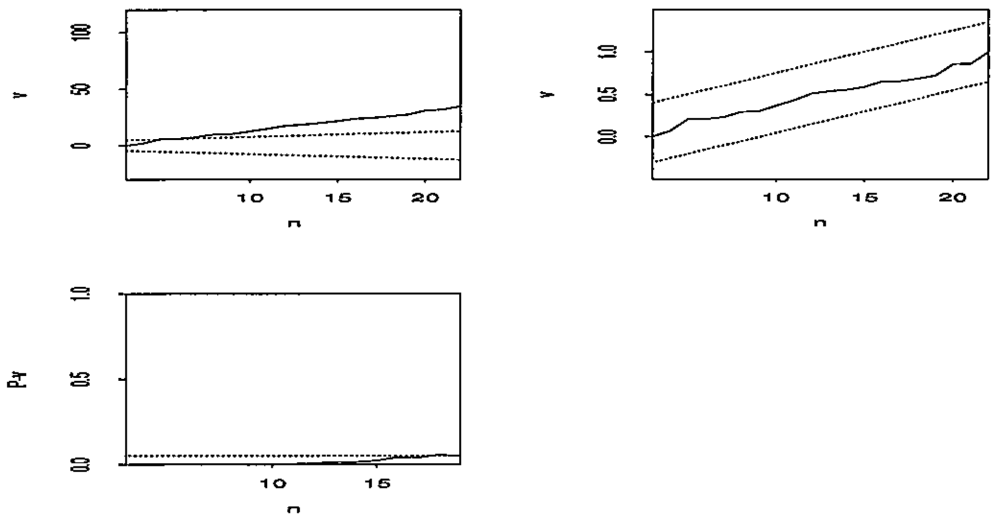

Figure 3 Cusum, CusumSq and SCT (Solow, OECD)

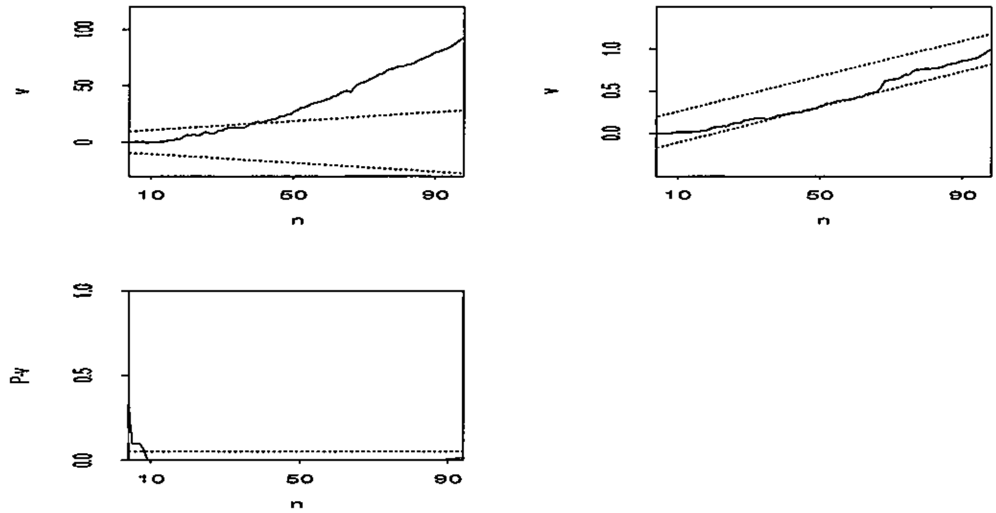

Figure 4 Cusum, CusumSq and SCT (augmented, non-oil)

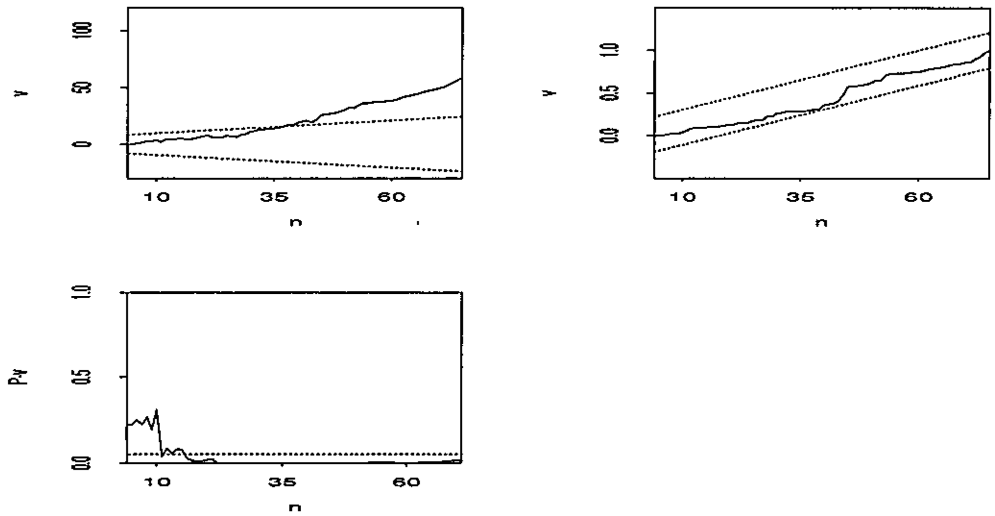

Figure 5 Cusum, CusumSq and SCT (augmented, intermed.)

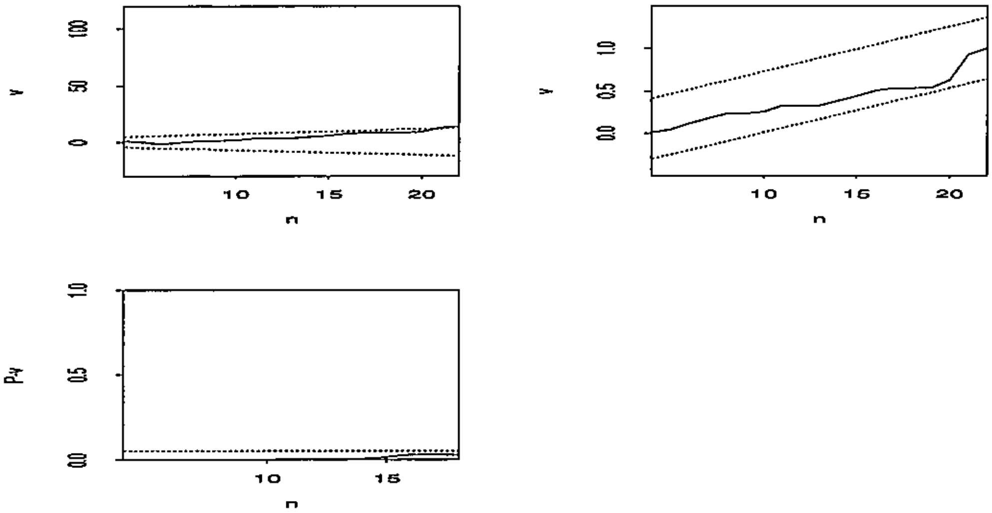

Figure 6 Cusum, CusumSq and SCT (augmented, OECD)

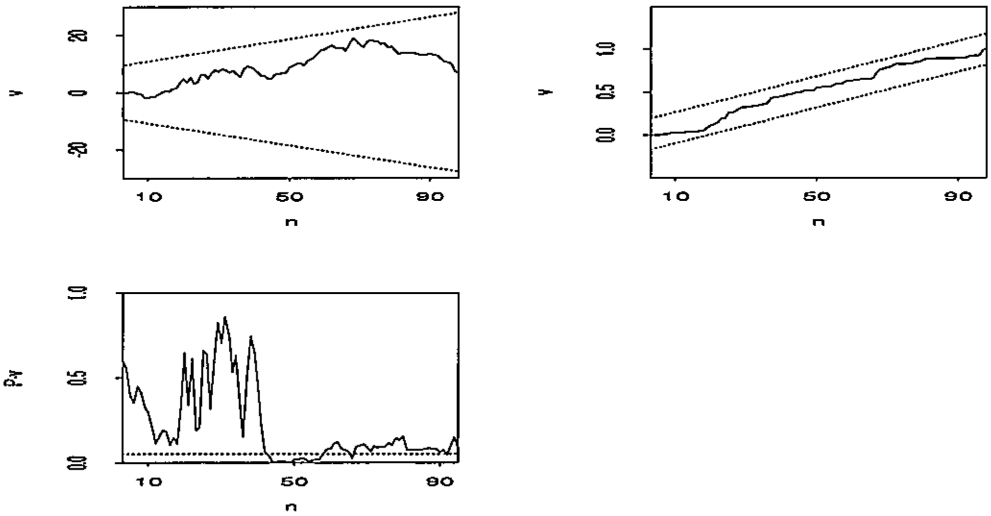

Figure 7 Cusum, CusumSq and SCT (unconditional convergence, non-oil)

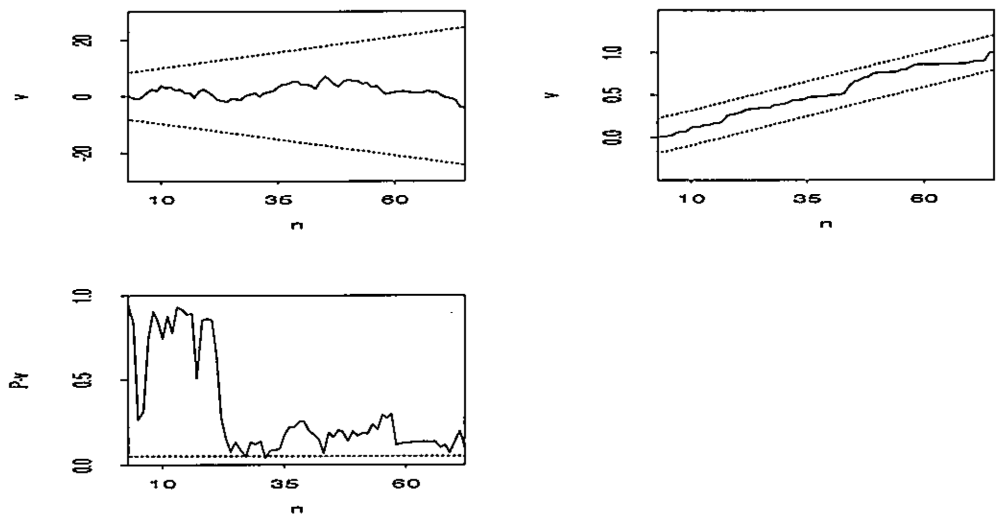

Figure 8 Cusum, CusumSq and SCT (unconditional convergence, interm.)

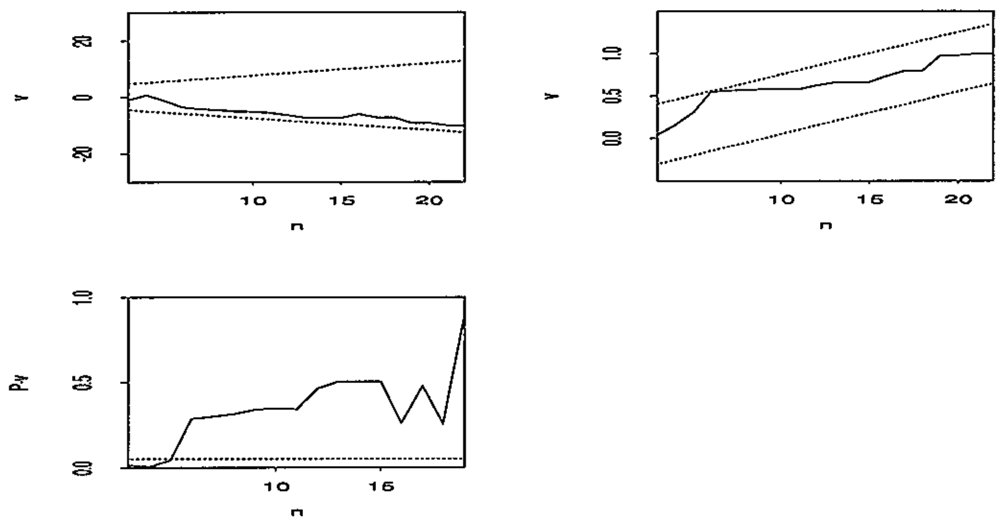

Figure 9 Cusum, CusumSq and SCT (unconditional convergence, OECD)

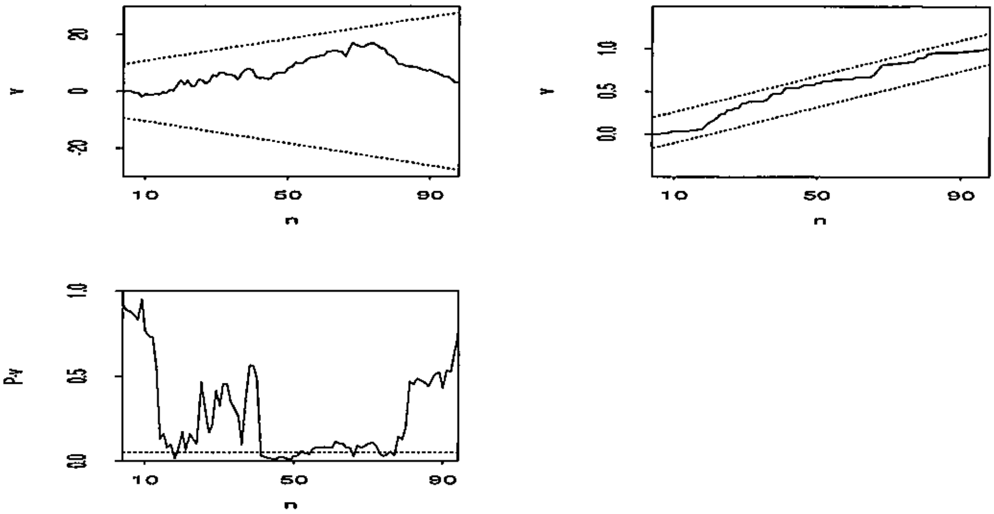

Figure 10 Cusum, CusumSq and SCT (convergence, Solow, non-oil)

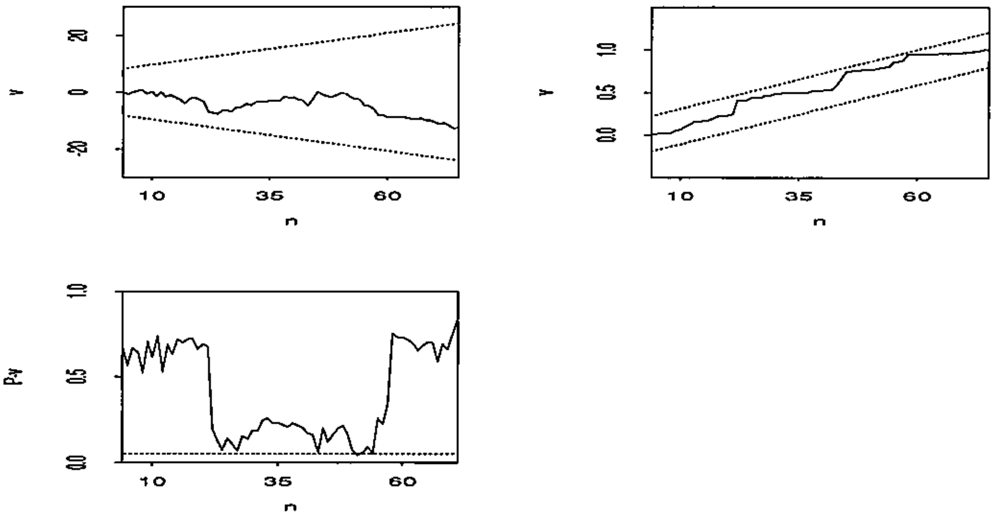

Figure 11 Cusum, CusumSq and SCT (convergence, Solow, interm.)

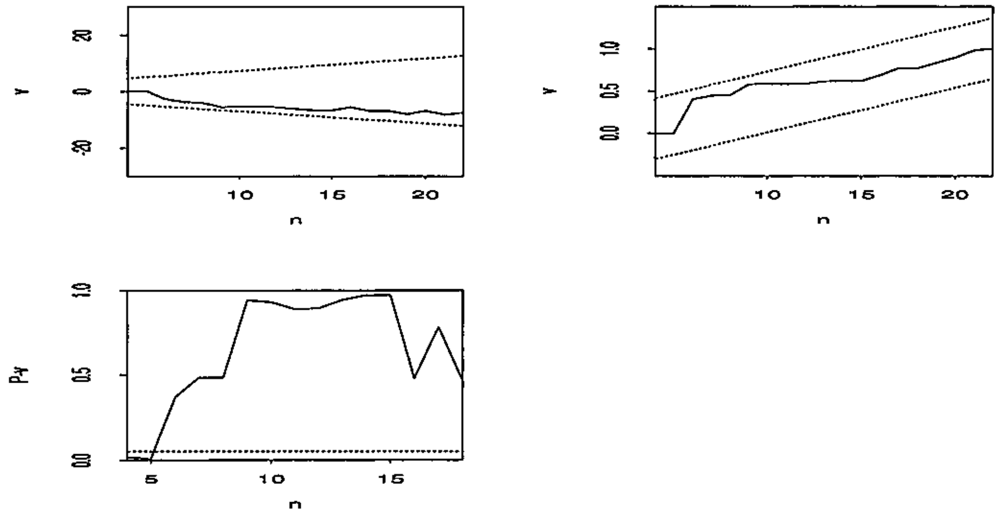

Figure 12 Cusum, CusumSq and SCT (convergence, Solow, OECD)

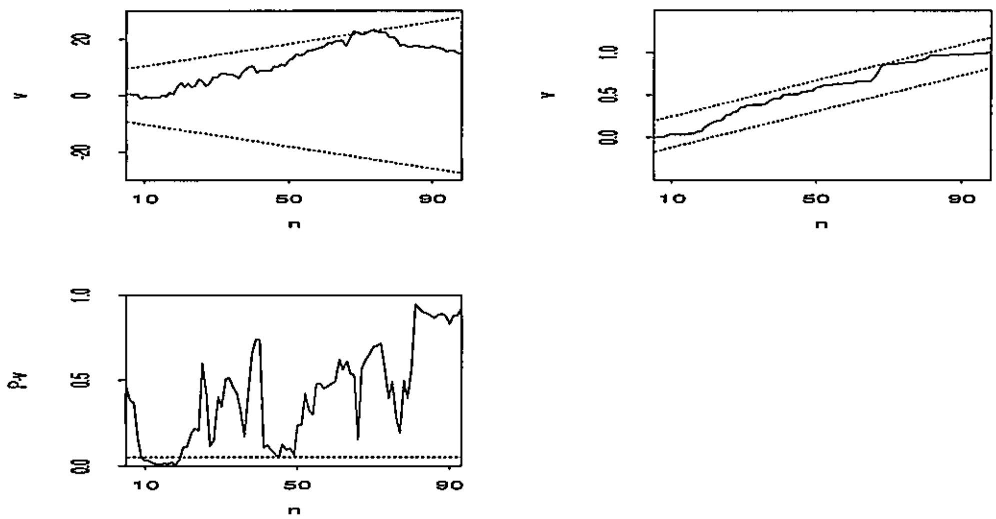

Figure 13 Cusum, CusumSq and SCT (convergence, augmented, non-oil)

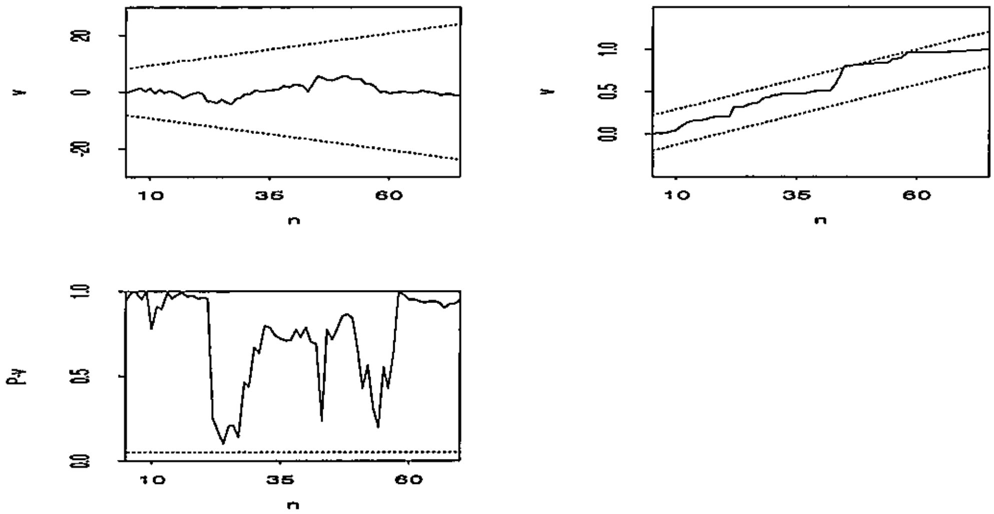

Figure 14 Cusum, CusumSq and SCT (convergence, augmented, interm.)

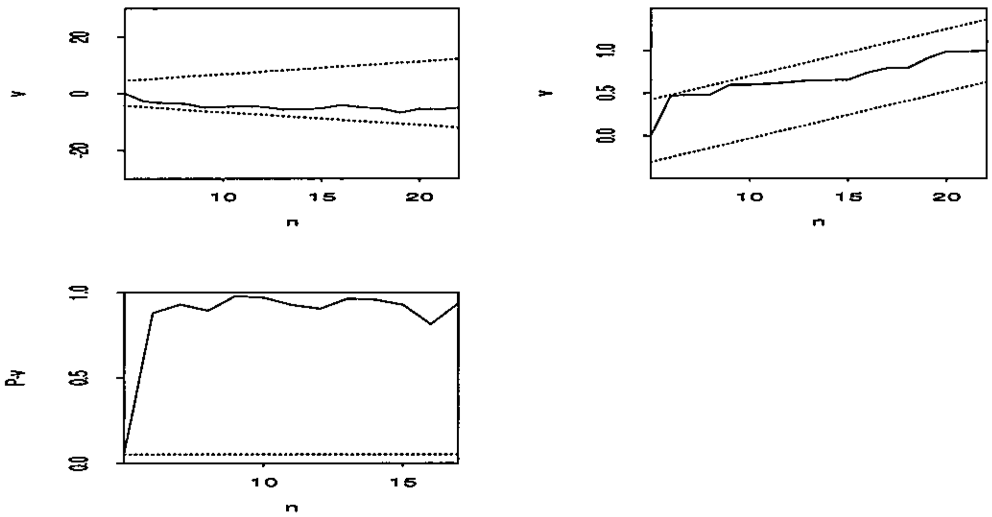

Figure 15 Cusum, CusumSq and SCT (convergence, augmented, OECD)