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Economic Growth and Exchange Rate Systems

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Introduction

The relation between economic growth and exchange rate system is an important issue, but it has been scarcely discussed in a fully dynamic model.[1] If a developing country, receiving sufficient foreign aid to intervene in the foreign exchange market, has economic growth as the primary target, should the exchange rate be overvalued or undervalued? What type of economy can successfully "take off" and maintain steady economic growth? Is it possible for a country to achieve both economic growth and the balance-of-payment surplus? Do the fluctuations of the domestic sector disturb the stability of the foreign exchange market? These are the main problems which we are going to discuss and give answers in this paper.

In Chapter I, we present a simple one-country growth model with the small country assumption. In the following chapters, the policy discussion and deduction of stability condition are developed in terms of a suggested taxonomy of an economy. Chapter II, III, and IV deal with the fixed exchange rate system, the managed-float exchange rate system, and the flexible exchange rate system respectively.

Footnote

[1] See Appendix.

foreign currency and the current level of imports is confined within the current earnings of foreign currency through exports. On the other hand, if $\delta_1=0$ and $0<\delta_2<1$, the country can easily finance her necessary imports.[3] We assume $1>\delta_1, \delta_2>0$ henceforth.

Our model is deduced to the following differential equation;

$$\dot{Y}=\rho Y+\beta G\left[\delta_1 \frac{Pe(0)}{Pm(0)} r^{b-1} \exp(\gamma t)-r^b \exp(\epsilon t)\right] \qquad (8)$$

where $\rho \equiv (1-\alpha+\delta_2)\beta>0$ and $\gamma \equiv \lambda+\epsilon$.

The balance of payments $(\dot{R})$ is defined as

$$\dot{R} \equiv \frac{PeE}{r} - PmM \ . \qquad (9)$$

We shall ignore capital movement throughout this paper.

Footnotes to Chapter I

(1) Export function (5) is deduced from

$$E=d_0 Yw^{d_1}\left(\frac{Pe}{rPw}\right)^{-d_2}$$

where Yw and Pw are world income and world price respectively. Assuming that $Yw=Yw(0)\exp(h_1 t)$ and $Pw=Pw(0)\exp(h_2 t)$, we obtain equation (5) by writing $G\equiv d_0 Yw(0)^{d_1}\left(\frac{Pe(0)}{Pw(0)}\right)^{-d_2}$, $b=d_2$, and $\epsilon\equiv h_1 d_1+(h_2-\mu-\lambda)d_2$.

(2) The rate of change of terms of trade is λ minus the rate of change of exchange rate.

(3) In some countries where import substitution has a great effect on imports, δ_2 can be negative.

Chapter II / Fixed Exchange Rate System

Section 1 / Conditions for Sustained Growth

From a long-run point of view, the maintenance of sustained growth is a key issue. We give the following definitions.

Definition: Successful-Sustained Growth (SSG) as

$$\lim_{t\to\infty} Y(t)>0$$

Definition: Successful Balance of Payments(SBP) as

$$\lim_{t\to\infty} \dot{R}(t)>0$$

SSG means that an economy can eventually "take off". Let us consider the conditions for SSG and SBP.

Under the fixed exchange rate system, the full solution for Yand $\dot{R}$ based on the initial value of capital stock (K(0)) becomes;

$$Y=\beta\left[A\exp(\rho t)+B\exp(\epsilon t)+D\exp(\gamma t)\right] \quad (10)$$

$$\dot{R}=\exp(\mu t)\left[\xi GPe(0)r^{b-1}\exp(\gamma t)-Pm(0)\delta_2\beta\left(A\exp(\rho t)+B\exp(\epsilon t)\right)\right] \quad (11)$$

where $A\equiv K(0)-B-D$, $B\equiv\frac{Gr^b}{\rho-\epsilon}$, $D\equiv\delta_1 G\frac{Pe(0)}{Pm(0)}r^{b-1}/(\gamma-\rho)$, and $\xi\equiv1-\delta_1-\frac{\delta_1\delta_2\beta}{\gamma-\rho}$. The parameters γ and ρ can be interpreted as the growth rate of foreign currency earnings deflated by Pm and the growth rate of domestic sector respectively.

According to the value of the different parameters, we can classify economies into several types.(1)

Definition: We call an economy as "X type" iff X=max(X,Y,Z), and denote it by the symbol $(\hat{X},Y,Z)$.

Hence our economies are basically classified into three categories; γ type, ρ type and ϵ type.

From (10) and (11), the next theorem follows.

Theorem 1 (Successful Growth)

(a) SSG occurs when (i) $(\hat{\gamma},\rho,\epsilon)$, or (ii) $(\gamma,\hat{\rho},\epsilon)$ and $A>0$.

(b) SBP occurs when (i) $(\widehat{\gamma+\mu},\rho+\mu,\epsilon+\mu)>0$ and $\xi>0$, or (ii) $(\gamma+\mu,\widehat{\rho+\mu},\epsilon+\mu)>0$ and $A<0$, or (iii) $(\gamma+\mu,\rho+\mu,\widehat{\epsilon+\mu})>0$.

Hence both SSG and SBP appear when $(\widehat{\gamma+\mu},\rho+\mu,\epsilon+\mu)>0$ and $\xi>0$. This might be the cases of Japan and West Germany in the late 1960s and the early 1970s.

Section 2 / Immiserizing Export Growth

An increase in exports has effects in two ways on future GNP. While its direct effect on capital accumulation is negative as mentioned in Chapter I, it increases imports through its effect on the foreign currency earnings, and thus stimulates capital accumulation. Thus the overall effect of the growth of export on future GNP depends on the types of economy.

Definition: Immiserizing Export Growth (IEG) as

$$\lim_{t\to\infty}\frac{\partial Y(t)}{\partial \epsilon}<0.$$

Theorem 2 (Immiserizing Export Growth)

IEG occurs if (i) $(\gamma,\hat{\rho},\epsilon)$ and $r<r_I$, where $r_I\equiv \delta_1\frac{Pe(0)}{Pm(0)}\left(\frac{\rho-\epsilon}{\rho-\gamma}\right)^2$, or (ii) $(\gamma,\rho,\hat{\epsilon})$.

Proof: Differentiating (10) with respect to ϵ, we have

$$\frac{\partial Y}{\partial \epsilon} = t\beta(B\exp(\epsilon t) + D\exp(\gamma t)) + \beta\left(\frac{\partial A}{\partial \epsilon}\exp(\rho t) + \frac{\partial B}{\partial \epsilon}\exp(\epsilon t) + \frac{\partial D}{\partial \epsilon}\exp(\gamma t)\right)$$

where $\frac{\partial A}{\partial \epsilon} = \frac{Gr^{b-1}}{(\rho-\epsilon)^2}(r_I - r)$. ∎

Section 3 / Policy Effects

Let us consider the current and long-run effects of the exchange rate devaluation. Differentiating (11) with respect to r, we can deduce the following theorem.

Theorem 3 (Marshall-Lerner Condition and Long-run Effect of Currency Devaluation on The Balance of Payments)

(a) $\frac{\partial \dot{R}(0)}{\partial r} > 0$ iff $b-1 > 0$.

(b) $\lim_{t\to\infty}\frac{\partial \dot{R}(t)}{\partial r} > 0$ if (i) $(\widehat{\gamma+\mu}, \rho+\mu, \epsilon+\mu) > 0$ and $(b-1)\xi > 0$; or (ii) $(\gamma+\mu, \widehat{\rho+\mu}, \epsilon+\mu) > 0$ and $r > r_\pi$, where $r_\pi \equiv C_1\left(\frac{\rho-\epsilon}{\rho-\gamma}\right)$ and $C_1 \equiv \delta_1\frac{(b-1)}{b}\frac{Pe(0)}{Pm(0)}$; or (iii) $(\gamma+\mu, \rho+\mu, \widehat{\epsilon+\mu}) > 0$.

Theorem 3 (a) corresponds to the Marshall-Lerner condition.[2]

We assume this condition is satisfied hereafter.

Assumption I: The Marshall-Lerner condition is met. Namely $b-1 > 0$.

Similarly, differentiating (10) with respect to r, the next theorem follows.

Theorem 4 (Long-run Effect of Currency Devaluation on GNP)

Under Assumption I, $\lim_{t\to\infty}\frac{\partial Y(t)}{\partial r} > 0$ if (i) $(\hat{\gamma}, \rho, \epsilon)$, or (ii) $(\gamma, \hat{\rho}, \epsilon)$ and $r < r_\pi$.

In the late 1950s and the early 1960s, although Japanese economy enjoyed high rate of growth, her balance of trade, on the whole, turned into red. In our model this situation can be interpreted as either $(\gamma+\mu, \rho\hat{+}\mu, \epsilon+\mu)>0$ and $A>0$, or $(\gamma\hat{+}\mu, \rho+\mu, \epsilon+\mu)>0$ and $\xi<0$. Hence Yen devaluation in this period must have deteriorated either Y or $\dot{R}$ in the long run. In other words, high economic growth and the balance-of-payment surplus were in a trade off relation as far as the exchange rate policy is concerned.

It is also interesting to note, considering from Theorem 1 to Theorem 4, that in a country where people enjoy both SSG and SBP -- Japan and West Germany in the late 1960s and the early 1970s seem to have represented this case --, currency appreciation will decrease both GNP and the balance-of-payment surplus in the long run.

Section 4 / Optimum-Fixed Exchange Rate

We define the long-run policy target as the discounted accumulation of GNP.

Definition: National Wealth as

$$V \equiv \lim_{T\to\infty} \int_0^T Y(t)\exp(-nt)\,dt$$

The parameter n is an adequate discount rate to secure the existence of V. Integrating (10) with respect to time, and taking a limit, we have

$$V \equiv \beta\left[\frac{K(0)}{n-\rho} - \frac{Gr^b}{(n-\epsilon)(n-\rho)} + \frac{\delta_1 Gr^{b-1}}{(n-\gamma)(n-\rho)}\frac{Pe(0)}{Pm(0)}\right] \qquad (12)$$

Definition: Optimum-Fixed Exchange Rate (r_v) as a fixed exchange rate which maximizes V,

Theorem 5 (Optimum-Fixed Exchange Rate)

Under Assumption I, there exists Optimum-Fixed Exchange Rate.

Proof: Differentiating (12) with respect to r, and setting it equal to zero, we have

$$r_v = C_1\left(\frac{n-\epsilon}{n-\gamma}\right).$$

If b>1, r_v is positive and gives a global maximum of V.∎

It is worth noticing that r_v is not necessarily associated with the balance-of-payment deficit.

Section 5 / Equilibrium-Fixed Exchange Rate

Definition: Future Foreign-Exchange Reserves as

$$F \equiv \lim_{T\to\infty}\int_0^T \dot{R}(t)\exp(-mt)\,dt$$

The parameter m is an adequate discount rate to secure the existence of F. Integrating (11) with respect to time, and taking a limit, we have

$$F \equiv \frac{\xi G Pe(0) r^{b-1}}{m-\mu-\gamma} + \frac{Pm(0)\delta_2\beta}{(m-\mu-\rho)}\left[-K(0) + \frac{Gr^b}{m-\mu-\epsilon} + \frac{\delta_1 G r^{b-1}}{(\gamma-\rho)}\frac{Pe(0)}{Pm(0)}\right] \quad (13)$$

Definition: Equilibrium-Fixed Exchange Rate (r_H) as a fixed exchange rate which gearantees $\dot{R}(0)=0$ and $F=0$. [3]

Theorem 6 (Equilibrium-Fixed Exchange Rate)

There exists Equilibrium-Fixed Exchange Rate iff $\gamma<\rho$ or $\xi<0$.

Proof: From $\dot{R}(0)=0$ we have

$$K(0) = \delta_1 G\frac{Pe(0)}{Pm(0)} r^{b-1}\left(\frac{1-\delta_1}{\delta_1\delta_2\beta}\right) \quad (14)$$

Substituting (14) into (13) and setting F=0, we obtain

$$r_H = \delta_1 \frac{Pe(0)}{Pm(0)} \left[1 - \left(\frac{1-\delta_1}{\delta_1 \delta_2 \beta}\right)(\gamma - \rho)\right]\left(\frac{m-\epsilon-\mu}{m-\gamma-\mu}\right). \qquad (15)$$

$r_H > 0$ iff $\gamma < \rho$ or $\xi < 0$. ∎

It must be noted that r_v is not necessarily undervalued than r_H. In some economies, if a country aims to maximize V, the exchange rate must be overvalued than the Equilibrium-Fixed Exchange Rate.[4] Also it is interesting to note that r_H does not exist in a country where people enjoy both SSG and SBP. Hence the fixed exchange rate system was not suitable to achieve the balance-of-payment equilibrium in Japan as well as in West Germany in the late 1960s and the early 1970s.[5]

Footnotes to Chapter II

[1] The taxonomy presented here is used only in this chapter. Another taxonomy will be offered in the following chapters. Taxonomy of economies and the results obtained in this chapter are summarized in Table 1.

[2] Differentiating $\dot{R}(0)$ with respect to r, and evaluating at the equilibrium, we obtain

$$\frac{\partial R(0)}{\partial r} = \frac{Pe(0)E(0)}{r^2}\left[\frac{r}{E(0)}\frac{\partial E(0)}{\partial r} - \frac{r}{M(0)}\frac{\partial M(0)}{\partial r} - 1\right]$$
$$= \frac{Pe(0)E(0)}{r^2}\left[b - \delta_1(b-1) - 1\right].$$

[3] The condition $\dot{R}(0)=0$ is necessary to obtain unique solution. By this condition, we implicitly assumed the initial adjustment of K(0) such that $K(0) = \delta_1 \frac{Pe(0)}{Pm(0)} Gr_H^{\,b-1}\left(\frac{1-\delta_1}{\delta_1\delta_2\beta}\right)$. This is possible either by raising foreign loans and importing foreign capital or by laying idle the excess capital.

[4] See Theorem 9.

[5] Cf. Theorem 12.

Table 1

		Parameters				SSG	SBP	Policy Effect		$r_H > 0$	IEG
		A	ξ	$r-r_\pi$	$r-r_I$			$\dot{Y}_r(\infty)$	$\dot{R}_r(\infty)$		
γ			+			○	○	+	+	×	×
			−			○	×	+	−	○	×
ρ		+		+	+	○	×	−	+	○	×
		+		+	−	○	×	−	+	○	○
		+		−	+	○	×	+	−	○	×
		+		−	−	○	×	+	−	○	○
		−		+	+	×	○	−	+	○	×
		−		+	−	×	○	−	+	○	○
		−		−	+	×	○	+	−	○	×
		−		−	−	×	○	+	−	○	○
ϵ	$\gamma > \rho$		+			×	○	−	+	×	○
						×	○	−	+	○	○

Key: ○ ; Satisfaction, × ; Not satisfied.

As for SBP, both ○ and × include the case $\lim_{t\to\infty} \dot{R}(t)=0$.

Chapter III / Managed-Float Exchange Rate System

Section 1 / Equilibrium Exchange Rate

We shall call an exchange rate which changes by the identical rate with the market-clearing exchange rate as Managed-Float Exchange Rate.[1]

To begin with, we define Equilibrium Exchange Rate and will study it in this section.

Definition: Equilibrium Exchange Rate (re) as an exchange rate which secures $\dot{R}(t)=0$ for all t.

Setting $\dot{R}(t)=0$ in equation (9), and solving it for Y(t), we have

$$Y(t)=\left(\frac{1-\delta_1}{\delta_2}\right)\frac{Pe(0)}{Pm(0)}\ Gr^{b-1}\exp(\gamma t). \qquad (16)$$

We assume re to be an exponential form and write

$$re(t)=C_2\exp\ (kt). \qquad (17)$$

Since re(t) must satisfy both (8) and (16), we obtain

$$C_2=\delta_1\frac{Pe(0)}{Pm(0)}\ \tilde{q} \qquad (18a)$$

$$k=\lambda \qquad (18b)$$

where $\tilde{q}\equiv 1-\frac{1-\delta_1}{\delta_1\delta_2\beta}(\gamma^*-\rho)$ (18c)

and $\gamma^*\equiv(b-1)\lambda+\gamma=b\lambda+\epsilon.$ (18d)

Hence from (16),(17) and (18), we obtain

$$Y(t,re(t))=\left(\frac{1-\delta_1}{\delta_2}\right)\frac{Pe(0)}{Pm(0)}\ GC_2^{\ b-1}\exp(\gamma^*t) \qquad (19)$$

and $re(t)=\delta_1\frac{Pe(0)}{Pm(0)}\ \tilde{q}\exp(\lambda t).$ (20)

By assuming an exponential form, we implicitly presupposed the initial adjustment of K(0) such that

$$K(0)=\delta_1\left(\frac{1-\delta_1}{\delta_1\delta_2\rho}\right)\frac{Pe(0)}{Pm(0)}\ GC_2^{\,b-1}\cdot[2] \qquad (21)$$

We will show in Chapter IV that, even though this initial condition does not hold, Equilibrium Exchange Rate coincides with re(t) in equation (20) for sufficiently large t. We shall assume that the initial condition (21) is met when we talk about re(t) henceforth.

γ^* in equation (18d) can be interpreted as the rate of growth of foreign-currency earnings deflated by Pm under the managed-float exchange rate regime. Similarly ρ can be regarded as the rate of growth of domestic sector as mentioned in Chapter II. We redefine the types of economy as follows.

Definition: We call an economy as Outward-Biased-Growth type (OBG type) when $\rho+\frac{\delta_1\delta_2\beta}{1-\delta_1}>\gamma^*>\rho$.

Definition: We call an economy as Inward-Biased-Growth type (IBG type) when $\gamma^*<\rho$.[3]

The following theorem holds.

Theorem 7 (Equilibrium Exchange Rate)

Equilibrium Exchange Rate (re) exists if a country is one of the two types; OBG or IBG.[4]

Section 2 / Optimum Maneged-Float Exchange Rate

Definition: Optimum Managed-Float Exchange Rate (rd) as Managed-Float Exchange Rate which maximizes

$$Y(T)\equiv\int_0^T \dot{Y}(t)\,dt$$

subject to equation (8).

Define the Hamiltonian (H) as

$$H \equiv -\dot{Y}(t) = -\left[\rho Y(t) + \beta G\left\{\delta_1 \frac{Pe(0)}{Pm(0)} r(t)^{b-1} \exp(\gamma t) - r(t)^b \exp(\epsilon t)\right\}\right]. \qquad (22)$$

Solving the necessary condition, we have

$$rd(t) = \delta_1\left(\frac{b-1}{b}\right)\frac{Pe(0)}{Pm(0)} \exp(\lambda t). \text{[5]} \qquad (23)$$

It is easily verified that rd(t) also maximizes

$$V(T) \equiv \int_0^T Y(t)\exp(-nt)dt. \text{[6]}$$

Theorem 8 (Optimum Managed-Float Exchange Rate)

Under Assumption I, there exists Optimum Managed-Float Exchange Rate.

It is intuitively clear that rd(t) is the most efficient exchange rate for the growth target.

Substituting (23) into (8), and integrating, we have

$$Y(t, rd(t)) = \beta\left[\tilde{A}\exp(\rho t) + \tilde{B}\exp(\gamma^* t)\right] \qquad (24)$$

where $\tilde{A} \equiv K(0) - \tilde{B}$, $\tilde{B} \equiv \frac{\delta_1 G C_1{}^{b-1} Pe(0)}{b(\gamma^* - \rho) Pm(0)}$ and $C_1 \equiv \delta_1\left(\frac{b-1}{b}\right)\frac{Pe(0)}{Pm(0)}$.

Similarly, substituting (23) and (24) into (9), we obtain

$$\dot{R}(t, rd(t)) = \exp(\mu t)\left[G C_1{}^{b-1} Pe(0)\xi^* \exp(\gamma^* t) - \delta_2 \beta \tilde{A} Pm(0)\exp(\rho t)\right] \qquad (25)$$

where $\xi^* \equiv 1 - \delta_1 - \frac{\delta_1 \delta_2 \beta}{(\gamma^* - \rho)b}$.

Section 3 / Biased Evaluation Theorem

We classify OBG type economy further into Strong-OBG type and Weak-OBG type.

Definition: We call an economy as Strong-OBG type (SOBG type) when $\rho+\frac{\delta_1\delta_2\beta}{1-\delta_1}>\gamma^*>\rho+\frac{\delta_1\delta_2\beta}{(1-\delta_1)b}$.

Definition: We call an economy as Weak-OBG type (WOBG type) when $\rho+\frac{\delta_1\delta_2\beta}{(1-\delta_1)b}>\gamma^*>\rho$.

With these definitions, we have the following theorem.

Theorem 9 (Biased Evaluation Theorem)

In WOBG and IBG (in SOBG) economies, if economic growth is the primary target, the exchange rate must be overvalued (undervalued) than Equilibrium Exchange Rate.

Proof: $re(t)\gtrless rd(t)\Longleftrightarrow\tilde{q}\lessgtr\frac{b-1}{b}\Longleftrightarrow\gamma^*\lessgtr\rho+\frac{\delta_1\delta_2\beta}{(1-\delta_1)b}$ ∎

This theorem supports the evidences of overvaluation in some IBG economies.[7]

From (19),(24) and(25), the next theorem follows.

Theorem 10 (Successful Growth)

(a) If rd(t) is adopted, the economy enjoys SSG.

(b) If re(t)>0 is adopted and if $\gamma^*>0$, then the economy enjoys SSG.

(c) In SOBG (in WOBG and IBG) economies, if a country adopts rd(t), she will enjoy (suffer from) perpetual balance-of-payment surplus and SBP (deficit).

Proof: From (21) and (25), we have

$$\text{sign }\dot{R}(t,rd(t))=\text{sign}\left[\left(\frac{b-1}{b}\right)^{b-1}\left(1-\tilde{q}-\frac{1}{b}\right)\left(\frac{\exp(\gamma^*t)-\exp(\rho t)}{\gamma^*-\rho}\right)+\left(\frac{1-\tilde{q}}{\gamma^*-\rho}\right)\left[\left(\frac{b-1}{b}\right)^{b-1}-\tilde{q}^{b-1}\right]\exp(\rho t)\right]$$

Therefore $\dot{R}(t,rd(t))\gtrless 0\Longleftrightarrow\tilde{q}\lessgtr\frac{b-1}{b}\Longleftrightarrow\gamma^*\gtrless\rho+\frac{\delta_1\delta_2\beta}{(1-\delta_1)b}$. [8] ∎

Theorem 10 suggests that if a country is SOBG type, she can realize both growth target and perpetual balance-of-payment surplus simultaneously by adopting rd(t). On the other hand, IBG and WOBG countries will often confront with severe foreign-exchange shortage when they try to attain growth target by overvaluation.

Section 4 / Immiserizing Export Growth

Let us define Dual-Structure Exchange Rate as

$$ru(t) \equiv \delta_1 \frac{Pe(0)}{Pm(0)} \exp(\lambda t). \tag{26}$$

Then, using ru(t), we can write Managed-Float Exchange Rate as

$$r(t;x) \equiv x \cdot ru(t) = \delta_1 x \frac{Pe(0)}{Pm(0)} \exp(\lambda t), \quad x > 0. \tag{27}$$

Substituting (27) into (8), and integrating, we have

$$Y(t, r(t;x)) = \beta \left[\bar{A}\exp(\rho t) + \bar{B}\exp(\gamma^* t)\right] \tag{28}$$

where $\bar{A} \equiv K(0) - \bar{B}$ and $\bar{B} \equiv G\left(\delta_1 \frac{Pe(0)}{Pm(0)}\right)^b \frac{x^{b-1}(1-x)}{\gamma^* - \rho}$.

When ru(t) is adopted, GNP grows at the rate ρ, and the foreign trade $\left(\frac{PeE}{r}\right)$ grows at the rate γ^*. Moreover the growth rate of GNP is totally independent of the external growth rates; λ, ϵ and γ^*. This is why we named ru(t) Dual-Structure Exchange Rate.

From (28),

$$\frac{\partial Y(t, r(t;x))}{\partial \epsilon} = \beta \bar{B}\left[\left(t - \frac{1}{\gamma^* - \rho}\right)\exp(\gamma^* t) + \frac{1}{\gamma^* - \rho}\exp(\rho t)\right]. \tag{29}$$

From (29), follows the next theorem.

Theorem 11 (Immiserizing Export Growth)

IEG occurs iff the exchange rate is undervalued than Dual-Structure Exchange Rate.

Proof: $r(t;x) \gtrless ru(t) \Longleftrightarrow x \gtrless 1 \Longleftrightarrow \lim_{t\to\infty} \frac{\partial Y(t)}{\partial \epsilon} \lessgtr 0$ ∎

Theorem 11 suggests that an expansion of exports does decelerate economic growth in the long run, if $r(t;x)$ is held higher than $ru(t)$.[9]

Section 5 / Sustained Growth and Policy Effects

Using the initial condition (21), equation (28) can be rewritten as

$$Y(t,r(t;x)) = \beta\left[\bar{A}^{*}\exp(\rho t) + \bar{B}\exp(\gamma^{*}t)\right] \tag{30}$$

where $\bar{A}^{*} \equiv G\left(\delta_1 \frac{Pe(0)}{Pm(0)}\right)^{b}\left(\frac{1}{\gamma^{*}-\rho}\right)\left[\tilde{q}^{b-1}(1-\tilde{q}) - x^{b-1}(1-x)\right]$.

Similarly, substituting (27) and (30) into (9), we have

$$\dot{R}(t,r(t;x)) = N\Big[x^{b-1}(x-\tilde{q})\left(\frac{\exp(\gamma^{*}t)-\exp(\rho t)}{\gamma^{*}-\rho}\right) + (x^{b-1}-\tilde{q}^{b-1})\left(\frac{1-\tilde{q}}{\gamma^{*}-\rho}\right)\exp(\rho t)\Big] \tag{31}$$

where $N \equiv \exp(\mu t)\left(\delta_1 \frac{Pe(0)}{Pm(0)}\right)^{b} Pm(0)\delta_2\beta G$.

From (30) and (31), follows the next theorem.

Theorem 12 (Successful Growth)

(SSG)(a) The OBG countries enjoy SSG if they hold the exchange rate below $ru(t)$.

(b) The IBG countries enjoy SSG if they keep the exchange rate below $re(t)$.

(SBP) Both OBG countries and IBG countries enjoy (suffer from) the balance-of-payment surplus (deficit) iff the exchange rate is undervalued (overvalued) than $re(t)$.

Proof: (SBP) $r(t;x) \gtrless re(t) \Longleftrightarrow x \gtrless \tilde{q} \Longleftrightarrow \dot{R}(t,r(t;x)) \gtrless 0$ [10]

Theorem 12 does not mean that the devaluation always improves the balance of payments. As for the effects of currency devaluation, we have the following theorem.

Theorem 13 (Effects of Currency Devaluation)

(a) $\frac{\partial Y(t)}{\partial x} \gtrless 0$ for all t iff $x \lessgtr \frac{b-1}{b}$.

(b) In OBG countries, $\lim_{t\to\infty} \frac{\partial \dot{R}(t)}{\partial x} \gtrless 0$ if $x \gtrless \frac{b-1}{b}\tilde{q}$.

In IBG countries, $\lim_{t\to\infty} \frac{\partial \dot{R}(t)}{\partial x} \gtrless 0$ if $x \gtrless \frac{b-1}{b}$.

Proof: (a) $$\frac{\partial Y(t)}{\partial x} = \beta G\left(\delta_1 \frac{Pe(0)}{Pm(0)}\right)^b x^{b-2}(b-1-bx)\left(\frac{\exp(\gamma^* t)-\exp(\rho t)}{\gamma^* - \rho}\right) \quad (32)$$

(b) $$\frac{\partial \dot{R}(t)}{\partial x} = \frac{N}{\gamma^* - \rho} x^{b-2}\left[(bx-(b-1)\tilde{q})\exp(\gamma^* t)+(b-1-bx)\exp(\rho t)\right] \quad (33)$$

These results are derived without assuming (21). Note that $K(0)$ in equation (21) is independent of x. Hence this theorem does not depend on the initial condition. [11]

Theorem 12 suggests that both SSG and SBP are concurrent in the OBG countries, if the exchange rate is held such that $ru(t)>r(t;x)>re(t)$. Moreover, $re(t)$ does exist in these countries in contrast with the fixed exchange rate regime. [12]

Footnotes to Chapter III

[1] Managed-Float Exchange Rate does not diverge infinitely from the market clearing level.

[2] See footnote [3] to Chapter II. Given the initial condition K(0), the exchange rate which makes $\dot{R}(0)=0$ (denoted by rf) becomes

$$rf=\left[\frac{\delta_2}{(1-\delta_1)}\frac{Pm(0)}{Pe(0)}\frac{\beta K(0)}{G}\right]^{\frac{1}{b-1}}.$$

[3] The distinction between OBG and IBG type depends on the long-run leading sector of the economy. The similar ideas are proposed in Balassa [2] , Fukuchi and Imagawa [7] , and Kaldor [8] . Also Lubitz [12] suggests consumption-led growth and export-led growth.

[4] In the case when re(t) is adopted, E and Y expand at the same rate and that

$$\frac{E(t,re(t))}{Y(t,re(t))}=\frac{\delta_1\delta_2}{1-\delta_1}\tilde{q}.$$

[5] Since H is a concave function of the state variable Y(t), rd(t) satisfies the sufficient condition as well.

[6] In this case the Hamiltonian becomes

$$H\equiv Y(t)+\eta(t)\Big[\rho Y(t)+\beta G\Big(\delta_1\frac{Pe(0)}{Pm(0)}r(t)^{b-1}\exp(\gamma t)$$
$$-r(t)^{b}\exp(\epsilon t)\Big)\Big]$$

where $\eta(t)$ is a Lagrange multiplier or a costate variable. The sufficient condition is met here, too.

[7] See, for example, Balassa [2] ,pp.39.

[8] By assuming that the initial condition (21) holds, we also presuppose $\tilde{q}>0$ and that $re(t)>0$, because, otherwise, $K(0)<0$. Theorem 10 (c) and Theorem 12 heavily depend on this condition.

[9] Note that when $ru(t)$ is adopted,

$$\frac{\partial Y(t)}{\partial \epsilon}=\frac{\partial Y(t)}{\partial \lambda}=\frac{\partial Y(t)}{\partial \gamma^*}=0 \quad \text{for all } t.$$

[10] When the initial condition (21) does not hold, $\dot{R}(t)$ becomes

$$\dot{R}(t)=\exp(\mu t)Pm(0)\Big[U\big(1-\frac{1-x}{1-\tilde{q}}\big)\exp(\gamma^* t)+ + \Big\{U\Big(\frac{1-x}{1-\tilde{q}}\Big)-\delta_2\beta K(0)\Big\}\exp(\rho t)\Big]$$

where $U\equiv G\left(\delta_1\frac{Pe(0)}{Pm(0)}\right)^b x^{b-1}\left(\frac{1-\delta_1}{\delta_1}\right)$.

When a country is neither OBG nor IBG type, we use this equation and (28) to obtain the conditions for SSG and SBP. See Table 2.

[11] The following figure shows the graph of equation (31). $\bar{x}$ stands for $\frac{b-1}{b}\tilde{q}$ in the case of OBG countries and represents $\frac{b-1}{b}$ in the case of IBG economies.

[12] The results obtained in this chapter are summarized in Table 2.

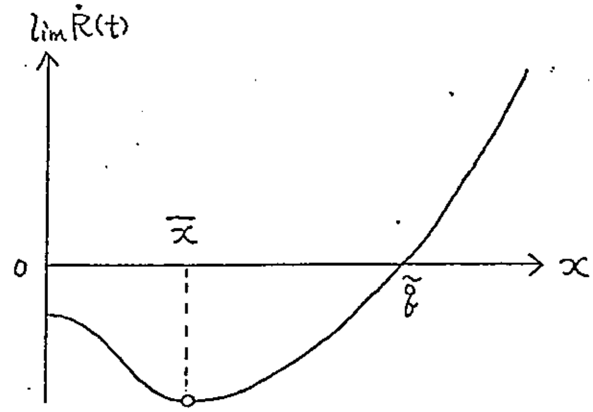


Table 2

Types of Economy		The level of $r(\tau;x)$ currently employed	SSG	SBP	Policy Effect $Y_x(t)$	Policy Effect $\dot{R}_x(\infty)$	$r_e>0$	IEG
$\gamma^* > \rho + \frac{\delta_1\delta_2\beta}{1-\delta_1}$		$x > 1$	×	○	−	+	×	○
		$1 > x > \frac{b-1}{b}$	○	○	−	+	×	×
		$\frac{b-1}{b} > x$	○	○	+	+	×	×
OBG		$x > 1$	×	○	−	+	○	○
	SOBG	$1 > x > \frac{b-1}{b}$	○	○	−	+	○	×
		$\frac{b-1}{b} > x > \tilde{q}$	○	○	+	+	○	×
		$\tilde{q} > x > \frac{b-1}{b}\tilde{q}$	○	×	+	+	○	×
	WOBG	$1 > x > \tilde{q}$	○	○	−	+	○	×
		$\tilde{q} > x > \frac{b-1}{b}$	○	×	−	+	○	×
		$\frac{b-1}{b} > x > \frac{b-1}{b}\tilde{q}$	○	×	+	+	○	×
		$\frac{b-1}{b}\tilde{q} > x$	○	×	+	−	○	×
IBG		$x > \tilde{q}$	×	○	−	+	○	○
		$\tilde{q} > x > 1$	○	×	−	+	○	○
		$1 > x > \frac{b-1}{b}$	○	×	−	+	○	×
		$\frac{b-1}{b} > x$	○	×	+	−	○	×

Key: ○; Satisfaction, × ; Not satisfied.

Chapter IV / Flexible Exchange Rate System

We call an exchange rate which is totally entrusted to market mechanism as flexible exchange rate, and fix the rule as $\dot{r} \gtrless 0$ if $\dot{R} \lessgtr 0$. Thus we have two differential equations; one that shows the growth of GNP and the other which indicates the adjustment mechanism of the foreign exchange market with a certain adjustment speed. In this chapter we will examine the stability conditions of the system.

The equation (8) can be rewritten as

$$\dot{y}=(\rho-\gamma^*)y+q^{b-1}(1-q) \tag{34}$$

where y and q are defined as follows.

$$y\equiv\frac{Y\exp(-\gamma^* t)}{\rho G\left(\delta_1\frac{Pe(0)}{Pm(0)}\right)^b} \tag{35}$$

$$q\equiv\frac{r\exp(-\lambda t)}{\delta_1\frac{Pe(0)}{Pm(0)}} \tag{36}$$

In order to remove the exponential trend, Y and r are devided by $\exp(\gamma^* t)$ and $\exp(\lambda t)$ respectively. Recall that λ is the growth rate of Equilibrium Exchange Rate, re(t), and that γ^* is the growth rate of the corresponding GNP, Y(t, re(t)).[1] Thus if $\frac{\dot{y}}{y}>0$, GNP grows faster than its equilibrium growth rate (γ^*), and if $\frac{\dot{y}}{y}<0$, GNP grows slower than γ^*. In order to exclude the possibility that $y<0$, i.e. $Y<0$, we assume the following.

||Assumption II: $\dot{y} \geq 0$ if $y=0$, and $\dot{y} \gtreqless 0$ otherwise.

Under Assumption II, equation (34) must be reformulated as

$$\dot{y}=\begin{cases} \max\left[(\rho-\gamma^{*})y+q^{b-1}(1-q),\ 0\right] & \text{if } y=0 \\ (\rho-\gamma^{*})y+q^{b-1}(1-q) & \text{if } y\neq 0\ . \end{cases} \quad (37)$$

We suppose that the rate of change of exchange rate is proportional to the relative size of the excess demand for foreign currency. Since $\mu+\gamma^{*}$ is the equilibrium growth rate of foreign trade $\left(\frac{PeE}{r}\right)$, $-\dot{R}\exp(-(\mu+\gamma^{*})t)$ shows the relative size of the excess demand for foreign currecy. Therefore we write

$$\dot{q}=h^{*}[-\dot{R}]\exp(-(\mu+\gamma^{*})t) \quad (38)$$

where h^{*} is the speed of adjustment assumed to be constant. Defining $\tilde{C}$ and $\tilde{A}$ as

$$\tilde{C}\equiv h^{*}Pm(0)\delta_2\beta G\left[\delta_1\frac{Pe(0)}{Pm(0)}\right]^{b}>0$$

and $\tilde{A}\equiv\dfrac{1-\delta_1}{\delta_1\delta_2\beta}>0$,

(38) can be written as

$$\dot{q}=\tilde{C}(y-\tilde{A}q^{b-1}). \quad (39)$$

Now we have two differential equations, (37) and (39).

Let us define the internal equilibrium line by

$$\dot{y}=0 \text{ or } y=\frac{q^{b-1}(1-q)}{\gamma^{*}-\rho} \quad (40)$$

and the external equilibrium line by

$$\dot{q}=0 \text{ or } y=\tilde{A}q^{b-1}. \quad (41)$$

The equilibrium point, $P\equiv(\tilde{y},\tilde{q})$, is specified as

$$\tilde{y}=\tilde{A}\tilde{q}^{b-1} \tag{42}$$

$$\tilde{q}=1-\tilde{A}(\gamma^{*}-\rho) \tag{43}$$

Hence if positive $\tilde{y}$ and $\tilde{q}$ exist, we have a unique equilibrium point. At a moving equilibrium point, $r(t)=re(t)$ and $Y(t)=Y(t,re(t))$.

Fig. 1 and Fig. 2 show the phase diagrams of the system when positive $\tilde{q}$ exists.[2] By Assumption II, when a time path hits the q-axis, it moves along the q-axis, and once q=1, it returns back to the positive quadrant. It is clear from the phase diagrams that the system cannot diverge infinitely. Any divergent trajectory may eventually hit the q-axis and return to the point (1,0). In other words, as long as positive $\tilde{q}$ exists, there is no globally unstable case and the system is either globally stable or it has at least one limit cycle.

In order to get further insight, let us examine the local stability condition of the system. Expanding (37) and (39) in the neighbourhood of P and taking the first order approximation, we have

$$\begin{bmatrix} (\dot{y}-\tilde{y}) \\ (\dot{q}-\tilde{q}) \end{bmatrix} = \begin{bmatrix} \rho-\gamma^{*}, & (b-1-b\tilde{q})\tilde{q}^{b-2} \\ \tilde{C}\ , & \tilde{C}\tilde{A}(1-b)\tilde{q}^{b-2} \end{bmatrix} \begin{bmatrix} y-\tilde{y} \\ q-\tilde{q} \end{bmatrix} \tag{44}$$

Since the determinant of the coefficient matrix is positive, we have the following necessary and sufficient condition for local stability.

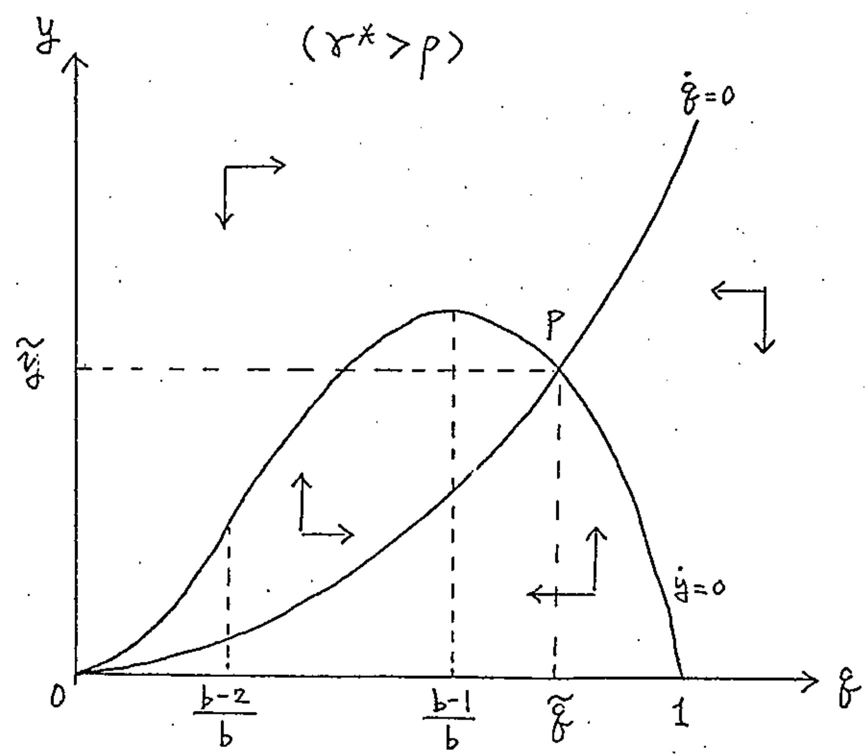


Fig. 1

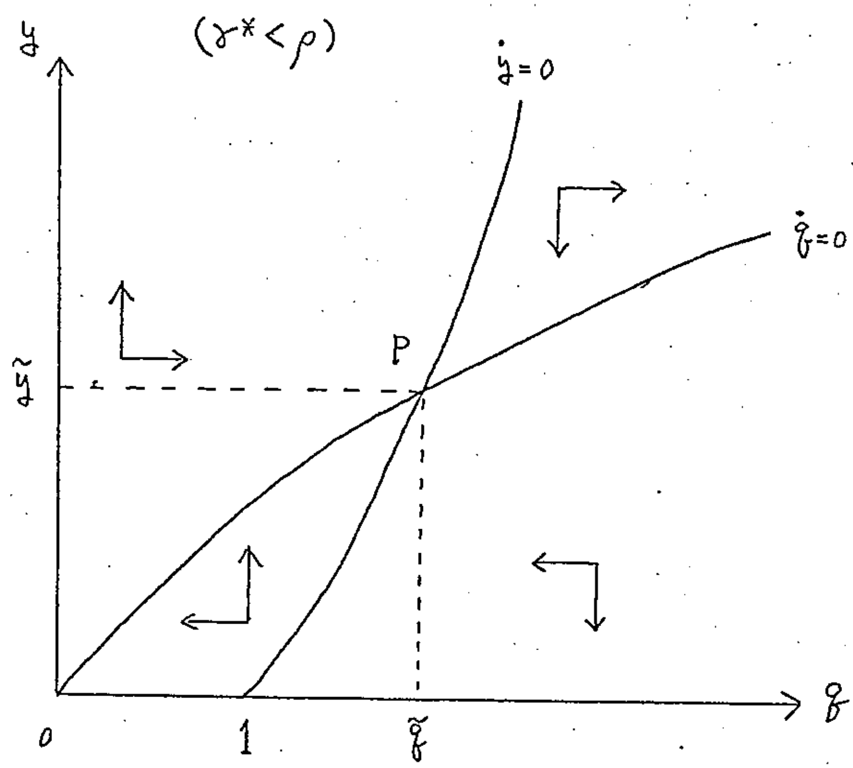


Fig. 2

$$\tilde{C} > \frac{\rho - \gamma^*}{\tilde{A}(b-1)\tilde{q}^{b-2}} \tag{45}$$

In an OBG country, (45) always holds and the system is locally stable. We will prove later that there exists no limit cycle in this case and hence the system is globally stable as well. In an IBG country, if (45) is not satisfied, the system is not stable and thus there must be at least one limit cycle, whereas when (45) is met, there remains ambiguity.

In addition, the system of (45) moves cyclically, if

$$(\rho - \gamma^* + \tilde{C}\tilde{A}(b-1)\tilde{q}^{b-2})^2 + 4\tilde{C}\tilde{q}^{b-2}(b-1-b\tilde{q}) < 0 \tag{46}$$

Therefore, if $\frac{b-1}{b} \geqq \tilde{q}$, or equivalently, if the country is of SOBG type, P is nodal. To assure, see Fig.3.

Theorem 14 (Global Stability)

(a) The system of (37) and (39) is globally stable, if a country is of OBG type.

(b) The system has at least one limit cycle if a country is of IBG type and $\tilde{C} < \frac{\rho - \gamma^*}{\tilde{A}(b-1)\tilde{q}^{b-2}}$.

(c) P is a locally nodal point if a country is of SOBG type.

Proof: We prove that there is no limit cycle when $\gamma^* > \rho$.

We can see that there are two possible types of limit cycle. They are shown in Fig. 4 and Fig. 5. The second type (Fig. 5) is an ordinary limit cycle,

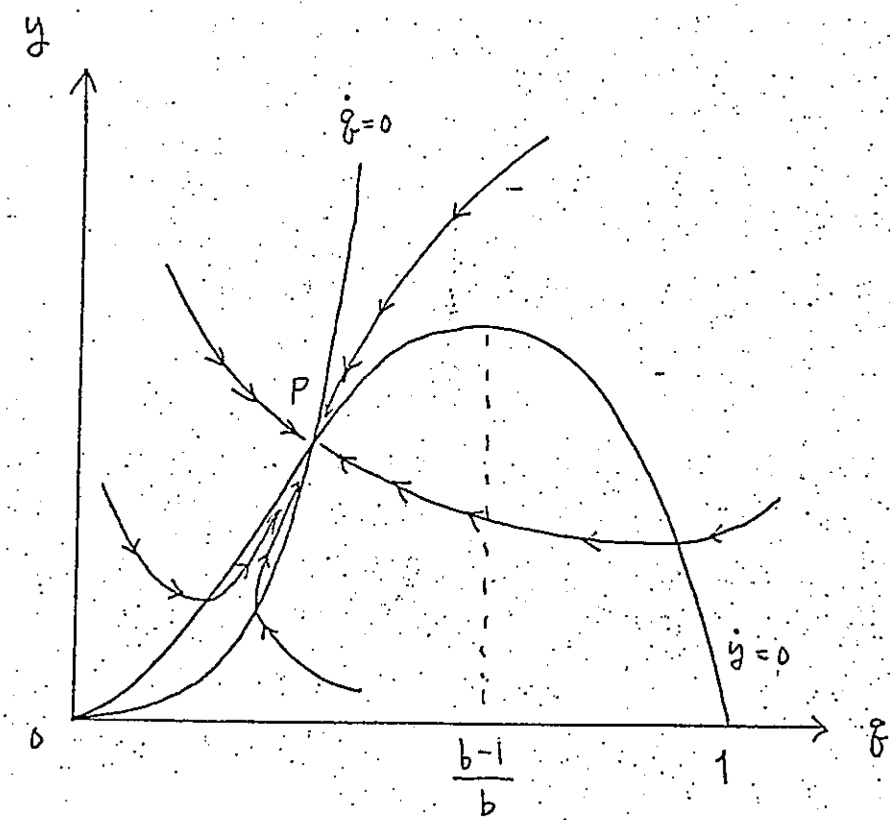


Fig. 3

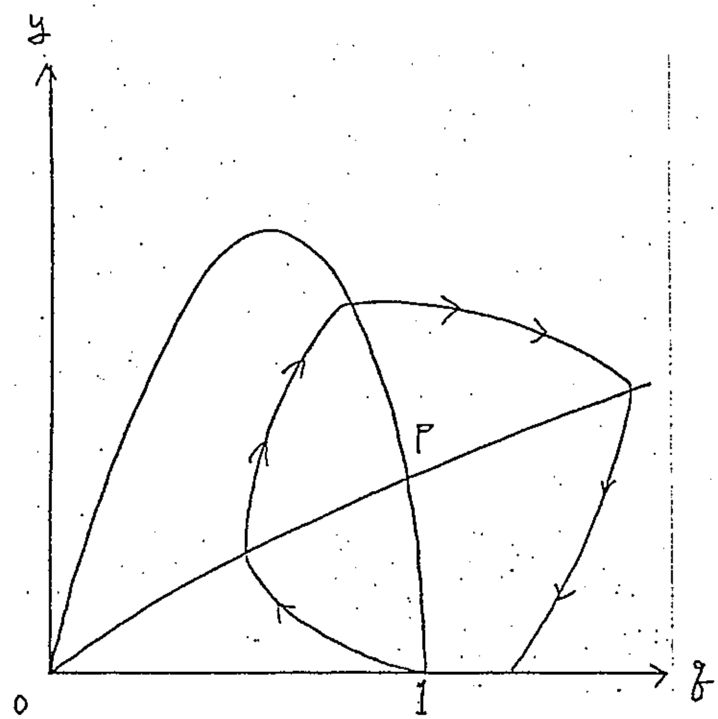


Fig. 4

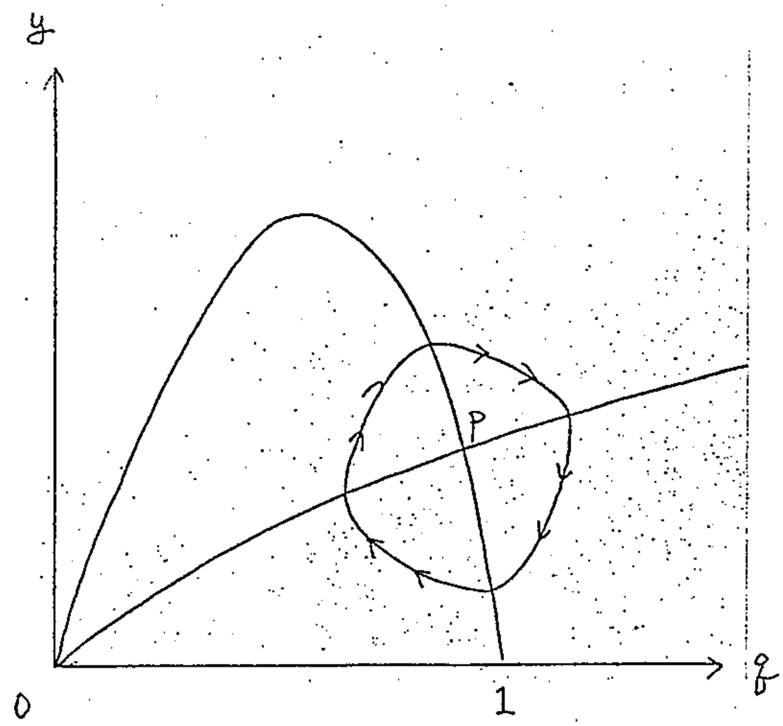


Fig. 5

whereas the first one (Fig. 4) appears because of Assumption II. In order for the first type of limit cycle to exist, there must be at least one ordinary type of limit cycle, because, in an OBG-type country, the system is always locally stable; see Fig. 6. Therefore we only need to prove that there is no limit cycle of the second type in an OBG-type country.

First of all, let us construct a closed square $(N_1N_2N_3N_4)$ in non-negative orthant of (q,y) as in Fig. 7. The closed square contains two corners, one at the origin and the other on the external equilibrium line. $\overset{*}{q}$ in Fig. 7 is given by

$$\overset{*}{q}=\frac{b-1}{b}\left(\frac{1}{b\tilde{A}(\gamma^{*}-\rho)}\right)^{\frac{1}{b-1}}.$$

Note that, under the system of (37) and (39), any time path will eventually come into the square $(N_1N_2N_3N_4)$ and no time path will go out of it. Hence a limit cycle can appear only inside of this closed domain.

We will prove that a limit cycle, in fact, does not exist within this region.

From (37) and (39), we have

$$\frac{\partial\dot{y}}{\partial y}+\frac{\partial\dot{q}}{\partial q}=\rho-\gamma^{*}-(b-1)\tilde{C}\tilde{A}q^{b-2}. \tag{47}$$

If $\gamma^{*}>\rho$, $\frac{\partial\dot{y}}{\partial y}+\frac{\partial\dot{q}}{\partial q}<0$ for all (q,y) within the closed square $(N_1N_2N_3N_4)$. Therefore, if $\gamma^{*}>\rho$, no limit cycle exists within this area by Bendixson's negation theorem.

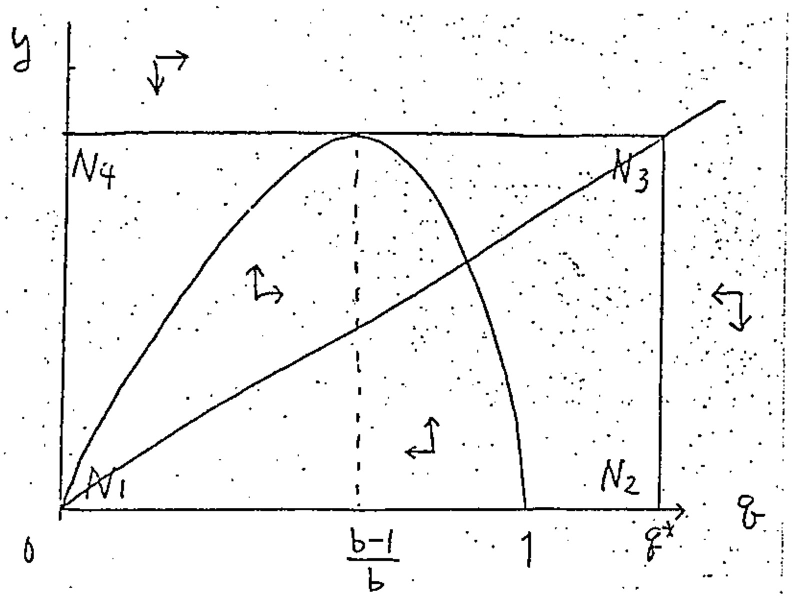


Fig. 7

Hence, by the preceding discussion, the system of (37) and (39) has neither the first type nor the second type of limit cycle. ∎

Theorem 14 suggests that the flexible exchange rate system is successful in OBG-type countries. IBG countries must be very cautious in adopting the flexible exchange rate system.[3]

It is interesting to note that when the foreign exchange market is always in equilibrium, that is, $\widetilde{C} \to \infty$, the system of (37) and (39) is always stable. This is shown by Fig. 8 and Fig. 9. Therefore, although Equilibrium Exchange Rate in strict sense may differ from re(t) in the short run, it approaches re(t) as $t \to \infty$. This corresponds to the remark which we made in Chapter III Section 1.

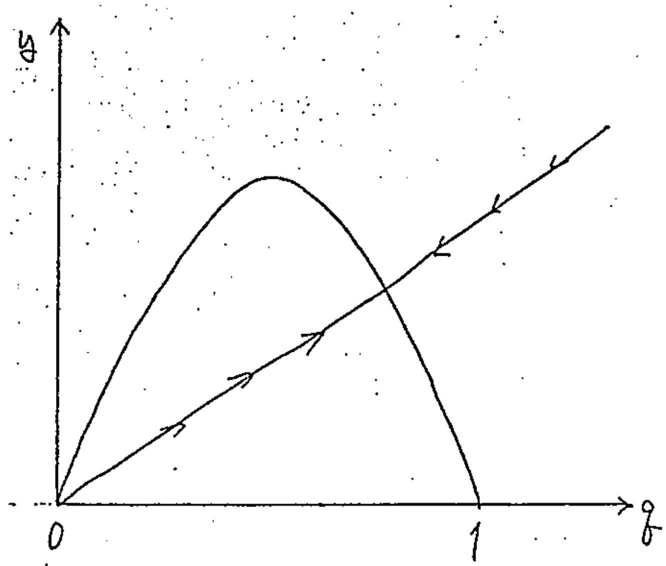


Fig. 8

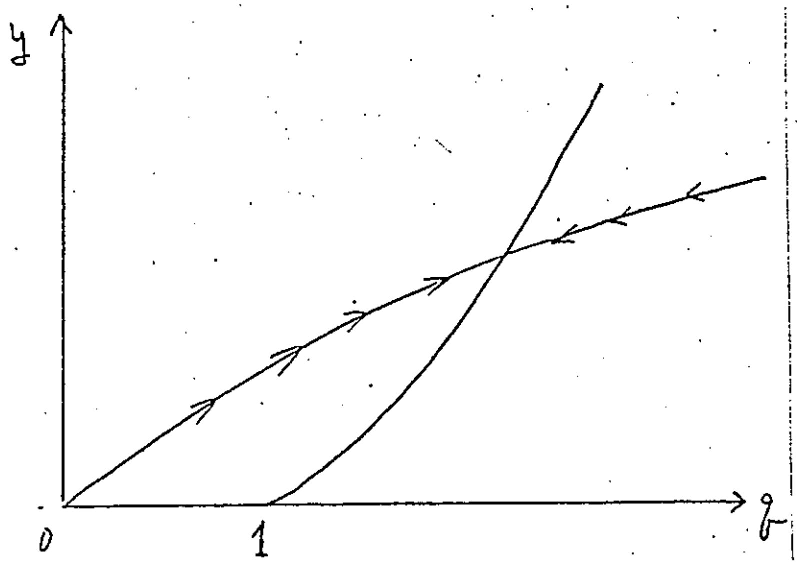
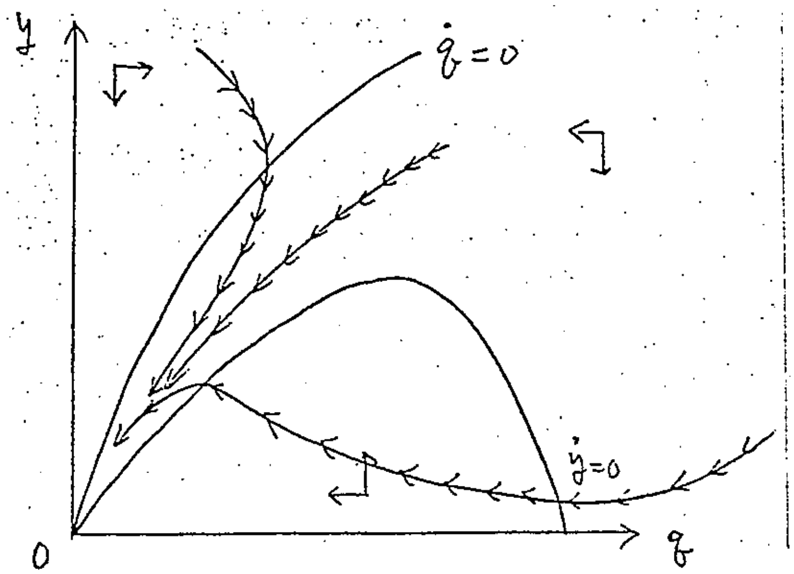


Fig. 9

Footnotes to Chapter IV

〔1〕 You could normalize y and q by deviding Y by Y(t,re(t)) and r by re(t) respectively if you wish.

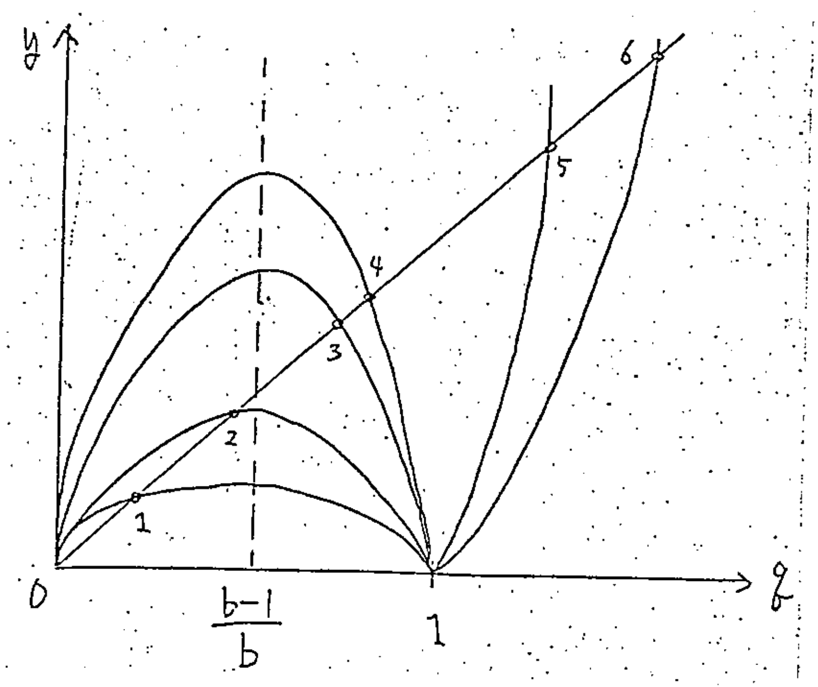
〔2〕 When a country is more outward-biased than SOBG, there will be no positive equilibrium point. See Theorem 7. In this country, if she adopts the flexible exchange rate system, the exchange rate appreciates infinitely and she suffers from economic decline. See the following figure.



〔3〕 Suppose α represents the fiscal policy instrument. Since

$$\frac{\partial \tilde{y}}{\partial \alpha} = -\beta(b-1)\tilde{A}^2\tilde{q}^{b-2} < 0,$$

a government can facilitate economic growth by decreasing α, provided the time path is stable. When α is decreased, the equilibrium point P moves along the external equilibrium line to the northeast (see the figure below), showing that y is increased. However, it is apparent that the risk of the instability of the system grows as well. Thus there is a trade off between economic growth and stability of the flexible exchange rate system as far as the fiscal policy is concerned.



Appendix

Table 3 makes a comparison between the model of this paper and some other relevant models. The table does not list many econometric or simulation studies. The classical partial-equilibrium analysis of the stability of the foreign exchange market (Marshall-Lerner, and Robinson [16]) was to an extent extended to a partially dynamic system by Britton [4] , and was combined with Keynesian multiplier analysis (Laursen-Metzler [11]) by Savosnik [17] and Alexander [1] . On the other hand, the dynamic nature of investment was taken into account by many authors (for example, Khang [9] , Moriguchi [13] , Bardhan-Lewis [3] and Fukuchi-Imagawa [7]), but they usually assumed that the balance of payments is in equilibrium. This paper has integrated these two streams into a fully dynamic model.

Table 3

	Model	Trade Sector	Income Sector	Capital Accumulat'n	Exchange Rate Determination
Balance of Payments; Either Balanced or Unbalanced	Laursen-Metzler (1950)		$Y=\alpha Y+\bar{I}+\bar{E}-mY$		
	Robinson Marshall-Lerner (1947)	$E=E(r,\frac{P_e}{P_m},Y_w)$ $M=M(r,\frac{P_e}{P_m},Y)$			
	Savosnik (1950) Alexander (1958)	$E=E(r,\frac{P_e}{P_m},Y_w)$ $M=M(r,\frac{P_e}{P_m},Y)$	$Y=\alpha Y+\bar{I}+E-M$		
	Britton (1970) Kogane-Kawana (1972)	$E=E(r,\frac{P_e}{P_m},Y_w)$ $M=M(r,\frac{P_e}{P_m},Y)$	$Y=\alpha Y+\bar{I}+E-M$		$\dot{r}=h(\frac{P_eE}{r}-P_mM)$
	The present model	$E=E(r,\frac{P_e}{P_w},Y_w)$ $M=M(r,\frac{P_e}{P_m},E,Y)$	$Y=\alpha Y+I+E-M$	$Y=\beta K$ or $I=\frac{1}{\beta}\dot{Y}$	$\dot{r}=h(\frac{P_eE}{r}-P_mM)$ or $r=r(t)$ or $r=F$
Balance of Payments; Balanced	Fukuchi-Imagawa (1975)	$E=E(r,Y_w)$ $M=\frac{P_eE}{rP_m}$	$Y=\alpha Y+I+E-M$	$Y=\beta K$ or $I=\frac{1}{\beta}\dot{Y}$	
	Bardhan-Lewis (1970)	$E=E(\frac{P}{rP_m},Y_w)$ $\dot{K}_f=\frac{PE}{rP_m}$	$Y=f(K_d,K_f,\bar{L})$ $\frac{P}{rP_m}=g(K_d,K_f,\bar{L})$	$\dot{K}_d=sY-\frac{\dot{K}_f rP_m}{P}$	
	Khang (1968)	$E=E(\frac{P_e}{rP_m},Y_w)$ $M=\frac{P_eE}{rP_m}, \frac{P_m}{P_e}=\gamma(\frac{Y}{M})$	$Y=f(K,\bar{L},M)$.	$\dot{K}=s(Y)$	
	Moriguchi (1974)	$E=E(\frac{P_e}{P_w},Y_w)$ $M=mY$	$Y=f(K,\bar{L},t)$	$\dot{K}=s(Y-E)$ $\frac{W}{P_e}=f_L[1-\frac{m}{\frac{P_er}{P_m}}]$	$r=\frac{P_eE}{P_mM}$

References

[1] Alexander, S.S., "Effects of a Devaluation: A Simplified Synthesis of Elasticities and Absorption Approaches," American Economic Review, March, 1959.

[2] Balassa, B., "Growth Strategies in Semi-Industrial Countries," Quarterly Journal of Economics, Vol.LXXXIV, No.1, Febuary, 1964.

[3] Bardhan, P. and S.Lewis, "Models of Growth with Imported Inputs," Economica, November, 1970.

[4] Britton, A.J.C., "The Dynamic Stability of the Foreign Exchange Market," The Economic Journal, Vol.LXXX, March, 1970.

[5] Emery, R.F., "The Relation of Exports and Economic Growth," Kyklos, Vol.XX ,1967.

[6] Fukuchi, T., "The One Gap Approach versus the Two Gap Approach," The Developing Economies, September, 1968.

[7] Fukuchi, T. and T.Imagawa, "Long-term Effects of Outward-Looking and Inward-Looking Growth Policies," The Economic Studies Quartely, Vol.XXVI, No.1, April, 1975.

[8] Kaldor, N., "Conflicts in National Objectives," The Economic Journal, March, 1971.

[9] Khang, C., "A Neoclassical Growth Model of a Resource-Poor Open Economy," International Economic Review, Vol.9, October, 1968.

[10] Kogane, Y. and E.Kawana,"Exchange Rate Fluctuation and the Balance of Payments under the Flexible Exchange Rate System," Keizaibunseki, Vol.40, June,1972(in Japanese).

[11] Laursen,S. and L.A.Metzler,"Flexible Exchange Rates and the Theory of Employment," Review of Economics and Statistics, November,1950.

[12] Lubitz,R.,"Export-Led Growth in Industrial Economies," Kyklos, Vol.XXVI, 1973.

[13] Moriguchi,C.,"Terms-of-Trade and Full Capacity Growth," Discussion Paper (Kyoto University),March,1974 (in Japanese).

[14] Mundell,R.A.,"The Monetary Dynamics of International Adjustment Under Fixed and Flexible Exchange Rates," Quarterly Journal of Economics, May, 1960.

[15] Mundell,R.A.,"The Appropriate Use of Monetary and Fiscal Policy for Internal and External Stability," IMF Staff Papers, March, 1962.

[16] Robinson,J.,"The Foreign Exchanges,"in Essays in the Theory of Employment (Second edition), 1947.

[17] Savosnick,K.M.,"National Income, Exchange Rate and the Balance of Trade," Economica, May,1950.

[18] Shinkai, Y.,"Problems Related to Measurement of Price Elasticities of Export and Import Demand," The Economic Studies Quartely, Vol.XXIV, No.2, August, 1973 (in Japanese).