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An Operational Approach to a Worldwide Temporal Food Allocation and Price Determination Problem

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It is well known that until several years ago, U.S., Canada, Australia, and some other countries had suffered from overproduction of wheat and had to make efforts to reduce the production. Then they wanted to share with importing countries high costs invoked by storing the excess wheat. In order to raise the consumers' price of wheat even a little higher, some of them gave surpluses of wheat as free economic aids to developing countries which needed more food. However, recent worldwide abnormal weather has caused serious floods and droughts on various parts of the earth, which have disturbed the world food market together with other factors such as rapidly-growing world population and changes in food tastes toward animal protein. These phenomena have clearly started making people, especially in food-importing countries, worried about serious food crises in the near future. Droughts in Africa, floods in Bangladesh, and poor harvests in the Soviet Union may seem to be the symptoms of food crises.

To challenge a worldwide food problem, we economists need an operational method by which we can evaluate the impacts of a change in the food situation of an influential country or countries on the worldwide food situation. Word "operational" here implies "unabstract and usable only with ordinary statistics".

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The purpose of this paper is to provide an operational method for optimal food allocation and equilibrium price determination in the free world over the planned time horizon based on worldwide cooperation. The framework is characterized by the objective function of net social quasi-welfare (or equivalent to net social payoff), realistic constraints, and a quadratic programming technique [5, 8, 13, 15, 16]. Here realistic constraints are export and import quota systems, government-directed food reserve policies, (long-term) contracts, storage and international transportation availabilities, minimum farm supply price support policies, fixed farm supply price policies, and free economic aids in kind, in addition to the constraints on production, spatial and/or temporal transfer, and consumption concerned with capitalistic (or mixed economy) and planned economy countries. Various kinds of taxes and subsidies are treated as given in the model. The paper consists of two parts. In PART I, we would like to refer to a free competition case. Then we proceed to a constrained free competition case in PART II where we utilize the results of PART I as a basis for formulating a realistic food problem.

PART I. Free Competition in the Free World

Before taking into consideration realistic food situations of countries, we would like to refer to a free competition case in which there are no constraints given from outside except for the trade of capitalistic countries with planned economy countries which will be treated as one country. Then, by employing the results of a free competition case, we will refer to the worldwide realistic food situation in the next part. This part consists of six sections. The first two sections will be used for notation and basic assumptions, and then the main sections of PART I will follow.

I.1. Notation and Minor Assumptions

To express compactly worldwide temporal food allocation and efficient pricing, we introduce the following notation with minor assumptions:

h and k : numbered food items or numbered goods (hereafter the word goods will be used), where $1 \leqq h, k \leqq H$;

i and j : numbered countries (or numbered regions in a country), where $1 \leqq i, j \leqq J$, and country J denotes the planned economy block while countries 1, 2,..., J-1 denote capitalistic countries;

m : numbered transportation mode available to domestic and international trade within exporting and importing countries as well as between these countries or to domestic trade within producing country (see Basic Assumption (L) in Section I.2);

t: numbered time or numbered time period of the planning time horizon consisting of T times or T time periods, where $0 \leqq t \leqq T$, and $t = 0$ implies the initial time;

$T \equiv \{0, 1, 2,..., T\}$: set consisting of the numbers of all times within the planning time horizon;

$\Theta \equiv \{1, 2, 3,..., H\}$: set consisting of the numbers of all goods unpolished, unground, and unprocessed, where we call all goods belonging to Θ primary goods (see Basic Assumption (J) in Section I.2);

$\Theta^\circ \equiv \{1, 2, 3,..., H\}$: set consisting of the numbers of all goods which are polished and/or ground if necessary, where Θ and Θ° have one-to-one correspondence and we call all goods belonging to Θ° consumers' goods (see Basic Assumption (J) in Section I.2);

$\Delta_W \equiv \{1, 2, 3,..., J-1, J\}$: set consisting of the numbers of all countries;

$\Delta \equiv \Delta_W - \{J\}$: set consisting of the numbers of all capitalistic countries;

Δ_{hW} for any $h \in \Theta$: set consisting of the numbers of all countries which can produce good h;

$\Delta_h \equiv \Delta_{hW}$ if $J \notin \Delta_{hW}$ or $\Delta_{hW} - \{J\}$ if $J \varepsilon \Delta_{hW}$ for any $h \varepsilon \Theta$;

Θ_i for any $i \varepsilon \Delta$: set consisting of the numbers of all goods which country i can produce, where it is assumed that there are H_i elements in Θ_i;

$\Phi_{hij} \equiv \{1, 2, \ldots, M_{hij}\}$ for any $h \varepsilon \Theta$, $i \varepsilon \Delta_{hW}$, and $j \varepsilon \Delta_W$: set consisting of the numbers of all transportation modes by which good h can be carried within exporting country i and importing country j and between these countries if $i \neq j$, or only within producing country i if $i = j$, where we assume, for simplicity, that there is only one average transportation mode for each good in each country, but there are M_{hij} distinct transportation means between countries i and j if $i \neq j$, so that $\Phi_{hij} = \{1\}$ if $i = j$ and $\Phi_{hij} = \{1, 2, \ldots, M_{hij}\}$ if $i \neq j$;

$* \equiv hijm$ for any $h \varepsilon \Theta$, $i \varepsilon \Delta_h$, $j \varepsilon \Delta$, and $m \varepsilon \Phi_{hij}$: * will be used as subscript hijm;

$\# \equiv hJjm$ for any $h \varepsilon \Delta_J$, $j \varepsilon \Delta$, and $m \varepsilon \Phi_{hJj}$: # will be used as subscript hJjm;

$\hat{s}_h$ for each $h \varepsilon \Theta$: maximum storable time length of good h technically given by its own unique storage technique, where $0 \leqq \hat{s}_h \leqq T$ for any $h \varepsilon \Theta$;

$\hat{w}_* \equiv \hat{w}_{hijm}$ for any $h \varepsilon \Theta$, $i \varepsilon \Delta_h$, $j \varepsilon \Delta$, and $m \varepsilon \Phi_{hij}$: fixed transportation time length required for good h to be shipped from country i to country j by transportation mode m and received by country j, where domestic transportation time length of each good in each country is assumed to be zero, so that $\hat{w}_{hii1} = 0$ and $0 \leqq \hat{w}_* \leqq \hat{s}_h$ if $i \neq j$;

$\Lambda_{*t} \equiv \{t, t+1, t+2, \ldots, t + \hat{s}_h - \hat{w}_*\}$ or $\Lambda_{*t} \equiv \{t\}$ for any $h \varepsilon \Theta$, $i \varepsilon \Delta_h$, $j \varepsilon \Delta - \{i\}$ or $j = i$, $m \varepsilon \Phi_{hij}$, and $t \varepsilon T$, respectively : subset of T consisting of the numbers of all times between production time t and maximum possible exportation time $t + \hat{s}_h - \hat{w}_*$ inclusive;

$u_* \equiv u_{hijm}$ for any $h \varepsilon \Theta$, $i \varepsilon \Delta_h$, $j \varepsilon \Delta$, and $m \varepsilon \Phi_{hij}$: export time of good h produced in country i and shipped from country i to country j by transportation mode m, where if good h is produced during time t, then $u_* \varepsilon \Lambda_{*t}$;

$\Xi_{*tu_*} \equiv \{u_* + \hat{w}_*, u_* + \hat{w}_* + 1, \ldots, t + \hat{s}_h\}$ for any $h\varepsilon\Theta$, $i\varepsilon\Delta_h$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hij}$, $t\varepsilon T$, and $u_*\varepsilon\Lambda_{*t}$: subset of T consisting of the numbers of all times between importation time $u_* + \hat{w}_*$ and maximum possible consumption time $t + \hat{s}_h$ inclusive, where it is assumed that good h was produced during time t;

$v_* \equiv v_{hijm}$ for any $h\varepsilon\Theta$, $i\varepsilon\Delta_h$, $j\varepsilon\Delta$, and $m\varepsilon\Phi_{hij}$: consumption time of good h produced in country i, shipped from country i to country j by transportation mode m, and received by country j, where if good h is produced during time t, shipped during time $u_*\varepsilon\Lambda_{*t}$, and received during time $u_* + \hat{w}_*$, then $v_*\varepsilon\Xi_{*tu_*}$;

$\hat{w}_\# \equiv \hat{w}_{hJjm}$, $\Lambda_{\#t} \equiv \{t, t + 1, t + 2, \ldots, t + \hat{s}_h - \hat{w}_\#\}$, $u_\# \equiv u_{hJjm}$, $\Xi_{\#tu_\#} \equiv \{u_\# + \hat{w}_\#, u_\# + \hat{w}_\# + 1, \ldots, t + \hat{s}_h\}$, and $v_\# \equiv v_{hJjm}$ for any $h\varepsilon\Theta_J$, $j\varepsilon\Delta$, and $m\varepsilon\Phi_{hJm}$ will be used in the same way as $\hat{w}_*$, Λ_{*t}, u_*, Ξ_{*tu_*}, and v_*;

$x_{hi}(t)$ and $p_{hi}(t)$ for any $h\varepsilon\Theta$, $i\varepsilon\Delta_h$, and $t\varepsilon T$: variable farm supply (production) quantity and nondiscounted farm supply price of good h prevailing in country i during time t, respectively;

$y_{hj}(t)$ and $q_{hj}(t)$ for any $h\varepsilon\Theta°$, $j\varepsilon\Delta$, and $t\varepsilon T$: variable demand (consumption) quantity and nondiscounted demand price of good h prevailing in country j during time t, respectively;

$z_*(t, u_*, v_*)$ and $\hat{e}_*(t, u_*, v_*)$ for any $h\varepsilon\Theta$, $i\varepsilon\Delta_h$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hij}$, $t\varepsilon T$, $u_*\varepsilon\Lambda_{*t}$, $v_*\varepsilon\Xi_{*tu_*}$: variable spatial and/or temporal "transfer" quantity and given unit transfer cost of good h, defined in Basic Assumption (G), which is produced in country i during time t, shipped to country j by transportation mode m during time u_*, received during time $u_* + \hat{w}_*$, and then consumed during time v_* by country j, where it is postulated that if $t < 0$ or $v_* > T$, $z_*(t, u_*, v_*)$ will be treated as predetermined;

$\hat{x}_{hi}(t)$ for any $h\varepsilon\Theta$, $i\varepsilon\Delta_h$, and $t\varepsilon T$: given quantity of good h to be used as domestic consumption by farmers and as feed for farmers' animals in country i during time t;

$\hat{y}_{hj}(t)$ for any $h\varepsilon\Theta°$, $j\varepsilon\Delta$, and $t\varepsilon T$: given quantity of good h to be used as feed for the animals of stock holders who buy feed in country j during time t, converted into goods which consumers consume at demand level;

$\hat{\eta}_h$ for any $h\varepsilon\Theta$: given conversion rate of primary good $h\varepsilon\Theta$ into consumers' good $h\varepsilon\Theta°$, where $0 < \hat{\eta}_h \leqq 1$;

$\hat{\sigma}(t)$ for any $t\varepsilon T$: given appropriate discount factor applied to all capitalistic countries (see Basic Assumption (F) in Section I.2);

$T_{\#} \equiv T_{hJjm}$ for any $h\varepsilon\Theta_J$, $j\varepsilon\Delta$, and $m\varepsilon\Phi_{hJj}$: subset of T consisting of the numbers of all production times of good h produced in country J which is determined to be shipped to country j by transportation mode m some time;

$\Gamma_{\#t} \equiv \Gamma_{hJjmt}$ for any $h\varepsilon\Theta_J$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hJj}$, and $t\varepsilon T_{\#}$: subset of $\Lambda_{\#t}$ consisting of the numbers of the exportation times of good h produced in country J during time t which is determined to be shipped to country j by transportation mode m, where $\Gamma_{\#t} = \phi$ if $t\notin T_{\#}$;

$\hat{z}_{\#}(t, u_{\#}) \equiv \hat{z}_{hJjm}(t, u_{hJjm})$ for any $h\varepsilon\Theta_J$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hJj}$, $t\varepsilon T_{\#}$, and $u_{\#}\varepsilon\Gamma_{\#t}$: predetermined transfer quantity of good h produced in country J during time t and shipped to country j by transportation mode m during time $u_{\#}$, where $\hat{z}_{\#}(t, u_{\#}) > 0$;

$z_{\#}(t, u_{\#}, v_{\#})$ for any $h\varepsilon\Theta_J$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hJj}$, $t\varepsilon T_{\#}$, $u_{\#}\varepsilon\Gamma_{\#t}$, and $v_{\#}\varepsilon\Xi_{\#tu_{\#}}$: defined in the same way that $z_*(t, u_*, v_*)$ is defined above;

$\hat{p}_{hJE}(t)$ for each $h\varepsilon\Theta_J$ and $t\varepsilon T$: predetermined exportation price of good h prevailing in country J during time t;

$\hat{e}_{\#}(u_{\#}, v_{\#}) \equiv \hat{e}_{hJjm}(u_{hJjm}, v_{hJjm})$ for any $h\varepsilon\Theta_J$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hJj}$, $u_{\#}\varepsilon\Gamma_{\#t}$, and $v_{\#}\varepsilon\Xi_{\#tu_{\#}}$ if $t\varepsilon T_{\#}$: given unit transfer cost, evaluated at time $u_{\#}$, which consists of transportation charge between country J and country j, storage, polishing and/or milling charges incurred in country j, and related insurance charges, of good h produced in country J, shipped to country j by transportation mode m during time $u_{\#}$, and consumed in country j during time $v_{\#}$;

$\hat{z}_{hiJm}(t, u_{hiJm})$ for any $h\varepsilon\Theta$, $i\varepsilon\Delta_h$, $m\varepsilon\Phi_{hiJ}$, $t\varepsilon T$, and $u_{hiJm}\varepsilon\Lambda_{hiJm}$:
predetermined transfer quantity of good h produced in country i during time t and shipped to country J by transportation mode m during time u_{hiJm}, where $m\varepsilon\Phi_{hiJ}$ does not specify the domestic transportation means in country J from the capitalistic countries' viewpoint.

I.2. Basic Assumptions

We should like to postulate Basic Assumptions (A) to (N) for Basic Problem to be formulated in the next section. However, some of the basic assumptions will be modified and additional assumptions will be added for Realistic Problem in section II.3. We shall assume that:

(A) Each farm supply price function is well estimated in the linear form of the farm supply quantities of all goods producible in each country during each time with integrability condition, given algebraically as

(1) $p_{hi}(t) = \hat{a}_{hi}(t) + \sum_{k\varepsilon\Theta_i} \hat{b}_{hki}(t)x_{ki}(t)$ for each $i\varepsilon\Delta$, $h\varepsilon\Theta_i$, and $t\varepsilon T$;[1/]

(B) Each demand price function is well estimated in the linear form of the demand quantities of all goods in question in each country during each time with integrability condition, given algebraically as

(2) $q_{hj}(t) = \hat{c}_{hj}(t) - \sum_{k\varepsilon\Theta^\circ} \hat{d}_{hkj}(t)y_{kj}(t)$ for each $h\varepsilon\Theta^\circ$, $j\varepsilon\Delta$, and $t\varepsilon T$;

(C) The following conditions are satisfied: technical coefficient matrices $B_i(t)$ and $D_j(t)$ defined below are strictly positive definite if $H_i \geqq 1$ and $H \geqq 1$, and

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1/ One of the ways to obtain (1) is as follows: Estimate $x_{hi}(t)$ as a linear function of the prices of all goods as well as the prices of all production factors, substitute predicted prices of all production factors into the production factor terms in the estimated $x_{hi}(t)$ function, sum up all of the production factor terms and the original constant term, and obtain the reverse functions with all estimated $x_{hi}(t)$ functions.

symmetric, because of integrability condition, if $H_i \geqq 2$ and $H \geqq 2$, respectively:

$B_i(t) \equiv [\hat{b}_{hki}(t)$ for row indicator $h\varepsilon\Theta_i$ and column indicator $k\varepsilon\Theta_i]$ 2/ and

$D_j(t) \equiv [\hat{d}_{hkj}(t)$ for row indicator $h\varepsilon\Theta°$ and column indicator $k\varepsilon\Theta°]$

for each i, $j\varepsilon\Delta$, and $t\varepsilon T$; and $\hat{a}_{hi}(t) \gtreqless 0$ and $\hat{c}_{kj}(t) > 0$ for each $h\varepsilon\Theta$,

$i\varepsilon\Delta_h$, $j\varepsilon\Delta$, $k\varepsilon\Theta°$, and $t\varepsilon T$;

(D) At the stage of supply of goods to middlemen by farmers, possibility of overproduction and free disposal of each good are accepted in each country during each time, given algebraically as

(3) $$f_{hi}(t) \equiv x_{hi}(t) - \sum_{j\varepsilon\Delta} \sum_{m\varepsilon\Phi_{hij}} \sum_{u_*\varepsilon\Lambda_{*t}} \sum_{v_*\varepsilon\Xi_{*tu_*}} z_*(t, u_*, v_*)$$

$$- \sum_{m\varepsilon\Phi_{hiJ}} \sum_{u_{hiJm}\varepsilon\Lambda_{hiJm}} \hat{z}_{hiJm}(t, u_{hiJm}) - \hat{x}_{hi}(t) \geqq 0$$

for each $h\varepsilon\Theta$, $i\varepsilon\Delta_h$, and $t\varepsilon T$;

(E) At the stage of supply of goods to consumers by middlemen, possibility of oversupply and free disposal of each good are accepted in each country during each time, given algebraically as

(4) $$g_{hj}(t) \equiv \sum_{i\varepsilon\Delta_h} \sum_{m\varepsilon\Phi_{hij}} \sum_{\tau_*\varepsilon\Lambda_{*t}} \sum_{\xi_*\varepsilon\Xi_{*t\tau_*}} \hat{\eta}_h z_*(\tau_*-\hat{s}_h, \xi_*-\hat{s}_h-\hat{w}_*, t)$$

$$+ \sum_{m\varepsilon\Phi_{hJj}} \sum_{\tau_\#\varepsilon\Lambda_{\# t}} \sum_{\xi_\#\varepsilon\Gamma_{\#(\tau_\#-\hat{s}_h)}} \hat{\eta}_h z_\#(\tau_\#-\hat{s}_h, \xi_\#, t) - y_{hj}(t) - \hat{y}_{hj}(t) \geqq 0$$

for each $h\varepsilon\Theta°$, $j\varepsilon\Delta$, and $t\varepsilon T$, and

(5) $$\ell_\#(t, u_\#) \equiv \hat{z}_\#(t, u_\#) - \sum_{v_\#\varepsilon\Xi_{\# tu_\#}} z_\#(t, u_\#, v_\#) \geqq 0$$

for each $h\varepsilon\Theta_J$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hJj}$, $t\varepsilon T_\#$, and $u_\#\varepsilon\Gamma_{\# t}$;

(F) Given appropriate time discount factor $\hat{\delta}(t)$ between time t and the initial time is utilized, regardless of capitalistic countries, where, for instance, $\hat{\delta}(t) = \{1 + \sum_{j\varepsilon\Delta} \hat{\alpha}_j(-1)\hat{r}_j(0)\}^{-t}$ may be appropriate where $\hat{r}_j(0)$ and $\hat{\alpha}_j(-1)$ denote the predicted interest rate of the initial time and the market share of international trade of H goods under study of time -1 of country j, respectively.

2/ If there are no externalities, $B_i(t)$ will be diagonal. This case is most often expected.

(G) The payment system concerning good $h \varepsilon \Theta$ (and $h \varepsilon \Theta^\circ$) characterized by production country $i \varepsilon \Delta_{hW}$, production time $t \varepsilon T$, consumption country $j \varepsilon \Delta$, transportation mode $m \varepsilon \Phi_{hij}$, export time $u_* \varepsilon \Lambda_{*t}$, and consumption time $v_* \varepsilon \Xi_{*tu_*}$ and the components of $\hat{e}_*(t, u_*, v_*)$ and $\hat{e}_\#(u_\#, v_\#)$ are as follows:

Table 1. The Payment System in Terms of U.S. Dollars Per Unit of Good h, (i and j denote country i and country j, respectively, where $i \neq j$)

Price and Cost Items	Who Pays	Who Receives	When Paid
Farm Supply Price, $p_{hi}(t)$	Middlemen of i and Government of i	Farmers of i	t
Subsidy on Production, $\hat{p}_{hiS}(t)$ out of $p_{hi}(t)$	Government of i	Farmers of i	t
Substantial Farm Supply Price, $p_{hi}(t) - \hat{p}_{hiS}(t)$	Middlemen of i	Farmers of i	t
Domestic Transport Cost from Farm to Storage Area, $\hat{e}_{hiFS}(t)$	Middlemen of i	Owners of Domestic Transport Means in i	t
Storage Cost, $\hat{e}_{hi}(t, u_*)$	Middlemen of i	Owners of Storage Facilities in i	t
Export Tariff, $\hat{e}_{hiE}(u_*)$	Middlemen of i	Government of i	u_*
International Transport Cost, $\hat{e}_*(u_*)$	Middlemen of i	Owners of International Transport Means Specified by $m \varepsilon \Phi_{hij}$	u_*
Import Cost, $e_{*I}(t, u_* + \hat{w}_*)$	Middlemen of j	Middlemen of i	$u_* + \hat{w}_*$
Margin of Middlemen of i, $\hat{e}_{hiM}(u_*)$ out of $e_{*I}(t, u_* + \hat{w}_*)$	Middlemen of j	Middlemen of i	$u_* + \hat{w}_*$
Import Tariff, $\hat{e}_{hjI}(u_* + \hat{w}_*)$	Middlemen of j	Government of j	$u_* + \hat{w}_*$
Storage Cost, $\hat{e}_{hj}(u_* + \hat{w}_*, v_*)$	Middlemen of j	Owners of Storage Facilities in j	$u_* + \hat{w}_*$
Transforming Cost, $\hat{e}_{hjT}(v_*)$	Middlemen of j	Polishers and/or Grinders of j	v_*
Domestic Transport Cost from Storage to Consumption Area, $\hat{e}_{hjSC}(v_*); h \varepsilon \Theta^\circ$	Middlemen of j	Owners of Domestic Transport Means in j	v_*

Price and Cost Items	Who Pays	Who Receives	When Paid
Demand Price Minus Consumption Tax, $q_{hj}(v_*)-\hat{q}_{hjC}(v_*);h\varepsilon\Theta°$	Consumers of j	Middlemen of j	v_*
Consumption Tax, $\hat{q}_{hjC}(v_*);h\varepsilon\Theta°$	Consumers of j	Government of j	v_*
Margin of Middlemen of j, $\hat{e}_{hjM}(v_*)$ out of $q_{hj}(v_*)-\hat{q}_{hjC}(v_*);h\varepsilon\Theta°$	Consumers of j	Middlemen of j	v_*

Where Import Cost $e_{*I}(t, u_*+\hat{w}_*) \equiv \{\hat{\sigma}(t)/\hat{\sigma}(u_*+\hat{w}_*)\}\{p_{hi}(t) - \hat{p}_{hiS}(t) + \hat{e}_{hiFS}(t) + \hat{e}_{hi}(t, u_*)\} + \{\hat{\sigma}(u_*)/\hat{\sigma}(u_*+\hat{w}_*)\}\{\hat{e}_{hiE}(u_*) + \hat{e}_*(u_*) + \hat{e}_{hiM}(u_*)\}$.

We define: $\hat{e}_*(t, u_*, v_*) \equiv \hat{e}_{hiFS}(t) + [\hat{e}_{hi}(t, u_*) + \{\hat{\sigma}(u_*)/\hat{\sigma}(t)\}x \{\hat{e}_{hiE}(u_*) + \hat{e}_*(u_*) + \hat{e}_{hiM}(u_*)\} + \{\hat{\sigma}(u_*+\hat{w}_*)/\hat{\sigma}(t)\}\hat{e}_{hjI}(u_*+\hat{w}_*)] + \{\hat{\sigma}(u_*+\hat{w}_*)/\hat{\sigma}(t)\}\hat{e}_{hj}(u_*+\hat{w}_*, v_*) + \{\hat{\sigma}(v_*)/\hat{\sigma}(t)\}\{\hat{e}_{hjT}(v_*) + \hat{\eta}_h\hat{e}_{hjSC}(v_*) + \hat{\eta}_h\hat{e}_{hjM}(v_*)\}$ for each $h\varepsilon\Theta$, $i\varepsilon\Delta_h$, $j\varepsilon\Delta-\{i\}$, $m\varepsilon\Phi_{hij}$, $t\varepsilon T$, $u_*\varepsilon\Lambda_{*t}$, and $v_*\varepsilon\Xi_{*tu_*}$. If i = j, then the cost items in [...] of $\hat{e}_*(t, u_*, v_*)$ above should be eliminated. $\hat{e}_\#(u_\#, v_\#) \equiv \hat{e}_\#(u_\#) + \{\hat{\sigma}(u_\#+v_\#)/\hat{\sigma}(u_\#)\}\{\hat{e}_{hjI}(u_\#+\hat{w}_\#) + \hat{e}_{hj}(u_\#+\hat{w}_\#, v_\#)\} + \{\hat{\sigma}(v_\#)/\hat{\sigma}(u_\#)\}\{\hat{e}_{hjT}(v_\#) + \hat{\eta}_h\hat{e}_{hjSC}(v_\#) + \hat{\eta}_h\hat{e}_{hjM}(v_\#)\}$. It should be noted that $\hat{p}_{hJE}(u_\#)$ roughly corresponds to $\{\hat{\sigma}(t)/\hat{\sigma}(u_\#)\}\{\hat{p}_{hJ}(t) + \hat{e}_{hJFS}(t) + \hat{e}_{hJ}(t, u_\#)\} + \hat{e}_{hJE}(u_\#)$, where $\hat{e}_{hJM}(u_\#)$ may be included in $\hat{e}_{hJE}(u_\#)$.

(H) Marginal social significance and marginal utility of money are unitary, regardless of countries and times.

(I) Economic units are farmers, middlemen who do domestic as well as international business, owners of storage facilities, owners of domestic and international transport means, owners of polishing and grinding machinery, consumers, government in each country, and an international organization, such as U.N., during the food planning time horizon. The functions of an international organization are to help all countries in the world including planned economy countries exchange necessary information, to ban any noncompetitive agreements such as international cartels and boycotts among capitalistic countries, and to prevent countries from dumping their goods on international markets.

(J) Goods covered with husks, hulls, shucks, or shells are transfered over space and time, if preferable for preservation. For example, unhulled rice and unhulled wheat which are called primary goods are sold to middlemen by farmers, while polished rice and wheat flour which are called consumers' goods are sold to consumers by middlemen. The conversion rates of, for example, unhulled rice and fresh potatoes into polished rice and potatoes immediately edible are 0.75 and 1.00, respectively. The values of by-products produced in conversion processes can be ignored.

(K) Only one storage technique is available to each good. Each good can be stored for the maximum storage time horizon given by its unique storage technique without any damage in quality as well as in quantity, but can not be stored beyond the maximum storage time horizon due to serious damage in quality.

(L) International transportation is emphasized. Transportation mode implies a combination of domestic as well as international transportation means.

(M) Net social quasi-welfare if $H \geqq 2$ and net social economic surplus if $H = 1$ can be discounted with an appropriate time discount factor.

(N) Free competition prevails in the free world, and perfect information is available. All subsidies and taxes cited in Table 1 should be ignored in the case of free competition (but they will be used in the case of constrained free competition). Any country and any group of countries can not behave like monopolist or monopsonist. Capitalistic countries' trade with the planned economy block are regarded as exogenous. Constraints (3) and (4) may be called not as exogenous but as endogenous constraints.

I.3. Definition of an Optimal Spatial and Temporal Goods Allocation and Efficient Pricing in Case of Free Competition

Let us define optimal goods (foods) allocation and equilibrium price determination over space and time of the free world. We would like to call an economic situation which satisfies all of the following 7 conditions (I) through (VII) an equilibrium of the free world over space and time:

(I) Unique farm supply price $\bar{p}_{hi}(t)$ of each good $h\varepsilon\Theta$ in each producing country $i\varepsilon\Delta_h$, and unique demand price $\bar{q}_{hj}(t)$ of each good $h\varepsilon\Theta°$ in each consuming country $j\varepsilon\Delta$ during each time of the planning time horizon $t\varepsilon T$;

(II) Possibility of overproduction and efficient pricing at the stage of farm supply of goods characterized as follows:

(6) $\bar{f}_{hi}(t) \geqq 0$, $\bar{p}_{hi}(t)\bar{f}_{hi}(t) = 0$, and $\bar{p}_{hi}(t) \geqq 0$ for each $h\varepsilon\Theta$, $i\varepsilon\Delta_h$, and $t\varepsilon T$;

(III) Possibility of free disposal and efficient pricing at the stage of consumption of goods characterized as follows:

(7) $\bar{g}_{hj}(t) \geqq 0$, $\bar{q}_{hj}(t)\bar{g}_{hj}(t) = 0$, and $\bar{q}_{hj}(t) \geqq 0$ for each $h\varepsilon\Theta°$, $j\varepsilon\Delta$, and $t\varepsilon T$;

(IV) Price differential between converted demand price and farm supply price plus given unit transfer cost, and the generation of (spatial and/or temporal) transfer quantity characterized as follows:

(8) $\bar{n}_*(t, u_*, v_*) \equiv - \hat{\eta}_h\{\hat{\sigma}(v_*)/\hat{\sigma}(t)\}\bar{q}_{hj}(v_*) + \bar{p}_{hi}(t) + \hat{e}_*(t, u_*, v_*) \geqq 0$

$\bar{z}_*(t, u_*, v_*)\bar{n}_*(t, u_*, v_*) = 0$, and $\bar{z}_*(t, u_*, v_*) \geqq 0$ for each $h\varepsilon\Theta$, $i\varepsilon\Delta_h$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hij}$, $t\varepsilon T$, $u_*\varepsilon\Lambda_{*t}$, and $v_*\varepsilon\Xi_{*tu_*}$;

(V) Possibility of generation of abnormal profits or losses in the trade with the planned economy block, and generation of transfer quantity to consumption through the trade with planned economy block characterized as follows:

Let $\bar{n}_{\#}(u_{\#}, v_{\#}) \equiv \hat{\eta}_h\{\hat{\sigma}(v_{\#})/\hat{\sigma}(u_{\#})\}\bar{q}_{hj}(u_{\#}) - \hat{p}_{hJE}(u_{\#}) - \hat{e}_{\#}(u_{\#}, v_{\#})$ and

$\bar{n}_{\#}(u_{\#}, \bar{v}_{\#}) \equiv \max_{v_{\#}\varepsilon\Xi_{\#tu_{\#}}} \bar{n}_{\#}(u_{\#}, v_{\#})$ for each $h\varepsilon\Theta_J$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hJj}$,

$u_{\#} \varepsilon \Gamma_{\# t}$, and $v_{\#} \varepsilon \Xi_{\# t u_{\#}}$ for $t \varepsilon T_{\#}$.

(9) If there exists at least one $\bar{v}_{\#}$ such $\bar{n}_{\#}(u_{\#}, \bar{v}_{\#}) > 0$, then $\bar{z}_{\#}(t, u_{\#}, v_{\#}) > 0$ but $\bar{z}_{\#}(t, u_{\#}, v_{\#}) = 0$ for other $v_{\#} \varepsilon \Xi_{\# t u_{\#}}$ with $\bar{\ell}_{\#}(t, u_{\#}) = 0$;

(10) If there exists at least one $\bar{v}_{\#}$ such that $\bar{n}_{\#}(u_{\#}, \bar{v}_{\#}) = 0$, then $\bar{z}_{\#}(t, u_{\#}, \bar{v}_{\#}) \geqq 0$ but $\bar{z}_{\#}(t, u_{\#}, v_{\#}) = 0$ for other $v_{\#} \varepsilon \Xi_{\# t u_{\#}}$ with $\bar{\ell}_{\#}(t, u_{\#}) \geqq 0$;

(11) If there exists no $\bar{v}_{\#}$ such that $\bar{n}_{\#}(u_{\#}, \bar{v}_{\#}) \geqq 0$, then $\bar{z}_{\#}(t, u_{\#}, v_{\#}) = 0$ for any $v_{\#} \varepsilon \Xi_{\# t u_{\#}}$, so that $\bar{\ell}_{\#}(t, u_{\#}) = \hat{z}_{\#}(t, u_{\#}) > 0$;

(VI) Identity of the above farm supply price to the farm supply price derived through farm supply price function (1), i.e.,

(12) $\bar{p}_{hi}(t) = \hat{a}_{hi}(t) + \sum_{k \varepsilon \Theta_i} \hat{b}_{hki}(t) \bar{x}_{ki}(t)$ for each $h \varepsilon \Theta$, $i \varepsilon \Delta_h$, and $t \varepsilon T$;

(VII) Identity of the above demand price to the demand price derived through demand price function (2), i.e.,

(13) $\bar{q}_{hj}(t) = \hat{c}_{hj}(t) - \sum_{k \varepsilon \Theta^\circ} \hat{d}_{hkj}(t) \bar{y}_{kj}(t)$ for each $h \varepsilon \Theta^\circ$, $j \varepsilon \Delta$, and $t \varepsilon T$.

Let us give brief economic explanations to the equilibrium conditions (I) through (VII). (I) is clear. (II) states that if farmers' production quantity exceeds middlemen's demand quantity, implying $\bar{f}_{hi}(t) > 0$, then farm supply price $\bar{p}_{hi}(t)$ must be zero, while if farmers' production quantity is completely bought by middlemen, implying $\bar{f}_{hi}(t) = 0$, then farm supply price is nonnegative and usually positive. (III) states that if middlemen's supply quantity exceeds consumers' demand quantity, implying $\bar{g}_{hj}(t) > 0$, then demand price $\bar{q}_{hj}(t)$ must be zero, while if middlemen's supply quantity is completely demanded by consumers, implying $\bar{g}_{hj}(t) = 0$, then demand price can be positive. (IV) states that if the sum of farm supply price and the given transfer cost of spatial and/or temporal transfer quantity $\bar{z}_*(t, u_*, v_*)$ exceeds converted discounted demand price $\hat{\eta}_h \{\hat{\sigma}(v_*)/\hat{\sigma}(t)\} \bar{q}_{hj}(v_*)$, implying $\bar{n}_*(t, u_*, v_*) > 0$, then transfer quantity must be zero, but if the above sum is exactly equal to converted discounted demand price, implying $\bar{n}_*(t, u_*, v_*) = 0$, then transfer quantity can be positive.

(V) states the following (1), (2), and (3): (1) If there is at least one (consumption) time $\bar{v}_{\#}$ such that price differential between exportation time $u_{\#}$ and consumption time $\bar{v}_{\#}$, defined as $\bar{n}_{\#}(u_{\#}, \bar{v}_{\#})$, is positive and largest among all price differentials $\bar{n}_{\#}(u_{\#}, v_{\#})$'s such that $u_{\#}+\hat{\omega}_{\#} \leq v_{\#} \leq t+\hat{s}_h$, then transfer quantity $\bar{z}_{\#}(t, u_{\#}, \bar{v}_{\#})$ is positive but all other $\bar{z}_{\#}(t, u_{\#}, v_{\#})$'s for $v_{\#}\varepsilon\Xi_{\#tu_{\#}}-\{\bar{v}_{\#}\}$ are zero, even if $\bar{n}_{\#}(u_{\#}, \bar{v}_{\#}) > \bar{n}_{\#}(u_{\#}, v_{\#}) > 0$. Imported quantity $\hat{z}_{\#}(t, u_{\#})$ must be cleared. (2) If the maximum price differential is zero, implying $\bar{n}_{\#}(u_{\#}, \bar{v}_{\#}) = 0$, then $\bar{z}_{\#}(t, u_{\#}, \bar{v}_{\#})$ can be positive, but all other $\bar{z}_{\#}(t, u_{\#}, v_{\#})$'s for $v_{\#}\varepsilon\Xi_{\#tu_{\#}}-\{\bar{v}_{\#}\}$ are zero. Imported quantity is not necessarily cleared. (3) If the maximum price differential is negative, implying $\bar{n}_{\#}(u_{\#}, \bar{v}_{\#}) < 0$, then all transfer quantities are zero, and imported quantity is not consumed at all. In case (1), abnormal profits will be generated. In case (2), if imported quantity is completely consumed, only normal profits included in $\hat{e}_{\#}(u_{\#}, \bar{v}_{\#})$ are earned by middlemen, but if not completely consumed, middlemen sustain abnormal losses. In case (3), middlemen must sustain abnormal losses. Finally, (VI) and (VII) state that farm supply and demand prices mentioned above are equivalent to the farm supply and demand prices derived through farm supply and demand price functions, respectively. These explanations are used for any good, any capitalistic country, any transportation mode, and any time of the planning time horizon.

I.4. Net Social Quasi-Welfare Maximization Problem in Case of Free Competition

To find the optimal values of all economic variables which satisfy all of the equilibrium conditions (I) to (VII) in the previous section, we would like to propose a Net Social Quasi-Welfare Maximization Problem in the case of free competition (Basic Problem) here and a Realistic Net Social Quasi-Welfare Maximization Problem in the case of restricted free competition (Realistic Problem) in Section II.3. Let us define the sum of the present values of net social quasi-welfares generated through all goods in all capitalistic countries during the planning time horizon as follows:

$$(14)\quad \sum_{t\varepsilon T}\sum_{h\varepsilon\Theta^\circ}\sum_{j\varepsilon\Delta}\int_0^{y_j(t)}\hat{\sigma}(t)q_{hj}(t)\,dy_{hj}(t) - \sum_{t\varepsilon T}\sum_{h\varepsilon\Theta}\sum_{i\varepsilon\Delta_h}\int_0^{x_i(t)}\hat{\sigma}(t)p_{hi}(t)\,dx_{hi}(t)$$

$$-\sum_{h\varepsilon\Theta}\sum_{i\varepsilon\Delta_h}\sum_{j\varepsilon\Delta}\sum_{m\varepsilon\Phi_{hij}}\sum_{t\varepsilon T}\sum_{u_*\varepsilon\Lambda_{*t}}\sum_{v_*\varepsilon\Xi_{*tu_*}}\hat{\sigma}(t)\hat{e}_*(t, u_*, v_*)z_*(t, u_*, v_*)$$

$$-\sum_{h\varepsilon\Theta_J}\sum_{j\varepsilon\Delta}\sum_{m\varepsilon\Phi_{hJj}}\sum_{t\varepsilon T_\#}\sum_{u_\#\varepsilon\Gamma_{\#t}}\sum_{v_\#\varepsilon\Xi_{\#tu_\#}}\hat{\sigma}(u_\#)\{\hat{p}_{hJE}(u_\#) + \hat{e}_\#(u_\#, v_\#)\}z_\#(t, u_\#, v_\#)$$

where $\int$ denotes line integral, and $y_j(t) \equiv (y_{1j}(t), y_{2j}(t),\ldots, y_{Hj}(t))$, $x_i(t) \equiv (x_{hi}(t)$ for all $h\varepsilon\Theta_i)$ and 0's are H- and H_i- zero vectors, depending on $y_j(t)$ and $x_i(t)$.

Now we are in a position to formulate Basic Problem as follows:

Basic Problem: Find an economic situation $\{\bar{x}_{hi}(t), \bar{y}_{kj}(t), \bar{z}_*(t, u_*, v_*), \bar{p}_{hi}(t),$ $\bar{q}_{kj}(t)$ for all $h\varepsilon\Theta$, $k\varepsilon\Theta^\circ$, $i\varepsilon\Delta_h$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hij}$, $t\varepsilon T$, $u_*\varepsilon\Lambda_{*t}$, and $v_*\varepsilon\Xi_{*tu_*}$ and $\bar{z}_\#(t, u_\#, v_\#)$ for all $h\varepsilon\Theta_J$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hJj}$, $t\varepsilon T_\#$, $u_\#\varepsilon\Gamma_{\#t}$, and $v_\#\varepsilon\Xi_{\#tu_\#}\} \geqq 0$, if it exists, such that maximizes (14) subject to (3), (4), (5), and

(15) $x_{hi}(t) \geqq 0$, $y_{kj}(t) \geqq 0$, $z_*(t, u_*, v_*) \geqq 0$ for each $h\varepsilon\Theta$, $k\varepsilon\Theta^\circ$, $i\varepsilon\Delta_h$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hij}$, $t\varepsilon T$, $u_*\varepsilon\Lambda_{*t}$, and $v_*\varepsilon\Xi_{*tu_*}$, and $z_\#(t, u_\#, v_\#) \geqq 0$ for each $h\varepsilon\Theta_J$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hJj}$, $t\varepsilon T_\#$, $u_\#\varepsilon\Gamma_{\#t}$, and $v_\#\varepsilon\Xi_{\#tu_\#}$, while maintaining price-quantity relationships (1) and (2).

To solve the Basic Problem, we introduce the following Saddle Value Problem:

the Saddle Value Problem : Find a saddle point $\{\bar{x}_{hi}(t), \bar{y}_{kj}(t), \bar{z}_*(t, u_*, v_*)$ for all $h\varepsilon\Theta$, $k\varepsilon\Theta^\circ$, $i\varepsilon\Delta_h$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hij}$, $t\varepsilon T$, $u_*\varepsilon\Lambda_{*t}$, and $v_*\varepsilon\Xi_{*tu_*}$, $\bar{z}_\#(t, u_\#, v_\#)$ for all $h\varepsilon\Theta_J$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hJj}$, $t\varepsilon T_\#$, $u_\#\varepsilon\Gamma_{\#t}$, and $v_\#\varepsilon\Xi_{\#tu_\#}$, $\bar{\mu}_{hi}(t)$ and $\bar{\gamma}_{kj}(t)$ for all $h\varepsilon\Theta$, $k\varepsilon\Theta^\circ$, $i\varepsilon\Delta_h$, $j\varepsilon\Delta$, and $t\varepsilon T$, and $\bar{\lambda}_\#(t, u_\#)$ for all $h\varepsilon\Theta_J$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hJj}$, $t\varepsilon T_\#$, and $u_\#\varepsilon\Gamma_{\#t}\} \geqq 0$, if it exists, such that satisfies $\Omega(X, Y, Z_*, Z_\#, \bar{\mu}, \bar{\gamma}, \bar{\lambda}) \leqq \Omega(\bar{X}, \bar{Y}, \bar{Z}_*, \bar{Z}_\#, \bar{\mu}, \bar{\gamma}, \bar{\lambda}) \leqq$ $\Omega(\bar{X}, \bar{Y}, \bar{Z}_*, \bar{Z}_\#, \mu, \gamma, \lambda)$ for any $(X, Y, Z_*, Z_\#, \mu, \gamma, \lambda) \geqq 0$, where

(16) $\Omega(X, Y, Z_*, Z_\#, \mu, \gamma, \lambda) \equiv (14) + \sum_{t\varepsilon T} \sum_{h\varepsilon\Theta} \sum_{i\varepsilon\Delta_h} \hat{\sigma}(t)\mu_{hi}(t)f_{hi}(t)$

$+ \sum_{t\varepsilon T} \sum_{h\varepsilon\Theta^\circ} \sum_{j\varepsilon\Delta} \hat{\sigma}(t)\gamma_{hj}(t)g_{hj}(t) + \sum_{h\varepsilon\Theta_J} \sum_{j\varepsilon\Delta} \sum_{m\varepsilon\Phi_{hJj}} \sum_{t\varepsilon T_\#} \sum_{u_\#\varepsilon\Gamma_{\# t}}$

$\hat{\sigma}(u_\#)\lambda_\#(t, u_\#)\ell_\#(t, u_\#)$,

and X, Y, Z_*, $Z_\#$, μ, γ, and λ are vectors (their definitions are omitted).

We solve the Basic Problem by solving the Saddle Value Problem by means of a quadratic programming technique.

I.5. Existence and Uniqueness of a Solution

First of all, we would like to check the uniqueness of the components in a solution. Let us express in the matrix form the sum of the present values of the gross social quasi-welfares generated through all goods in all capitalistic countries during the planning time horizon in (14). Then, we have

(17) $\sum_{t\varepsilon T} \hat{\sigma}(t)\{\sum_{h\varepsilon\Theta^\circ} \sum_{j\varepsilon\Delta} \int_0^{y_j(t)} q_{hj}(t)\, dy_{hj}(t) - \sum_{h\varepsilon\Theta} \sum_{i\varepsilon\Delta_h} \int_0^{x_i(t)} p_{hi}(t)\, dx_{hi}(t)\}$

$= \sum_{t\varepsilon T} \hat{\sigma}(t)[\sum_{j\varepsilon\Delta} \{c_j(t)y_j(t)' - (1/2)y_j(t)D_j(t)y_j(t)'\}$

$- \sum_{i\varepsilon\Delta} \{a_i(t)x_i(t)' + (1/2)x_i(t)B_i(t)x_i(t)'\}]$,

where $c_j(t) \equiv (\hat{c}_{1j}(t), \hat{c}_{2j}(t), \ldots, \hat{c}_{Hj}(t))$ and $a_i(t) \equiv (\hat{a}_{hi}(t)$ for all $h\varepsilon\Theta_i)$ for each i, $j\varepsilon\Delta$ and $t\varepsilon T$.

Thus, (17) can be expressed in a quadratic form. Taking Basic Assumption (C) into consideration, we can know that the objective function (14) is strictly concave with respect to $x_{hi}(t)$ and $y_{kj}(t)$ for each $h\varepsilon\Theta$, $k\varepsilon\Theta^\circ$, $i\varepsilon\Delta_h$, $j\varepsilon\Delta$, and $t\varepsilon T$, and concave with respect to $z_*(t, u_*, v_*)$ for each $h\varepsilon\Theta$, $i\varepsilon\Delta_h$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hij}$, $t\varepsilon T$, $u_*\varepsilon\Lambda_{*t}$, and $v_*\varepsilon\Xi_{*tu_*}$, and $z_\#(t, u_\#, v_\#)$ for each $h\varepsilon\Theta_J$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hJj}$, $t\varepsilon T_\#$, $u_\#\varepsilon\Gamma_{\# t}$, and $v_\#\varepsilon\Xi_{\# tu_\#}$. Hence, if there exists a solution to Basic Problem, all $\bar{x}_{hi}(t)$'s, $\bar{y}_{kj}(t)$'s, $\bar{p}_{hi}(t)$'s, and $\bar{q}_{kj}(t)$'s are unique, but all $\bar{z}_*(t, u_*, v_*)$'s and $\bar{z}_\#(t, u_\#, v_\#)$'s are not necessarily unique.

Let us prove the existence of a solution. We put $p_i(t)' = a_i(t)' + B_i(t)x_i(t)'$, where $p_i(t) \equiv (p_{hi}(t)$ for all $h\varepsilon\Theta_i)$ for each $i\varepsilon\Delta$ and $t\varepsilon T$. Following P. Gordan [4, 7, 10], there always exists an $x_i^\circ(t) > 0$, by Basic Assumption (C), such that $a_i(t)' + B_i(t)x_i^\circ(t)' \equiv p_i^\circ(t)' > 0$ for each $i\varepsilon\Delta$ and $t\varepsilon T$. Therefore, for a sufficiently large $x_i^\circ(t) >> 0$, we can always find a point represented as $\{x_{hi}^\circ(t)$, $y_{kj}^\circ(t)$, $z_*^\circ(t, u_*, v_*)$ for all $h\varepsilon\Theta$, $k\varepsilon\Theta^\circ$, $i\varepsilon\Delta_h$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hij}$, $t\varepsilon T$, $u_*\varepsilon\Lambda_{*t}$, and $v_*\varepsilon\Xi_{*tu_*}$, and $z_\#^\circ(t, u_\#, v_\#)$ for all $h\varepsilon\Theta_J$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hJj}$, $t\varepsilon T_\#$, $u_\#\varepsilon\Gamma_{\#t}$, and $v_\#\varepsilon\Xi_{\#tu_\#}\}$ such that $x_{hi}^\circ(t) >> 0$, $y_{kj}^\circ(t) = 0$, $z_*^\circ(t, u_*, v_*) > 0$ for each $h\varepsilon\Theta$, $k\varepsilon\Theta^\circ$, $i\varepsilon\Delta_h$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hij}$, $t\varepsilon T$, $u_*\varepsilon\Lambda_{*t}$, and $v_*\varepsilon\Xi_{*tu_*}$, and $z_\#^\circ(t, u_\#, v_\#) = 0$ for each $h\varepsilon\Theta_J$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hJj}$, $t\varepsilon T_\#$, $u_\#\varepsilon\Gamma_{\#t}$, and $v_\#\varepsilon\Xi_{\#tu_\#}$ satisfying $f_{hi}^\circ(t) > 0$ and $g_{kj}^\circ(t) > 0$ for each $h\varepsilon\Theta$, $k\varepsilon\Theta^\circ$, $i\varepsilon\Delta_h$, $j\varepsilon\Delta$, and $t\varepsilon T$,
which always result in $p_{hi}^\circ(t) > 0$ and $q_{kj}^\circ(t) > 0$ for each $h\varepsilon\Theta$, $k\varepsilon\Theta^\circ$, $i\varepsilon\Delta_h$, $j\varepsilon\Delta$, and $t\varepsilon T$, and $\ell_\#^\circ(t, u_\#) > 0$ for each $h\varepsilon\Theta_J$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hJj}$, $t\varepsilon T_\#$, $u_\#\varepsilon\Gamma_{\#t}$.
Then, we can know that such a point represents nonnegative quantities and prices and meets Slater condition [1]. Since the feasibility set consisting of constraints (3), (4), (5), and (15) is nonempty, closed, and convex, and includes in itself a point which meets Slater condition, we can conclude that there always exists a solution to Basic Problem.

I.6. Kuhn-Tucker Necessary Optimal Conditions for Basic Problem

The assumption that $B_i(t)$ and $D_j(t)$ for each i, $j\varepsilon\Delta$ and $t\varepsilon T$ are both strictly positive definite guarantees the sufficient conditions for Basic Problem. Hence, we need to find the necessary conditions for optimality. By Kuhn-Tucker necessary condition [6], we can derive the following conditions:

(18) $\partial\bar{\Omega}/\partial(t)\partial x_{hi}(t) = \bar{\mu}_{hi}(t) - \bar{p}_{hi}(t) \leqq 0$, $\bar{x}_{hi}(t)\{\partial\bar{\Omega}/\partial(t)\partial x_{hi}(t)\} = 0$, and $\bar{x}_{hi}(t) \geqq 0$ for each $h\varepsilon\Theta$, $i\varepsilon\Delta_h$, and $t\varepsilon T$;

(19) $\partial\bar{\Omega}/\partial(t)\partial y_{hj}(t) = \bar{q}_{hj}(t) - \bar{\gamma}_{hj}(t) \leqq 0$, $\bar{y}_{hj}(t)\{\partial\bar{\Omega}/\partial(t)\partial y_{hj}(t)\} = 0$, and $\bar{y}_{hj}(t) \geqq 0$ for each $h\varepsilon\Theta^\circ$, $j\varepsilon\Delta$, and $t\varepsilon T$;

(20) $\partial\bar{\Omega}/\partial(t)\partial z_*(t, u_*, v_*) = \hat{\eta}_h\{\partial(v_*)/\partial(t)\}\bar{\gamma}_{hj}(v_*) - \bar{\mu}_{hi}(t) - \hat{e}_*(t, u_*, v_*) \leqq 0$,

$\bar{z}_*(t, u_*, v_*)\{\partial\bar{\Omega}/\partial(t)\partial z_*(t, u_*, v_*)\} = 0$, and $\bar{z}_*(t, u_*, v_*) \geqq 0$ for each

$h\varepsilon\Theta$, $i\varepsilon\Delta_h$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hij}$, $t\varepsilon T$, $u_*\varepsilon\Lambda_{*t}$, and $v_*\varepsilon\Xi_{*tu_*}$;

(21) $\partial\bar{\Omega}/\partial(u_\#)\partial z_\#(t, u_\#, v_\#) = \hat{\eta}_h\{\partial(v_\#)/\partial(u_\#)\}\bar{\gamma}_{hj}(v_\#) - \hat{p}_{hJE}(u_\#) - \hat{e}_\#(u_\#, v_\#)$

$- \bar{\lambda}_\#(t, u_\#) \leqq 0$, $\bar{z}_\#(t, u_\#, v_\#)\{\partial\bar{\Omega}/\partial(u_\#)\partial z_\#(t, u_\#, v_\#)\} = 0$, and

$\bar{z}_\#(t, u_\#, v_\#) \geqq 0$ for each $h\varepsilon\Theta_J$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hJj}$, $t\varepsilon T_\#$, $u_\#\varepsilon\Gamma_{\#t}$, and $v_\#\varepsilon\Xi_{\#tu_\#}$;

(22) $\partial\bar{\Omega}/\partial(t)\partial\mu_{hi}(t) = \bar{f}_{hi}(t) \geqq 0$, $\bar{\mu}_{hi}(t)\bar{f}_{hi}(t) = 0$, and $\bar{\mu}_{hi}(t) \geqq 0$

for each $h\varepsilon\Theta$, $i\varepsilon\Delta_h$, and $t\varepsilon T$;

(23) $\partial\bar{\Omega}/\partial(t)\partial\gamma_{hj}(t) = \bar{g}_{hj}(t) \geqq 0$, $\bar{\gamma}_{hj}(t)\bar{g}_{hj}(t) = 0$, and $\bar{\gamma}_{hj}(t) \geqq 0$ for each $h\varepsilon\Theta°$,

$j\varepsilon\Delta$, and $t\varepsilon T$;

(24) $\partial\bar{\Omega}/\partial(u_\#)\partial\lambda_\#(t, u_\#) = \bar{\ell}_\#(t, u_\#) \geqq 0$, $\bar{\lambda}_\#(t, u_\#)\bar{\ell}_\#(t, u_\#) = 0$, and

$\bar{\lambda}_\#(t, u_\#) \geqq 0$ for $h\varepsilon\Theta_J$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hJj}$, $t\varepsilon T_\#$, and $u_\#\varepsilon\Gamma_{\#t}$.

A solution to Basic Problem which satisfies condition (18) through (24) also satisfies the equilibrium conditions (I) through (VII) of spatial and temporal goods allocation and efficient pricing in the free world. It may suffice to expound the following case: $\bar{x}_{hi}(t) > 0$, $\bar{y}_{kj}(t) > 0$, $\bar{z}_*(t, t, t+\hat{w}_*) > 0$, $\bar{z}_*(t, u_*, v_*) = 0$ for each $h\varepsilon\Theta$, $k\varepsilon\Theta°$, $i\varepsilon\Delta_h$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hij}$, $t\varepsilon T$, $u_*\varepsilon\Lambda_{*t}-\{t\}$ and $v_*\varepsilon\Xi_{*tu_*}$, or $u_*=t$ and $v_*\varepsilon\Xi_{*tu_*}-\{t+\hat{w}_*\}$, and $\bar{z}_\#(t, u_\#, u_\#+\hat{w}_\#) > 0$ and $\bar{z}_\#(t, u_\#, v_\#) = 0$ for each $h\varepsilon\Theta_J$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hJj}$, $t\varepsilon T_\#$, $u_\#\varepsilon\Gamma_{\#t}$, and $v_\#\varepsilon\Xi_{\#tu_\#}-\{u_\#+\hat{w}_\#\}$

In this case, we have, since $\bar{x}_{hi}(t) > 0$ and $\bar{y}_{hj}(t) > 0$ in (18) and (19),

(a) $\hat{a}_{hi}(t) + \sum_{k\varepsilon\Theta_i} \hat{b}_{hki}(t)\bar{x}_{ki}(t) = \bar{p}_{hi}(t) = \bar{\mu}_{hi}(t)$, and

(b) $\hat{c}_{hj}(t) - \sum_{k\varepsilon\Theta°} \hat{d}_{hkj}(t)\bar{y}_{kj}(t) = \bar{q}_{hj}(t) = \bar{\gamma}_{hj}(t)$.

Substituting (a) and (b) into (22), (23), (20), and (21), respectively, we have

(c) $\bar{f}_{hi}(t) \geqq 0$, $\bar{f}_{hi}(t)\bar{p}_{hi}(t) = 0$, and $\bar{p}_{hi}(t) \geqq 0$,

(d) $\bar{g}_{hj}(t) \geqq 0$, $\bar{g}_{hj}(t)\bar{q}_{hj}(t) = 0$, and $\bar{q}_{hj}(t) \geqq 0$,

(e) $-\bar{n}_*(t, u_*, v_*) \leqq 0$, $\bar{z}_*(t, u_*, v_*)\bar{n}_*(t, u_*, v_*) = 0$, and $\bar{z}_*(t, u_*, v_*) \geqq 0$,

(f) $\bar{n}_\#(u_\#, v_\#) - \bar{\lambda}_\#(t, u_\#) \leqq 0$, $\bar{z}_\#(t, u_\#, v_\#)\{\bar{n}_\#(u_\#, v_\#) - \bar{\lambda}_\#(t, u_\#)\} = 0$, and

$\bar{z}_\#(t, u_\#, v_\#) \geqq 0$,

where the notation in the equilibrium conditions (IV) and (V) is used in the above (e) and (f), respectively.

The uniqueness of $\bar{p}_{hi}(t)$ and $\bar{q}_{hj}(t)$, (c), (d), (e), (a), and (b) correspond to the equilibrium conditions (I), (II), (III), (IV), (VI), and (VII), respectively. From (e) and $\bar{z}_*(t, t, t+\hat{w}_*) > 0$, we must have $\hat{\eta}_h\{\hat{\sigma}(t+\hat{w}_*)/\hat{\sigma}(t)\}\bar{q}_{hj}(t+\hat{w}_*) = \bar{p}_{hi}(t) + \hat{e}_*(t, t, t+\hat{w}_*)$. For $j = i$ for $i\varepsilon\Delta_h$, we must have $\hat{\eta}_h\bar{q}_{hi}(t) = \bar{p}_{hi}(t) + \hat{e}_{hiil}(t, t, t)$ Let us see (f) in detail. If $\bar{\lambda}_{\#}(t, u_{\#}) > 0$, then we regard $\bar{\lambda}_{\#}(t, u_{\#})$ as an abnormal profit per unit of good $h\varepsilon\Theta$ generated by the rigidity of the supply cost peculiar to the trade with the planned economy block. Since we assumed that only $\bar{z}_{\#}(t, u_{\#}, u_{\#}+\hat{w}_{\#})$ is positive but all other $\bar{z}_{\#}(t, u_{\#}, v_{\#})$'s where $u_{\#}+\hat{w}_{\#}+1 \leqq v_{\#} \leqq t+\hat{s}_h$ are zero, the middlemen who import $\hat{z}_{\#}(t, u_{\#})$ quantity of good $h\varepsilon\Theta_J$ from country J by transportation mode $m\varepsilon\Phi_{hJj}$ can earn the unit abnormal profit $\hat{\eta}_h\{\hat{\sigma}(u_{\#}+\hat{w}_{\#})/\hat{\sigma}(u_{\#})\}\bar{q}_{hj}(u_{\#}+\hat{w}_{\#}) - \hat{p}_{hJE}(u_{\#}) - \hat{e}_{\#}(u_{\#}, v_{\#})$, if $\bar{\lambda}_{\#}(t, u_{\#}) > 0$. Of course, if $\bar{\lambda}_{\#}(t, u_{\#}) = 0$, then no abnormal profit is generated so that $\hat{\eta}_h\{\hat{\sigma}(u_{\#}+\hat{w}_{\#})/\hat{\sigma}(u_{\#})\}\bar{q}_{hj}(u_{\#}+\hat{w}_{\#}) = \hat{p}_{hJE}(u_{\#}) + \hat{e}_{\#}(u_{\#}, u_{\#}+\hat{w}_{\#})$. And also we have $\hat{z}_{\#}(t, u_{\#}) \geqq \bar{z}_{\#}(t, u_{\#}, u_{\#}+\hat{w}_{\#}) > 0$. Consequently, it can be known that (f) is equivalent to the equilibrium condition (V).

Accordingly, a Net Social Quasi-Welfare Maximization Approach in case of free competition can give a good basis for a worldwide temporal foods allocation and efficient pricing problem in reality.

PART II. Restricted Free Competition in the Free World

In PART I, we have referred to a case in which free competition prevails in the free world. Here we would like to utilize the economic theories developed in PART I as a basis for PART II in which realistic constraints will be taken into consideration. There are many restrictions on production, international trade, consumption, and storage activities concerned with foods in the world. Among these restricitions, we would like to select some important constraints which seem to affect the determination of equilibrium food prices and quantities. Restricted free competition should be understood to imply that any economic units are not monopolists or monopsonists and can not behave as a group which pursues their common goals (for example, OPEC), but individually and freely behave within the framework set by all countries' economic environments concerned with foods.

II.1. Additional Notation and Minor Assumptions

Θ_I and Θ_E : subsets of Θ consisting of the numbers of all goods which are affected by import quota systems and by export quota systems of at least one country, respectively;

Θ_C and Θ_A : subsets of Θ consisting of the numbers of all goods which are purchased through (long-term) contracts and of all goods which are used as free economic aids (gifts) in kind, respectively;

Δ_{hI} for any $h\varepsilon\Theta_I$: subset of Δ consisting of the numbers of all countries which impose import quota on good h;

Δ_{hjI} for any $h\varepsilon\Theta_I$ and $j\varepsilon\Delta_{hI}$: subset of Δ_h consisting of the numbers of all countries exporting good h to country j which imposes import quota on good h;

$\hat{\omega}_{hij}(t)$ for any $h\varepsilon\Theta_I$, $j\varepsilon\Delta_{hI}$, $i\varepsilon\Delta_{hjI}$, and $t\varepsilon T$: specified import quota on good h imported from country i which country j imposes during time t;

Δ_{hE} for any $h\varepsilon\Theta_E$: subset of Δ_h consisting of the numbers of all countries which impose export quota on good h;

Δ_{hiE} for any $h\varepsilon\Theta_E$ and $i\varepsilon\Delta_{hE}$: subset of $\Delta-\{i\}$ consisting of the numbers of all countries whose importation of good h from country i is under export quota of country i;

$\hat{\vartheta}_{hij}(t)$ for any $h\varepsilon\Theta_E$, $i\varepsilon\Delta_{hE}$, $j\varepsilon\Delta_{hiE}$, and $t\varepsilon T$: specified export quota on country j's importation of good h from country i imposed by country i during time t;

Δ_R : subset of Δ consisting of the numbers of all countries whose governments take food reserve policies, e.g., for keeping in stock some amounts of goods which assure several months' emergency consumption of H goods in question;

r : any of calorie numbered as 1 and necessary nutrients, such as protein and carbohydrate, numbered as 2, 3,...;

$\Pi \equiv \{1, 2,..., r,...\}$: set consisting of the numbers of calorie and all necessary nutrients;

$\hat{\pi}_{rj}(t)$ for any $r\varepsilon\Pi$, $j\varepsilon\Delta_R$, and $t\varepsilon T$: some specified minimum requirements of calories or necessary nutrients for the population of country j during time t which can be used as emergency provision and is considered to remove or lower social unease that is easily triggered from lack of food if the amounts of goods (foods) satisfying all $\hat{\pi}_{rj}(t)$'s are stored in cuntry j during time t;

Δ_{hSC} for any $h\varepsilon\Theta_C$: subset of Δ_h consisting of the numbers of all countries which have sales contracts of good h newly produced;

Δ_{hiBC} for any $h\varepsilon\Theta_C$ and $i\varepsilon\Delta_{hSC}$: subset of $\Delta-\{i\}$ consisting of the numbers of all countries which have purchase contracts of good h newly produced with country i;

$\tilde{r}$: either of weight numbered as 1 and bulk numbered as 2;

$\nabla \equiv \{1, 2\}$: set consisting of the numbers of weight and bulk;

$\hat{\delta}_{hr}$ for any $h\varepsilon\Theta^\circ$ and $r\varepsilon\Pi$: given calories or given amounts of necessary nutrients indicated by r which a unit of good h, of course, measured by $\tilde{r} = 1$, generates or contains;

$\hat{\rho}_{h\tilde{r}}$ for any $h\varepsilon\Theta$ and $\tilde{r}\varepsilon\nabla$: $\hat{\rho}_{h1} \equiv 1$ and $\hat{\rho}_{h2}$ denotes the bulk of a unit of good h measured by $\tilde{r} = 1$;

Θ_{SS} and Θ_{FS} : subsets of Θ consisting of the numbers of all goods whose farm supply prices are supported from below and fixed by at least one country, respectively;

Δ_{hSS} for any $h\varepsilon\Theta_{SS}$: subset of Δ_h consisting of the numbers of all countries which have policies for supporting farm supply price of good h;

Δ_{hFS} for any $h\varepsilon\Theta_{FS}$: subset of Δ_h consisting of the numbers of all countries which have policies for fixing farm supply price of good h;

$\hat{\zeta}_{hij}(t)$ for any $h\varepsilon\Theta_C$, $i\varepsilon\Delta_{hSC}$, $j\varepsilon\Delta_{hiBC}$, and $t\varepsilon T$: specified quantity of good h to be sold through (long-term) contracts to country j by country i during time t;

$\hat{\kappa}_{h\tilde{r}j}(t)$ for any $h\varepsilon\Theta$, $\tilde{r}\varepsilon\nabla$, $j\varepsilon\Delta$, and $t\varepsilon T$: given capacity of storage facilities allocated to good h, measured by $\tilde{r}$, in country j during time t;

$\tilde{\kappa}_{*\tilde{r}}(t) \equiv \tilde{\kappa}_{hijm\tilde{r}}(t)$ for any $h\varepsilon\Theta$, $i\varepsilon\Delta_h$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hij}$, $\tilde{r}\varepsilon\nabla$, and $t\varepsilon T$: given capacity of international transportation means specified in m, measured by $\tilde{r}$, avaiblable to good h sold to country j by country i during time t, where we assume that the international transportation means is chartered by country j;

Δ_{hAG} for any $h\varepsilon\Theta$: subset of Δ_h consisting of the numbers of all countries which produce good h and give it to at least one country as free economic aids;

Δ_{hiAG} for any $h\varepsilon\Theta$ and $i\varepsilon\Delta_{hAG}$: subset of $\Delta - \{i\}$ consisting of the numbers of all countries to which country i gives some predetermined quantities of good h as free economic aids;

Δ_{hAR} for any $h\varepsilon\Theta$: subset of Δ consisting of the numbers of all countries which receive good h as free economic aids from at least one foreign country;

Δ_{hjAR} for any $h\varepsilon\Theta$ and $j\varepsilon\Delta_{hAR}$: subset of Δ_{hAG} consisting of the numbers of all countries which give country j some predetermined quantities of good h as free economic aids;

Φ_{hijA} for any $h\varepsilon\Theta_A$, $i\varepsilon\Delta_{hAG}$, and $j\varepsilon\Delta_{hiAG}$: set of the numbers of all transportation modes specified by country i by which good h is carried to country j for free economic aids, where we assume that any $m\varepsilon\Phi_{hijA}$ does not compete with any $m\varepsilon\Phi_{hij}$.

$T_{*A} \equiv T_{hijmA}$ for any $h\varepsilon\Theta_A$, $i\varepsilon\Delta_{hAG}$, $j\varepsilon\Delta_{hiAG}$, and $m\varepsilon\Phi_{hijA}$: subset of T consisting of the numbers of all t's which indicate the production times of good h produced in country i and planned to be shipped to country j by transportation mode m as free economic aids (gifts) by country i some future time;

$\Gamma_{*tA} \equiv \Gamma_{hijmtA}$ for any $h\varepsilon\Theta_A$, $i\varepsilon\Delta_{hAG}$, $j\varepsilon\Delta_{hiAG}$, $m\varepsilon\Phi_{hijA}$ and $t\varepsilon T_{*A}$: subset of T consisting of the numbers of all (exportation) times when good h produced in country i during time t is planned to be shipped to country j by transportation mode m as free economic adis;

$\hat{z}_{*A}(t, u_*)$ for any $h\varepsilon\Theta_A$, $i\varepsilon\Delta_{hAG}$, $j\varepsilon\Delta_{hiAG}$, $m\varepsilon\Phi_{hijA}$, $t\varepsilon T_{*A}$, and $u_*\varepsilon\Gamma_{*tA}$: given quantity of good h produced in country i during time t and given freely to country j by country i during time u_* after carried by transportation mode m, where $\hat{z}_{*A}(t, u_*) > 0$;

$z_{*A}(t, u_*, v_*)$ and $\hat{e}_{*A}(u_*+\hat{w}_*, v_*)$ for any $h\varepsilon\Theta_A$, $i\varepsilon\Delta_{hAG}$, $j\varepsilon\Delta_{hiAG}$, $m\varepsilon\Phi_{hijA}$, $t\varepsilon T_{*A}$, $u_*\varepsilon\Gamma_{*tA}$, and $v_*\varepsilon\Xi_{*tu_*}$: variable quantity out of $\hat{z}_{*A}(t, u_*)$ consumed during time v_* and given unit allocation cost usually consisting of domestic transportation and storage costs, respectively;

II.2. Modification of Net Social Quasi-Welfare and Some Important Realistic Constraints

Here we take into consideration realistic economic restrictions and conditions which are actually imposed on international as well as domestic activities concerned with food. In finding the equilibrium prices and quantities of food in each capitalistic country during each time, we may have to emphasize conditions (A') through (Q') shown below. However, appropriate constraints should be selected, depending on each country's food situation, between (B') and (C'), between (D') and (E'), between (H') and (I'), and among (J') through (M').

(A') Subsidies on production have strong effects on the increases in production quantities of goods, while taxes on consumption affect consumption quantities. Among various kinds of subsidies and taxes, we would like to handle constant subsidies given to production of each unit of goods and constant taxes imposed on consumption of each unit of goods. Since subsidies denoted as $\hat{p}_{hiS}(t)$ and taxes denoted as $\hat{q}_{hjC}(t)$ have effects on reducing unit production costs by $\hat{p}_{hiS}(t)$ and demand prices received by middlemen by $\hat{q}_{hjC}(t)$, respectively, we need to modify the previous objective function (14) into (25) as follows:

$$(25)\quad (14) - \sum_{t\varepsilon T}\sum_{h\varepsilon\Theta^\circ}\sum_{j\varepsilon\Delta}\int_0^{y_j(t)} \hat{\sigma}(t)\hat{q}_{hjC}(t)\ dy_{hi}(t)$$

$$+ \sum_{t\varepsilon T}\sum_{h\varepsilon\Theta}\sum_{i\varepsilon\Delta_h}\int_0^{x_i(t)} \hat{\sigma}(t)\hat{p}_{hiS}(t)\ dx_{hi}(t).$$

To understand the terms added to (14), we factorize the first added term and the term implying total gross welfare attainable from consumption in (14), and the second added term and the term implying total (variable) production costs in (14), respectively. Then we can get, omitting the transport and storage cost term, $\sum_{t\varepsilon T}\sum_{h\varepsilon\Theta^\circ}\sum_{j\varepsilon\Delta}\int_0^{y_j(t)} \hat{\sigma}(t)\{q_{hi}(t) - \hat{q}_{hjC}(t)\}\ dy_{hj}(t) - \sum_{t\varepsilon T}\sum_{h\varepsilon\Theta}\sum_{i\varepsilon\Delta_h}$ $\int_0^{x_i(t)} \hat{\sigma}(t)\{p_{hi}(t) - \hat{p}_{hiS}(t)\}\ dx_{hi}(t)$. This implies that farm supply and demand price planes are shifted downward by $\hat{p}_{hiS}(t) > 0$ and $\hat{q}_{hjC}(t) > 0$, respectively.

(B')The constraint on the importation of good h from country i imposed by country j during time t may be expressed as follows:

$$(26)\quad \psi_{hij}(t) \equiv \hat{\omega}_{hij}(t) - \sum_{m\varepsilon\Phi_{hij}}\ \sum_{\tau_*=t-\hat{s}_h}^{t-\hat{w}_*}\ \sum_{v_*=t}^{\tau_*+\hat{s}_h} z_*(\tau_*,\ t-\hat{w}_*,\ v_*) \geqq 0$$

for each $h\varepsilon\Theta_I$, $j\varepsilon\Delta_{hI}$, $i\varepsilon\Delta_{hjI}$, and $t\varepsilon T$.

(C')The constraint on the importation of good h from all exporting countries imposed by country j during time t may be expressed as follows:

$$(27)\quad \psi_{hj}(t) \equiv \sum_{i\varepsilon\Delta_{hjI}} \psi_{hij}(t) \geqq 0 \text{ for each } h\varepsilon\Theta_I,\ j\varepsilon\Delta_{hI}, \text{ and } t\varepsilon T,$$

where each $\hat{\omega}_{hij}(t)$ is not specified but $\sum_{i\varepsilon\Delta_{hjI}} \hat{\omega}_{hij}(t)$ is specified.

(D')The constranint on the exportation of good h from country i to country j imposed by country i during time t may be expressed as follows:

(28) $\xi_{hij}(t) \equiv \hat{\vartheta}_{hij}(t) - \sum_{m\varepsilon\Phi_{hij}} \sum_{\tau_*=t-\hat{s}_h+\hat{w}_*}^{t} \sum_{v_*=t+\hat{w}_*}^{\tau_*+\hat{s}_h} z_*(\tau_*,\ t,\ v_*) \geqq 0$

for each $h\varepsilon\Theta_E$, $i\varepsilon\Delta_{hE}$, $j\varepsilon\Delta_{hiE}$, and $t\varepsilon T$.

(E')The constraint on the exportation of good h from country i to all importing countries imposed by country i during time t may be expressed as follows:

(29) $\xi_{hi}(t) \equiv \sum_{j\varepsilon\Delta_{hiE}} \xi_{hij}(t) \geqq 0$ for each $h\varepsilon\Theta_E$, $i\varepsilon\Delta_{hE}$, and $t\varepsilon T$,

where each $\hat{\vartheta}_{hij}(t)$ is not specified but $\sum_{j\varepsilon\Delta_{hiE}} \hat{\vartheta}_{hij}(t)$ is specified.

(F')The constraint on government-directed food reserve of goods which assures several months' emergency provision of minimum requirements for calories and necessary nutrients for the population of country j during time t may be expressed as follows: 3/

(30) $\sum_{h\varepsilon\Theta^\circ} \sum_{i\varepsilon\Delta_{hW}} \sum_{m\varepsilon\Phi_{hij}} \sum_{\tau_*=t-\hat{s}_h+1}^{t-\hat{w}_*-1} \sum_{u_*=\tau_*}^{t-\hat{w}_*-1} \sum_{v_*=t+1}^{\tau_*+\hat{s}_h} \hat{\delta}_{hr}\hat{n}_h z_*(\tau_*,\ u_*,\ v_*) - \hat{\pi}_{rj}(t) \geqq 0$

for each $r\varepsilon\Pi$, $j\varepsilon\Delta_R$, and $t\varepsilon T$, where $z_\#(\tau_\#,\ u_\#,\ v_\#) \equiv 0$ if $\tau_\#\notin T_\#$ or $u_\#\notin\Gamma_{\#\tau_\#}$ even though $\tau_\#\varepsilon T_\#$.

3/ Country $j\varepsilon\Delta_R$ must possess storage facilities big enough to reserve foods derived from the government-directed food reserve policies. If $\hat{\pi}_{rj}(t)$'s for all $j\varepsilon\Delta_R$ are too large, we may have to introduce these countries' as well as exporting countries' production frontier constraints based on limitation of agricultural land, labor, and other scarce factors of production.
The government-directed food reserve policy implies here that at any price levels of all H food items, the government tries to reserve enough quantities of foods which guarantee the minimum requirements for calories and necessary nutrients for the population.

(G')The constraint on the purchase of good h through contracts from country i by country j during time t may be expressed as follows:

$$(31)\quad \sum_{m\varepsilon\Phi_{hij}} \sum_{u_*\varepsilon\Lambda_{*t}} \sum_{v_*\varepsilon\Xi_{*tu_*}} z_*(t,\ u_*,\ v_*) - \hat{\zeta}_{hij}(t) \geqq 0$$

for each $h\varepsilon\Theta_C$, $i\varepsilon\Delta_{hSC}$, $j\varepsilon\Delta_{hiBC}$, and $t\varepsilon T$.

(H')The constraint on storage of good h stored in country j during time t by the limited capacity of storage facilities allocated to good h may be expressed as follows:

$$(32)\quad \beta_{h\tilde{r}j}(t) \equiv \hat{\kappa}_{h\tilde{r}j}(t) - \sum_{i\varepsilon\Delta_{hW}} \sum_{m\varepsilon\Phi_{hij}} \sum_{\tau_*=t-\hat{s}_h}^{t-\hat{w}_*} \sum_{u_*=\tau_*}^{t-\hat{w}_*} \sum_{v_*=t}^{\tau_*+\hat{s}_h} \hat{\rho}_{h\tilde{r}} z_*(\tau_*,\ u_*,\ v_*) \geqq 0$$

for each $h\varepsilon\Theta$, $j\varepsilon\Delta - \Delta_h$, $\tilde{r}\varepsilon\nabla$, and $t\varepsilon T$; and

$$(33)\quad \beta_{h\tilde{r}j}(t) \equiv (32) - \sum_{i\varepsilon\Delta-\{j\}} \sum_{m\varepsilon\Phi_{hji}} \sum_{\tau_\S=t-\hat{s}_h+\hat{w}_\S}^{t} \sum_{u_\S=t}^{\tau_\S+\hat{s}_h-\hat{w}_\S} \sum_{v_\S=u_\S+\hat{w}_\S}^{\tau_\S+\hat{s}_h} \hat{\rho}_{h\tilde{r}} z_\S(\tau_\S,\ u_\S,\ v_\S)$$

$$- \sum_{m\varepsilon\Phi_{hjJ}} \sum_{\tau_\dagger=t-\hat{s}_h+\hat{w}_\dagger}^{t} \sum_{u_\dagger=t}^{\tau_\dagger+\hat{s}_h-\hat{w}_\dagger} \hat{\rho}_{h\tilde{r}} \hat{z}_\dagger(\tau_\dagger,\ u_\dagger) \geqq 0$$ for each $h\varepsilon\Theta$, $j\varepsilon\Delta_h$, $\tilde{r}\varepsilon\nabla$,

and $t\varepsilon T$,

where subscripts § and † denote subscripts hjim and hjJm, respectively.

(I')The constraint on storage of goods stored in country j during time t by the limited capacity of storage facilities in which some goods can be stored together may be expressed as follows:

$$(34)\quad \beta_{\tilde{h}\tilde{r}j}(t) \equiv \sum_{h\varepsilon\tilde{\Theta}_{\tilde{h}}} \beta_{h\tilde{r}j}(t) \geqq 0$$ for each $\tilde{h}\varepsilon\tilde{\Theta}$, $j\varepsilon\Delta$, $\tilde{r}\varepsilon\nabla$, and $t\varepsilon T$,

where $\tilde{\Theta}$ and $\tilde{\Theta}_{\tilde{h}}$ denote sets consisting of the numbers of all storage techniques, e.g., various combinations of temperature, humidity, etc. and of all goods which can be stored in the facilities with storage technique $\tilde{h}\varepsilon\tilde{\Theta}$, where we assume that $\tilde{\Theta}_{\tilde{h}} \cap \tilde{\Theta}_{\tilde{k}} = \phi$ for any $\tilde{h}\varepsilon\tilde{\Theta}$ and $\tilde{k}\varepsilon\tilde{\Theta}-\{\tilde{h}\}$, respectively.

and each $\hat{\kappa}_{h\tilde{r}j}(t)$ is not specified but $\sum_{h\varepsilon\Theta_{\tilde{h}}} \hat{\kappa}_{h\tilde{r}j}(t)$ is specified.

(J')The constraint on international trade of good h between each exporting and importing countries i and j during time t by the limited capacity of international transport means for good h may be expressed as follows:

$$(35)\quad \chi_{*\tilde{r}}(t) \equiv \tilde{\kappa}_{*\tilde{r}}(t) - \sum_{u_*=t-\hat{w}_*}^{t} \sum_{\tau_*=u_*-\hat{s}_h+\hat{w}_*}^{u_*} \sum_{v_*=u_*+\hat{w}_*}^{\tau_*+\hat{s}_h} \hat{\rho}_{h\tilde{r}} z_*(\tau_*, u_*, v_*) \geqq 0$$

for each $h\varepsilon\Theta$, $i\varepsilon\Delta_h$, $j\varepsilon\Delta - \{i\}$, $m\varepsilon\Theta_{hij}$, $\tilde{r}\varepsilon\nabla$, and $t\varepsilon T$.

(K')The constraint on international trade of goods between each exporting and importing countries i and j during time t by the limited capacity of international transportation means which can carry some goods together may be expressed as follows:

$$(36)\quad \chi_{ijm\tilde{r}}(t) \equiv \sum_{h\varepsilon\Theta_{ijm}} \chi_{*\tilde{r}}(t) \geqq 0 \text{ for each } i\varepsilon\Delta,\ j\varepsilon\Delta - \{i\},\ \tilde{r}\varepsilon\nabla,\ m\varepsilon\Phi_{ij}, \text{ and } t\varepsilon T,$$

where Φ_{ij} and Θ_{ijm} denote sets consisting of the numbers of all transportation means for international trade between each exporting and importing countries i and j and of all goods which can be carried by transportation means $m\varepsilon\Phi_{ij}$, where we assume that $\Theta_{ijm} \cap \Theta_{ijm'} = \phi$ for any $m\varepsilon\Phi_{ij}$ and $m'\varepsilon\Phi_{ij}-\{m\}$, respectively.

(L')The constraint on international trade of good h from all exporting countries to country j during time t by the limited capacity of the international transportation means for good h may be expressed as follows:

$$(37)\quad \chi_{hjm\tilde{r}}(t) \equiv \sum_{i\varepsilon\Delta_h} \chi_{*\tilde{r}}(t) \geqq 0 \text{ if } j\notin\Delta_h, \text{ or } \sum_{i\varepsilon\Delta_h-\{j\}} \chi_{*\tilde{r}}(t) \geqq 0 \text{ if } j\varepsilon\Delta_h,$$

for each $h\varepsilon\Theta$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hj}$, $\tilde{r}\varepsilon\nabla$, and $t\varepsilon T$,

where Φ_{hj} denotes a set consisting of the numbers of all international transportation means for good h available to country j.

(M')The constraint on international trade of goods from all exporting countries to country j during time t by the limited capacity of international transportation means which can carry some goods together may be expressed as follows:

$$(38)\quad \chi_{jm\tilde{r}}(t) \equiv \sum_{i\varepsilon\Delta-\{j\}} \sum_{h\varepsilon\Theta_{ijm}} \chi_{*\tilde{r}}(t) \geqq 0 \text{ for each } j\varepsilon\Delta,\ m\varepsilon\Phi_j,\ \tilde{r}\varepsilon\nabla, \text{ and } t\varepsilon T,$$

where Φ_j denotes a set consisting of the numbers of all international transportation means which can carry some goods together to country j.

(N')The constraint on farm supply price of good h by country i's governmental policies for guaranteeing the minimum farm supply price denoted as $\hat{p}_{hi}(t)$ during time t may be expressed as follows:

$$(39)\quad p_{hi}(t) - \hat{p}_{hi}(t) = \sum_{k\varepsilon\Theta_i} \hat{b}_{hki}(t)x_{ki}(t) - \{\hat{p}_{hi}(t) - \hat{a}_{hi}(t)\} \geqq 0$$

for each $h\varepsilon\Theta_{SS}$, $i\varepsilon\Delta_{hSS}$, and $t\varepsilon T$.

(O')The constraint on fixed farm supply price of good h by country i's governmental policies for fixing the farm supply price at $\tilde{p}_{hi}(t)$ during time t may be expressed as follows:

$$(40)\quad p_{hi}(t) - \tilde{p}_{hi}(t) = \sum_{k\varepsilon\Theta_i} \hat{b}_{hki}(t)x_{ki}(t) - \{\tilde{p}_{hi}(t) - \hat{a}_{hi}(t)\} = 0$$

for each $h\varepsilon\Theta_{FS}$, $i\varepsilon\Delta_{hFS}$, and $t\varepsilon T$.

(P')The constraint on free economic aids in kind and the modification of (3), (4), and (25):

$$(41)\quad \hat{z}_{*A}(t, u_*) - \sum_{v_*\varepsilon\Xi_{*tu_*}} z_{*A}(t,u_*, v_*) \geqq 0 \text{ with } z_{*A}(t, u_*, v_*) \geqq 0$$

for each $h\varepsilon\Theta_A$, $i\varepsilon\Delta_{hAG}$, $j\varepsilon\Delta_{hiAG}$, $m\varepsilon\Phi_{hijA}$, $t\varepsilon T_{*A}$, and $u_*\varepsilon\Gamma_{*tA}$;

$$(42)\quad (3) - \sum_{j\varepsilon\Delta_{hiAG}} \sum_{m\varepsilon\Phi_{hijA}} \sum_{u_*\varepsilon\Gamma_{*tA}} \hat{z}_{*A}(t, u_*) \geqq 0$$

for each $h\varepsilon\Theta_A$, $i\varepsilon\Delta_{hAG}$, and $t\varepsilon T$;

$$(43)\quad (4) + \sum_{i\varepsilon\Delta_{hjAR}} \sum_{m\varepsilon\Phi_{hijA}} \sum_{\tau_*\varepsilon\Lambda_{*t}} \sum_{u_*\varepsilon\Gamma_{*(\tau_*-\hat{s}_h)A}} \hat{n}_h z_{*A}(\tau_*-\hat{s}_h, u_*, t) \geqq 0$$

for each $h\varepsilon\Theta_A$, $j\varepsilon\Delta_{hAR}$, and $t\varepsilon T$.

$$(44)\quad (25) - \sum_{h\varepsilon\Theta_A} \sum_{i\varepsilon\Delta_{hAG}} \sum_{j\varepsilon\Delta_{hiAG}} \sum_{m\varepsilon\Phi_{hijA}} \sum_{t\varepsilon T_{*A}} \sum_{u_*\varepsilon\Gamma_{*tA}} \sum_{v_*\varepsilon\Xi_{*tu_*}} \hat{\sigma}(u_*+\hat{w}_*)\hat{e}_{*A}(u_*+\hat{w}_*, v_*)z_{*A}(t, u_*, v_*).$$

(Q') All economic units such as farmers, middlemen, governments, etc. do not or can not behave to maximize their monetary goals such as their net profits by manipulating market prices at any stage of transactions. Free competition prevails only within the economic activity field allowable by constraints (3) or (42), (4) or (43), (5), (15), and a proper combination of additional constraints which can well reflect our current competition concerning foods.

II.3. Realistic Problem

We have referred to some important additional constraints together with the minor modification of the objective function in Section I.4. There may be many other constraints on a worldwide food economy. However, we will ignore them except for foreign reserves constraints to be referred to later.

We may be able to formulate the following Realistic Problem:

Realistic Problem : Find an economic situation $\{\bar{x}_{hi}(t)$, $\bar{y}_{kj}(t)$, $\bar{p}_{hi}(t)$, $\bar{q}_{kj}(t)$, and $\bar{z}_*(t, u_*, v_*)$ for all $h\varepsilon\Theta$, $k\varepsilon\Theta°$, $i\varepsilon\Delta_h$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hij}$, $t\varepsilon T$, $u_*\varepsilon\Lambda_{*t}$, and $v_*\varepsilon\Xi_{*tu_*}$, $\bar{z}_\#(t, u_\#, v_\#)$ for all $h\varepsilon\Theta_J$, $j\varepsilon\Delta$, $m\varepsilon\Phi_{hJj}$, $t\varepsilon T_\#$, $u_\#\varepsilon\Gamma_{\#t}$, and $v_\#\varepsilon\Xi_{\#tu_\#}$, and $\bar{z}_{*A}(t, u_*, v_*)$ for all $h\varepsilon\Theta_A$, $i\varepsilon\Delta_{hAG}$, $j\varepsilon\Delta_{hiAG}$, $m\varepsilon\Phi_{hijA}$, $t\varepsilon T_{*A}$, $u_*\varepsilon\Gamma_{*tA}$, and $v_*\varepsilon\Xi_{*tu_*}\} \geqq 0$, if it exists, such that maximizes (44) subject to (3) for each $h\varepsilon\Theta$, $i\varepsilon\Delta_h - \Delta_{hAG}$, and $t\varepsilon T$, (42), (4) for each $h\varepsilon\Theta°$, $j\varepsilon\Delta - \Delta_{hAR}$, and $t\varepsilon T$, (43), (5), (41), (15), any combination of (26) and (27), any combination of (28) and (29), (30), (31), (39), (40), and proper constraints among constraints (32) through (38).

The above Realistic Problem may be able to be solved by solving its Saddle Value Problem which can be formulated in the same way as in Section I.4. Then the Wolfe's or Beale's solution algorithm is available [2,18]

Especially, in a phase of a worldwide recession like the current one, the foreign reserves, say, in terms of U.S. dollars which are allowed to be spent only for the purpose of the importation over the exportation of foods may play a significant role. From a purely mathematical viewpoint, we have not included foreign reserves constraints, since the set permitted by them will not be convex but concave. We may be able to express foreign reserves constraints as follows:

$$(45)\quad \tilde{\omega}_j(t) + \sum_{h\varepsilon\Theta_j} \sum_{i\varepsilon\Delta-\{j\}} \sum_{m\varepsilon\Phi_{hji}} \sum_{\tau_§=t-\hat{s}_h}^{t-\hat{w}_§} \sum_{v_§=t}^{\tau_§+\hat{s}_h} e_{§I}(\tau_§,\ t)z_§(\tau_§,\ t-\hat{w}_§,\ v_§)$$

$$- \sum_{h\varepsilon\Theta} \sum_{i\varepsilon\Delta_{hj}} \sum_{m\varepsilon\Phi_{hij}} \sum_{\tau_*=t-\hat{s}_h}^{t-\hat{w}_*} \sum_{v_*=t}^{\tau_*+\hat{s}_h} e_{*I}(\tau_*,\ t)z_*(\tau_*,\ t-\hat{w}_*,\ v_*) \geq 0$$

for each $j\varepsilon\Delta$ and $t\varepsilon T$.

where subscript § denotes subscript hjim, $\Delta_{hj}\equiv\Delta_h$ if $j\notin\Delta_h$ or $\Delta_h - \{j\}$ if $j\varepsilon\Delta_h$, $\tilde{\omega}_j(t)$ stands for a given amount of foreign reserves allowed only for the net payments for foods, and the foreign reserves concerned with the trade with the planned economy block are already taken into consideration in $\tilde{\omega}_j(t)$.

If we solve a Realistic Problem with additional constraint (45), a solution can not necessarily guarantee global optimality. Therefore, in the case where foreign reserves are working effectively in some countries' food trade, we have to solve Realistic Problem without foreign reserves constraints and then investigate whether a solution can satisfy constraints(45). If the solution satisfies constraint (45), it will be an accecptable solutuion. But if not, the countries whose constraints on foreign reserves allowed for food work effectively must reconsider their foreign trade policies regarding food. Some possible international trade policies are to raise $\tilde{\omega}_j(t)$ or $\hat{e}_{hjI}(t)$ and to reduce $\sum_{i\varepsilon\Delta_{hjI}} \hat{\omega}_{hij}(t)$ or $\hat{e}_{hjE}(t)$. On the other hand, some possible agricultural policies are to raise $\hat{p}_{hjS}(t)$, $\hat{p}_{hj}(t)$, or $\tilde{p}_{hj}(t)$ and to make efforts to shift downward farm supply price surfaces and lower the slopes of these surfaces.

II.4 Brief Explanation of Lagrangean Multipliers in Realistic Problem

It is quite important for economists to understand the economic meaning of Lagrangean multipliers for constraints. For if the optimal values of some Lagrangean multipliers are positive, we need to know how these Lagrangean multipliers will be treated in our real economic activities. We would like to give briefly the economic meaning of the Lagrangean multipliers only for constraints (26), (28), (30), (31), (33), (35), (39), (40), and (41) in relation to the definition of a worldwide temporal goods allocation and efficient pricing in Section I.3. The other Lagrangean multipliers can be easily understood.

The Lagrangean multiplier for (26), denoted as $\hat{\sigma}(t)\psi^L_{hij}(t)$, may be able to be understood as unit extra import tariff on good $h\varepsilon\Theta_I$ imported from country $i\varepsilon\Delta_{hjI}$ imposed by the government of country $j\varepsilon\Delta_{hI}$ during time t if the government wants to do so, or as unit abnormal profit of good $h\varepsilon\Theta_I$ generated during time t and earned by middlemen (importers) of country j when the good is consumed if the government does not absorb abnormal profits through imposition of unit extra import tariff on that good. In the latter case, the strong demand for that good as well as governmental import quota system are considered to give middlemen a chance to earn abnormal profits. If we regard $\bar{\psi}^L_{hij}(u_*+\hat{w}_*) + \hat{e}_{hjI}(u_*+\hat{w}_*)$ as $\hat{e}_{hjI}(u_*+\hat{w}_*)$ of Table 1 in the case of unit extra import tariff, or $\hat{n}^{-1}_h\{\hat{\sigma}(u_*+\hat{w}_*)/\hat{\sigma}(v_*)\}\bar{\psi}^L_{hij}(u_*+\hat{w}_*) + \hat{e}_{hjM}(v_*)$ as $\hat{e}_{hjM}(v_*)$ of Table 1 in the case of unit abnormal profit concerning related h, i, j, m, t, u_*, and v_*, then the definition of a worldwide temporal goods allocation and efficient pricing can be utilized even in Realistic Problem.

The Lagrangean multiplier for (28), denoted as $\hat{\sigma}(t)\xi^L_{hij}(t)$, may be able to be understood as unit extra export tariff on good $h\varepsilon\Theta_E$ imported by country $j\varepsilon\Delta_{hiE}$ imposed by the government of country $i\varepsilon\Delta_{hE}$ during time t if the government wants to do so, or as unit abnormal profit of good $h\varepsilon\Theta_E$ generated during time t and earned by middlemen (exporters) of country i if the government does not absorb abnormal profits. If we regard $\bar{\xi}^L_{hij}(u_*) + \hat{e}_{hiE}(u_*)$ as $\hat{e}_{hiE}(u_*)$ of Table 1 in the case of unit extra export tariff, or $\{\hat{\sigma}(u_*)/\hat{\sigma}(u_*+\hat{w}_*)\}\bar{\xi}^L_{hij}(u_*) + \hat{e}_{hiM}(u_*+\hat{w}_*)$ as $\hat{e}_{hiM}(u_*+\hat{w}_*)$ of Table 1 in the case of unit abnormal profit concerning related h, i, j, m, and u_*, then the definition of a worldwide temporal equilibrium in Section I.3 can be made valid.

Let $\hat{\sigma}(t)\pi^L_{rj}(t)$ be the Lagrangean multiplier for (30). We may be able to say that the government of country $j\varepsilon\Delta_R$ must reduce given unit transfer cost of good $h\varepsilon\Theta$, especially storage cost, by unit subsidy $\sum_{r\varepsilon\Pi} \hat{\delta}_{hr}\hat{\eta}_h\bar{\pi}^L_{rj}(t)$ to encourage reservation of foods in its country. However, if $\sum_{r\varepsilon\Pi} \hat{\delta}_{hr}\hat{\eta}_h\bar{\pi}^L_{rj}(t)$ is considerably larger, compared with $\hat{e}_*(\tau_*, u_*, v_*)$'s concerned, then we must introduce additional constraints $x_{hj}(t) - \hat{x}^\circ_{hj}(t) \geqq 0$ for sufficiently large $\hat{x}^\circ_{hj}(t)$ given for each $h\varepsilon\Theta$, where $\hat{x}^\circ_{hj}(t)$'s are considered to satisfy $\hat{\pi}_{rj}(t)$'s. In this situation, the goverment of country $j\varepsilon\Delta_R$ must give its farmers producing good h extra unit subsidy corresponding to the optimal value of the Lagrangean multiplier for $x_{hj}(t) - \hat{x}^\circ_{hj}(t) \geqq 0$ in order to encourage farmers to produce more foods. If the upper bound of $x_{hj}(t)$ is not high, the government must reduce $\hat{\pi}_{rj}(t)$'s. Otherwise, the government of country $j\varepsilon\Delta_R$ may want to give farmers of other countries subsidies for promotion of more foods through agreements with these governments and farmers.

Let $\hat{\sigma}(t)\zeta^L_{hij}(t)$ be the Lagrangean multiplier for (31). So, if $\bar{\zeta}^L_{hij}(t) > 0$, we can consider that middlemen of country $j\varepsilon\Delta_{hiBS}$ must pay unit subsidy equivalent to $\bar{\zeta}^L_{hij}(t)$ to farmers producing good h in country $i\varepsilon\Delta_{hSC}$ in order to compensate the loss $\bar{\zeta}^L_{hij}(t)$ caused by the (long-term) contracts. The compensation happens only when the demands covering time t through time v_* are so low that the value of the first term of (31) under the assumption of no contracts is smaller than $\hat{\zeta}_{hij}(t)$. If we regard $\hat{e}_{hjM}(v_*) - \hat{\eta}^{-1}_h\{\hat{\sigma}(t)/\hat{\sigma}(v_*)\}\bar{\zeta}^L_{hij}(t)$ as $\hat{e}_{hjM}(v_*)$ in Table 1, then we can use the definition of a worldwide temporal equilibrium.

Let $\hat{\sigma}(t)\beta^{L}_{h\tilde{r}j}(t)$ denote the Lagrangean multiplier for (33). If we assume without loss of generality that only $\bar{\beta}^{L}_{h2j}(t) > 0$, where $\tilde{r} = 2$, then storage facilities measured by bulk must be completely filled with stocks of good h during time t. In this case, owners of storage facilities for good h could earn unit abnormal profit $\hat{\rho}_{h2}\bar{\beta}^{L}_{h2j}(t)$ as a kind of quasi-rent. If we regard $\hat{\rho}_{h2}\bar{\beta}^{L}_{h2i}(t) + \hat{e}_{hi}(t, u_*)$ or $\hat{\rho}_{h2}\bar{\beta}^{L}_{h2j}(u_*+\hat{w}_*) + \hat{e}_{hj}(u_*+\hat{w}_*, v_*)$ as $\hat{e}_{hi}(t, u_*)$ or $\hat{e}_{hj}(u_*+\hat{w}_*, v_*)$ in Table 1, respectively, the definition in Section I.3 would be acceptable.

Let $\hat{\sigma}(t)\chi^{L}_{*\tilde{r}}(t)$ be the Lagrangean multiplier for (35). Assuming without loss of generality that only $\bar{\chi}^{L}_{*1}(t) > 0$, where $\tilde{r} = 1$, then we may be able to consider $\bar{\chi}^{L}_{*}(t)$ as unit abnormal profit which owners of international transportation means m specified by * can earn by carrying good h from country i to country j during time t. Of course, even if the owners of this international transportation means m really raise given unit transportation cost by $\bar{\chi}^{L}_{*1}(t)$, no changes will happen in the optimal values of all quantity and price variables. If we regard $\{\hat{\sigma}(t)/\hat{\sigma}(u_*)\}\bar{\chi}^{L}_{*1}(t) + \hat{e}_*(u_*)$ for any t such that $u_* \leqq t \leqq u_*+\hat{w}_*$ as $\hat{e}_*(u_*)$ in Table 1, the definition of a worldwide temporal equilibrium would be valid even in Realistic Problem.

Let $\hat{\sigma}(t)\alpha^{L}_{hi}(t)$ be the Lagrangean multiplier for (39), and $\bar{\alpha}_{hi}(t) \equiv \sum_{k\in\Theta_i}\hat{b}_{khi}(t)\bar{\alpha}^{L}_{ki}(t) = \sum_{k\in\Theta_i}\hat{b}_{hki}(t)\bar{\alpha}^{L}_{ki}(t)$ due to integrability condition. For simplicity, we assume that $\Theta_i \subset \Theta_{SS}$, namely, the farm supply prices of all goods producible in country i are supported from below by the minimum farm supply prices guarantee policies. Then $\bar{\alpha}_{hi}(t) > 0$ would be viewed as goverment's unit loss which is attributable to unit subsidy on consumption of good h to encourage the consumption which would be otherwise too small to clear the equilibrium quantity of production $\bar{x}_{hi}(t)$. However, if we regard $\bar{\alpha}_{hi}(t) + \hat{p}_{hiS}(t)$ as unit subsidy paid for production of good h to farmers by the government of country i during time t in order to make farmers supply more food at lower substantial farm supply price, we do not need to modify the definition in Section I.3.

The economic meaning of the Lagrangean multiplier for (40) is very similar to that of (39). Here what is different from the previous is that this multiplier can take a negative value, implying unit tax on production of good $h \varepsilon \Theta_i \subset \Theta_{FS}$ to shift the farm supply price plane upward.

The Lagrangean multiplier for (41) can be explained exactly in the same way that $\ell_{\#}(t, u_{\#})$ was explained in Section I.4. However, it should be noted that $\hat{z}_{*A}(t, u_*)$'s do not require their import costs, but invoke domestic transportation costs and some other costs. If all of $\hat{z}_{*A}(t, u_*)$'s are freely given to very poor people in country $j \varepsilon \Delta_{hAR}$ whose demands for foods were not reflected in the estimated deman price functions of country j, then constraint (41) should be ignored, and constraint (4) and objective function (25) should be used in place of constraint (43) and objective function (44), respectively, but constraint (42) must still be used.

Concerning the constraints imposed by governments, we can regard the optimal values of the Lagrangean multipliers as the optimal subsidies or taxes. We may be able to find good governmental agricultural policies by giving various values to the parameters, solving the Realistic Problems, and selecting the best set among sets of optimal production, consumption, and spatial and/or temporal quantities, and equilibrium farm supply and demand prices.

II.5. Conclusion

In recent books, journals, magazines, newspapers, and so on, a food crisis is often cited. This crisis is not mentioned about a particular country, but about many countries which have higher net birth rates than increasing rates of food production. High net birth rates, abnormal weather, inefficient production methods, civil wars, and tastes toward more animal protein are some of the factors which are making people feel that food crises are impending. Some food reserve plans were proposed, for instance, in U.N. conferences, and food reserve research has been taken up as a topic of the U.N. university in Tokyo. Now, we need not abstract but operational methods by which we can evaluate proposed food reserve policies and estimate possible impacts of unexpected poor harvests in main food exporting countries or large purchase of wheat by the planned economy countries on farm supply prices, consumers' prices, consumption quantities, international trade quantities, etc. of mixed economy countries. It is important to judge whether the proposed food reserve policies are reasonable and acceptable by the countries concerned. Nowadays every country has important relationships with many countries in trade, business, and other areas. We can not ignore a country's food problem as an unrelated and unimportant matter.

It is necessary to develop and further economic methodologies and theories for worldwide economic problems including worldwide food and energy problems. An operational method is here proposed for finding optimal allocation of food over space and time and the equilibrium prices derived. The following conditions are taken into consideration: barriers to trade of food by import tariffs, import quota, export tariffs, and export quota, food reserve policies, contracts, free economic aids in kind, trade with the planned economy block, minimum farm supply price support policies, fixed farm supply price policies, limited capacities of storage facilities and international transportation means,lump sum taxes and subsidies.

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