

No.598

**The Tragedy of the Commons Revisited:
Identifying Behavioral Principles[†]**

by

Masaru Ito*
Tatsuyoshi Saijo**
and
Masashi Une***

May 1994/Revised August 1994

[†] Research was partially supported by the Inamori Foundation, Grant in Aid for General Research 04451111 of the Ministry of Education in Japan, and the Zengin Foundation for Studies on Economics and Finance. The authors thank Saburo Ikeda, Yoshifusa Kitabatake, Yuji Kubo, Mutsuhide Matsumura, Joseph Sicilian, Stephen Turnbull, Jun Wako and Jong-Shin Wei for their help and comments. The authors are also grateful to two anonymous referees and the editor of this journal for comments that greatly improved this paper. The views expressed herein are those of the authors and not those of the Bank of Japan.

* Graduate School of Management Sciences and Public Policy Studies, University of Tsukuba, Tsukuba, Ibaraki 305, Japan

** Institute of Socio-Economic Planning, University of Tsukuba, Tsukuba, Ibaraki 305, Japan and Institute of Social and Economic Research, Osaka University, 6-1 Mihogaoka, Ibaraki, Osaka 567, Japan

*** Research and Statistics Department, Bank of Japan, Chuo, Tokyo 103, Japan

Abstract

The theme of Hardin's renowned "The Tragedy of the Commons" is that the common-pool resource is dissipated to the level where the average value of extraction equals the wage rate in the long run when the number of appropriators is unlimited. Yet experimental results and rapid deforestation in tropical countries follow a contrasting pattern: even though the number of appropriators is limited, the common-pool resource is dissipated more than the Nash equilibrium predicts. First, given these market or experimental observations, we identify two behavioral principles: share maximization and difference maximization. Second, we characterize these principles at Nash equilibrium.

Journal of Economic Literature classification codes: C92, D74, H82, L13 and Q20.

Correspondent:

Tatsuyoshi Saijo
Institute of Socio-Economic Planning
University of Tsukuba
Tsukuba, Ibaraki 305
Japan

Phone 01181-298 (country & area codes)
53-5379 (office)/53-5579 (experimental lab.)/53-5182 (institute)/55-3849 (fax)

E-mail: saijo@shako.sk.tsukuba.ac.jp

1. Introduction

Garrett Hardin's (1968) classic work on "the tragedy of the commons" describes the degradation of scarce resources that are open to all appropriators in society. Since all appropriators have equal access to the commons, they can enjoy the value of the *average* product from the commons. As long as this average value is greater than the *marginal* value, new appropriators aiming at the difference come into the commons with the result that resources are dissipated.

A real-world example is rapid deforestation in tropical countries. According to Repetto (1988), "once leading exporters like the Philippines have already virtually exhausted their lowland productive forests."¹ Although the reasons for the rapid deforestation in tropical countries are complex, the character of the problem is that a few foreign firms, from countries such the U.S.A. and Japan, who want to obtain timber concessions, compete with each other to exploit all the profitable areas.² Our paper may be interpreted as the analysis of situation where a few big outsiders exploit the commons that has been shared by many members in a traditional society.

In a laboratory experiment on the common-pool resource problem by Walker, Gardner, and Ostrom (1990b), the common-pool resource was dissipated more than the Nash equilibrium prediction even though the number of subjects was fixed throughout the experiment according to Davis and Holt (1992). Holt (1992) reported a similar phenomenon in oligopoly experiments where the theoretical structure was essentially the same as the common-pool resource model: subjects produced more product than in the Cournot-Nash equilibrium outcome. In a slightly different context, Saijo and Nakamura (1993) found in the voluntary contribution mechanism that subjects did not contribute all of their endowments even though the marginal return of the contribution was greater than one. That is, contributing everything is the dominant strategy. A common factor in these experiments is that no communication was allowed. Since in a traditional society communication among

¹ The tragedy of rapid deforestation of tropical areas embraces the loss of many animal and plant species including genes for possible future medical research as well as commercial materials. See Boado (1988) and Repetto (1988).

² Outsiders occasionally destroy a commons that is supported by some institutional arrangement among the natives. See Berkes, Feeny, McCay, and Acheson (1989) and Ostrom (1990).

appropriators is an important factor in coping with the dilemma, we believe the setting of these experiments better represents a few big outsiders who do not talk to each other.

Another real-world example is the panic hoarding of consumption goods³: a toilet paper panic occurred in Japan in 1973, triggered by the rumor of the tight demand and supply of paper due to the first oil shock.⁴ Recently, consumers in Japan rushed into rice shops to maintain their supplies because of the bad crop of rice in 1993, even though enough rice was supplied by import. Both toilet paper and rice are relatively cheap commodities although they might be more expensive in Japan than in other countries. The actual cost of buying these commodities is searching the shops and standing in line. This queuing externality makes the structure of the problem very similar to the tragedy of the commons.

Why is deforestation in tropical areas so rapid? Why didn't subjects in the above experiments follow Nash or dominant strategy predictions? Why did consumers buy up toilet paper and rice? For concreteness, we consider fish as the common-pool resource. First, in the special circumstance that all appropriators have the same endowment and spend the same hours for fishing, we show that appropriators can increase their shares or the differences between their incomes and the average income by spending more of their time to catch fish if and only if the value of average product of fish is greater than the wage rate determined by outside industries. This preliminary observation partly drives the difference between non-income and income-maximizing Nash equilibrium behaviors of appropriators in the above experiments. Second, we identify the class of behavioral principles that equate the value of *average* product from the common with the outside opportunity cost at Nash equilibrium even though the number of appropriators is finite.⁵ Although this class of behavioral principles is relatively large, we identify two important classes of behavioral principles imposing reasonable restrictions on the class of utility profiles: share maximizing and difference maximizing behaviors. Third, after identifying the two behavioral principles,

³ We thank Professor Wako who pointed out these examples to us.

⁴ See Hirose (1985).

⁵ Our approach identifying behavioral principles through the allocation has its origin in Houthakker's revealed preference theory (1950) and the integrability approach to demand functions originated by Hurwicz and Uzawa (1971).

we characterize them. Share maximizing behavior is equivalent to equality between the value of average product and the wage rate together with a share condition related to the distribution of endowments. Difference maximizing behavior is equivalent to equality between the value of average product and the wage rate together with a symmetry condition on labor input. That is, in both cases the commons can be wiped out even though the access to the commons is limited, or the number of appropriators is small.⁶ Fourth, although in the difference maximizing Nash equilibrium every appropriator's labor input is the same without assuming symmetry in the distribution of endowments, in the share maximizing Nash equilibrium allocation a larger endowment holder exerts a bigger labor input and receives a larger share. That is, big companies dominate the commons. Finally, if appropriators put weight to both the monetary benefit from the commons and the share, then the aggregate labor input level realized is between the one when appropriators care about the monetary benefit only and the one when they care about the share only. Further, this setting explains certain asymmetric behavior in experiments. For comparison, we develop similar analyses for two polar cases: the value of marginal product equals the wage rate (i.e., the socially optimal case) and the outcome when each appropriator employs the income maximizing behavioral principle.

Section 2 characterizes income maximizing Nash equilibria and proposes the share maximizing and difference maximizing behaviors as preliminary hypotheses. Section 3 identifies the behavioral principles based upon market data. In section 4, we characterize the behavioral principles obtained in section 3 and a numerical example is presented in section 5. Concluding remarks are in section 6.

2. Income maximizing Nash equilibrium: a preliminary

We employ a simple static model adopted, for example, by Weitzman (1974), Sandler (1992) and Roemer and Silvestre (1993).⁷ There are n appropriators (n is fixed) in a society,

⁶ The alleged "export flooding" by some Japanese firms that might result in wiping out another country's industry may be partly explained by this share maximizing behavior in our static model.

⁷ Our model is a special case of Roemer and Silvestre. Weitzman used a slightly different "many lake" model.

and appropriator i faces the decision to split i 's endowment, say 24 hours, into catching fish and leisure time.

Let w_i be appropriator i 's initial endowment, that is the possible total leisure time, and x_i be the labor input for catching fish. Let the production function be $f(x)$ with $f(0)=0$ and $f'(0) < \infty$, where $x = \sum x_j$. Assume that the average product is always greater than the marginal product, that is, $f'(x)x - f(x) < 0$.⁸ Appropriator i 's share of the total catch equals i 's share of total labor inputs. We normalize the price of fish as one and denote the wage rate by p .⁹ Then appropriator i 's income is defined by

$$m_i(\mathbf{x}) = f(x) \frac{x_i}{x} + p(w_i - x_i)$$

where $\mathbf{x} = (x_1, x_2, \dots, x_n)$. Define $m_i(0) = pw_i$. $m_i(\mathbf{x})$ is continuous since $f'(0)$ is finite. Then in the usual analysis of the tragedy of the commons, m_i is the objective function. We will generalize the objective function so that other appropriators' incomes as well as i 's own are arguments of the objective function and denote appropriator i 's utility function $u_i(m_1(\mathbf{x}), \dots, m_n(\mathbf{x}))$.¹⁰ We also write i 's utility function by $u_i(\mathbf{m}(\mathbf{x})) = u_i(\mathbf{x}) = u_i(x_i, x_{-i})$. Assume that $u_i(\mathbf{m})$ is continuous for all i . Then $u_i(\mathbf{x})$ is continuous for all i . Further, assume that $u_i(x_i, x_{-i})$ is concave with respect to x_i and $u_i(\cdot) > 0$ for all i for analytical convenience. The optimization problem of appropriator i is

$$\max_{x_i} u_i(\mathbf{m}(\mathbf{x})) \text{ subject to } x_i \in [0, w_i]. \quad (1)$$

A list of labor inputs $\mathbf{x}^* = (x_1^*, x_2^*, \dots, x_n^*)$ is a *Nash equilibrium* if for all i , $u_i(\mathbf{m}(\mathbf{x}^*)) \geq u_i(\mathbf{m}(x_i, x_{-i}^*))$ for all $x_i \in [0, w_i]$. The existence of a Nash equilibrium can be obtained by applying a standard argument.¹¹

Proposition 1. A Nash equilibrium exists.

⁸ If $f''(x) < 0$, $f'(x)x - f(x) < 0$ follows immediately.

⁹ Our interpretation is that the opportunity cost of leisure is the wage rate of the best possible job other than fishing.

¹⁰ Instead of introducing the vector of all appropriators' income, the amount of fish that appropriator i catches ($f(x)x_i/x$) is a candidate for the objective function. We did not take this approach since appropriator i spent the entire endowment for catching fish under this objective function.

¹¹ See, for example, Ichiishi (1983).

Proof. Let $W = \times_{i=1}^n [0, w_i]$. Define

$$r_i(x_{-i}) = \operatorname{argmax}_{x_i} \{u_i(x_i, x_{-i}) | x_i \in [0, w_i]\}.$$

Since W is compact and u_i is continuous for all i , r_i is upper semi-continuous by Berge's Maximum theorem. Notice further that r_i is non-empty-valued, closed-valued and convex-valued, the correspondence $(r_1, \dots, r_n): W \rightarrow W$ has a fixed-point by Kakutani's fixed-point theorem. Clearly, this fixed point is a Nash equilibrium. ■

In what follows, we will assume that the solution x_i in (1) is in the interior of $[0, w_i]$, and then consider conditions that validate the interior assumption. Since $\sum m_i(x) = f(x) + p(\sum w_i - x)$, define $m(x) = \sum m_i(x)$ and let \hat{x} be a solution of $dm(x)/dx = 0$. Apparently, \hat{x} is the socially optimal total labor input.

Although $(1/n)f'(x) + ((n-1)/n)(f(x)/x) = p$ is well known as a necessary condition for the income maximizing Nash equilibrium, we found that this condition together with $x_i = (1/n)x$ is necessary and sufficient for the equilibrium condition.

Proposition 2. $\frac{\partial m_i(x_i^m, x_{-i}^m)}{\partial x_i} = 0$ for all i if and only if $\frac{1}{n}f'(x^m) + \frac{n-1}{n}\frac{f(x^m)}{x^m} = p$ and $x_i^m = \frac{1}{n}x^m$ for all i .

Proof. Only if part: Notice that

$$\frac{\partial m_i(x)}{\partial x_i} = \frac{1}{x^2} \{ (f'(x)x_i + f(x))x - f(x)x_i \} - p = \frac{x_i}{x} \left\{ f'(x) - \frac{f(x)}{x} \right\} + \frac{f(x)}{x} - p. \quad (2)$$

Summing up the first order conditions, we have

$$\sum \frac{\partial m_i(x_i^m, x_{-i}^m)}{\partial x_i} = f'(x^m) + (n-1)\frac{f(x^m)}{x^m} - np = 0. \quad (3)$$

That is,

$$f'(x^m) - \frac{f(x^m)}{x^m} = -n \left\{ \frac{f(x^m)}{x^m} - p \right\} < 0 \quad (\because f'(x)x - f(x) < 0). \quad (4)$$

Since

$$\frac{\partial m_i(x_i^m, x_{-i}^m)}{\partial x_i} = \left\{ 1 - n \frac{x_i^m}{x^m} \right\} \left\{ \frac{f(x^m)}{x^m} - p \right\} = 0,$$

we have $x_i^m = \frac{1}{n}x^m$ for all i .

If part: By assumption, we have (4). Since $x_i^m = \frac{1}{n}x^m$, it is clear that $\frac{\partial m_i(x_i^m, x_{-i}^m)}{\partial x_i} = 0$ for all i .

■

Differentiating $\frac{1}{n}f'(x) + \frac{n-1}{n}\frac{f(x)}{x}$, we have $\frac{1}{n}f''(x) + \frac{n-1}{n}\frac{f'(x)x - f(x)}{x^2}$. Hence, if the production function is strictly concave, x^m is determined uniquely. Consequently, x_i^m is also unique for all i . That is, no asymmetric equilibrium labor input profile exists under the strictly concave production technology.¹² Therefore, Proposition 2 provides a simple algorithm to find the income maximizing Nash equilibrium allocation. In order to check the second order condition, consider the second derivative of m_i at x^m . A simple computation gives

$$\frac{\partial^2 m_i(x_i^m, x_{-i}^m)}{\partial x_i^2} = \frac{2}{x^m} \{f'(x^m) - p\} + \frac{x_i^m}{x^m} f''(x^m).$$

Since $f'(x^m) < \frac{f(x^m)}{x^m}$ and $\frac{1}{n}f'(x^m) + \frac{n-1}{n}\frac{f(x^m)}{x^m} = p$, $f'(x^m) < p$. Assuming that $f''(x^m) < 0$, we have $\frac{\partial^2 m_i(x_i^m, x_{-i}^m)}{\partial x_i^2} < 0$. In order for x^m to be an interior allocation, $\frac{1}{n}x^m < \min w_i$ and $x^m < w = \sum w_i$ assuming that $f'(0) > p$ and $f'' < 0$. As is well known, if $n = 1$, then $x^m = \hat{x}$, and $x^m \uparrow x^s$ as $n \rightarrow \infty$ which is obvious from Propositions 2, where $f(x^s)/x^s = p$.

In order to explain the fact that the appropriators' labor input in Walker, Gardner, and Ostrom's experiments (1990b) tends to be greater than the income maximizing Nash equilibrium labor input,¹³ we introduce two tentative behavioral principles for the illustration of the problem. In the following sections, we will consider the justification of two principles and more general behavioral principles.

Let $\frac{m_i(x)}{\sum m_j(x)}$ be appropriator i 's share of total income and $m_i(x) - \sum_j \frac{m_j(x)}{n}$ be appropriator i 's difference from the average income. We say that appropriator i increases i 's share (difference) at x with $\Delta x_i > 0$ if $\frac{m_i(x; \Delta x_i)}{\sum m_j(x; \Delta x_i)} > \frac{m_i(x)}{\sum m_j(x)} \left(m_i(x; \Delta x_i) - \sum_j \frac{m_j(x; \Delta x_i)}{n} > m_i(x) - \sum_j \frac{m_j(x)}{n} \right)$ respectively, where $(x; \Delta x_i) = x + (0, \dots, 0, \Delta x_i, 0, \dots, 0)$. Consider a situation

¹² See Sandler (1992, pp. 117-119) and Ostrom, Gardner and Walker (1994, p.111).

¹³ See Davis and Holt (1992, pp. 350-55).

where appropriator i maximizes i 's income m_i at a Nash equilibrium. Although appropriator i decreases i 's income when appropriator i increases i 's labor input from x_i by Δx_i , this increment Δx_i reduces all other appropriators' income at the same time. If the sum of all other appropriators' reduction of their income is sufficiently large, appropriator i increases i 's share and difference. The following proposition shows that in the symmetric case, the value of average product of fish is greater than the wage rate if and only if appropriator i can increase i 's share and difference.

Proposition 3. Suppose that $w_i = w$ for all i and $\mathbf{x} = (a, a, \dots, a)$. Then appropriator i increases i 's share and difference at \mathbf{x} with $\Delta a_i > 0$ if and only if $\frac{f(na + \Delta a_i)}{na + \Delta a_i} > p$.

Proof. First, consider the share case. Suppose that appropriator i increases i 's share at \mathbf{x} with $\Delta a > 0$. Then

$$\begin{aligned} \frac{m_i(\mathbf{x}; \Delta a_i)}{\sum_j m_j(\mathbf{x}; \Delta a_i)} &> 1/n. \text{ Since} \\ \frac{m_i(\mathbf{x}; \Delta a_i)}{\sum_j m_j(\mathbf{x}; \Delta a_i)} &= \frac{f(na + \Delta a_i) \frac{a + \Delta a_i}{na + \Delta a_i} + p(w - a - \Delta a_i)}{f(na + \Delta a_i) + p(nw - na - \Delta a_i)} \\ &= \frac{1}{n} + \frac{1}{\sum_j m_j(\mathbf{x}; \Delta a_i)} \left\{ f(na + \Delta a_i) \frac{a + \Delta a_i}{na + \Delta a_i} + p(w - a - \Delta a_i) - f(na + \Delta a_i) \frac{a + \Delta a_i/n}{na + \Delta a_i} - p(w - a - \Delta a_i/n) \right\} \\ &= \frac{1}{n} + \frac{1}{\sum_j m_j(\mathbf{x}; \Delta a_i)} \left\{ \left(\frac{f(na + \Delta a_i)}{na + \Delta a_i} \right) \left(\Delta a_i - \frac{\Delta a_i}{n} \right) - p \left(\Delta a_i - \frac{\Delta a_i}{n} \right) \right\}, \\ \text{we have } \frac{f(na + \Delta a_i)}{na + \Delta a_i} &> p. \text{ This shows the equivalence.} \end{aligned}$$

Consider the difference case. Similar computation gives

$$m_i(\mathbf{x}; \Delta a) - \sum_j \frac{m_j(\mathbf{x}; \Delta a)}{n} - \left(m_i(\mathbf{x}) - \sum_j \frac{m_j(\mathbf{x})}{n} \right) = \Delta a \left(1 - \frac{1}{n} \right) \left(\frac{f(na + \Delta a_i)}{na + \Delta a_i} - p \right),$$

which shows the result. ■

Since every appropriator inputs the same amount of labor at the income maximizing Nash equilibrium, let $\mathbf{x} = (a, a, \dots, a)$ be the labor input vector. That is, by Proposition 2 and $\frac{f(na)}{na} > f'(na)$, we have $\frac{f(na)}{na} > \frac{1}{n} f'(na) + \frac{n-1}{n} \frac{f(na)}{na} = p$. Hence, if appropriator i does care about i 's share or difference, appropriator i would increase i 's labor input ($\Delta a_i > 0$) by

Proposition 3. Similar overproduction phenomena are also reported in oligopoly experiments [see Holt]. In voluntary contribution mechanism experiments in which the marginal return is greater than one, Saijo and Nakamura found that subjects did not contribute their entire endowments even though contributing all of the endowment was every subject's dominant strategy and the prisoners' dilemma situation was not present. In their design, reducing their contributions make their income less but it increases their shares and differences.

3. Identifying the class of behavioral principles

3.1 Necessary and Sufficient Conditions

Given observations such as $\frac{f(x^*)}{x^*} = p$, $\frac{1}{n}f'(x^*) + \frac{n-1}{n}\frac{f(x^*)}{x^*} = p$ and $f'(x^*) = p$, it is a challenge to identify the behavioral principles of appropriators or their utility functions.

Although our focus is on the case with which the value of average product of fish equals the wage rate determined by outside industries at the equilibrium, we will consider three cases together for the purpose of comparisons.

First, we derive the necessary and sufficient conditions regarding the shapes of utility function profiles for each case at equilibrium.

Proposition 4. Suppose that $\frac{\partial u_i(x^)}{\partial x_i} = 0$ for all i . Then*

- (i) $\frac{f(x^*)}{x^*} = p$ if and only if $\sum_{j=1}^n \frac{\partial u_i(m^*)}{\partial m_j} x_j^* = 0$ for all i ;
- (ii) $\frac{1}{n}f'(x^*) + \frac{n-1}{n}\frac{f(x^*)}{x^*} = p$ if and only if $\sum_{j=1}^n \frac{\partial u_i(m^*)}{\partial m_j} x_j^* = \frac{\partial u_i(m^*)}{\partial m_i} \frac{x^*}{n}$ for all i ; and
- (iii) $f'(x^*) = p$ if and only if $\sum_{j=1}^n \frac{\partial u_i(m^*)}{\partial m_j} x_j^* = \frac{\partial u_i(m^*)}{\partial m_i} x^*$ for all i .

Proof. (i) By (2) and $\frac{\partial m_j(x)}{\partial x_i} = x_j \frac{f'(x)x - f(x)}{x^2}$ for $i \neq j$, we have

$$\begin{aligned}
 \frac{\partial u_i(x^*)}{\partial x_i} &= \sum_{j \neq i} \frac{\partial u_i(m^*)}{\partial m_j} \frac{\partial m_j(x^*)}{\partial x_i} + \frac{\partial u_i(m^*)}{\partial m_i} \frac{\partial m_i(x^*)}{\partial x_i} \\
 &= \frac{f'(x^*)x^* - f(x^*)}{x^{*2}} \sum_{j \neq i} \frac{\partial u_i(m^*)}{\partial m_j} x_j^* + \frac{\partial u_i(m^*)}{\partial m_i} \left(x_i^* \frac{f'(x^*)x^* - f(x^*)}{x^{*2}} + \frac{f(x^*)}{x^*} - p \right) \\
 &= \frac{f'(x^*)x^* - f(x^*)}{x^{*2}} \sum_{j \neq i} \frac{\partial u_i(m^*)}{\partial m_j} x_j^* + \frac{\partial u_i(m^*)}{\partial m_i} \left(\frac{f(x^*)}{x^*} - p \right)
 \end{aligned} \tag{5}$$

$$= 0. \quad (6)$$

Since $f'(x^*)x^* - f(x^*) \neq 0$, $\frac{f(x^*)}{x^*} - p = 0$ implies $\sum \frac{\partial u_i}{\partial m_j} x_j^* = 0$. On the other hand, if

$$\sum \frac{\partial u_i}{\partial m_j} x_j^* = 0, \text{ we have } \frac{f(x^*)}{x^*} - p = 0.$$

(ii) and (iii) See appendix. ■

In the following subsection, we argue that the class of utility profiles satisfying the simultaneous partial differential equations in Proposition 4 is quite large even though we impose the relation expressed by the equations globally, not merely at the equilibrium point.

3.2 Difficulties in Identifying Behavioral Principles

Let $\nabla u(\mathbf{m}) = \left[\frac{\partial u_i(\mathbf{m})}{\partial m_j} \right]$ be an $n \times n$ Jacobian matrix. Consider the case with which the

value of average product of fish equals the wage rate. From Proposition 4-(i), to identify the class of behavioral principles is equivalent to identify the following class of utility function profiles:

$$\Lambda' = \left\{ (u_1, \dots, u_n) \mid \frac{\partial u_i(x^*)}{\partial x_i} = 0, \nabla u(\mathbf{m}^*)x^* = 0, x^* \geq 0 \text{ and } x^* \neq 0 \right\},$$

where $\mathbf{m}^* = \mathbf{m}^*(x^*)$. Consider a utility function profile $(u_1, \dots, u_n) \in \Lambda'$. At the equilibrium x^* (i.e., $\partial u_i(x^*)/\partial x_i = 0$), $\nabla u(\mathbf{m}^*)x^* = 0, x^* \geq 0$ and $x^* \neq 0$ must be satisfied. Now consider another utility function profile $(\tilde{u}_1, \dots, \tilde{u}_n)$ where the slope of indifference curve at x_i^* (and m_i^*) of \tilde{u}_i coincide with the slope of indifference curve at x_i^* (and m_i^*) of u_i for all i . If this is the case, $(\tilde{u}_1, \dots, \tilde{u}_n)$ is also an element of Λ' . That is, even though the behavior of $(\tilde{u}_1, \dots, \tilde{u}_n)$ is totally different from (u_1, \dots, u_n) other than x^* , these two utility function profiles have the same equilibrium. In other words, Λ' just describes only the local properties at x^* .

Therefore, in order to find behavioral principles, we restrict the class of utility function profiles further.

$$\Lambda'' = \left\{ (u_1, \dots, u_n) \mid \text{for each } \mathbf{m} \text{ there exists an } x^* \text{ such that } \nabla u(\mathbf{m})x^* = 0, x^* \geq 0 \text{ and } x^* \neq 0 \right\}$$

which includes the simultaneous partial differential equations $\nabla \mathbf{u}(\mathbf{m})\mathbf{x}^* = 0$. Λ'' extends the conditions in Λ' to the entire domain of \mathbf{m} . Let $\Lambda = \Lambda' \cap \Lambda''$. Although Λ is not a linear space,¹⁴ an element $\mathbf{u} \in \Lambda$ generates an infinite-dimensional linear space that is a subset of Λ . As an example, we consider the case where the utility functions are expressed by the difference between i 's own income and the average income. Let $u_i(\mathbf{m}) = m_i - \frac{\sum m_j}{n}$ for all i .

First, we show that $\mathbf{u} = (u_1, \dots, u_n) \in \Lambda$. Consider

$$\nabla \mathbf{u}(\mathbf{m}) = \begin{bmatrix} 1 - \frac{1}{n} & -\frac{1}{n} & \dots & -\frac{1}{n} \\ -\frac{1}{n} & 1 - \frac{1}{n} & \dots & -\frac{1}{n} \\ \vdots & \vdots & \ddots & \vdots \\ -\frac{1}{n} & -\frac{1}{n} & \dots & 1 - \frac{1}{n} \end{bmatrix}.$$

Clearly, we have $\nabla \mathbf{u}(\mathbf{m})(1, 1, \dots, 1)' = 0$ for all \mathbf{m} . That is, $\mathbf{u} \in \Lambda''$. Then there exists a $c > 0$ such that $\frac{f(\mathbf{x}^*)}{x^*} = p$ where $\mathbf{x}^* = (c, c, \dots, c)$ and $x^* = nc$. Furthermore, by (6), we have

$\frac{\partial u_i(\mathbf{x}^*)}{\partial x_i} = 0$ for all i . That is, $\mathbf{u} \in \Lambda'$. Now consider $[u_i(\mathbf{m})]^k = \left[m_i - \frac{\sum m_j}{n} \right]^k$ where k is a natural number and let $\mathbf{u}^k = ([u_1(\mathbf{m})]^k, \dots, [u_n(\mathbf{m})]^k)$. Then applying the same argument as above, we have $\alpha \mathbf{u}^k + \beta \mathbf{u}^m \in \Lambda$ where $\alpha, \beta \in \mathbb{R}$. Let $\Lambda_{\mathbf{u}} = \{ \alpha \mathbf{u}^k + \beta \mathbf{u}^m \mid k, m \in \mathbb{N} \text{ and } \alpha, \beta \in \mathbb{R} \}$, where \mathbb{N} is the set of all natural numbers. Then $(\mathbf{u}^1, \mathbf{u}^2, \dots)$ becomes the basis of $\Lambda_{\mathbf{u}}$ and hence $\Lambda_{\mathbf{u}}$ is a linear space. Since the cardinal number of the basis is not finite, we have the following proposition.

Proposition 5. $\Lambda_{\mathbf{u}}$ is an infinite-dimensional linear space for each $\mathbf{u} \in \Lambda$.

Although $\Lambda_{\mathbf{u}}$ is *mathematically* infinitely dimensional for each \mathbf{u} , it is not *economically* infinitely dimensional since each element of $(\mathbf{u}^1, \mathbf{u}^2, \dots)$ in the above proof represents essentially the same utility function profile.

¹⁴ Notice that the convex combination of two singular matrices is not necessary singular.

3.3 Restricting the Class of Utility Profiles

The cause of the difficulty identifying behavioral principles is due to the fact that we must find n appropriators' utility function profiles from thin information, i.e., from just one equation, for example, $\frac{f(x^*)}{x^*} = p$. To handle this problem, we restrict the class of utility functions by a few parameters. Since the class of one parameter utility functions is still meager to express important behavioral principles such as share and difference maximizing principles, we consider a class of utility functions with two basic parameters to find economically meaningful behavioral principles: $u_i(x) = (\alpha_{ii}m_i)^{\beta_i} \left(\sum_j \alpha_{ij}m_j \right)^{\gamma_i}$, where β_i and γ_i are the two basic parameters and the α_{ij} are auxiliary parameters. Notice that this class of utility functions is general enough to contain utility functions such as $u_i(x) = m_i$, $u_i(x) = m_i$, $u_i(x) = m_i - \frac{\sum_j m_j}{n}$ and $u_i(x) = \frac{m_i}{\sum_j m_j}$. Under appropriate conditions, the following proposition shows that the equality of the value of average product with the wage rate gives rise to two types of behavioral principles: appropriator i 's utility function is expressed by the (generalized) share of i 's income or the difference of incomes.

Proposition 6. Suppose that $\frac{\partial u_i(x_i^, x_{-i}^*)}{\partial x_i} = 0$ for all i .*

(i) *Assume that $\beta_i > 0$ and $\frac{\alpha_{ii}x_i^*}{\sum_j \alpha_{ij}x_j^*} = \frac{\alpha_{ii}m_i(x_i^*, x_{-i}^*)}{\sum_j \alpha_{ij}m_j(x_j^*, x_{-j}^*)}$ for all i . Then $\frac{f(x^*)}{x^*} = p$ if and only if*

$$u_i(x) = \left(\frac{\alpha_{ii}m_i}{\sum_j \alpha_{ij}m_j} \right)^{\beta_i}; \text{ and}$$

(ii) *Assume that $\beta_i = 0$ and $\alpha_{ij} = \bar{\alpha}_i$ for all j with $j \neq i$ and $x_i^* = \frac{1}{n}x^*$. Then $\frac{f(x^*)}{x^*} = p$ if and only if*

$$u_i(x) = n(-\bar{\alpha}_i) \left(m_i - \frac{\sum_j m_j}{n} \right) \text{ for all } i.$$

Proof. (i) Only if part: By Proposition 4-(i), if $\frac{\partial u_i(x_i^, x_{-i}^*)}{\partial x_i} = 0$ and $\frac{f(x^*)}{x^*} = p$, we have*

$$\sum_j \frac{\partial u_i}{\partial m_j} x_j^* = 0. \text{ That is,}$$

$$\left(\frac{\sum_j \alpha_{ij}x_j^*}{\sum_j \alpha_{ij}m_j^*} \gamma_i + \frac{x_i^*}{m_i^*} \beta_i \right) u_i(x^*) = 0. \quad (7)$$

Since $\frac{\alpha_{ii}x_i^*}{\sum_j \alpha_{ij}x_j^*} = \frac{\alpha_{ii}m_i^*}{\sum_j \alpha_{ij}m_j^*}$, we have $\gamma_i + \beta_i = 0$. Since $\beta_i > 0$, $\gamma_i = -\beta_i (< 0)$. Hence

$$u_i(x) = \left(\frac{\alpha_{ii}m_i}{\sum_j \alpha_{ij}m_j} \right)^{\beta_i}.$$

If part: For $u_i(x)$, if $\gamma_i = -\beta_i$, then $u_i(x) = \left(\frac{\alpha_{ii}m_i}{\sum_j \alpha_{ij}m_j} \right)^{\beta_i}$. Using $\frac{\alpha_{ii}x_i^*}{\sum_j \alpha_{ij}x_j^*} = \frac{\alpha_{ii}m_i^*}{\sum_j \alpha_{ij}m_j^*}$, we have $\sum_j \frac{\partial u_i}{\partial m_j} x_j^* = \frac{x_i^*}{m_i^*} (\gamma_i + \beta_i) u_i(x^*) = 0$. By Proposition 4-(i), $\frac{f(x^*)}{x^*} = p$.

(ii) Only if part: From Proposition 4-(i), if $\frac{\partial u_i(x^*)}{\partial x_i} = 0$ for all i and $\frac{f(x^*)}{x^*} = p$, we then have

$$\sum_j \frac{\partial u_i(m^*)}{\partial m_j} x_j^* = \sum_j \alpha_{ij} x_j^* = 0. \text{ Since } x_i^* = \frac{1}{n} x^* \text{ for all } i, \text{ we have } \sum_j \alpha_{ij} = 0. \text{ Using } \sum_j \alpha_{ij} = 0,$$

and $\alpha_j = \bar{\alpha}_i$ for all j with $j \neq i$, then $\sum_j \alpha_{ij} = (n-1)\bar{\alpha}_i + \alpha_{ii} = 0$. Hence, $\alpha_{ii} = -(n-1)\bar{\alpha}_i$.

Therefore,

$$u_i(x) = \alpha_{ii}m_i + \bar{\alpha}_i \sum_{j \neq i} m_j = -(n-1)\bar{\alpha}_i m_i + \bar{\alpha}_i \left(\sum_{j=1}^n m_j - m_i \right) = n(-\bar{\alpha}_i) \left(m_i - \frac{\sum_j m_j}{n} \right).$$

If part: Since $\sum_j \alpha_{ij} = 0$ and $x_i^* = \frac{1}{n} x^*$, $\sum_j \frac{\partial u_i(m^*)}{\partial m_j} x_j^* = \sum_j \alpha_{ij} x_j^* = \frac{x^*}{n} \sum_j \alpha_{ij} = 0$. Using $\frac{\partial u_i(x^*)}{\partial x_i} = 0$, from Proposition 4-(i), we have $\frac{f(x^*)}{x^*} = p$. ■

Notice that $u_i(x) = n(-\bar{\alpha}_i) \left(m_i - \frac{\sum_j m_j}{n} \right) = \alpha_{ii} \left(m_i - \frac{\sum_{j \neq i} m_j}{n-1} \right)$ in Proposition 6-(ii). That is, the

utility function expressed by the difference between i 's income and the average of all is actually the same as the one expressed by the difference between i 's income and the average of the rest.

The following proposition characterizes the observation $\frac{1}{n} f'(x^*) + \frac{n-1}{n} \frac{f(x^*)}{x^*} = p$. As might have been expected, this observation is equivalent to income maximization by all appropriators within the limited class of utility functions.

Proposition 7. Suppose that $\alpha_{ij} \geq 0$, $\sum_j \alpha_{ij} > 0$, $\frac{\partial u_i(x_i^*, x_{-i}^*)}{\partial x_i} = 0$, and $x_i^* = \frac{1}{n} x^*$ for all i . Then

$$\frac{1}{n} f'(x^*) + \frac{n-1}{n} \frac{f(x^*)}{x^*} = p \text{ if and only if } u_i(x) = (\alpha_{ii} m_i)^{\beta_i + \gamma_i}.$$

Proof. See appendix. ■

Proposition 8 is for the observation $f'(x^*) = p$. This observation is equivalent to total income maximization by all appropriators.

Proposition 8. Suppose that $\frac{\partial u_i(x_i^*, x_{-i}^*)}{\partial x_i} = 0$, $\sum_j \alpha_{ij} = \sum_j \alpha_{kj} = \sum_i \alpha_{ii}$ for all i and k , $\alpha_{ij} = \bar{\alpha}_i$ for all j with $j \neq i$, and $\gamma_i \neq 0$, $x_i^* = \frac{1}{n} x^*$ and $w_i = \bar{w}$ for all i . Then $f'(x^*) = p$ if and only if $u_i(x) = (\alpha_{ii} m_i)^{\beta_i + \gamma_i}$.

Proof. See appendix. ■

In order to identify behavioral principles for three observations, we impose some assumptions for each of Propositions 6, 7, and 8. Without these assumptions, we might worry about, for example, a situation where $\frac{f(x^*)}{x^*} = p$ is observed, but the principle is income maximization. The next proposition shows that these situations would not happen.

Proposition 9. Suppose that $\frac{\partial u_i(x_i^*, x_{-i}^*)}{\partial x_i} = 0$. Then

- (i) If $\frac{f(x^*)}{x^*} = p$, then $u_i(x) \neq m$ for some i and $u_i(x) \neq m_i$ for some i ;
- (ii) If $n > 1$ and $\frac{1}{n} f'(x^*) + \frac{n-1}{n} \frac{f(x^*)}{x^*} = p$, then $u_i(x) \neq m$ for some i , $u_i(x) \neq \frac{m_i}{m}$ for some i , and $u_i(x) \neq m_i - \frac{\sum_j m_j}{n}$ for some i ; and
- (iii) If $n > 1$ and $f'(x^*) = p$, then $u_i(x) \neq m_i$ for some i , $u_i(x) \neq \frac{m_i}{m}$ for some i , and $u_i(x) \neq m_i - \frac{\sum_j m_j}{n}$ for some i .

Proof. See appendix. ■

4. Share and difference maximizing Nash equilibria

Since we identify several important behavioral principles, we characterize

equilibrium conditions given behavioral principles.¹⁵ First, given $u_i^s(x) = \frac{m_i}{m}$ for all i , we obtain that the equality between the value of average product and the wage rate together with a share condition is equivalent to the share maximizing Nash equilibrium.

Proposition 10. Suppose $n > 1$. Then $\frac{\partial u_i^s(x_i^*, x_{-i}^*)}{\partial x_i} = 0$ for all i if and only if $\frac{f(x^*)}{x^*} = p$ and $\frac{x_i^*}{x^*} = \frac{m_i(x_i^*, x_{-i}^*)}{m(x_i^*, x_{-i}^*)} = \frac{w_i}{w}$ for all i .

Proof. Only if part: First, we prove $\frac{f(x^*)}{x^*} = p$. By (5), we have

$$\frac{\partial u_i^s}{\partial x_i} = \frac{1}{x^*} \left(f'(x^*) - \frac{f(x^*)}{x^*} \right) \left(\frac{m - m_i}{m^2} x_i^* - \frac{m_i}{m^2} \sum_{j \neq i} x_j^* \right) + \frac{m - m_i}{m^2} \left(\frac{f(x^*)}{x^*} - p \right).$$

Since $\frac{\partial u_i^s(x_i^*, x_{-i}^*)}{\partial x_i} = 0$ for all i , $\sum_i \frac{\partial u_i^s}{\partial x_i} = 0$. Hence

$$\begin{aligned} \sum_i \frac{\partial u_i^s}{\partial x_i} &= \sum_i \left\{ \frac{1}{x^*} \left(f'(x^*) - \frac{f(x^*)}{x^*} \right) \left(x_i^* - \frac{m_i}{m} x^* \right) \frac{1}{m} + \frac{1}{m} \left(1 - \frac{m_i}{m} \right) \left(\frac{f(x^*)}{x^*} - p \right) \right\} \\ &= \frac{n-1}{m} \left(\frac{f(x^*)}{x^*} - p \right) = 0. \end{aligned} \quad (8)$$

Since $n > 1$, we have $\frac{f(x^*)}{x^*} = p$. Then by Proposition 4-(i), $\sum_j \frac{\partial u_i^s}{\partial m_j} x_j^* = \left(x_i^* - \frac{m_i}{m} x^* \right) \frac{1}{m} = 0$.

Since $x^* \neq 0$, we have $\frac{x_i^*}{x^*} = \frac{m_i}{m}$. Moreover, since $\frac{f(x^*)}{x^*} = p$, we have

$$\frac{m_i}{m} = \frac{\frac{f(x^*)}{x^*} x_i^* + p(w_i - x_i^*)}{\frac{f(x^*)}{x^*} + p(w - x^*)} = \frac{w_i}{w}.$$

If part: Since $\frac{f(x^*)}{x^*} = p$, $\frac{x_i^*}{x^*} = \frac{m_i}{m}$ and by (5), $\frac{\partial u_i^s(x_i^*, x_{-i}^*)}{\partial x_i} = 0$. ■

Since $f'(x)x - f(x) < 0$, x^* is determined uniquely, and hence x_i^* is also unique for all i . That is, Proposition 10 also provides a simple algorithm to find the share maximizing Nash equilibrium allocation. In contrast to Proposition 2, the share maximizing Nash equilibrium allocation depends on the distribution of endowments although the total labor input is invariant. If the endowment of appropriator i increases, then i 's labor input for fishing increases and at the same time the labor input for fishing of the rest of the appropriators decreases. In order to check the second order condition, consider the second derivative of

¹⁵ Proposition 3 identifies the direction of share (or difference) increase, but it does not characterize the share maximizing (or difference) maximizing equilibrium.

m_i at x^* . A simple computation gives

$$\frac{\partial^2 m_i(x_i^*, x_{-i}^*)}{\partial x_i^2} = \frac{1}{x^*} \left(f'(x^*) - \frac{f(x^*)}{x^*} \right) \left\{ \left(1 - \frac{x_i^*}{x^*} \right) + \left(1 - \frac{m_i}{m} \right) \right\}.$$

Since $f'(x^*) - \frac{f(x^*)}{x^*} < 0$, $1 - \frac{x_i^*}{x^*} > 0$, and $1 - \frac{m_i}{m} > 0$, we have $\frac{\partial^2 m_i(x_i^*, x_{-i}^*)}{\partial x_i^2} < 0$. In order for

x^* to be an interior allocation, $0 < x^* < w$.

Second, consider the case with difference maximizing behavior. Let $u_i^d(x) = m_i - \frac{\sum_j m_j}{n}$

for all i .

Proposition 11. Suppose $n > 1$. Then $\frac{\partial u_i^d(x_i^*, x_{-i}^*)}{\partial x_i} = 0$ for all i if and only if $\frac{f(x^*)}{x^*} = p$ and $x_i^* = \frac{1}{n}x^*$ for all i .

Proof. Only if part: Since $\frac{\partial u_i^d(x_i^*, x_{-i}^*)}{\partial x_i} = 0$ for all i , $\sum_i \frac{\partial u_i^d}{\partial x_i} = 0$. By (5),

$$\begin{aligned} \sum_i \frac{\partial u_i^d}{\partial x_i} &= \sum_i \left\{ \frac{1}{x^*} \left(f'(x^*) - \frac{f(x^*)}{x^*} \right) \left(x_i^* - \frac{1}{n} \sum_j x_j^* \right) + \left(1 - \frac{1}{n} \right) \left(\frac{f(x^*)}{x^*} - p \right) \right\} \\ &= (n-1) \left(\frac{f(x^*)}{x^*} - p \right) = 0. \end{aligned}$$

Since $n > 1$, we have $\frac{f(x^*)}{x^*} = p$. Then by Proposition 4-(i), $\sum_j \frac{\partial u_i^d}{\partial m_j} x_j^* = x_i^* - \frac{1}{n} \sum_j x_j^* = 0$. Hence, $x_i^* = \frac{1}{n}x^*$.

If part: Since $\frac{f(x^*)}{x^*} = p$ and $x_i^* = \frac{1}{n}x^*$, by (5), $\frac{\partial u_i^d(x_i^*, x_{-i}^*)}{\partial x_i} = 0$. ■

The difference between Propositions 10 and 11 is important. Proposition 11 says that all appropriators input the same hours of labor *regardless* of their initial endowments as long as the behavioral principle is the difference maximizing one. On the other hand, in Proposition 10, equilibrium labor inputs are determined by the ratio of one's endowment to the total endowment as long as the behavioral principle is the share maximizing one. By the same reason as in Proposition 10, x^* is determined uniquely. Consequently, x_i^* is also unique for all i . A similar computation shows that the second order condition at the equilibrium is

satisfied.¹⁶ In order for x^* to be an interior allocation, $0 < x^* < w$ and $x^* / n < \min w_i$.

Third, consider the case with total income maximizing behaviors. Let $u_i^{so}(x) = m$.

Proposition 12. $\frac{\partial u_i^{so}(x^*)}{\partial x_i} = 0$ for all i if and only if $f'(x^*) = p$.

Proof. See appendix. ■

The fact that $\hat{x} < x^m < x^s$ is well known where \hat{x} is a solution of $f'(x) = p$, x^m is a solution of $(1/n)f'(x) + ((n-1)/n)(f(x)/x) = p$, and x^s is a solution of $f(x)/x = p$. As we noticed before, both income maximizing and difference maximizing behaviors give rise to symmetric equilibria. On the other hand, experimental results such as Walker, Gardner, and Ostrom (1990b) show non-symmetric patterns of labor inputs. It is share maximizing behavior that gives a non-symmetric equilibrium in our class of restricted utility profiles. Based upon these facts, we propose a more general form of utility functions: let appropriator i 's utility function be $v_i(m_i, s_i)$ where $s_i = m_i / m$. Assume that $\frac{\partial v_i}{\partial m_i} > 0$ and $\frac{\partial v_i}{\partial s_i} > 0$. In order to show that $x^m < x^v < x^s$, we give the following lemma showing that each appropriator's best response curve shifts to the right hand side as the parameter moves from m to v and from v to s (see Figure 1).

Lemma 1. $\frac{\partial m_i(x_i^v, x_{-i})}{\partial x_i} < 0$ and $\frac{\partial s_i(x_i^v, x_{-i})}{\partial x_i} > 0$ for all x_{-i} .

Proof. Since x_i^v is a solution of $\frac{\partial v_i(x_i, x_{-i})}{\partial x_i} = 0$, $\frac{\partial v_i(x_i^v, x_{-i})}{\partial m_i} \frac{\partial m_i(x_i^v, x_{-i})}{\partial x_i} + \frac{\partial v_i(x_i^v, x_{-i})}{\partial s_i} \frac{\partial s_i(x_i^v, x_{-i})}{\partial x_i} = 0$. Since $\frac{\partial s_i(x)}{\partial x_i} = \frac{1}{m^2} \left\{ \frac{\partial m_i(x)}{\partial x_i} m - \sum_{j=1}^n \frac{\partial m_j(x)}{\partial x_i} m_j \right\}$, solving with respect to $\frac{\partial m_i(x)}{\partial x_i}$, we have

¹⁶ A simple computation gives that $\frac{\partial^2 m_i(x_i^*, x_{-i}^*)}{\partial x_i^2} = \frac{1}{x^*} \left(f'(x^*) - \frac{f(x^*)}{x^*} \right) \left(\left(1 - \frac{x_i^*}{x^*} \right) + \left(1 - \frac{1}{n} \right) \right)$. Since

$1 - \frac{1}{n} > 0$, the second order condition is satisfied.

$$\left\{ \frac{\partial v_i}{\partial m_i} + \frac{\partial v_i}{\partial s_i} \frac{1}{m} \left(1 - \frac{m_i}{m} \right) \right\} \frac{\partial m_i(x)}{\partial x_i} = \frac{\partial v_i}{\partial s_i} \frac{1}{m^2} \sum_{j \neq i} \frac{\partial m_j(x)}{\partial x_i} m_i.$$

Due to the interior solution assumption, $m > 0$ and $1 - \frac{m_i}{m} > 0$. Since $\frac{\partial m_j(x)}{\partial x_i} < 0$ for all i and j with $i \neq j$, $\frac{\partial v_i}{\partial m_i} > 0$ and $\frac{\partial v_i}{\partial s_i} > 0$, $\frac{\partial m_i(x_i^v, x_{-i})}{\partial x_i} < 0$. Since $\frac{\partial v_i}{\partial m_i} > 0$ and $\frac{\partial v_i}{\partial s_i} > 0$, $\frac{\partial s_i(x_i^v, x_{-i})}{\partial x_i} > 0$. ■

Proposition 13. (i) If the production function is strictly concave, then $\hat{x} < x^m < x^v$.

(ii) $x^v < x^s$.

Proof. (i) Define $G(x) = \sum \frac{\partial m_i(x_i, x_{-i})}{\partial x_i}$. Then $G(x) = f'(x) + (n-1) \frac{f(x)}{x} - np$ (see (3)). Since $f''(x) < 0$, $G'(x) = f''(x) + (n-1) \frac{f'(x)x - f(x)}{x^2} < 0$. First, we show $G(\hat{x}) > 0$. By (2), we have

$$\frac{\partial m_i(x)}{\partial x_i} = \frac{x_i}{x} (f'(x) - p) + (1 - \frac{x_i}{x}) \left(\frac{f(x)}{x} - p \right).$$

Since $f'(\hat{x}) = p < \frac{f(\hat{x})}{\hat{x}}$, $G(\hat{x}) > 0$. By Lemma 1, $\frac{\partial m_i(x_i^v, x_{-i})}{\partial x_i} < 0$ for all x_{-i} . Hence $G(x^v) < 0$.

Since $G(x^m) = 0$, we have $\hat{x} < x^m < x^v$.

(ii) Define $Q(x) = \frac{f(x)}{x} - p$. Then $Q'(x) < 0$. Since $\sum \frac{\partial s_i(x^v)}{\partial x_i} = \frac{n-1}{m} \left\{ \frac{f(x^v)}{x^v} - p \right\}$ (see (8)) and

$\frac{\partial s_i(x_i^v, x_{-i})}{\partial x_i} > 0$ for all i by Lemma 1, we have $Q(x^v) > 0$. Since $Q(x^s) = 0$ by Proposition 10,

we have $x^v < x^s$. ■

5. A numerical example

In this short section, a numerical example is provided. Let $f(x) = \alpha \sqrt{x}$, where we set $\alpha = 6.57$. Let $n = 3$, $w_i = 20$ for all i , and let $p = 1$. When appropriators care about both m_i and s_i , we suppose that every appropriator has a utility function expressed by $v_i = m_i s_i$. Figure 1 shows three best response curves and three symmetric equilibria. The horizontal line stands for the sum of labor inputs other than appropriator i and the vertical line is for appropriator i 's labor input. Hence, the diagonal line shows symmetric labor inputs. The three symmetric equilibrium labor inputs are

$$x_i^m = 9.99, x_i^v = 11.7 \text{ and } x_i^s = 14.4.$$

On the other hand, the symmetric social benefit maximizing solution is $\hat{x}_i = \hat{x} / 3 = 3.58$.

<<Figure 1 is around here>>

Consider now an asymmetric case. Suppose that $w_1 = 40$, $w_2 = w_3 = 20$. By Proposition 1, x_i^m is still 9.99 for all i , but $x_1^s = 21.6$, $x_2^s = x_3^s = 10.8$.

6. Concluding remarks

We have shown that even though the number of appropriators is limited (or even in a short-run static model), a common-pool resource is dissipated completely up to the point where the average value equals the wage rate when appropriators compete for shares or differences. This contrasts with Hardin's theme, where the number of appropriators is unlimited.

Our analysis explains a part of the experimental results, but some of the problems are still unsolved. Walker, Gardner, and Ostrom (1990a) observed that by raising each appropriator's endowment sufficiently high, the labor inputs are made high enough to exceed the point where the average value equals the wage rate. Although the convergence of contribution in the voluntary contribution mechanism is a relatively strong property, Gardner, Ostrom, and Walker (1990) and Walker, Gardner, and Ostrom (1990a and 1990b) found that the average labor input does not converge to a certain point in common-pool resource experiments. These issues must be addressed by theoretical models.

References

- Berkes, F., D. Feeny, B.J. McCay, and J.M. Acheson, 1989, The benefit of the commons, *Nature* 340, 91-93.
- Boado, Eufresina L., 1988, Incentive policies and forest use in the Philippines, in: Robert Repetto and Malcolm Gillis, eds., *Public policies and the misuse of forest resources* (Cambridge University Press, Cambridge) 165-203.
- Davis, Douglas D. and Charles A. Holt, 1992, *Experimental economics* (Princeton University Press, Princeton).
- Gardner, Roy, Elinor Ostrom and James M. Walker, 1990, The nature of common-pool resource problems, *Rationality and Society* 2, 335-58.
- Hardin, Garrett, 1968, The tragedy of the commons, *Science* 162, 1243-8.
- Hirose, S., 1985, A tentative model of consumer's decision making in the hoarding panic, *Shakai Shinrigaku Kenkyu* 1, 45-53 (in Japanese with an English summary).
- Holt, C., 1992, Industrial organization: a survey of laboratory research, forthcoming in: Alvin E. Roth and John H. Kagel eds., *Handbook of experimental economics* (Princeton University Press, Princeton).
- Houthakker, H.S., 1950, Revealed preference and the utility function, *Economica* 7, 159-174.
- Hurwicz, Leonid and Hirofumi Uzawa, 1971, On the integrability of demand functions, in: John Chipman, Leonid Hurwicz, Marcel Ket Richter, and Hugo Sonnenschein, eds., *Preferences, utility, and demand* (Harcourt Brace Jovanovich, Cambridge, New York) 114-148.
- Ichiishi, Tatsuro, 1983, *Game theory for economic analysis* (New York, Academic Press).
- Ostrom, Elinor, 1990, *Governing the commons: the evolution of institutions for collective action*, (Cambridge University Press, Cambridge).
- Ostrom, Elinor, Roy Gardner and James M. Walker, 1994, *Rules, games, and common-pool resources*, (The University of Michigan Press, Ann Arbor).
- Repetto, Robert, 1988, Overview, in: Robert Repetto and Malcolm Gillis, eds., *Public policies and the misuse of forest resources* (Cambridge University Press, Cambridge) 1-41.
- Roemer, John E. and Joaquim Silvestre, 1993, The proportional solution for economies with both private and public ownership, *Journal of Economic Theory* 59, 426-444.
- Saijo, Tatsuyoshi and Hideki Nakamura, 1993, The 'Spite' dilemma in voluntary contribution mechanism experiments, ISEP Discussion Paper 520, University of Tsukuba.
- Sandler, Todd, 1992, *Collective action: theory and application* (New York: Harvester Wheatsheaf).
- Walker, James M., Roy Gardner and Elinor Ostrom, 1990a, Rent dissipation in a limited-access common-pool resource: experimental evidence, *Journal of Environmental Economics and Management* 19, 203-211.

Walker, James M., Roy Gardner and Elinor Ostrom, 1990b, Rent dissipation and balanced deviation disequilibrium in common pool resources: experimental evidence, in: Reinhard Selten, ed., *Game equilibrium models II: methods, morals, and markets* (Berlin, Springer-Verlag).

Weitzman, Martin L., 1974, Free access vs private ownership as alternative systems for managing common property, *Journal of Economic Theory* 8, 225-234.

Appendix: Proofs of Propositions

Proof of Proposition 4-(ii). Since $\frac{1}{n}f'(x^*) + \frac{n-1}{n}\frac{f(x^*)}{x^*} = p$, that is,

$$\frac{f(x^*)}{x^*} - p = -\frac{1}{n}\left(f'(x^*) - \frac{f(x^*)}{x^*}\right) \text{ and by (5), we have}$$

$$\frac{\partial u_i(x^*)}{\partial x_i} = \frac{1}{x^*}\left(f'(x^*) - \frac{f(x^*)}{x^*}\right)\left(\sum \frac{\partial u_i}{\partial m_j} x_j^* - \frac{\partial u_i}{\partial m_i} \frac{x^*}{n}\right) = 0.$$

Since $f'(x^*) - \frac{f(x^*)}{x^*} \neq 0$, $\sum_{j=1}^n \frac{\partial u_i(m^*)}{\partial m_j} x_j^* = \frac{\partial u_i(m^*)}{\partial m_i} \frac{x^*}{n}$. On the other hand, using

$$\sum_{j=1}^n \frac{\partial u_i}{\partial m_j} x_j^* = \frac{\partial u_i}{\partial m_i} \frac{x^*}{n} \text{ and (5), we have}$$

$$\frac{\partial u_i(x^*)}{\partial x_i} = \frac{1}{n} \frac{\partial u_i}{\partial m_i} \left(f'(x^*) - \frac{f(x^*)}{x^*}\right) + \frac{\partial u_i}{\partial m_i} \left(\frac{f(x^*)}{x^*} - p\right) = \frac{\partial u_i}{\partial m_i} \left\{\frac{1}{n}\left(f'(x^*) - \frac{f(x^*)}{x^*}\right) + \left(\frac{f(x^*)}{x^*} - p\right)\right\}.$$

Since $\frac{\partial u_i(x^*)}{\partial x_i} = 0$, we have $\frac{1}{n}f'(x^*) + \frac{n-1}{n}\frac{f(x^*)}{x^*} = p$.

Proof of Proposition 4-(iii). By (5) and $p = f'(x)$,

$$\frac{\partial u_i(x^*)}{\partial x_i} = \frac{1}{x^*}\left(f'(x^*) - \frac{f(x^*)}{x^*}\right)\left(\sum \frac{\partial u_i}{\partial m_j} x_j^* - \frac{\partial u_i}{\partial m_i} x^*\right) = 0.$$

Since $f'(x^*) - \frac{f(x^*)}{x^*} \neq 0$, we have $\sum_{j=1}^n \frac{\partial u_i(m^*)}{\partial m_j} x_j^* = \frac{\partial u_i(m^*)}{\partial m_i} x^*$. On the other hand, using

$$\sum_{j=1}^n \frac{\partial u_i}{\partial m_j} x_j^* = \frac{\partial u_i}{\partial m_i} x^* \text{ and (5), we have } \frac{\partial u_i(x^*)}{\partial x_i} = \frac{\partial u_i(m^*)}{\partial m_i} (f'(x^*) - p). \text{ Since } \frac{\partial u_i(x^*)}{\partial x_i} = 0,$$

we have $f'(x^*) = p$. ■

Proof of Proposition 7. Only if part: By Proposition 4-(ii), if $\frac{\partial u_i(x_i^*, x_{-i}^*)}{\partial x_i} = 0$ and

$$\frac{1}{n}f'(x^*) + \frac{n-1}{n}\frac{f(x^*)}{x^*} = p, \text{ we have } \sum_j \frac{\partial u_i}{\partial m_j} x_j^* = \frac{\partial u_i}{\partial m_i} \frac{x^*}{n}. \text{ That is,}$$

$$\sum_j \frac{\partial u_i}{\partial m_j} x_j^* - \frac{\partial u_i}{\partial m_i} \frac{x^*}{n} = \left\{ \frac{\gamma_i}{\sum_j \alpha_{ij} m_j} \left(\sum_j \alpha_{ij} x_j^* - \frac{\alpha_{ii} x^*}{n} \right) + \frac{\beta_i}{m_i} \left(x_i^* - \frac{x^*}{n} \right) \right\} u_i(x^*) = 0. \quad (A1)$$

Since $x_i^* = \frac{1}{n}x^*$, we have $\frac{\gamma_i}{\sum_j \alpha_{ij} m_j} \left(\sum_j \alpha_{ij} x_j^* - \frac{\alpha_{ii} x^*}{n} \right) = 0$. Since $\sum_j \alpha_{ij} > 0$, $m_j^* > 0$, $x^* > 0$ and $n > 0$,

we have $\gamma_i = 0$ or $\sum_{j \neq i} \alpha_{ij} = 0$. If $\gamma_i = 0$, then we have $u_i(x) = (\alpha_{ii} m_i)^{\beta_i}$. Consider the case with

$\sum_{j \neq i} \alpha_{ij} = 0$. Since $\alpha_{ij} \geq 0$, $\sum_{j \neq i} \alpha_{ij} = 0$ implies $\alpha_{ij} = 0$ for all j with $j \neq i$. Therefore,

$$u_i(x) = (\alpha_{ii} m_i)^{\beta_i + \gamma_i}.$$

If part: Suppose that $u_i(x) = (\alpha_i m_i)^{\beta_i + \gamma_i}$. Then we have

$$\sum_j \frac{\partial u_i}{\partial m_j} x_j^* - \frac{\partial u_i}{\partial m_i} \frac{x^*}{n} = \frac{\beta_i + \gamma_i}{m_i} \left(x_i^* - \frac{x^*}{n} \right) u_i(x^*) = 0. \quad (\because x_i^* = \frac{1}{n}x^*)$$

By Proposition 4-(ii), we have $\frac{1}{n}f'(x^*) + \frac{n-1}{n}\frac{f(x^*)}{x^*} = p$. ■

Proof of Proposition 8. Only if part: By Proposition 4-(iii), if $\frac{\partial u_i(x_i^*, x_{-i}^*)}{\partial x_i} = 0$ and $f'(x^*) = p$, we have $\sum_j \frac{\partial u_i}{\partial m_j} x_j^* = \frac{\partial u_i}{\partial m_i} x^*$. That is,

$$\sum_j \frac{\partial u_i}{\partial m_j} x_j^* - \frac{\partial u_i}{\partial m_i} x^* = \left\{ \frac{\gamma_i}{\sum_j \alpha_{ij} m_j} (\sum_j \alpha_{ij} x_j^* - \alpha_{ii} x^*) + \frac{\beta_i}{m_i} (x_i^* - x^*) \right\} u_i(x^*) = 0. \quad (A2)$$

Since $u_i(x^*) \neq 0$, we have

$$\frac{\sum_j \alpha_{ij} x_j^*}{\sum_j \alpha_{ij} m_j} m_i^* - \frac{\alpha_{ii} x^*}{\sum_j \alpha_{ij} m_j} m_i^* = \frac{\beta_i}{\gamma_i} (x_i^* - x^*). \quad (A3)$$

Since $x_i^* = \frac{1}{n}x^*$ and $w_i = \bar{w}$, we have $m_i^* = \frac{m^*}{n}$. Then (A3) becomes $\frac{x^*}{n} - \frac{\alpha_{ii} x^*}{\sum_j \alpha_{ij}} = \frac{\beta_i}{\gamma_i} (x^* - \frac{x^*}{n})$.

Summing up this for all i , we have $x^* - \frac{x^*}{\sum_j \alpha_{ij}} \sum_i \alpha_{ii} = \sum_i \frac{\beta_i}{\gamma_i} (x^* - \frac{x^*}{n})$ ($\because \sum_j \alpha_{ij} = \sum_j \alpha_{ji}$). By assumption, $\sum_j \alpha_{ij} = \sum_i \alpha_{ii}$. Hence we obtain $\sum_i \frac{\beta_i}{\gamma_i} (1 - \frac{1}{n}) x^* = 0$. Therefore, $n=1$ or $\beta_i = 0$

for all i . If $n=1$, then $u_i(x) = (\alpha_{ii} m_i)^{\beta_i + \gamma_i} = (\alpha_{ii} m)^{\beta_i + \gamma_i}$. Consider now $n > 1$. If $\beta_i = 0$ for all i , then $u_i(x) = (\sum_j \alpha_{ij} m_j)^{\gamma_i}$. That is, $\sum_j \frac{\partial u_i}{\partial m_j} x_j^* - \frac{\partial u_i}{\partial m_i} x^* = \frac{\gamma_i}{\sum_j \alpha_{ij} m_j} (\sum_j \alpha_{ij} x_j^* - \alpha_{ii} x^*) u_i(x^*) = 0$.

Since $\gamma_i \neq 0$, $\sum_j \alpha_{ij} x_j^* - \alpha_{ii} x^* = 0$. Utilizing $\alpha_{ij} = \bar{\alpha}_i$ for $j \neq i$, we have

$$\sum_{j=1}^n \alpha_{ij} x_j^* - \alpha_{ii} x^* = (\sum_j \bar{\alpha}_i x_j^* - \bar{\alpha}_i x_i^* + \alpha_{ii} x_i^*) - \alpha_{ii} x^* = (\bar{\alpha}_i - \alpha_{ii})(x^* - x_i^*) = 0.$$

Hence $\alpha_{ii} = \bar{\alpha}_i$ or $x_i^* = x^*$. If $\alpha_{ii} = \bar{\alpha}_i$, then $u_i(x) = (\sum_j \alpha_{ij} m_j)^{\gamma_i} = (\alpha_{ii} m)^{\gamma_i}$. If $x_i^* = x^*$, summing up for all participants, we have $x^* = n x^*$. That is, $n=1$. This gives $u_i(x) = (\alpha_{ii} m_i)^{\gamma_i} = (\alpha_{ii} m)^{\gamma_i}$.

If part: If $u_i(x) = (\alpha_{ii} m)^{\beta_i + \gamma_i} = [\alpha_{ii} \{f(x) + p(w-x)\}]^{\beta_i + \gamma_i}$, we have

$$\frac{\partial u_i(x^*)}{\partial x_i} = (\beta_i + \gamma_i) (\alpha_{ii} m)^{\beta_i + \gamma_i - 1} \alpha_{ii} \{f'(x^*) - p\} = 0.$$

That is, $f'(x^*) = p$. ■

Proof of Proposition 9. (i) Suppose by way of contradiction that $u_i(x) = m$. Then

$\alpha_{ij} = 1$, $\beta_i = 0$, and $\gamma_i = 1$. Substituting these values in (7), we have then $x^* = 0$, which contradicts to the interior solution assumption. Similarly, supposing that $u_i(x) = m_i$ (i.e., $\alpha_{ij} = 1$, $\beta_i = 1$, and $\gamma_i = 0$), we have $x_i^* = 0$, which again contradicts to the interior solution assumption.

(ii) Suppose that $u_i(x) = m$ (i.e., $\alpha_{ij} = 1$, $\beta_i = 0$, and $\gamma_i = 1$). Then substituting these values in (A1), we have $\left(1 - \frac{1}{n}\right)x^* = 0$, which again contradicts the interior solution assumption.

Next, suppose $u_i(x) = \frac{m_i}{m}$ (i.e., $\alpha_{ij} = 1$, $\beta_i = 1$, and $\gamma_i = -1$). Then we have

$\left\{-\frac{m_i}{m}\left(1 - \frac{1}{n}\right)x^* + \left(x_i^* - \frac{x^*}{n}\right)\right\} \frac{1}{m} = 0$. Summing up for all i , $\left(1 - \frac{1}{n}\right)\frac{x^*}{m} = 0$, which contradicts the

assumption. Next, suppose $u_i(x) = m_i - \frac{\sum_j m_j}{n}$ (i.e., $\alpha_{ii} = 1 - \frac{1}{n}$, $\alpha_{ij} = -\frac{1}{n}$ ($i \neq j$), $\beta_i = 0$, and

$\gamma_i = 1$). Then we have $x_i^* - \frac{x^*}{n} - \left(1 - \frac{1}{n}\right)\frac{x^*}{n} = 0$. Summing up for all i , $\left(1 - \frac{1}{n}\right)x^* = 0$, which

contradicts the assumption.

(iii) Suppose that $u_i(x) = m_i$ (i.e., $\alpha_{ij} = 1$, $\beta_i = 1$, and $\gamma_i = 0$). Substituting these values in (A2), we have $x_i^* = x^*$. That is, $\sum_{j \neq i} x_j^* = 0$, which contradicts the interior solution assumption.

Suppose that $u_i(x) = \frac{m_i}{m}$ (i.e., $\alpha_{ij} = 1$, $\beta_i = 1$, and $\gamma_i = -1$). Then $(x_i^* - x^*)\frac{1}{m} = 0$. Hence

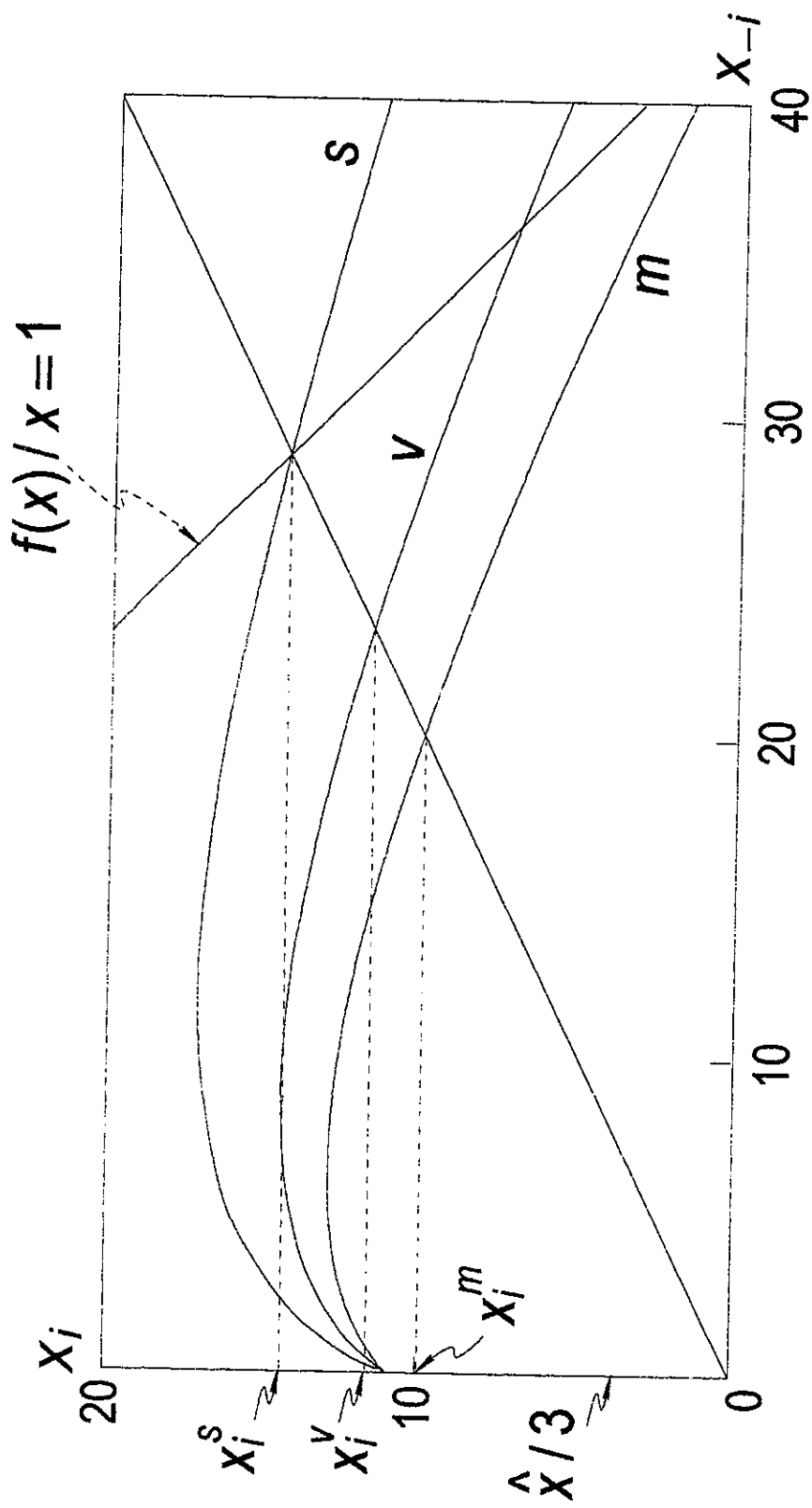
$x_i^* = x^*$, which contradicts the assumption. Suppose that $u_i(x) = m_i - \frac{\sum_j m_j}{n}$ (i.e., $\alpha_{ii} = 1 - \frac{1}{n}$,

$\alpha_{ij} = -\frac{1}{n}$ ($i \neq j$), $\beta_i = 0$, and $\gamma_i = 1$). Then $x_i^* - \frac{x^*}{n} - \left(1 - \frac{1}{n}\right)\frac{x^*}{n} = 0$. Summing up for all i ,

$(n-1)x^* = 0$, which contradicts the assumption. ■

Proof of Proposition 12. Notice that $u_i^{so}(x) = f(x) + p(w-x)$. Then $\frac{\partial u_i^{so}(x^*)}{\partial x_i} = f'(x^*) - p$.

If $\frac{\partial u_i^{so}(x^*)}{\partial x_i} = 0$, then $f'(x^*) = p$. On the other hand, $f'(x^*) = p$ implies $\frac{\partial u_i^{so}(x^*)}{\partial x_i} = 0$. ■



m : income maximizing best response curve
 v : best response curve when $v_i = m_i$
 s : share maximizing best response curve

Figure 1. Three best response curves and symmetric equilibria.