No.560

On the Definition of Favorableness*

by

Masashi Une** and Tatsuyoshi Saijo***

November, 1993

* Research was partially supported by the Inamori Foundation, the Nomura Foundation for the Social Sciences, and Grant in Aid for General Research 04451111 of the Ministry of Education in Japan.
** College of Socio-Economic Planning, University of Tsukuba, Tsukuba, Ibaraki 305, Japan
*** Institute of Socio-Economic Planning, University of Tsukuba, Tsukuba, Ibaraki 305, Japan

This is a technical note on the definition of "favorableness" introduced by Laffont-Tirole (page 77, 1993) in the principal-agent analysis with both moral hazard and adverse selection. Consider a firm whose technology is expressed as a number between an interval $[\underline{\beta},\overline{\beta}]$, where the lower number is the better technology. The technology cannot be observed by the regulator, and hence the regulator has some distribution on the interval. Laffont-Tirole introduced the following concept of the comparison between two distributions G and F defined on $[\underline{\beta},\overline{\beta}]$ such that $G'(\beta)$, $F'(\beta) > 0$ for all $\beta \in [\underline{\beta},\overline{\beta}]$.

Definition. The distribution G on $[\underline{\beta},\overline{\beta}]$ is more *favorable* than the distribution F on the same interval if

(i) $G(\beta) \geq F(\beta)$ for all $\beta \in [\underline{\beta},\overline{\beta}]$ (first order stochastic dominance); and

(ii) $G'(\beta)\, F(\beta) \leq F'(\beta)\, G(\beta)$ for all $\beta \in [\underline{\beta},\overline{\beta}]$ (hazard rate dominance).

We will show that condition (i) is unnecessary since condition (ii) implies condition (i). Furthermore, we will show that condition (i) is clearly much weaker than condition (ii) by finding a condition that implies condition (ii) assuming condition (i).

Proposition. (a) $G'(\beta)\, F(\beta) \leq F'(\beta)\, G(\beta)$ for all $\beta \in [\underline{\beta},\overline{\beta}]$ implies $G(\beta) \geq F(\beta)$ for all $\beta \in [\underline{\beta},\overline{\beta}]$.

(b) If $G(\beta) \geq F(\beta)$ and $[F'(\beta) \leq G'(\beta)$ implies $F'(\beta)\,(G(\beta) - F(\beta)) \geq F(\beta)\,(G'(\beta) - F'(\beta))]$ for all $\beta \in$ $[\underline{\beta},\overline{\beta}]$, then $G'(\beta)\, F(\beta) \leq F'(\beta)\, G(\beta)$ for all $\beta \in [\underline{\beta},\overline{\beta}]$.

Proof. (a) If $\beta = \underline{\beta}$ ($\beta = \overline{\beta}$), then $G(\underline{\beta}) = F(\underline{\beta}) = 0$ ($G(\overline{\beta}) = F(\overline{\beta}) = 1$) respectively, which shows the result. Consider the case with $\beta \in (\underline{\beta},\overline{\beta})$. Since $G(\beta) \neq 0$ and $F(\beta) \neq 0$,

$$G'(\beta)\, F(\beta) \leq F'(\beta)\, G(\beta) \Leftrightarrow \frac{G'(\beta)}{G(\beta)} \leq \frac{F'(\beta)}{F(\beta)} \Leftrightarrow (\ln G(\beta))' \leq (\ln F(\beta))'.$$

Since $0 \leq \frac{G'(\beta)}{G(\beta)} \leq \frac{F'(\beta)}{F(\beta)}$, we have

(1) $$\int_{\beta}^{\overline{\beta}} (\ln G(\hat{\beta}))' d\hat{\beta} \leq \int_{\beta}^{\overline{\beta}} (\ln F(\hat{\beta}))' d\hat{\beta} \quad \text{for all } \beta \in (\underline{\beta},\overline{\beta}).$$

Since $\int_{\beta}^{\overline{\beta}} (\ln G(\hat{\beta}))' d\hat{\beta} = \ln G(\overline{\beta}) - \ln G(\beta) = -\ln G(\beta)$, (1) becomes

$$\ln F(\beta) \leq \ln G(\beta) \text{ for all } \beta \in (\underline{\beta}, \overline{\beta}).$$

Furthermore, since $\ln(\cdot)$ is monotone increasing, we have $F(\beta) \leq G(\beta)$ for all $\beta \in (\underline{\beta}, \overline{\beta})$.

(*b*) Since $G(\beta) \geq F(\beta)$ for all β, the result is immediate if $F'(\beta) > G'(\beta)$. If $F'(\beta) \leq G'(\beta)$, $F'(\beta)\,(G(\beta) - F(\beta)) \geq F(\beta)\,(G'(\beta) - F'(\beta))$ is nothing but $G'(\beta)\,F(\beta) \leq F'(\beta)\,G(\beta)$. ∎

As (*b*) in the Proposition shows, condition (i) is much weaker than condition (ii).

Reference

Jean-Jacques Laffont and Jean Tirole, 1993, A Theory of Incentives in Procurement and Regulation (The MIT Press, Cambridge).