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A Note on No Arbitrage Condition for International Financial Market

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Abstract: We consider an international financial market model consists of N currencies. The purpose is to derive the no arbitrage condition which is not affected by the choice of numeraire between the N currencies. As a result, we show that a finiteness condition for an arbitrary chosen currency and the no arbitrage condition for the basket currency are necessary and sufficient for the no arbitrage propetry of all the N currencies.
Keywords: Multi-currency, Basket currency, No arbitrage, Numeraire.

1 Introduction

After the pioneer work by Harrison-Kreps [9], many researchers have studied on the relationship between the existence of equivalent martingale measure and the no arbitrage property. However in most studies, the existence of riskless asset with finite variation is assumed and used as numeraire. But in the case of international economy model, a propri choice of numeraire has some problem to characterize the no arbitrage property.

Suppose that an international financial market consists of N currencies. Then from the viewpoint of economic efficiency, we should claim the no arbitrage property for all the portfolio value processes defined for each currency. This means that we have to satisfy the no arbitrage property regardless of the choice of numeraire between the N currencies. However, Delbaen-Schachermayer [5] have shown a example of two currencies trading model which satisfies no arbitrage procerty for one currency but not for another. The purpose of this note is to derive a compact equivalent condition which guaranties the no

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arbitrage property of all the currencies. As a result, we show that a finiteness condition for an arbitrary chosen currency and the no arbitrage condition with respect to the basket currency are necessary and sufficient for the no arbitrage propetry of all the N currencies.

This note is organized as follows. In section 2, we summarize the fundamental results in Delbaen-Schachermayer [4]–[8] used in this note. In section 3, we explain the multi-currency international financial market model and see the restriction of trading policy caused by the choice of numeraire. In section 4, we define the finiteness condition which is invariant for the choice of numeraire. Finally in section 5, we derive the compact no arbitrage condition of all the currencies.

2 Summary of Results by Delbaen-Schachermayer

Without loss of generality, we consider the finite time horizon model on [0,1]. Let $(\Omega, \mathcal{F}, \boldsymbol{P})$ be a probability space and $(\mathcal{F}_t)_{0\le t\le 1}$ be a filteration on $[0, 1]$. This filteration is supposed to satisfy the usual conditions. That is, the filteration is right continuous and contains all negligible sets : if $B \subset A \in \mathcal{F}$ and $\boldsymbol{P}\{A\} = 0$ then $B \in \mathcal{F}_0$. We also suppose that the σ-algebra $\mathcal{F}$ is generated by $\cup_{0\le t\le 1}\mathcal{F}_t$.

We consider a financial market consist of N assets numbered from 1 to N. The price of asset k, $1 \le k \le N$, at time t is denoted by S_t^k. S=$(S^1, \cdots, S^N)$ is supposed to be continuous, strictly positive vector semi-martingale. For convenience, asset 1 is supposed to be constant price process with $S_t^1 = 1$. That is, we use asset 1 as numeraire. Let us consider a class of $\boldsymbol{S}$ integrable, predictable process $\boldsymbol{H}$ as trading policy. We abbrevbiate $\boldsymbol{H}.\boldsymbol{S}$ as vector stochastic integral in the sense of Chatelain and Stricker [3]. Under the choice of asset 1 as numeraire, we claim the following condition as the economic efficieny criterion.

Definition 2.1 *Let a be a positive real number. A $\boldsymbol{S}$-integrable predictable vector process $\boldsymbol{H}$=$(H^1, \cdots, H^N)$ is called a-admissible if $\boldsymbol{H}_0 = \mathbf{o}$ and $(\boldsymbol{H}.\boldsymbol{S})_t \ge -a$, P-a.s. for all $0 \le t \le 1$. $\boldsymbol{H}$ is called admissible if it is a-admissible for some $a \in \boldsymbol{R}$.*

Definition 2.2 *We say that the vector semi-martingale $\boldsymbol{S}$ satisfies the no arbitrage (NA) condition for general admissible integrand $\boldsymbol{H}$ if*

$$(\boldsymbol{H}.\boldsymbol{S})_1 \ge 0 \Rightarrow (\boldsymbol{H}.\boldsymbol{S})_1 = 0, \ \boldsymbol{P}\text{-}a.s.. \tag{2.1}$$

Theorem 2.3 *If $\boldsymbol{S}$ is a continuous vector semi-martingale decomposed as $d\boldsymbol{S}_t = d\boldsymbol{M}_t + d\boldsymbol{A}_t$*

where M is a continuous local vector martingale, A is a continuous of bounded variation (the Doob Meyer decomposition of S), then

(a) *if S satisfies (NA) for general admissible integrands, there is a predictable vector process h such that*

$$dA_t = d < M, M >_t h_t, \tag{2.2}$$

where $d < M, M >$ is matrix measure and h is a vector process.

(b) *Under the same hypothesis,*

$$\tau = \inf\left\{t; \int_0^t h'_u d < M, M >_u h_u = \infty\right\} > 0, P\text{-a.s.} \tag{2.3}$$

(c) *A necessary and sufficient condition for the existence of an equivalent local martingale measure for S is*

(i) *S has (NA) property for general admissible integrands.*

(ii)

$$\int_0^1 h'_u d < M, M >_u h_u < \infty, \ P\text{-a.s.}, \tag{2.4}$$

where h is given by (2.2)

Remark 2.4 *The local martingale process*

$$L_t = \exp\left(-\int_0^t h'_u dM_u - \frac{1}{2}\int_0^t h'_u d < M, M > h_u\right) \tag{2.5}$$

is not necessary a martingale and hence the obvious Girsanov-Maruyama transformation does not give an equivalent local martingale measure for S (see Delbaen-Schachermayer [6] and Schachermayer [11]).

Let $\mathcal{M}_e(P)$ be the set of equivalent measures to P under which S becomes vector *local* martingale and

$$\mathcal{K}_1 = \{(H.S)_1; H \text{ is 1-admissible}\}, \tag{2.6}$$
$$\mathcal{K}' = \{(H.S)_1; H \text{ is admissible}\}. \tag{2.7}$$

We can easily see that $\mathcal{K}_1 \subset \mathcal{K}$ and $\mathcal{K} = \cup_{a>0}\mathcal{K}_a = \cup_{\lambda>0}\lambda\mathcal{K}_1$. The following theorem is proved in Delbaen-Schachermayer [7] (see also Jacka [10], Ancel-Stricker [1]).

Theorem 2.5 *If $f \in \mathcal{K}_1$, then the followings are equivalent.*

(a) f is a maximal element in $\mathcal{K}_1$.

(b) f is a maximal element in $\mathcal{K}$.

(c) $\exists Q \in \mathcal{M}_e(\boldsymbol{P})$ such that $\boldsymbol{E}_Q\{f\} = 0$.

(d) $\exists$ 1-admissible integrand $\boldsymbol{H}$ and $\exists Q \in \mathcal{M}_e(\boldsymbol{P})$ such that $f = (\boldsymbol{H}.\boldsymbol{S})_1$ and $\boldsymbol{H}.\boldsymbol{S}$ is a $\boldsymbol{Q}$-uniformly integrable martingale.

If $V_t = c + (\boldsymbol{H}.\boldsymbol{S})_t > 0$, then we also have the equivalence of

(a') $f = (\boldsymbol{H}.\boldsymbol{S})_1$ is a maximal element in $\mathcal{K}_1$.

(b') $f = (\boldsymbol{H}.\boldsymbol{S})_1$ is a maximal element in $\mathcal{K}$.

(c') $\exists Q \in \mathcal{M}_e(\boldsymbol{P})$ such that $\sup\{\boldsymbol{E}_R\{V_1\}; \boldsymbol{R} \in \mathcal{M}_e(\boldsymbol{P})\} = \boldsymbol{E}_Q\{V_1\} < \infty$.

(d') $\exists Q \in \mathcal{M}_e(\boldsymbol{P})$ such that V is a $\boldsymbol{Q}$-uniformly integrable martingale.

(e') $\frac{\boldsymbol{S}}{V}$ has an equivalent local martingale measure.

The results in Delbaen-Schachermayer [7] also show that if $f_1, \cdots, f_n \in \mathcal{K}$ are maximal then $f_1 + \cdots + f_n$ also maximal. From this, it follows that

Theorem 2.6 *If $V^j = c^j + (\boldsymbol{H}^j\boldsymbol{S}) > 0$, $j = 1, \cdots, J < \infty$ are stochastic integral such that for $\forall$ j, $\exists \boldsymbol{Q}^j \in \mathcal{M}_e(\boldsymbol{P})$ with V^j is a $\boldsymbol{Q}^j$-uniformly integrable martingale, then $\exists \boldsymbol{Q} \in \mathcal{M}_e(\boldsymbol{P})$ such that for all $j \leq J$, V^j is a $\boldsymbol{Q}$-uniformly integrable martingale.*

3 International Financial Market Model

We consider an international financial market model consists of N currencies numbered from 1 to N. For each currency k there is a positive interest rate r^k such that

$$\int_0^1 r_u^k du < \infty, \boldsymbol{P}\text{-}a.s.,\ 1 \leq k \leq N. \tag{3.1}$$

Without loss of generality, we assume the currency 1 is domestic currency which is used as numeraire to express another value. The exchange rate of currency k for the *domestic* currency 1 is descrived by E_t^k. From the definition, we have $E_t^1 = 1$. E^k and r^k are supposed to be adapted processes and E^k is continuous, strictly positive semi-martingale.

Following Harrison-Kreps [9] and Artzner-Delbaen [2], we define the following discounted exchange rates

$$S_t^1 = 1, \tag{3.2}$$
$$S_t^k = \exp\left(-\int_0^t r_u^1 du\right) \exp\left(\int_0^t r_u^k du\right) E_t^k,\ 2 \le k \le N. \tag{3.3}$$

S^k implies a relative value of unit deposit in currency k under continuous accumulation with interest rate r_k compared to currency 1. From the definition, $S^k > 0$, $\boldsymbol{P}$-a.s., $1 \le k \le N$. If we choose currency k as the numeraire, the discounted vector value process becomes $(\frac{S^1}{S^k}, \cdots, \frac{S^N}{S^k})$. More generally, for a positive constant weight vector $\alpha = (\alpha^1, \cdots, \alpha^N)$, $\alpha^j > 0$, define the basket currency B by

$$B_t = \sum_{k=1}^{N} \alpha^k S_t^k. \tag{3.4}$$

When the basket currency is used as numeraire, the discounted vector value process is expressed by $(\frac{S^1}{B}, \cdots, \frac{S^N}{B})$. Here notice that an admissible strategy $\boldsymbol{H}$ for $\boldsymbol{S}$, i.e. for the domestic currency, is not necessary admissible for currency k. That is, $\boldsymbol{H}$ is not admissible for the vector process $\frac{1}{S^k}\boldsymbol{S}$. It follows that (NA) depends on the currency used as numeraire (see also Artzner-Delbaen [8] and Delbaen-Schachermayer [5]).

4 Finiteness Property

We shall define the following condition for $\boldsymbol{S}$ which claims the existence of integrable risk premium process $\boldsymbol{h}$.

Definition 4.1 *Let us say that $\boldsymbol{S}$ with Doob Meyer decomposition $\boldsymbol{M} + \boldsymbol{A}$ under $\boldsymbol{P}$ satisfies the finiteness condition if there exists a predictable vector process $\boldsymbol{h}$ such that*

$$d\boldsymbol{A}_t = d < \boldsymbol{M}, \boldsymbol{M} >_t \boldsymbol{h}_t,\ \boldsymbol{P}\text{-a.s.}, \tag{2.2}$$

and

$$\int_0^1 \boldsymbol{h}_u' d < \boldsymbol{M}, \boldsymbol{M} >_u \boldsymbol{h}_u < \infty,\ \boldsymbol{P}\text{-a.s.}. \tag{4.2}$$

Lemma 4.2 *The finiteness condition does not depend on the choice of numeraire, i.e. if $\boldsymbol{S}$ satisfies the finiteness condition and if $\rho = c + (\boldsymbol{H}.\boldsymbol{S}) > 0$ is a stochastic integral, then $\frac{\boldsymbol{S}}{\rho}$ also satisfies the finiteness condition.*

Proof. From the definition, $d\rho = \boldsymbol{H}'d\boldsymbol{S}$. Then from the genelarized Ito's lemma and $d\boldsymbol{S} = d\boldsymbol{M} + d\boldsymbol{A}$,

$$
\begin{aligned}
d\left(\frac{1}{\rho}\right) &= -\frac{1}{\rho^2}d\rho + \frac{1}{\rho^3}(d\rho)^2 \\
&= -\frac{1}{\rho^2}\boldsymbol{H}'d\boldsymbol{S} + \frac{1}{\rho^3}\boldsymbol{H}'d < \boldsymbol{M}, \boldsymbol{M} > \boldsymbol{H}. \qquad (4.3)
\end{aligned}
$$

Hence

$$
\begin{aligned}
& d\left(\frac{\boldsymbol{S}}{\rho}\right) \\
= \;& \frac{1}{\rho}d\boldsymbol{S} + \boldsymbol{S}d\left(\frac{1}{\rho}\right) + d < \boldsymbol{S}, \frac{1}{\rho} > \\
= \;& \frac{1}{\rho}d\boldsymbol{M} + \frac{1}{\rho}d < \boldsymbol{M}, \boldsymbol{M} > \boldsymbol{h} - \frac{1}{\rho^2}\boldsymbol{S}\boldsymbol{H}'d\boldsymbol{S} \\
& + \frac{1}{\rho^3}\boldsymbol{S}\boldsymbol{H}'d < \boldsymbol{M}, \boldsymbol{M} > \boldsymbol{H} - \frac{1}{\rho^2}d < \boldsymbol{M}, \boldsymbol{M} > \boldsymbol{H} \\
= \;& \frac{1}{\rho}d\boldsymbol{M} - \frac{1}{\rho^2}\boldsymbol{S}\boldsymbol{H}'d\boldsymbol{M} - \frac{1}{\rho^2}\boldsymbol{S}\boldsymbol{H}'d < \boldsymbol{M}, \boldsymbol{M} > \boldsymbol{h} + \frac{1}{\rho}d < \boldsymbol{M}, \boldsymbol{M} > \boldsymbol{h} \\
& + \frac{1}{\rho^3}\boldsymbol{S}\boldsymbol{H}'d < \boldsymbol{M}, \boldsymbol{M} > \boldsymbol{H} - \frac{1}{\rho^2}d < \boldsymbol{M}, \boldsymbol{M} > \boldsymbol{H}. \qquad (4.4)
\end{aligned}
$$

The martingale part $\{\boldsymbol{N}_t\}$ of (4.4) is given by

$$
d\boldsymbol{N} = \frac{1}{\rho}d\boldsymbol{M} - \frac{1}{\rho^2}\boldsymbol{S}\boldsymbol{H}'d\boldsymbol{M}. \qquad (4.5)
$$

Then we have

$$
\begin{aligned}
& d\boldsymbol{N}d\boldsymbol{N}' \\
= \;& \left(\frac{1}{\rho}d\boldsymbol{M} - \frac{1}{\rho^2}\boldsymbol{S}\boldsymbol{H}'d\boldsymbol{M}\right)\left(\frac{1}{\rho}d\boldsymbol{M}' - \frac{1}{\rho^2}d\boldsymbol{M}'\boldsymbol{H}\boldsymbol{S}'\right) \\
= \;& \frac{1}{\rho^2}\left(d\boldsymbol{M}d\boldsymbol{M}' - \frac{1}{\rho}d\boldsymbol{M}d\boldsymbol{M}'\boldsymbol{H}\boldsymbol{S}' - \frac{1}{\rho}\boldsymbol{S}\boldsymbol{H}'d\boldsymbol{M}d\boldsymbol{M}' + \frac{1}{\rho^2}\boldsymbol{S}\boldsymbol{H}'d\boldsymbol{M}d\boldsymbol{M}'\boldsymbol{H}\boldsymbol{S}'\right) \\
= \;& \frac{1}{\rho^2}\begin{pmatrix} d < \boldsymbol{M}, \boldsymbol{M} > - \frac{1}{\rho}d < \boldsymbol{M}, \boldsymbol{M} > \boldsymbol{H}\boldsymbol{S}' \\ \frac{1}{\rho}\boldsymbol{S}\boldsymbol{H}'d < \boldsymbol{M}, \boldsymbol{M} > + \frac{1}{\rho^2}\boldsymbol{S}\boldsymbol{H}'d < \boldsymbol{M}, \boldsymbol{M} > \boldsymbol{H}\boldsymbol{S}' \end{pmatrix} \\
= \;& \frac{1}{\rho^2}\left(\boldsymbol{I} - \frac{1}{\rho}\boldsymbol{S}\boldsymbol{H}'\right)d < \boldsymbol{M}, \boldsymbol{M} > \left(\boldsymbol{I} - \frac{1}{\rho}\boldsymbol{H}\boldsymbol{S}'\right). \qquad (4.6)
\end{aligned}
$$

The bounded variation part $\{\boldsymbol{B}_t\}$ of (4.4) is given by

$$
\begin{aligned}
d\boldsymbol{B} &= -\frac{1}{\rho^2}\boldsymbol{S}\boldsymbol{H}'d < \boldsymbol{M}, \boldsymbol{M} > \boldsymbol{h} + \frac{1}{\rho}d < \boldsymbol{M}, \boldsymbol{M} > \boldsymbol{h} \\
& + \frac{1}{\rho^3}\boldsymbol{S}\boldsymbol{H}'d < \boldsymbol{M}, \boldsymbol{M} > \boldsymbol{H} - \frac{1}{\rho^2}d < \boldsymbol{M}, \boldsymbol{M} > \boldsymbol{H}. \qquad (4.7)
\end{aligned}
$$

Since $\rho > 0$, $U = \frac{1}{\rho}H$ is well defined. From the exponential formula, ρ_t is given by

$$\rho_t = \exp\left(\int_0^t U' dS - \int_0^t U' d < M, M > U\right). \tag{4.8}$$

Then (4.6) and (4.7) are rewritted as

$$d < N, N > = \frac{1}{\rho^2}(I - SU')\, d < M, M > (I - US'), \tag{4.9}$$

and

$$\begin{aligned} dB &= -\frac{1}{\rho}SU'd < M, M > h + \frac{1}{\rho}d < M, M > h \\ &\quad + \frac{1}{\rho}SU'd < M, M > U - \frac{1}{\rho}d < M, M > U \\ &= (I - SU')\, d < M, M > \frac{h}{\rho} - (I - SU')\, d < M, M > \frac{U}{\rho} \\ &= (I - SU')\, d < M, M > \left(\frac{h - U}{\rho}\right). \end{aligned} \tag{4.10}$$

Next we shall show that dB is given by the form $(I - SU')\, d < M, M > (I - SU')\, g$ for some g. Since $d < M, M >$ is a nonnegative definite symmetric matrix, we have a Choleski decomposition $d < M, M > = LL'$. Let $C = I - SU'$. From the definition, $\mathcal{R}(CLL'C') \subset \mathcal{R}(CLL') \subset \mathcal{R}(CL)$. However $\mathcal{R}(DD') \supset \mathcal{R}(D)$ for all matrices D. Indeed if $y \perp \mathcal{R}(DD')$, we have $y'DD' = o'$. Then $y'DD'y = 0$ and $y'D = o'$. This means that $\mathcal{R}(DD')^{\perp} \subset \mathcal{R}(D)^{\perp}$ which is equivalent to $\mathcal{R}(DD') \supset \mathcal{R}(D)$. Therefore $\mathcal{R}(CLL'C') = \mathcal{R}(CLL')$. This together with (4.9) and (4.10) implies that there exists a g such that $dB = d < N, N > g$. We should check the finiteness property for g.

$$\begin{aligned} g'd < N, N > g &= g'CLL'C'g = g'CLL'\left(\tfrac{h-U}{\rho}\right) \\ &\le (g'CLL'C'g)^{\frac{1}{2}}\left[\left(\tfrac{h-U}{\rho}\right)' LL' \left(\tfrac{h-U}{\rho}\right)\right]^{\frac{1}{2}}. \end{aligned}$$

The inequality follows from Chaucy-Schwartz's inequality. Hence we have

$$\begin{aligned} &\int_0^1 g_s' d < N, N >_s g_s \\ \le\; &\int_0^1 \left(\tfrac{h_s - U_s}{\rho_s}\right)' L_s L_s' \left(\tfrac{h_s - U_s}{\rho_s}\right) \\ =\; &\int_0^1 \left(\tfrac{h_s - U_s}{\rho_s}\right)' d < M, M >_s \left(\tfrac{h_s - U_s}{\rho_s}\right) \\ \le\; &\int_0^1 \frac{1}{\rho_s^2}(h_s' d < M, M >_s h_s + U_s' d < M, M >_s U_s) \end{aligned} \tag{4.11}$$

Since $\inf_{1 \le t \le 1} \rho_t < \infty$, we have $\int_0^1 \frac{1}{\rho_s}^2 U_s' d < M, M >_s U_s < \infty$ from (4.8). This together with the finiteness property (4.2) and $\inf_{0 \le t \le 1} \rho_t > 0$ yields the desired result. □

Remark 4.3 *From Lemma 4.2, the finiteness condition is invariant for the choice of numeraire. Hence from Theorem 2.3 (c), if the finiteness condition is satisfied under $\boldsymbol{P}$, (NA) condition is equivalent to the existence of equivalent local martingale measure.*

5 Theorem

We show that under the finiteness condition for an arbitrary chosen currency, no arbitrage condition for the basket currency is necessary and sufficient for the no arbitrage propetry of all the N currencies.

Theorem 5.1 *The followings are equivalent for $\boldsymbol{S}$ satisfying the finiteness condition for an arbitrary chosen currency under $\boldsymbol{P}$.*

(a) $\exists Q \in \mathcal{M}_e(\boldsymbol{P})$ *such that* $V = c + (\boldsymbol{H}.\boldsymbol{S}) > 0$ *is a* $\boldsymbol{Q}$*-uniformly integrable martingale.*

(b) $\forall j,\ 1 \leq j \leq N,\ \exists Q^j \in \mathcal{M}_e(\boldsymbol{P})$ *such that* S^j *is a* Q^j*-uniformly integrable martingale.*

(c) $\exists Q \in \mathcal{M}_e(\boldsymbol{P})$ *such that* $\boldsymbol{S}$ *is a* $\boldsymbol{Q}$*-uniformly integrable vector martingale.*

(d) $\forall j,\ 1 \leq j \leq N,\ \frac{\boldsymbol{S}}{S^j}$ *satisfies (NA).*

(e) $\frac{\boldsymbol{S}}{B}$ *satisfies (NA).*

Proof. $(a) \Rightarrow (b)$: obvious. $(b) \Rightarrow (c)$: Follows from Theorem 2.6. $(b) \Leftrightarrow (d)$: Follows from $(d') \Leftrightarrow (e')$ in Theorem 2.5 and Remark 4.3. $(c) \Rightarrow (e)$: From assumption, $\exists Q \in \mathcal{M}_e(\boldsymbol{P})$ such that B is a $\boldsymbol{Q}$-uniformly integrable martingale. Hence from Theorem 2.3 (c) and $(d') \Leftrightarrow (e')$ in Theorem 2.5, $\frac{\boldsymbol{S}}{B}$ satisfies (NA). $(e) \Rightarrow (a)$: If $\frac{\boldsymbol{S}}{B}$ satisfies (NA), from $(d') \Leftrightarrow (e')$ in Theorem 2.5 and Remark 4.3, there $\exists Q \in \mathcal{M}_e(\boldsymbol{P})$ such that B is a $\boldsymbol{Q}$-uniformly integrable martingale. Since each S^k is a $\boldsymbol{Q}$-local martingale and $S^k \leq \frac{B}{\min_{1 \leq j \leq N} \alpha^j}$, S^k is a $\boldsymbol{Q}$-uniformly integrable martingale. □

Remark 5.2 *Theorem 5.1 shows how important the exsistence of a martingale measure (instead of a local martingale measure) is when dealing with different currencies and numeraires. Another application of the theorem is that of different stocks and the use of an index as numeraire or hedging posibility.*

The (NA) properties for general admissible integrands follow, in the continuous case, from the no free lunch with vanishing risk (NFLVR) property for simple admissible integrands (see Delbaen-Schachermayer [6]). So the theorem can also be stated using

(NFLVR) for simple admissible integrands. In this case, the finiteness property follows from (NFLVR). We think this is not beautiful. So we prefer the other approach and should leave this a note.

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