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Price Strategic Economic Behaviour in an Exchange Economy — A General (Non-)Walrasian Prototype

PART 1

by

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Abstract

The traditional exchange economic model, which originates with Walras, Edgeworth, etc., is in an extensive concern. In Part 1, in order to shed a new light on the theory, and to propose a general (non-) Walrasian prototype, an exchange market is reformulated and reorganized under the hypothesis of a price strategic economic man, in stead of price-taking economic man and the Walrasian tatonnement process. An equilibrium of the reformulation is defined. With the concept of equilibrium, the Walrasian and the monopolistic competitive equilibria will be interpreted in the model.

1. A Price Formation Process Compatible with Strategic Behaviour

1.1 Introductory Remarks on the Hypothesis of Price Strategic Economic Man

The exchange economic models, which originate with Cournot, Walras, Edgeworth, etc., are in our extensive concern. In Part 1, in order to shed a new light on the theory, and, to propose a general (non-) Walrasian prototype, an exchange market is reformulated and reorganized under the hypothesis of price strategic (not price-taking) economic man. An equilibrium of the formulation is defined. In so doing, we shall pay special attentions to what follow below.

Firstly, in order to completely describe a strategic (not price taking) behaviour in each market, that is, to totally specify the method of exchange, which may replace the Walrasian tatonnement process for price taking behaviour, and to eventually consider the true dynamic process of trading and the effects of information, we shall here formulate a general process of price formation, which is compatible[1] with the strategic behaviour on the individual decision or coordinated decision basis. We wish to consider the cases in which the incentives for deviating from price taking behaviour by collusion hardly vanishes, even when the number of traders increases. We also wish to consider the incentives for price -taking behaviour in finite economies. In fact, in this and succeeding sections, we shall deal with the exchange markets with a finite number of traders whose demand or supply offers are discrete (demand or supply offer functions are discontinuous).

Secondly, in the general setup of price formation process, the reformulation and reorganization go only under the law of demand, as a decision rule revealed in markets. We need not make any rationality hypotheses, such as the utility hypothesis, in order to establish the rule and mechanizm of price formation. We shall take the weakly generalized law of demand as a behaviour property, to be exhibited *ceteris paribus*, which is common to all the preferences underlying decisions revealed in each market.

Thirdly, we shall pay a special attention to the *number* of traders (thus defined) who are actually involved in transaction and trade, so that they can produce effects in the determination of trading prices.

Lastly, we explicitly deal with only two degrees of (asymmetric or symmetric) information conditions; *information* about those strategic choice variables[2] which are not controlled by a trader, but, controlled by other traders, and about those which are jointly determined by these two categories of controlled and uncontrolled variables, such as the *trading prices* which are defined as the prices, at which actual transaction and trade occur in the markets.

To specify these respects in a model, we shall define an exchange market economy as a collection of traders, defined in terms of preferences, endowments, and an assignment to each trader of a correspondence (function) from prices (price intervals) into (demand and supply) offers of trade. For each such specification, a (possibly empty) set of market clearing prices, which we say trading prices, results, as we shall see in the sequels: if the offer correspondence, assigned to each trader is a Walrasian competitive demand (or supply) correspondence, then, these trading prices are the Walrasian competitive prices. In general, we think of, as we shall in fact see in our formulation, this offer correspondence as being able to influence the trading prices, at which actual exchange occurs. We shall determine the trading prices through the rule and mechanizm of price formation presented in seubsection 1.2, which may dispose the Walrasian tatonnement process. Thus, the trading prices are, in general, not competitive equilibrium prices. We are now able to define a new concept of trader as a price-strategic, not a price-taking economic man in the reformulation and reorganization.

We shall say; a trader is a price-strategic economic man, if the trader, losing the incentives to remain a price-taking economic man, can manipulate more or less the trading prices.

After making clear any functional dependencies of trading prices determined on the basis of decentralized decisions, we shall, in Subsection 2.2, introduce a partial condition of complete information about decisions due to others, of which a trader could take advantage in each market.

In Section 3, we shall formulate the continuous price formation process and see there essentially the same results as in the preceeding sections.

In Section 4, we shall establish, in the price-market regime; a general concept of equilibrium as a price strategic equilibrium, which may include the Walrasian competitive, the monopolistic equilibrium, the Keynsian disequilibrium, etc., as special cases.

1.2 A General Price Formation Process under the Law of Demand

The aim of this subsection is to study carefully how the trading prices are determined on the basis of individual decsion makings. We shall concern ourselves with how jointly the prices are determined by any behavioural decisions revealed in each market, which can be controlled by each trader, not controlled by each but controlled by other traders. We shall proceed to the formulation, without any comments on how are given to each trader any information about decisions due to others, the assignment of price expectation correspondence from past prices, etc., as well as on how are given the preference, endowments, and the assignment of offer correspondence from expected prices.

Suppose as usually all traders in the economy are required to trade each specifically designated commodity[3] in an organized market here called trading post, in which traders name both prices and quantities as their strategic decision. The named prices and quantities will precisely transmit a message that says each trader would buy (or sell) the quantity named by himself at the prices, named by himself or lower (or higher) than the named price, if they aje the prices at which any transaction and trade actually occur among the traders (involved in the following way). To present precisely the trading mechanism and rule which lead to the process of a price formation by strategic traders, we shall in full detail formulate the price-market regime' in our concern as follows : We shall assume no free goods and services possible.[4]

Assume there exist n traders who are endowed with initial endowment and (constant) preference, each denoted by i (where N is the countable set of indices of members in the economy), m kinds of commodities to be traded, each denoted by j (where M is the countable set of indices of commodities specifically designated) and m corresponding trading posts (one for each commodity so that j also represents the trading post for j th commodity). Assume N and M are finite except in case otherwise stated.

At each and every trading post $j \in M$, each individual $i \in N$, makes his bid and offer, <u>once and for all</u> so that his <u>strategy</u> is foursome (hereafter <u>strategic foursome</u>)

(1) $(p^i_{sj}, s^i_j, p^i_{dj}, d^i_j) \geqq 0,$ [5] [6] [7]

which satisfies two mutually related constraints, as will be shown later, determined by the initial endowment and by the prices bidden and the quantities offerred by trader i. The initial endowment is denoted by an $m+1$ dimensioanl vector $e^i = (e^i_1, e^i_2, \ldots, e^i_m, e^i_{m+1})$, which is assumed non negative, and whose $m+1$ th element e^i_{m+1}, assumed positive, possesses the general acceptability and serves as "money". Hence, for each individual i,

(2) $\sum_{j \in M} p^i_{dj}\, d^i_j \leqq e^i_{m+1}$, [8] $\qquad e^i_j \geqq s^i_j \geqq 0,$ [9]

where the price p^i_{sj} he bade is the minimum price at which trader i would sell s^i_j quanitity of commodity j so that $s^i_j = s^i_j\ (p_j)\quad p_j \geqq p^i_{sj}$, while the price p^i_j he bade for j th commodity is the maximum price at which trader i would buy d^i_j quantity so that $d^i_j = d^i_j\ (p_j)\quad p_j \leqq p^i_{dj}$.
Note that the Walrasian budget constraint is separated into two constraints; <u>cash constraint</u> (the former one in (2)) and <u>quantity constraint</u>. The quantity constraints are assumed for the purpose of always making feasible, on the supply side, any trade which depend on bids and offers. The cash constraint does exclude, from the item of cash balance, any possible cash receipts from the trade; this is because trader has no idea, in advance[10] about how much or none he can receive cash from buyers by actually selling the whole or part (possibly none) of the quantities he set up for sale out of his initial endowment at the prices bidden by himself or at the higher prices. He will not

sell any of them if the trading prices are lower than what he bade. Thus, the cash constraint is also made in such a way that any trade may be feasible on the demand side. Observe also the cash constraints of these traders are more restrictive than those budget constraints which restrict Walrasian traders who actually trade only at the equilibrium prices. Whenever trader i makes bid and offer, each trader is required to prepare to pay the maximum amount of money $p^i_{dj}\, d^i_j$, where the price p^i_{dj} is bidden by himself for quantity d^i_j offerred by himself, so <u>how much he would provide, depends on his own strategic bids and offers</u>. It may not necessarily be natural in the formulation, but plausible to assume that

(3) $p^i_{sj} \geqq p^i_{dj} \geqq 0$.[11]

It is high time to be specific about the trading mechanizm that leads to the formation of trading prices, at which barter and transaction actually occur in the trading posts.

Suppose j th price p_j , at which actual trade occurs in the j th post, is determined. This supposition is made to eventually specify a price formation process. Let

$$(4)\quad d^i_j(p_j) = \begin{cases} d^i_j(p^i_{dj}) & \text{if } p_j \leqq p^i_{dj} , \\ 0 & \text{otherwise (if } p_j > p^i_{dj}) . \end{cases}$$

Similarly, let

$$(5)\quad s^i_j(p_j) = \begin{cases} s^i_j(p^i_{sj}) & \text{if } p_j \geqq p^i_{sj} , \\ 0 & \text{otherwise (if } p_j < p^i_{sj}) . \end{cases}$$

The intended interpretation for the formulation (4) is that trader i actually buys his offered quantity $d_j^i(p_{dj}^i)$ at any price lower than what he bade for the commodity j so that he would buy his offered quantities $d_j^i(p_{dj}^i)$ at the trading price p_j, if it is lower than or equal to the maximum demand price p_{dj}^i bidden by himself, while otherwise he would not buy any. Likewise, for (5), trader i would sell his offered quantities $s_j^i(p_{sj}^i)$ at the trading price p_j if it is higher than or equal to the minimum supply price p_{sj}^i bidden by himself. Note that <u>these functions $d_j^i(.)$ and $s_j^i(.)$ are of step function type and not continuous; the discontinuity prevails at p_{dj}^i and at p_{sj}^i, respectively.</u>

Define a subset of N, denoted by N_{dj}, and its complement in N, denoted by cN_{dj}, as

(6) $N_{dj}(p_j) = [i \in N \; ; \; p_j \leqq p_{dj}^i] \; , \quad cN_{dj}(p_j) = N - N_{dj}(p_j) \; .$

Similarly,

(7) $N_{sj}(p_j) = [i \in N \; ; \; p_j \geqq p_{sj}^i] \; , \quad cN_{sj}(p_j) = N - N_{sj}(p_j) \; .$

Then, $N_{dj}(p_j) \cap N_{sj}(p_j) \neq \phi$ implies there exists at least one $i \in N$ such that $p_{sj}^i \leqq p_j \leqq p_{dj}^i$. Hence by (3) $p_{sj}^i = p_j = p_{dj}^i$. Thus, $p_{dj}^i < p_{sj}^i$ for all $i \in N$ implies $N_{dj}(p_j) \cap N_{sj}(p_j) = \phi$. Including this, the following remarks are immediate.

<u>Remark 1</u> : If the strategic foursome $[p_{sj}^i, s_j^i, p_{dj}^i, d_j^i] > 0 \;\; \forall \, i \in N, \forall \, j \in M$.

Then, $p_{dj}^i < p_{sj}^i$ implies (i) $\forall \, i \in N_{dj}(p_j) \;\; i \notin N_{sj}(p_j)$.

$\forall \, i \in N_{sj}(p_j) \;\; i \notin N_{dj}(p_j)$, so that $N_{dj}(p_j) \cap N_{sj}(p_j) = \phi$.

(ii) $N_{dj} \cup N_{sj} \subseteq N$,

(iii) $cN_{dj}(p_j) \cap cN_{sj}(p_j) \neq \phi$ if and only if $p^i_{dj} < p_j < p^i_{sj}$ for some $i \in N$.

(i) and (ii) are obvious, hence so is for (iii). Suppose $cN_{dj}(p_j) \neq \phi$ so that $p_j > p^i_{dj}$ for some $i \in N$. Then, $p^i_{sj} > p^i_{dj}$ implies $p_j \geqq p^i_{sj}$ or $p^i_{sj} > p_j > p^i_{dj}$ for all $i \in N_{dj}$. The former of the inequalities implies $i \in N_{sj}$, while the latter implies $i \notin N_{sj}$. The converse is also trivial.

This remark says; once the trading price is established, then, the assumption (3) with strict inequality implies that (i) the trading post is completely separated into two disjointed sides of the post, on one side of which traders are willing to sell but not to buy, while on the other side traders willing to buy but not to sell, at the common price, and (ii) traders who actually trade any at the post are smaller in number than all traders who made bid and offer, so that some do not trade any, and (iii) traders do not trade any, if and only if the trading price is so high for them that they will not buy any, and, the price is so low for them that they will not sell any.

We now define the trading price p^o_j at each trading post j to be so determined that

(8) $p^o_j = \min\{p_j; \ \ p_j \in P_j \ , \ 0 < d_j(p_j) \leqq s_j(p_j)\}$

where P_j is a subset of the real line R, which includes the minimum supply prices and the maximum demand prices bidden by all traders, such that

(9) $P_j = \{ p^i_{sj} \ ; p^i_{dj} \ ; i = 1, 2, \ldots, n\} \cup R_+ = R_+$,

and

(10) $d_j(p_j) = \sum_{i \in N} d^i_j(p_j)$, $s_j(p_j) = \sum_{i \in N} s^i_j(p_j)$.

Alternatively, we define it to be so determined that

$$(11)\quad p_j^o = \max\{p_j\,;\ p_j \in P_j\ ,\quad 0 < s_j(p_j) \leqq d_j(p_j)\}\ .$$

Or, alternatively, let $\bar{p}_j^o$ and $\underline{p}_j^o$ be such prices determined by (8) and (11), respectively, then, we define the price to be such that,

$$(12)\quad p_j^o = \begin{cases} \underline{p}_j^o & \text{if}\quad 0 \leqq s_j(\underline{p}_j^o) - d_j(\underline{p}_j^o) \leqq d_j(\bar{p}_j^o) - s_j(\bar{p}_j^o)\ , \\ \bar{p}_j^o & \text{if}\quad 0 \leqq d_j(\bar{p}_j^o) - s_j(\bar{p}_j^o) \leqq s_j(\underline{p}_j^o) - d_j(\underline{p}_j^o)\ . \end{cases}$$

Any one of the definitions[12] is different from another, because, in general, the discountinuity property of each offer function fails to achieve the equality relation between aggregate demand and supply offers, $d_j(.)$ and $s_j(.)$ at any p_j . However, in what follows, there are no differences among the above definitions of trading price.

<u>Assume</u> for each i e N, and, for each j e M, that $s_j^i > 0$ if $p_{sj}^i > 0$ and $d_j^i > 0$ if $p_{dj}^i > 0$. Then, for a positive price $p_j > 0$, since obviously $N_{dj}(p_j) = \phi \leftrightarrow d_j(p_j) = 0$ and $N_{sj}(p_j) = \phi \leftrightarrow s_j(p_j) = 0$ it follows that $N_{dj}(p_j^o) \neq \phi$ and $N_{sj}(p_j^o) \neq \phi$. This will be included in the following remark[13]: Suppose $p_{dj}^i < p_{sj}^i$, for every i in N ; then,

<u>Remark 2</u> : $N_{sj}(p_j) \neq \phi$ and $N_{dj}(p_j) \neq \phi \leftrightarrow p_j \geqq p_{sj}^i$ for some $i \in N$

and $p_j \leqq p_{dj}^i$ for some other $i \in N$.

Observe that this remark holds valid, irrespective of whether the assumption $p_{dj}^i < p_{sj}^i \ \forall\, i \in N$[14] is made or not. The remark says that actual trade occurs if and only if the trading price is established if and only if there exists a price such that some traders name the minimum supply prices equal to or lower than the price and some other traders name the maximum demand prices equal to or higher than the price. Let

$$(13)\quad \underline{p}_{dj} = \min\{p_{dj}^i;\ i \in N\},\ \bar{p}_{sj} = \max\{p_{sj}^i;\ i \in N\}\,.$$

Then, it is immediate from (4) that

(14) $N_{dj}(p_j) = N \quad p_j \leqq \underline{p}_{dj}$, $N_{sj}(p_j) = N \quad p_j \geqq \bar{p}_{sj}$

and from (3) with strict inequality and (4),

(15) $N_{sj}(p_j) = \phi \quad p_j \leqq \underline{p}_{dj}$, $N_{dj}(p_j) = \phi \quad p_j \geqq \bar{p}_{sj}$.

Since $N_{dj}(p_j) = \phi \leftrightarrow d_j(p_j) = 0$, $N_{dj}(p_j) \neq \phi \leftrightarrow d_j(p_j) > 0$, it follows that

(16) $d_j(p_j) = \sum_{i \in N_{dj}(p_j)} d_j^i(p_j) \geqq 0$.

Similarly,

(17) $s_j(p_j) = \sum_{i \in N_{sj}(p_j)} s_j^i(p_j) \geqq 0$.

Since each individual function $d_j^i(p_j)$ $(s_j^i(p_j))$ is of step function type and discontinuous, so is the aggregate function $d_j(p_j)$ (resp. $s_j(p_j)$), and discontinuity prevails at p_{dj}^i , $i \in N_{dj}(p_j)$ (resp. at p_{sj}^i, $i \in N_{sj}(p_j)$).

<u>Remark 3</u> : $d_j(p_j)$ is a non-increasing function in each such p_j and

$s_j(p_j)$ is a non-decreasing function in each such p_j .

Observe $N_{dj}(p_j) \supseteqq N_{dj}(p_j') \leftrightarrow p_j \leqq p_j'$, which is equivalent to $d_j(p_j) \geqq d_j(p_j')$. Similarly for $s_j(p_j)$. But, if $p_j < p_j'$, $N_{dj}(p_j) \supseteqq N_{dj}(p_j')$ alone follows. Let

(18) $\underline{p}_j = \sup\{p_j : N_{sj}(p_j) = \phi\}$, $\bar{p}_j = \inf\{p_j : N_{dj}(p_j) = \phi\}$.

Then, from Remark 2 it follows that trading price p_j^o must be between $\underline{p}_j$ and $\bar{p}_j$; that is, $p_j^o \in [\underline{p}_j, \bar{p}_j]$. If $\underline{p}_j < \bar{p}_j$, then, p_j^o is established by (8), (11) or (12). Thus, a sufficient condition for the trading price to be established is that

(19) $\underline{p}_j < \bar{p}_j$.

Suppose $\bar{p}_j < \underline{p}_j$, then, for some p_j such that $\bar{p}_j < p_j < \underline{p}_j$, $N_{sj}(p_j) = \phi$ since $p_j < \underline{p}_j$ and $N_{dj}(p_j) = \phi$ since $p_j > \bar{p}_j$. Hence, if any trading occur at all, it must hold that

(20) $\underline{p}_j \leqq \bar{p}_j$,[15]

hence, (19) follows only as a sufficient condition.

We can easily generalize the rule of bid and offer from only one strategic foursome for each individual trader to more than one foursome, without changing any essentials above.[16] For an easy and quick understanding this is true, one may think of an individual trader as representing a coalition[17] (subgroup) of traders, each of whom behaves as if he were the individual trader above characterized.

Formally, instead of (1) with the constraints (2) and (3), we may have

(21) $\{(p^{i\nu}_{sj}, s^{i\nu}_j)\ \nu = 1, 2, \ldots \lambda_i, \ (p^{i\nu}_{dj}, d^{i\nu}_j)_{\nu = 1, 2, 3, \ldots \kappa_i}\} \geqq 0$

for some integer λ_i and κ_i , which satisfies,

(22) $e^i_{m+1} \geqq \sum_{j \in M} \sum_{\nu=1}^{\kappa_i} p^{i\nu}_{dj}\ d^{i\nu}_j > 0$. $e^i_j \geqq \sum_{\nu=1}^{\lambda_i} s^{i\nu}_j \geqq 0$.

Let

(23) $\underline{p}^i_{sj} = \min_\nu p^{i\nu}_{sj}$, $\bar{p}^i_{sj} = \max_\nu p^{i\nu}_{sj}$, $\underline{p}^i_{dj} = \min_\nu p^{i\nu}_{dj}$, $\bar{p}^i_{dj} = \max_\nu p^{i\nu}_{dj}$.

Then, we can assume instead of (3),

(24) $\bar{p}^i_{sj} \geqq \bar{p}^i_{dj}$, $\underline{p}^i_{sj} \geqq \underline{p}^i_{dj}$,

in which the case of $\bar{p}^i_{dj} \geqq \underline{p}^i_{sj}$ implies trader i would be both a buyer and a seller if the trading price is determined between $\bar{p}^i_{dj}$ and $\underline{p}^i_{sj}$.

Observe also Remark 1 still remain true, if $\underline{p}^i_{sj} > \bar{p}^i_{dj}$. If not, trader i can be both a buyer and a seller at the same trading post.

Let

$$(25)\quad N_{dj}(p_j) = \{ i \in N \, ; \, p_j \leqq p^{i\nu}_{dj} \text{ for some } \nu \text{ such that } 1 \leqq \nu \leqq \lambda_i \},$$

$$N_{sj}(p_j) = \{ i \in N \, ; \, p_j \geqq p^{i}_{sj} \text{ for some } \nu \text{ such that } 1 \leqq \nu \leqq \kappa_i \}.$$

Then, $N_{dj}(p_j) \cap N_{sj}(p_j) \neq \phi$.

The individual offer functions are now defined as follow

$$(26)\quad d^i_j(p_j) = \begin{cases} \sum_{\nu=1}^{\lambda_i(p_j)} d^{i\nu}_j(p^{i\nu}_{dj}) = \sum \quad d^{i\nu}_j(p_j) \\ 0, \text{ if } p_j > \bar{p}^i_{dj} . \end{cases}$$

$$s^i_j(p_j) = \begin{cases} \sum_{\nu=1}^{\kappa_i(p_j)} s^{i\nu}_j(p^{i\nu}_{sj}) = \sum \quad s^{i\nu}_j(p_j) \\ 0, \text{ if } p < \underline{p}^i_{sj} . \end{cases}$$

where $\lambda_i(p_j)$ and $\kappa_i(p_j)$ are some integers less than or equal to λ_i and κ_i , respectively, which correspond to the price p_j . By definition each individual function $d^i_j(p_j)$(resp. $s^i_j(p_j)$) is a step function with more than one step and discontinuous at $p_j = p^{i\nu}_{dj}$, $\nu = 1, 2, 3\ldots, \lambda_i(p_j)$ (resp. at $p_j = p^{i\nu}_{sj}$, $\nu = 1, 2, \ldots, \kappa_i(p_j)$). Hence, the aggregate function $d_j(p_j) = \sum_{i \in N_{dj}(p_j)} d^i_j(p_j)$ for $N_{dj}(p_j)$ defined in (25)(resp. $s_j(p_j) = \sum_{i \in N_{sj}(p_j)} s^i_j(p_j)$ for $N_{sj}(p_j)$ defined in (25)) is also a step function with many steps and discontinuous at $p_j = p_{dj}^{i\nu}$, $\nu = 1, 2, 3, \ldots, \lambda_i(p_j)$, $i \in N_{dj}(p_j)$ (resp. at $p_j = p_{sj}^{i\nu}$, $\nu = 1, 2, 3, \ldots, \kappa_i(p_j)$, $i \in N_{sj}(p_j)$).

The cash constraint here to be employed however may deserve a special attension. Observe first that once the trading price p_j^o , at which trade actually occurs in the j th post, is determined, it always holds in this general case,

$$(27) \quad \sum_{\nu=1}^{\lambda_i} p_{dj}^{i\nu} d_j^i(p_{dj}^{i\nu}) \geqq p_j^o \{ \sum_{\nu=1}^{\lambda_i(p_j^o)} d_j^{i\nu}(p_{dj}^{i\nu}) \} = p_j^o d_j^i(p_j^o) ,$$

for this p_j^o . The right hand side of (27) shows the moneyvalue of trader i's actual payment, and the left hand side does the amount which he thought, before trade, might pay for $d_j^i(p_j)$, $p_j \leqq p_{dj}^i$, quantities of j th commodity. Trader i can calculate, in advance, the maximum amount of money that he might have to pay for $d_j^i(p_j)$ over p_j's ; that is, max $p_j d_j^i(p_j)$ over p_j such that $p_j \leqq \bar{p}_{dj}^i$. Since this maximum amount also satisfies the inequality (27), it follows that the cash constraint (22) is unnecessarily too restrictive, and a new more satisfactory constraint which is required is;

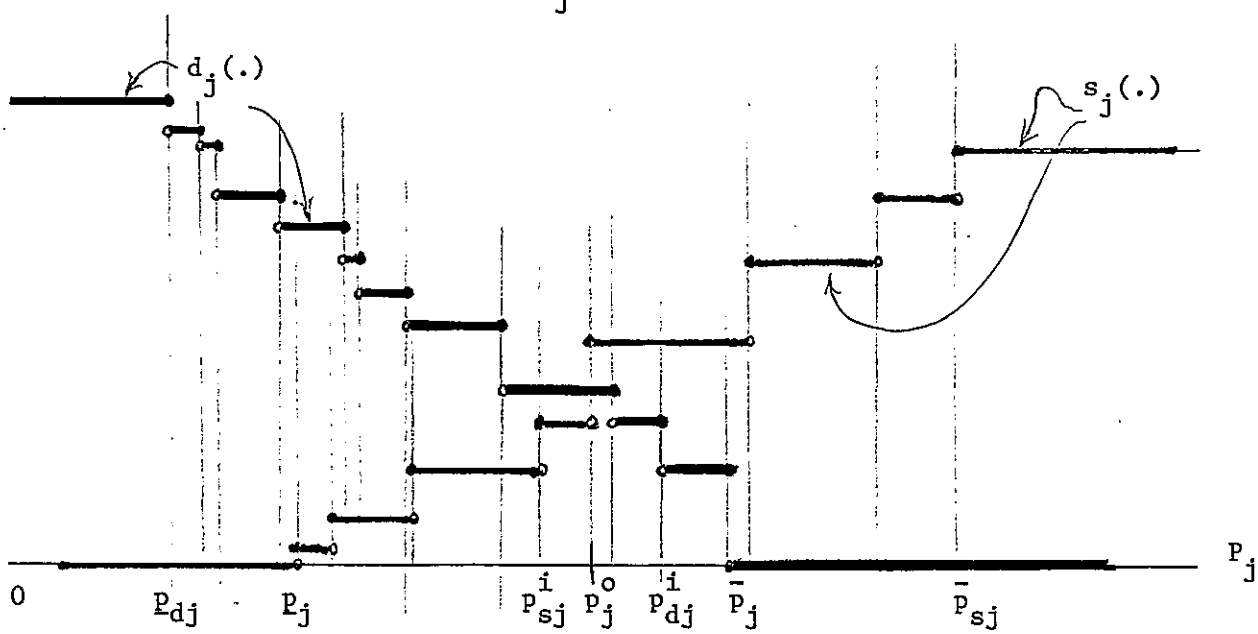
$$(22') \quad e_{m+1}^i \geqq \sum_{j \in M} \max_{p_j} \{ p_j \{ d_j^i(p_j) - s_j^i(p_j) \} \} , \; p_j \geqq 0.^{18}$$

This cash constraint however may be reduced to the cash constraint (2), in case the strategic foursome each trader can make is only one quartet (1), and therefore almost the same interpretation as for (2) may apply to this modified constraint, which is due to the generalization from one to more than one strategic foursome. In fact the maximum amount of money to be provided by each trader directly depends on <u>how the quantities offerred are related with the bidden prices, as well as the prices bidden by himself.</u>[19]

The three definitions (8), (10) and (12) of each trading price p_j^o may apply, as they stand, to this general formulation, $d_j(.)$ and $s_j(.)$ and it is easy to see that the sufficient condition for the trading price to exist is the inequality (19) for $\underline{p}_j$ and $\bar{p}_j$ defined for $N_{sj}(p_j)$ and $N_{dj}(p_j)$ in (25), in this general setting.

Figure 1 illustrates the content of the prescribed price formation.

Figure 1 , where p_j^o is defined by (8).



This general formulation will extensively be considered in a more analytical framework of the closed model economy in the sequels.

2. A Summary with Supplementary Specification and Economic Implications

2.1 Functional Relations and Dependencies.

Before proceeding to that extension, we had rather summarize the results obtained and the things observed so far, with supplementary specification which makes clear any functional dependencies.

We explicitly assume in making strategic foursome (1) (or (17)) that each quantity d_j^i (or s_j^i) offerred by each trader i depends directly on the price p_{dj}^i (resp. p_{sj}^i) bidden by himself or prices lower (resp. higher), so that $d_j^i(.)$ (resp. $s_j^i(.)$) is a real valued function of price[20] p_j. At the same time, this offer function $d_j^i(.)$ (resp. $s_j^i(.)$) also depends indirectly on the other prices bidden by himself at the other trading posts. This fact is mainly due, first, to the substitutability and complementarity[21] relationships among the commodities in his concern and, second, to the cash constraint. Hence, in (1), individual offer function may be described more precisely as follows :

For each $i \in N$ and each $j \in M$,

$$(28)\quad d_j^i(p_j) = d_j^i(p_j\ ;\ p_s^i\ ,\ p_d^i) = \begin{cases} d_j^i(p_{dj}^i) & p_j \leqq p_{dj}^i \\ 0 & \text{otherwise,} \end{cases}$$

$$(29)\quad s_j^i(p_j) = s_j^i(p_j\ ;\ p_s^i\ ,\ p_d^i) = \begin{cases} s_j^i(p_{sj}^i) & p_j \geqq p_{sj}^i \\ 0 & \text{otherwise,} \end{cases}$$

where $p_s^i = (p_{s_1}^i, p_{s_2}^i, \ldots, p_{sj}^i, \ldots, p_{sm}^i)$ and $p_d^i = (p_{d_1}^i, p_{d_2}^i, \ldots, p_{dj}^i, \ldots, p_{dm}^i)$.

Let $p^i = (p_s^i, p_d^i)$ $i = 1, 2, 3, \ldots, n$, then,

(30) $d_j(p_j) = \sum_{i \in N} d_j^i(p_j) = d_j(p_j; p^1, p^2, \ldots, p^n) = d_j(p_j ; (p^i \; i \in N))$,

in which one notes some of $(p^i \; i \in N)$ are dummy variables, since $d_j(p_j) = \sum_{i \in N_{dj}(p_j)} d_j^i(p_j)$ for a nonempty $N_{dj}(p_j) \subseteqq N$. Similarly for $s_j(p_j)$.

Let

(31) $d(p, p^1, p^2, \ldots, p^n) = (d_j(p_j ; (p^i \; i \in N)) \; j \in M)$

$s(p, p^1, p^2, \ldots, p^n) = (s_j(p_j ; (p^i \; i \in N)) \; j \in M)$,

then, the trading price vector p^o is determined by (8) for example as such an infimum (minimum) price vector that

(32) $0 < d(p^o, p^1, p^2, \ldots, p^n) \leqq s(p^o, p^1, p^2, \ldots, p^n)$.

Thus, trading price p^o thus determined, depends on the prices bidden by all the traders involved, i.e. all i such that $i \in N_{dj}(p_j) \cup N_{sj}(p_j)$, $j \in M$. More specifically,

(33) $p^o = \rho((p_s^1, p_d^1), (p_s^2, p_d^2), \ldots, (p_s^n, p_d^n))$.

where some may be dummy variables.

Reconsider this process of price formation in the general setup of bidds and offers. No essentials will be found to change in the general formulation.

Let in (17)

(34) $P_{dj}^i = \{p_{dj}^{i\nu}; \nu = 1, 2, \ldots, \lambda_i\}$, $P_{sj}^i = \{p_{sj}^{i\nu}; \nu = 1, 2, \ldots, \kappa_i\}$,

then, the fact that $d_j^{i\nu} = d_j^{i\nu}(p_{dj}^{i\nu})$ defines the maximum offer $d_j^i(P_{dj}^i)$ on the set P_{dj}^i of prices bidden by trader i such that

$$(35)\quad d^i_j(p^i_{dj}) = \sum_{\nu=1}^{\lambda_i} d^{i\nu}_j(p^{i\nu}_{dj}).$$

Since $d^{i\nu}_j(p_j) = \begin{cases} d^{i\nu}_j(p^{i\nu}_{dj}) & p_j \leqq p^{i\nu}_{dj} \\ 0 & \text{otherwise,} \end{cases}$

it follows that for a closed interval $[p_j, \bar{p}^i_{dj}]$, an individual offer <u>interval</u> function

$$(36)\quad D^i_j[p_j, \bar{p}_{dj}] = \sum_{\nu=1,2,\ldots,\lambda_i(p_j)} d^{i\nu}_j(p_j)$$

is determined and can be extended to the positive one dimensional cone R_+. It is a step function with many steps at p^i_{dj} $\nu = 1, 2, \ldots, \lambda_i(p_j)$, which we have seen.

Similarly for an individual (supply) offer function $S^i_j[\underline{p}^i_{sj}, p_j]$

$$(37)\quad S^i_j[\underline{p}^i_{sj}, p_j] = \sum_{\nu=1,2,\ldots,\kappa_i(p_j)} s^{i\nu}_j(p_j).$$

The cash constraint (22)' may imply that individual offer function $d^i_j(p_j)$ (or, $s^i_j(p_j)$) depends indirectly on the price intervals bidden at the other posts. The price intervals are determined by their boundaries and, after taking into account the substituability and complementarity relationships among commodities, we may write its indirect dependences in terms of their boundaries so that

$$(38)\quad d^i_j(p_j) = d^i_j(p_j, (\underline{p}^i_{sk}, \bar{p}^i_{sk}, \underline{p}^i_{dk}, \bar{p}^i_{dk}, \ k \in M)),$$

To see this more carefully, let $\bar{p}^i_k$ be the price such that

$$\max_{p^i_k \leqq \bar{p}^i_{dk}} p^i_k\, d^i_k(p^i_k) = \bar{p}^i_k\, d^i_k(\bar{p}^i_k).$$

Then, $d_j^i(p_j) = d_j^i(p_j, (\bar{p}_k^i; k \in M))$

but, each $\bar{p}_k^i$ may depend on $(\underline{p}_{sk}^i, \bar{p}_{sk}^i, \underline{p}_{dk}^i, \bar{p}_{dk}^i)$, and also depend on how individual offer function $d_k^i(.)$ is related with prices bidden. <u>Here the relation between quantities offerred and prices bidden is arbitrarily given, and any offer function is eligible, if it obeys the weakly generalized law of demand</u>. Aggregation over individual traders will define,

$$(39)\ \begin{cases} d_j(p_j) = d_j(p_j, (\underline{p}_s^i, \bar{p}_s^i, \underline{p}_d^i, \bar{p}_d^i ; i \in N)) \\ s_j(p_j) = s_j(p_j, (\underline{p}_s^i, \bar{p}_s^i, \underline{p}_d^i, \bar{p}_d^i ; i \in N)) \end{cases}$$

and trading price p^o is thus determined as the one which satisfies,

$$(40)\quad 0 < d(p^o; (\underline{p}_s^i, \bar{p}_s^i, \underline{p}_d^i, \bar{p}_d^i ; i \in N)) \leqq s(p^o; (\underline{p}_s^i, \bar{p}_s^i, \underline{p}_d^i, \bar{p}_d^i ; i \in N)).$$

Thus, trading price depends on the price intervals bidden by all the traders involved, as long as individual offer functions are given and fixed ;

$$(41)\quad p^o = \rho((\underline{p}_s^1, \bar{p}_s^1, \underline{p}_d^1, \bar{p}_d^1), (\underline{p}_s^2, \bar{p}_s^2, \underline{p}_d^2, \bar{p}_d^2), \ldots, (\underline{p}_s^n, \bar{p}_s^n, \underline{p}_d^n, \bar{p}_d^n)).$$

To make clear any economic effects which an individual trader may produce on the price formation process, suppose a trader i did not make any offers of trade in every post. Let us suppose that the trading price $p^o(i)$ is determined by (8) so that

$$(41)'\quad p^o(i) = \rho((\underline{p}_s^1, \bar{p}_s^1, \underline{p}_d^1, \bar{p}_d^1), \ldots, (\underline{p}_s^{(i-1)}, \bar{p}_s^{(i-1)}, \underline{p}_d^{(i-1)}, \bar{p}_d^{(i-1)}),$$
$$(\underline{p}_s^{(i+1)}, \bar{p}_s^{(i+1)}, \underline{p}_d^{(i+1)}, \bar{p}_d^{(i+1)}), \ldots, (\underline{p}_s^n, \bar{p}_s^n, \underline{p}_d^n, \bar{p}_d^n)).$$

Compare the trading price p^o with $p^o(i)$. Any difference would follow from the difference between the two definitional inequalities, in aggregation, one including trader i's offer functions and the other not. That is,

$$(40)' \quad \min_p \{ p \; ; \; 0 < d^i(p) + d(i \; ; p) \leqq s^i(p) + s(i \; ; p) \} = p^o,$$
$$\min_p \{ p \; ; \; 0 < d(i \; ; p) \leqq s(i \; ; p) \} = p^o(i),$$

where $d(i \; ; p) = {}_{k \neq i}\Sigma \, d^k(p)$, and $s(i \; ; p) = {}_{k \neq i}\Sigma \, s^k(p)$.

We conclude formulation of the general process, with the following remarks ;

Remark 4 : Suppose every trader thus defined is initially endowed with quantities of trade, so ' negligible '(not in the mathematical sense of Lebesgue) relative to the total endowment, that, under the constraints (any of (2), (22) or (22)'), the values of individual offer functions $s^i(.)$ and $d^i(.)$ at each price are small enough to keep satisfying the definitional relation (40) (in case of (8)) which defines the trading prices. Then, any trader behaves as if he were a price taker, not a price maker or any price strategic trader. Formally, $p^o = p^o(i)$, for every i e N.

This is with the perfect competitive case in discrete (discontinuous) version, whereas in continuous case we shall deal later ; see Remark 8. Remark 4 thus says, if individual trader i is not a price taker and produces any effects on the price formation so that $p_j^o \neq p_j^o(i)$ for some j e M, then, he is endowed with non-negligible quantities of trade. Conversely ;

Remark 5 : Any trader i, with non-negligible quantities of trade endowed with, can change the trading price $p^o(i)$ to what he wants the price to be, p^o, within a certain range.

Thus, we can say that being not a price taker is equivalent to being endowed with non-negligible quantities of trade. The incentives to deviate from a price-taking behaviour increases, in stead of staying a price-taking behaviour. Remark 5 thus can characterize the price-strategic behaviour we have defined at the end of Subsection 1.1.

Incidentally, the next remark has something to do with the quantity rationing problem, which seems to be essentially the same as the one raised in the Neo-walrasian(-Neokeynsian) disequilibrium framework.

Remark 6 : Suppose every trader is initially endowed with quantities of trade so negligible relative to the total endowment in the above sense. Then, each trader, involved in trade and transaction at the price which is determined by (8), (11) or (12), and, on the long side of post, must, with equiprobability, be quantitatively rationed.

Observe that the general price formation process always entails in general possibility of a quantity rationing in transaction at the price established. This is formally due to the discontinuity property of offer functions, which we shall precisely see in Section 3, whereas, in the Walrasian general equilibrium frameworks, it is because of transaction out of equilibrium. However, in our general setup which disposes the Walrasian auctioneer, a price taking competitive trader thus defined (in Remark 4) is rationed as if he were defined in the Neowalrasian-Neokeynesian disequilibrium analysis. It will be of some interest to start disequilibrium analysis from our approach.

2.2 New Allocation and the Individual Rationality under a Complete Information Condition — the Hypothesis of Strategic Economic Man.

In order to explicitly analyse any informational effects on the individual strategic behaviour and hence on the determination of trading prices, we shall, in the subsections which follow below, introduce the condition of a complete information about decisions due to others, of which a trader could take advantages. The information is given in terms of aggregate offer correspondences.

We shall see here a trader, who takes into account information given to him, making decision interdependently.

Once the trading price p_j^o at each post is thus established, each buyer i in $N_{dj}(p_j^o)$ purchases $d_j^i(p_j^o)$ quantity of j th commodity on sale, and pays money which amounts to $p_j^o\ d_j^i(p_j^o)$. Each seller i in $N_{sj}(p_j^o)$ receives money which amounts to $p_j^o\ s_j^i(p_j^o)$ for $s_j^i(p_j^o)$ out of $s_j^i(p_{sj}^i)$ quantity set up for a sale by himself. Thus, after transaction and trade, a new allocation x^i for each trader i involved, results in as follows ;

(42) $x_j^i\ (p_j^o) = e_j^i - s_j^i(p_j^o) + d_j^i(p_j^o),\quad j = 1, 2, \ldots, m$

(43) $x_{m+1}^i(p^o) = e_{m+1}^i - \sum_{j \in M} p_j^o\ d_j^i(p_j^o) + \sum_{j \in M} p_j^o\ s_j^i(p_j^o).$

Let us assume that each individual trader i is endowed with as well as his initial endowment e^i, his preference R_i which is representable by a real valued utility functional $u^i(.)$ of feasible (admissible) allocation x^i,

(44) $x^i \rightarrow u^i(x^i)\ ;\ u^i(x^i) \geqq u^i(y^i) \leftrightarrow x^i\ R_i\ y^i\ .$

Then, the utility of x^i may be interpretted as the welfare of the outcome brought by a strategic foursome made by trader i if every foursome made by other trader is known to him. Given strategic foursomes to be made by all the other traders, the individual rationality may be interpretted as the choice of an optimal strategic foursome from which an optimal outcome function maps into the new allocation that maximizes his numerical utility over the feasible strategic foursomes he could bid and offer.

Suppose trader i is given this information, as, in what follows, we shall call it a complete information condition.

To be more specific in view of (30)-(33) or (39)-(41), we shall explain stepwise what turns out to be the rational choice out of feasible strategies: <u>Given the aggregate (strategic) demand and supply offer functions, where the aggregation is made over all the traders but trader i</u> .

Then, find the boundary prices $(\underline{p}_s^i, \bar{p}_s^{-i}, \underline{p}_d^i, \bar{p}_d^{-i})$[22] which are arbitrary. One may think of the boundaries, within which the trading prices may individually be expected to be set up, or, the boundaries, between which the prices warranted for some or other reasons may be expected to be set up, or between which the prices the officers of the posts wish to establish may exist, and which are publicly informed to the traders, etc..

First, find m real valued functions χ_j^i, $j \in M$, defined on the closed interval with the boundaries in $2m$ dimensional space, such that each $\chi_j^i(\cdot)$ is the sum of real valued step functions on one dimensional (j th price) interval $[0, +)$, and such that the sum $\chi_j^i(\cdot) = \Sigma_{\nu=1}^{\lambda} \chi_j^{i\nu}(\cdot)$ maximizes utility u^i over the feasible individual allocation x^i's at each p_j^o such that $0 \leqq p_j^o \leqq \infty$. This is to choose $\sigma_j^i(\cdot)$ and $\phi_j^i(\cdot)$[23] such that

$$(45) \quad \chi_j^i(p_j^o) = e_j^i - \sigma_j^i(p_j^o) + \phi_j^i(p_j^o) \ , \quad e_j^i - \sigma_j^i(p_j^o) \geqq 0, \ j \in M,$$

$$(46) \quad \chi_{m+1}^i \ (p_j^o) = e_{m+1}^i + \sum_{j \in M} p_j^o \ \sigma_j^i(p_j^o) - \sum_{j \in M} p_j^o \ \phi_j^i(p_j^o) \geqq 0,$$

$$(47) \quad u^i((\chi_j^i(p_j^o) \ _{j \in M}), \ \chi_{m+1}^i \ (p_j^o)) \geqq u^i(x^i(p^o)).$$

Note here that if the trader i does not trade any, then, under the given conditions, the trading prices $p_j^o(i)$ is so determined that for each $j \in M$

$$(48) \quad p_j^o(i) = \min\{ \ p_j \ ; \ 0 < \sum_{k \neq i} d_j^k(p_j) \leqq \sum_{k \neq i} s_j^k(p_j) \ \}$$

if

$$(19)' \quad \underline{p}_j(i) < \bar{p}_j(i)$$

where $\underline{p}_j(i) = \inf\{p_j \; ; \; N(i) \supseteqq N_{sj}(p_j) = \phi\}$, $\bar{p}_j(i) = \sup\{p_j \; ; \; N(i) \supseteqq N_{dj}(p_j) = \phi\}$

and $N(i) = \{1, 2, 3, \ldots, i-1, i+1, \ldots, n\}$.

Thus, any choice of strategic foursome will change the trading price $p_j^o(i)$, if the trader i manipulates non negligible[24] quantity of trade (demand or supply). Second, choose the trading prices p^o_j such that, for each $j \in M$,

$$(49) \quad p_j^o = \min\{p_j^i \; ; \; 0 < \phi_j^i(p_j^i) + \sum_{k \neq i} d_j^k(p_j^i) \leqq \sigma_j^i(p_j^i) + \sum_{k \neq i} s_j^i(p_j^i)\},$$

where $p_j^i = p_j(\underline{p}_s^i, \bar{p}_s^{-i}, \underline{p}_d^i, \bar{p}_d^{-i})$, that is, p_j^i is determined by the strategic foursome chosen by the trader i. Thus, under the complete information condition, trader i can know what the trading prices and their corresponding demand and supply offer functions are, hence the cash constraint (22)' can be replaced by

$$(22)'' \quad e_{m+1}^i \geqq \sum_{j \in M} p_j^i \{d_j^i(p_j^i) - s_j^i(p_j^i)\}.$$

Furthermore, money is here assumed to give utility, and the individual rationality means to choose a new allocation for trader i, $(\chi_j^i(p_j^o)_{j \in M}, \chi_{m+1}^i(p^o))$, which maximizes utility $u^i(.)$.

We may say that <u>choosing an optimal strategy means here the optimal selection</u>[25] <u>of offer functions</u> $\sigma_j^i(.)$ <u>and</u> $\phi_j^i(.)$ <u>and of the trading price</u> p_j^o, <u>which satisfy</u> (45)-(47), <u>and</u> (49), <u>respectively, for every post</u> j.

Each such function $\chi_j^i(.)$ may be called <u>strategic demand or supply function</u>, so termed that it can be distinguished from the Walrasian demand or supply function of price taking trader. Variable interpretations for the hypothesis of strategic economic man are possible. Two extreme cases are pointed out.

Remark 7 ; In case transaction and trade occur at all :

(i) Suppose every trader i is initially endowed with quantity of trade, so "negligible"[26] relative to the total endowment, that, under the constraint (any of (2), (22) or (22')), the values of individual offer functions $\sigma_j^i(.)$ and $\phi_j^i(.)$ at each price are negligibly small. Then, any trader behaves as if he were a price taking individual trader, formally, $p_j^o(i) = p_j^o$ for every i and j, and this is with the Walrasian perfect competitive case.

(ii) Another extreme case to be pointed out is the case in which the number of traders is only two,[27] and a duopolistic or monopolistic exchange is in concern.

To preclude any ambiguity, we shall call hereinafter (individual) strategic demand (supply) function just strategic demand (supply) function (s.d. or s.s. function for short). Likewise, for the other functions. Let

$$(50) \quad \chi_j(.) = \sum_{i \in N_{sj}(.) \cup N_{dj}(.)} \chi_j^i(.)$$

$$(51) \quad \sigma_j(.) = \sum_{i \in N_{sj}(.)} \sigma_j^i(.)$$

$$(52) \quad \phi_j(.) = \sum_{i \in N_{dj}(.)} \phi_j^i(.) \ .$$

We shall call optimal (supply or demand) offer function $\sigma_j^i(.)$ or $\phi_j^i(.)$ <u>strategic supply or demand offer function</u> (s. s. (or d.) o. function for short), and call their aggregate function $\sigma_j(.)$ or $\phi_j(.)$ <u>aggregate strategic supply or demand offer function</u> (a. s. (or d.) o. function for short). Likewise, <u>aggregate strategic demand or supply function</u> $\chi_j(.)$ may be abbreviated to be a. s. d. (or s.) function.

3. A Continuous Formulation of the Price Formation.

3.1 The General Price Formation Process — Continuous Case.

To consider the preceding model of trade and transaction in a more analytical framework, we shall take some advantages of a well-behaved mathematical formulation below presented, which fully prescribes the essential parts of the price formation above outlined. One of the advantages is the continuity property. The discontinuity properties of individual and aggregate offer functions will be removed out and hence the definitive equations (8) (11) and (12) will be identical to one another since the continuity properties of offer functions $d_j^i(.)$ and $s_j^i(.)$ will eliminate the inequality cases from the definitions.

For each trading post $j \in M$, suppose that the set (the price set hereafter), denoted by P_j, of all the named supply and demand prices with lower boundary $\underline{p}_{dj}$ and upper boundary $\bar{p}_{sj}$ is a subset of the one dimensional euclidean space R, so that any countable price sets with the same boundaries may be contained in it.

Suppose the aggregate demand offer function $d_j(.)$ and the aggregate supply function $s_j(.)$ are continuous on P_j, respectively. Then, either of (8) (11) or (12) implies that the trading price p_j^o is so determined that

$$(53) \quad 0 < d_j(p_j^o) = s_j(p_j^o) \quad ; \quad \underline{p}_j < p_j^o < \bar{p}_j \ .$$

In order to specify these suppositions in case of a finite number of traders, each (at least one) individual trader i is required to make an infinite number of bids and offers once and for all. Assume in stead of (1) or (17) that each trader i names (supply and demand) prices and their corresponding (demand and supply) offers of quantity at each trading post j, so that they are one (or more than one) foursome of price intervals in R and quantity sets

$$(54)\ \{ P^i_{sj},\ S^i_j,\ P^i_{dj},\ D^i_j \}$$

where

$$(55)\quad P^i_{sj} = [\underline{p}^i_{sj},\ \bar{p}^i_{sj}]$$

$$(56)\quad \# S^i_j = S^i([\underline{p}^i_{sj},\ \bar{p}^i_{sj}])$$

here $\# S^i_j$ is the cardinal number of the set S^i_j and S^i is the so called interval function whose image for the interval P^i_{sj} shows the total quantity of supply offer of trader i, and similarly for demand price interval P^i_{dj} and its corresponding demand offer quantity set D^i_j,

$$(57)\quad P^i_{dj} = [\underline{p}^i_{dj},\ \bar{p}^i_{dj}]$$

$$(58)\quad \# D^i_j = D^i([\underline{p}^i_{dj},\ \bar{p}^i_{dj}])$$ where D^i is a real valued interval function.

Let $s^i_j(.)$ and $d^i_j(.)$ be both improperly integrable[28] on P^i_{sj} and P^i_{dj}, respectively. Then, the interval functions $S^i(.)$ and $D^i(.)$ are expressed as

$$(59)\quad S^i([\underline{p}^i_{sj},\ \bar{p}^i_{sj}]) = \int_{\underline{p}^i_{sj}}^{\bar{p}^i_{sj}} s^i_j(p^i_j)\ \partial p^i_j$$

and

$$(60)\quad D^i([\underline{p}^i_{dj},\ \bar{p}^i_{dj}]) = \int_{\underline{p}^i_{dj}}^{\bar{p}^i_{dj}} d^i_j(p^i_j)\ \partial p^i_j$$

The following condition (61) and (62), together with the other properties of the offer density functions, will preclude the discontinuity property of each offer function.

(61) $0 = s_j^i(\underline{p}_{sj}^i) \leqq s_j^i(p_j^i) < \infty, \quad p_j^i \in P_{sj}^i ,$

(62) $0 = d_j^i(\bar{p}_{dj}^i) \leqq d_j^i(p_j^i) < \infty, \quad p_j^i \in P_{dj}^i .$

We shall call the density function $s_j^i(.)$ (or, $d_j^i(.)$) supply (or demand) offer density function, which corresponds to the step function $s_j^{i\nu}(.)$ (or $d_j^{i\nu}(.)$) in the preceding countable case.

This foursome set (54) satisfies the quantity and cash constraints

(63) $\max_{p_j}\{ S^i([\underline{p}_{sj}, p_j]) - D^i([p_j, \bar{p}_{dj}^i])\} \leqq e_j^i , \quad j \in M,$

(64) $\sum_{j \in M} \max_{p_j \leqq \bar{p}_{dj}^i} p_j\{D^i([p_j, \bar{p}_{dj}^i]) - S^i([\underline{p}_{sj}^i, p_j]) \} \leqq e_{m+1}^i,$

which takes into account the restrictiveness of the constraint of (22) type and replaces

(65) $\sum_{j \in M} \int_{\underline{p}_{dj}^i}^{\bar{p}_{dj}^i} p_j^i \, d_j^i(p_j^i) \, \partial p_j^i \leqq e_{m+1}^i$

since,

(66) $\int_{\underline{p}_{dj}^i}^{\bar{p}_{dj}^i} p_j^i \, d_j^i(p_j^i) \, \partial p_j^i \geqq p_j \, D^i[p_j, \bar{p}_{dj}^i], \quad p_j \leqq \bar{p}_{dj}^i .$

Let

(67) $N_{dj}(p_j) = \{ i \in N \, ; \, p_j \leqq \bar{p}_{dj}^i\}, \; N_{sj}(p_j) = \{ i \in N \, ; \, p_j \geqq \underline{p}_{sj}^i\} \quad j = 1,2,\ldots,m,$

which correspond to (21), and let

(68) $\bar{p}_j = \sup\{p_j \, ; \, N_{dj}(p_j) \neq \phi\}, \quad \underline{p}_j = \inf\{p_j \, ; \, N_{sj}(p_j) \neq \phi\}.$

Assume that, in view of the condition (61) and (62), and instead of (24),

(69) $\bar{p}^i_{sj} > \bar{p}^i_{dj}, \quad \underline{p}^i_{sj} > \underline{p}^i_{dj}$.

Assume also that $p_j \in P^i_{sj}$, and $p_j > 0$ imply $s^i_j(p_j) > 0$, and $p_j \in P^i_{dj}$, and $p_j > 0$ imply $d^i_j(p_j) > 0$, for every $i \in N$, and for every $j \in M$. Then, it is easy to verify that, for two closed intervals P_{dj} and P_{sj} such that

(70) $P_{dj} = [\underline{p}_{dj}, \bar{p}_j] \qquad P_{sj} = [\underline{p}_j, \bar{p}_{sj}]$,

(71) $(P_{dj} \cap P_{sj})^o \neq \phi$

is the condition for the trading price to be established by (53) and is equivalent to the condition (19) under (69).

The individual (demand and supply) offer functions $D^i_j(.)$ and $S^i_j(.)$ may be expressed in terms of the interval functions,

(72) $D^i_j(p_j) = D^i([p_j, \bar{p}^i_{dj}]), \qquad S^i_j(p_j) = S^i([\underline{p}_{sj}, p_j])$,

and hence the aggregate offer functions $d_j(p_j)$ and $s_j(p_j)$ are, for each p_j, $\underline{p}_j < p_j < \bar{p}_j$,

(73) $d_j(p_j) = \sum_{i \in N_{dj}(p_j)} D^i_j(p_j), \qquad s_j(p_j) = \sum_{i \in N_{sj}(p_j)} S^i_j(p_j)$.

We can extend these $d_j(.)$ and $s_j(.)$ to $[0, \bar{p}_j]$ and $[\underline{p}_j, \infty)$, respectively, such that

(74) $D_j(p_j) = \begin{cases} d_j(p_j) & p_j \in P_{dj} \\ d_j(\underline{p}_{dj}) & p_j \in [0, \underline{p}_{dj}] \end{cases}$

(75) $S_j(p_j) = \begin{cases} s_j(p_j) & p \in P_{sj} \\ s_j(\bar{p}_{sj}) & p \in [\bar{p}_{sj}, \infty) \end{cases}$.

It is easily seen that, for each offer density functions $d_j^i(.)$ and $s_j^i(.)$,

(76) $\partial d_j(p_j)/\partial p_j = \sum_{i \in N_{dj}(p_j)} \{- d_j^i(p_j)\} \leqq 0$ for each $p_j \in P_{dj}^o$ [29]

(77) $\partial s_j(p_j)/\partial p_j = \sum_{i \in N_{sj}(p_j)} \{ s_j^i(p_j)\} \geqq 0$ for each $p_j \in P_{sj}^o$.[29]

Under (71), these two continuous functions intersect each other at the points $(p_j^o, D_j(p_j^o)) = (p_j^o, S_j(p_j^o))$, which is illustrated in Figure 2 below.

3.2 The Complete Information Condition and the Price Strategic Behaviour.

Given now the aggregate (strategic) demand and supply offer functions, where the aggregation is made over all the traders but trader i, that is,

(78) $d_j(i\ ; p_j) = \sum_{k \neq i} D_j^k(p_j)$, $s_j(i\ ; p_j) = \sum_{k \neq i} S_j^k(p_j)$, $j \in M$,

which are continuous in p_j.

Then, the rational choice of the strategic trader i is the choice $\{(p_j^o, \chi_j^i(p_j^o))\ j \in M\}$ over his admissible offer functions, such that

(79) $\chi_j^i(p_j^o) = e_j^i - \sigma_j^i(p_j^o) + \phi_j^i(p_j^o) \qquad e_j^i - \sigma_j^i(p_j^o) \geqq 0$,

$$\chi_{m+1}^i(p^o) = e_{m+1}^i - \sum_{j \in M} p_j^o \{\phi_j^i(p_j^o) - \sigma_j^i(p_j^o)\} \geqq 0 ,$$

$$u^i((\chi_j^i(p_j^o)\ _{j \in M}), \chi_{m+1}^i(p^o)) \geqq u^i(x^i(p^o)) ,$$

and

(80) $0 < \phi_j^i(p_j^o) + d_j(i\ ; p_j^o) = \sigma_j^i(p_j^o) + s_j(i\ ; p_j^o)$.

Recall that these functions are all interval functions though real valued. Also obseve that every other trader behaves as if he were a price taking trader. In fact, there is no need (no way) to change the strategic bidds and offers already made by himself.[30] He is actually taking a set of price and quantity at which he sell and/or buy whenever the trader i, who takes advantages, over all the others, of the complete information condition, changes the trading price p_j^o from $p_j^o(i)$ to whatever he may want the price to be, within a <u>certain range</u>. This range must be within the price interval with two boundary prices ; the price, at which his strategic demand functions $\chi_j^i(.)$ meets with his initial endowment e_j^i (or the price, at which he would not trade any), and, the price, at which any trade would occur among all others, if he does not enter the post (or the price at which no other traders would trade any). Within the range, any magnitude of the price manipulation or control would depend upon how many quantities of trade the price strategic trader, who takes the advantage of the complete infromation about the strategic foursomes due to the others, can handle, relatively to the total quantities of trade. A <u>new concept</u> could be introduecd in order to classify the degrees of price manipulability (controllability).

Assume that at $p_j^o(i)$ for every $j \in M$,

$$(81) \quad 0 < d_j(i\ ;\ p_j^o(i)) = s_j(i\ ;\ p_j^o(i)).$$

Then, since, according as whether $\phi_j^i(p_j^o) \gtreqless \sigma_j^i(p_j^o)$ $(\chi_j^i(p_j^o) \gtreqless e_j^i)$, $d_j(i\ ;\ p_j^o) \lesseqgtr s_j(i\ ;\ p_j^o)$, it follows that $\phi_j^i(p_j^o) \underset{(<)}{>} \sigma_j^i(p_j^o)$ $(\chi_j^i(p_j^o) \underset{(<)}{>} e_j^i)$ implies by the monotonicity property of $d_j(.)$ and $s_j(.)$ with respect to p_j,

$\bar{p}_j^o \underset{(<)}{>} p_j^o(i)$, which is illustrated in Figure 3 below.

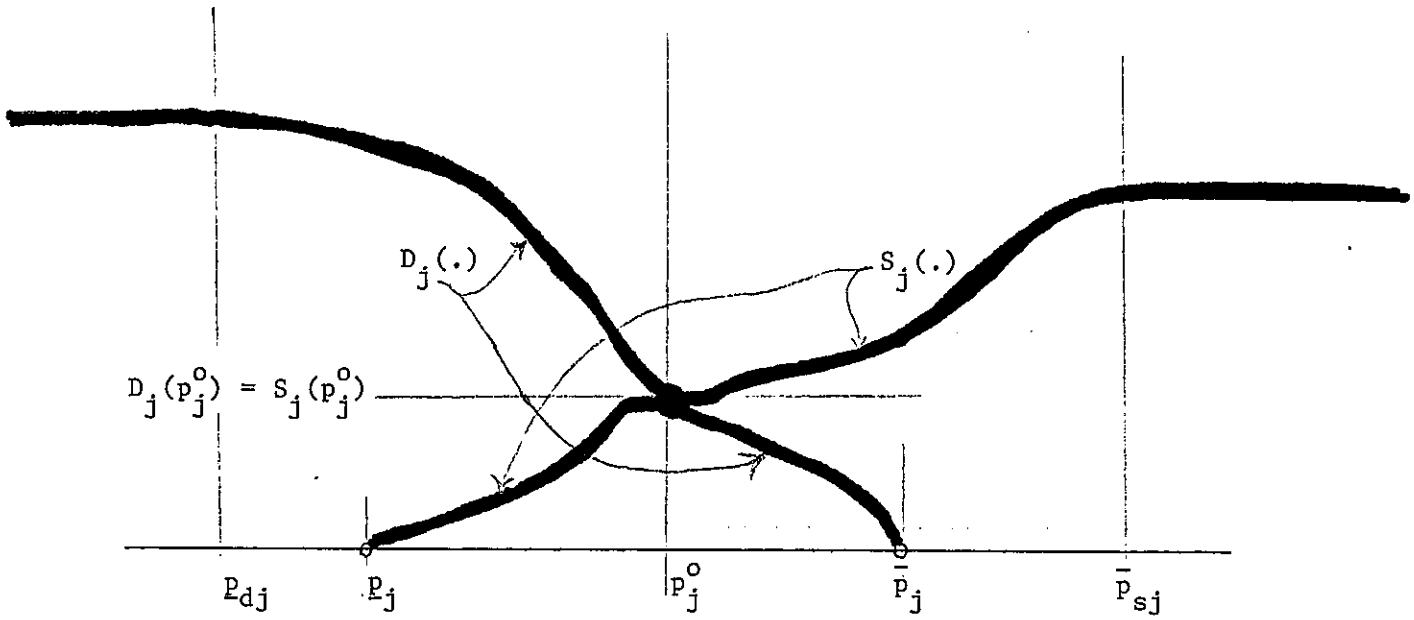


Figure 2 Continuous Case

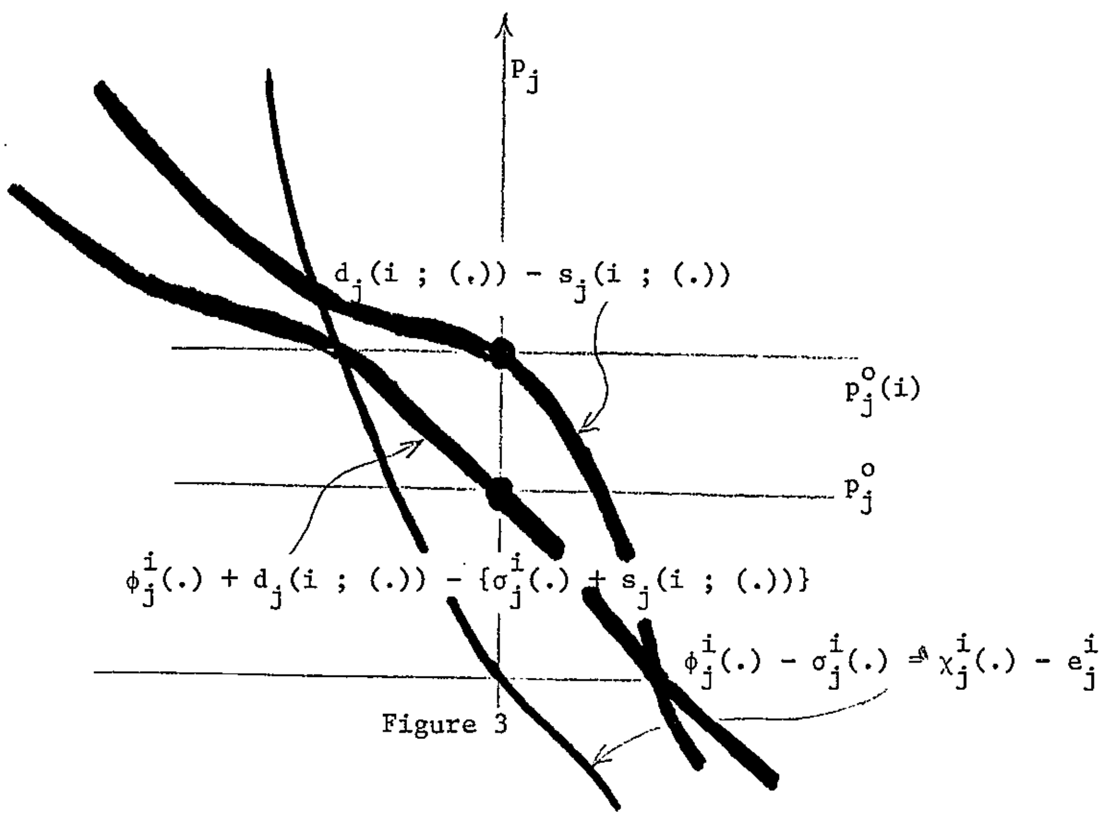


Figure 3

4. Mutual Interdependencies and a Price Strategic Equilibrium

Assume the complete information is given equally to every other trader k, which may be interpreted as a mutually equal information condition, that is, given the whole strategic bidds and offers, which are due to all others, to every trader. This is equivalent to the set of equations obtained by letting for each k, and for each j,

$$(81)\quad D_j^k(.) = \phi_j(.),\quad S^k(.) = \sigma_j^k(.),\quad d_j(i\ ;\ (.)) = \sum_{k\neq i} \phi_j^k(.)$$
$$s_j(i\ ;\ (.)) = \sum_{k\neq i} \sigma_j^k(.),$$

which satisfy, at a trading price p_j^i,

$$(82)\quad 0 < \phi_j(.) = \sigma_j(.) \quad (\text{hence, } \chi_j(.) = \sum_{i\, e\, N} e_j^i)\ .$$

Here, define strategic excess demand (or supply) function, denoted by $E_j^i(.)$, and the aggregate one, denoted by $E_j(.)$, to be, for each j e M,

$$(83)\quad E_j^i(.) = \chi_j^i(.) - e_j^i,\quad E_j(.) = \sum_{i\, e\, N_j(.)} E_j^i(.),\quad N_j(.) = N_{dj}(.) \cup N_{sj}(.),$$

Then, if there exists $p_j^o* > 0$, such that $E_j(p_j^o*) = 0$, $p_j^o* = p_j^i$, for every i e $N_j(p_j^o*)$, we may call such trading price an equilibrium price. In contrast with the Walrasian competitive equilibrium price, consider the trading price p_j^o*, $j = 1, 2, \ldots, m$, satisfying the condition (79) and (82) for every trader i. We shall say the price p^o* is a strategic equilibrium price, and the new allocation $\chi^i(p^o*)$, for trader i, which corresponds to the price p^o*, is a strategic equilibrium allocation for trader i, where

$$(84)\quad \chi^i(p^o*) = (\chi_j^i(p_j^o*)_{j\, e\, M},\ \chi_{m+1}^i(p^o*))$$

satisfying (79) and (82) for each i e N.

We shall say the set of the price and the allocation $(p^{o*}, \chi(p^{o*}))$ where $\chi(p^{o*}) = (\chi^{i}(p^{o*}))_{i \in N}$, is a price strategic equilibrium relative to the initial endowments and a constant profile of preferences, which are given to the strategic economic men in the closed economy.

Remark 8 :

(1) Suppose every trader is so negligible that he behaves as if he were a price taking economic man, that is, formally $p^{o*} = p^{o*}(i)$ for every $i \in N$.[31] Then, the strategic equilibrium price p^{o*} thus defined is the same with the Walrasian competitive equilibrium price.

(2) Suppose trader i, with non-negligible quantity of trade endowed with, can manipulate monopolistically trading price to whatever he may want the price to be, whereas every other trader[32] behaves as if he were the Walrasian competitive price taking individual. Then, the strategic equilibrium price p^{o*} thus defined is the same with the monopolistic equilibrium price.[33]

Both the equilibrium prices can be illustrated in the Edgeworth box diagram below in Figure 4, as what correspond to C.E. and M.E., respectively.

5. The Rudiments of Problems and Questions.

A few but rudimentary problems and questions to be solved and inquired into are, first, to establish an existential theorem of the price strategic equilibrium, second, to make clear any intimate relationships between the strategic equilibrium price and the other concept of equilibrium price, such as the Walrasian competitive equilibrium, or the monopolistic equilibrium, etc., it is third, but may be most interesting, to find the route, normative or descriptive, to the price strategic equilibrium allocation, the true dynamic

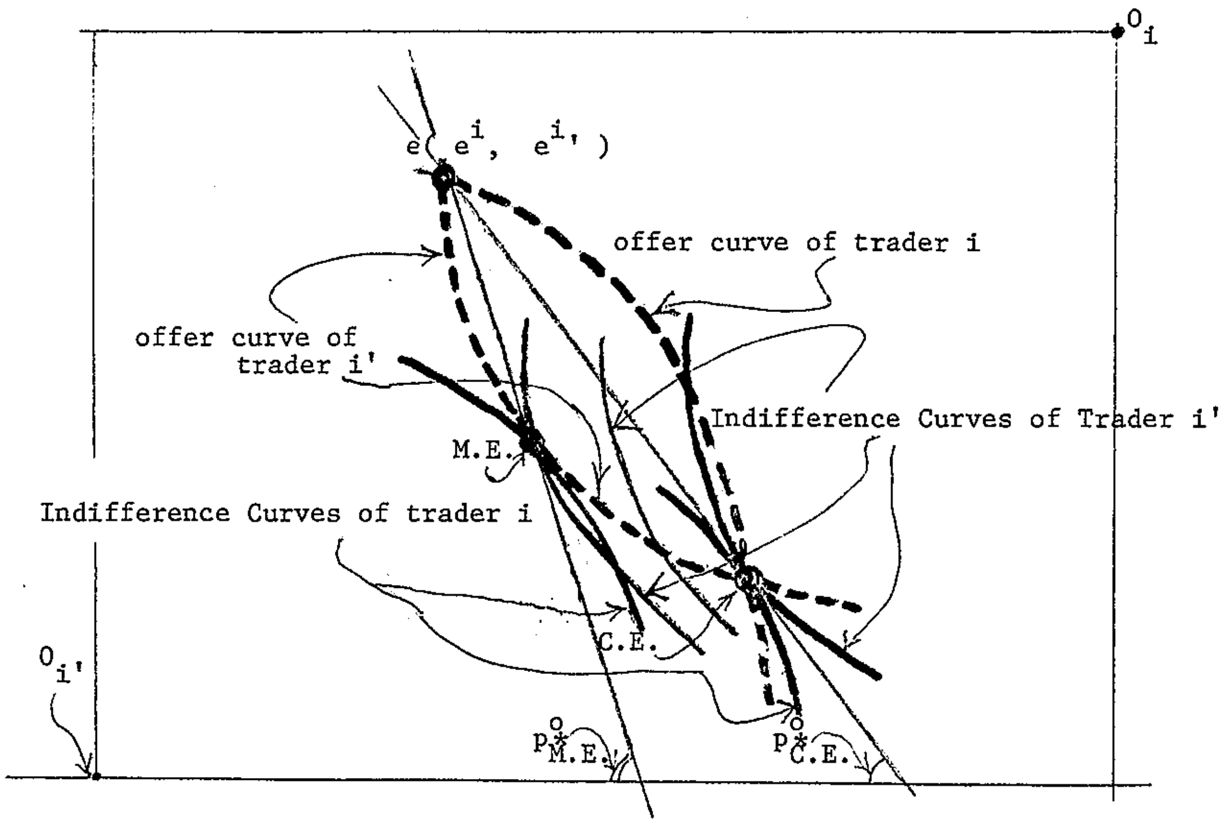


Figure 4

stability problem, etc.. We shall discuss these questions by making use of the continuous formulation of the price formation extended in Section 3 - 4, in Part 2 of the paper. We wish to see there how the sequence of the trading processes, at every process the trading prices being established, may converge to the process, at which the equilibrium prices are established , given, for instance, to each trader, an <u>expectation</u> correspondence (function) from realized prices and decisions due to others.

Footnotes

1. Also compatible with noncooperative or cooperative decision under uncertainty.

2. Interms of strategic demand or supply offer functions, which we shall define later.

3. Including goods and services.

4. The non-free good assumption need not be made in the sequel. But, we shall use the assumption for a convenience's sake in the analysis.

5. The foursome is a message expressed and given by trader i in the j th post for the purpose of trading.

6. We also allow every trader to make zero offer of trade with zero price bidden, that is, $p^i_{sj} = 0$, $s^i_j = 0$, and/or, $p^i_{dj} = 0$, $d^i_j = 0$. But, we shall assume that, whenever no trade is offerred, price is bidden zero ; $s^i_j = 0$ implies $p^i_{sj} = 0$, and $d^i_j = 0$ implies $p^i_{dj} = 0$.

7. Non-free good assumption does not, of course, forbid any trader i to make non-zero offer of trade by bidding zero price. It is possible that $p^i_{sj} = 0$, $s^i_j > 0$, and/or, $p^i_{dj} = 0$, $d^i_j > 0$, for some $i \in N$, but, not for every $i \in N$.

8. The constraints (2) imply the budget constraint of Walrasian type $\Sigma_j\, p^i_{dj}\, d^i_j - \Sigma_j\, p^i_{sj}\, s^i_j \leqq e^i_{m+1}$, but not vice versa. See also footnote 18.

   In case $p^i_{dj} \geqq p^i_{sj}$, the quantity constraints should read as

$$(2)' \quad e^i_j \geqq \max_{p_j} \{ s^i_j(p_j) - d^i_j(p_j) \}, \qquad j \in M_1(i),$$

and correspondingly, the cash constraint may be replaced by

$$(2)'' \max_{p_j} \{ \Sigma_{j \in M_2(i)}\, p_j [d^i_j(p_j) - s^i_j(p_j)] + \Sigma_{j \notin M_2(i)}\, p_j\, d^i_j(p_j) \} \leqq e^i_{m+1},$$

where we define $M_1(i)$ and $M_2(i)$ to be, respectively,

$$M_1(i) = [\, j \in M \,;\ p^i_{dj} \geqq p^i_{sj},\ s^i_j \geqq d^i_j \,],$$

$$M_2(i) = [\, j \in M \,;\ p^i_{dj} \geqq p^i_{sj},\ s^i_j < d^i_j \,].$$

9. Every trader here is not allowed to go bankruptcy, or to sell shorts and make a short contract.

10. Suppose trader i happens to know, in advance, what a trading price, at which actual trade occurs, turns out to be. Then, he needs not take this kind of exclusion into account. We shall see this situation in the subsection 2.2 etc., under the complete information condition.

11. This assumption (3) will be weakened so that a trader could buy at the prices higher than the price he sells at, or, he could sell at the prices lower than the price he buys at.

12. We shall see furthermore the equivalence among the three definitions in our setup of individual offer functions, if we define them by using inf and sup in stead of mim and max, respectively. In fact, either (8) or (11) determines the trading price p_j^o, so the definitional relation (12) always determines the price $p_j^o (= \underline{p}_j^o = \bar{p}_j^o)$.

13. This and the following remark are free from the assumption which has just been made.

14. Observe, however, since it is possible $p_{dj}^i \geqq p_{sj}^i$, a particular case $N_{sj}(p_j^o) \cup N_{dj}(p_j^o) = \{ i \}$ at a price p_j^o such that $p_{dj}^i = p_{sj}^i = p_j^o$, or $p_{dj}^i \geqq p_j^o \geqq p_{sj}^i$, may be possible.

15. In fact, this is a necessary and sufficient condition for the trading price p_j^o to be established here. To see this, note that $\bar{p}_j = \max [p_{dj}^i ; i \in N]$ and $\underline{p}_j = \min [p_{sj}^i ; i \in N]$, then, it follows from Remark 2 that $\underline{p}_j = \bar{p}_j$ determines p_j^o by (12).

16. A major difference, which deserves of an attention, is between the cash constraints, which will carefully be reconsidered later.

17. The coalition may be a syndicate, a cartel, or another decision coordinate such as coordintade decision by collusion.

18. (22)'' $e^{i}_{m+1} \geqq \sum_{j \in M_2(i)} \sum_{\nu \in M_2(i,j)} p^{i\nu}_{dj} (d^{i\nu}_{j} - s^{i\nu}_{j})$

$$+ \sum_{j \notin M_2(i)} \sum_{\nu \notin M_2(i,j)} p^{i\nu}_{dj}\, d^{i\nu}_{j},$$

and

$$e^{i}_{j} \geqq \sum_{\nu \in M_1(i,j)} (s^{i\nu}_{j} - d^{i\nu}_{j}) \qquad j \in M_1(i),$$

$$e^{i}_{j} \geqq \sum_{\nu \notin M_1(i,j)} s^{i\nu}_{j} \qquad j \notin M_1(i),$$

where $\mu_i = \min (\lambda_i, \kappa_i)$, and

let $M_1(i,j)$ and $M_2(i,j)$ be the maximal sets among the sets $M_1'(i,j)$ and $M_2'(i,j)$ defined as follow,

$$M_1'(i,j) = [\ \nu;\ \nu = 1,2,3,\ldots \mu_i,\ p^{i\nu}_{dj} \geqq p^{i\nu}_{sj},\ d^{i\nu}_{j} \leqq s^{i\nu}_{j}]$$

$$M_1(i) = [\ j \in M\ ;\ M_1(i,j) \neq \phi\ ],$$

$$M_2'(i,j) = [\nu\ ;\ \nu = 1,2,3,\ldots \mu_i,\ p^{i\nu}_{dj} \geqq p^{i\nu}_{sj},\ d^{i\nu}_{j} > s^{i\nu}_{j}]$$

and $M_2(i) = [\ j \in M\ ;\ M_2(i,j) \neq \phi\ ]$, respectively.

(27)' $\sum_{\nu \in M_2(i,j)} p^{i\nu}_{dj} (d^{i\nu}_{j} - s^{i\nu}_{j}) + \sum_{\nu \notin M_2(i,j)} p^{i\nu}_{dj}\, d^{i\nu}_{j}$

$$= p_j \left[\sum_{\nu \in M_2(i,j)} (d^{i\nu}_{j} - s^{i\nu}_{j}) + \sum_{\nu \notin M_2(i,j)} d^{i\nu}_{j} \right]$$

$$= p_j \left\{ \sum_{\nu \in M_2(i,\, p_j)} [\, d^{i\nu}_{j}(p_j) - s^{i\nu}_{j}(p_j) \,] \right.$$

$$\left. + \sum_{\nu \notin M_2(i,\, p_j)} d^{i\nu}_{j}(p_j) \right\}, \text{ for each } p_j > 0,$$

where $M_2(i,\, p_j) = [\ \nu \in M_2(i,j)\ ;\ \nu = 1,2,3,\ldots,\mu_i(p_j)\ ]$

and $\mu_i(p_j) = \min (\lambda_i(p_j),\ \kappa_i(p_j)),$

hence, (22)' follows.

In fact, $\max_{p_j} p_j [\, d^{i}_{j}(p_j) - s^{i}_{j}(p_j) \,] \geqq p_j\, d^{i}_{j}(p_j) \quad \forall\, p_j > 0.$

hence,

$$\sum_{j \in M} \max_{p_j} p_j [d_j^i(p_j) - s^i(p_j)] \geqq \sum_{j \in M} p_j d_j^i(p_j)$$

and, $j \in M$,

$$e_j^i \geqq \max_{p_j} [s_j^i(p_j) - d_j^i(p_j)] \geqq s_j^i(p_j) \geqq 0, \quad p_j \geqq 0.$$

19. There are many eligible offer functions, each of which satisfies the law of demand. We choose one and make it fixed. Each such offer function may be said to be an admissible function whenever a utility functional $u^i(.)$ of augment functions is introduced, in terms of Hadamard's terminology.

20. See footnote 19.

21. In Hicks's sense.

22. Of course, the case in which ($\underline{p}_s^i$, $\bar{p}_s^{-i}$, $\underline{p}_d^i$, $\bar{p}_d^{-i}$) = (0, ∞, 0, ∞), for all $i \in N$ may be possible, where 0 and ∞ are an m dimensional vector whose elements are all zero, and an m dimensional vector whose elements are all infinity.

23. Over the admissible offer functions $d_j^i(.)$ and $s_j^i(.)$ for each $j \in M$.

24. In the sense that is described in Remark 4.

25. An example of this is shown in a footnote given later ; see footnote 27.

26. In the sense of Remark 4.

27. With quantities of trade (very) large to the total endowment, suppose trader i can manipulte monopolistically trading price to whatever he may want the price to be, whereas all the other traders, with quantities of trade so negligible, behave as if they were competitive price takers. Such a case may also be included, and here two types of traders are classified as one being monopolistic and the other being price taking. See also footnote 33.

28. Even if $s_j^i(.)$ is not bounded on $[\underline{p}_{sj}^i, \ \bar{p}_{sj}^i)$, but, if bounded and integrable on any interval $[\underline{p}_{sj}^i, \ \bar{p}_{sj}^i - \varepsilon]$. Suppose

$$\lim_{\varepsilon \to 0+} \int_{\underline{p}_{sj}^i}^{p_{sj}^i - \varepsilon} \{ s_j^i(p_j^i)\} \partial \ p_j^i$$ exists as a finite limit. Then, the limit

denoted by $\int_{\underline{p}_{sj}^i}^{\bar{p}_{sj}^i} \{s_j^i(p_j^i)\} \partial \ \ p_j^i$ is called the improper (Riemann)

integral of $s_j^i(.)$ on $[\underline{p}_{sj}^i, \ \bar{p}_{sj}^i)$. The improper integral of $d_j^i(.)$ on $(\underline{p}_{dj}^i, \ \bar{p}_{dj}^i]$ may be defined in the same way, that is,

$$\lim_{\varepsilon \to 0+} \int_{\underline{p}_{dj}^i - \varepsilon}^{\bar{p}_{dj}^i} \{d_j^i(p_j^i)\} \ \partial \ p_j^i \quad = \int_{\underline{p}_{dj}^i}^{\bar{p}_{dj}^i} \{ d_j^i(p_j^i)\} \ \partial \ p_j^i \ .$$

Extensively, suppose $s_j^i(.)$ id defined on an interval $[\underline{p}_{sj}^i, \ \infty)$ and integrable on any interval $[\underline{p}_{sj}^i, \ p_j]$ for each $p_j > 0$. If

$$\lim_{p_j \to \infty} \int_{\underline{p}_{sj}^i}^{p_j} \{s^i(p_j^i)\} \partial p_j^i$$ exists and is denoted by

$$\int_{\underline{p}_{sj}^i}^{\infty} \{s_j^i(p_j^i)\} \partial p_j^i,$$ then, the limit is called the improper integral of $s_j^i(.)$ on $[\underline{p}_{sj}^i, \ \infty)$. The same is for the improper integral of $d_j^i(.)$ on $(0, \ \bar{p}_{dj}^i]$.

29. X^o means the interior of the set X.

30. Each other trader is so doing, as the format of his strategic message, described in (54)-(64), tells.

31. N may be an infinite set here.

32. See footnote 31.

33. An equilibrium with monopolistic behaviour in some trading posts. This monopolistic case is fully compatible with the case in which one group of traders can coordinate their actions, while the rest or other groups remain as individual price taking traders, so that the other groups are not able to establish <u>countervailing power</u>. See footnote 27.