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Abstract

Ambiguity aversion originally reported in conjunction with Ellsberg paradox appears to have established a law-like status in both empirical and mathematical researches that are mostly confined in the betting context. The present paper was designed to test its generality in a wider, natural context, using three psychological measures--belief, preference and willingness. Subjects responded to these measures in the choice problems of a hospital and a driving-school that differed in the locus of control over outcome uncertainties. The effects of framing and relativity of uncertainty were examined as the between-subjects factors. The results were in sharp contrast with the previous findings. That is, there is little evidence for ambiguity aversion, but ambiguity seeking and indifference were predominant. Ambiguity seeking was more pronounced in willingness than in belief. Through the MANOVA tests, the effects of framing and the relativity of uncertainty information were confirmed in belief and willingness, respectively. The problem effect was found not in these measures, but in qualitative willingness expressed in a binary mode. Finally, ambiguity seeking was also reflected in the preference rankings of the sites in which only the relativity effect was important. Further investigations are required to examine a) the dynamisms of the factors (externality-internality of uncertainty control, relativity of uncertainty information and positive-negative framing) upon belief, preference and willingness, and b) to what extent the present findings are attributable to the non-gambling, natural settings among other things.

Belief, Preference and Willingness under Ambiguity

The Ellsberg paradox (1961) has stimulated inquiries into the impact of uncertainty information on decision making, particularly that of nonspecific or ambiguous one. His subjects were asked to choose between two urns to draw a ball of a winning color. They both contained 100 balls of red and black, but with known (i.e., 50 each) and unknown proportions. Hence, the uncertainty evidence of the latter was ambiguous, implying maximally 51 possible combinations of the colors. Most of his subjects avoided the ambiguous urn when, say, red was a winning color despite color indifference initially expressed about both urns. According to the basic Savage's tenet (1954), they must have assigned a lower probability of drawing a red ball from the ambiguous urn than from the specific one. The ambiguity aversion persisted when a winning color was switched to black. Now, the same reasoning leads to a conclusion that the subjective probability of drawing a black ball from the ambiguous urn is also less than half. The complementary probabilities inferred from the choice behavior are apparently non-additive, i.e., a paradox.

The second setting of the Ellsberg's experiments demonstrated another violation of the Savage's theory. The urn used in this setting consisted of 30 red balls and 60 balls of black and yellow with unknown proportions. The Savage's sure-thing principle dictates that the choice between a red and a black ball (Bet 1) be uninfluenced by the (disjunctive) inclusion of a yellow ball to both alternative, i.e., red-or-yellow vs. black-or-yellow (Bet 2). Nevertheless, the data clearly contradicted the principle: While the majority preferred red to black in Bet 1, they reversed the choice in the second bet to bet on black-or-yellow with known chance (60 out of 90) than on red-or-yellow with partially specific chance (no less than 30 out of 90). Notice that the specificity of the alternatives comprised of red reduced from Bet 1 to 2 due to the inclusion of yellow, the opposite being true for black. Thus the violation occurred again in the direction of ambiguity aversion.

Ambiguity aversion seems to be fairly persistent. Even a sophisticated statistician like Raiffa (1975), an ardent proponent of the Savagean subjective theory, fell into another victim of the Ellsberg paradox. To make it worse, the tendency is hard to correct even when rational guidance is provided (Slovic and Tversky, 1974). Aversion to ambiguity is observable not only in the pair-wise choices, but in other response modes: preference ratings (Curley and Yates, 1985, 1989), premiums to play (Becker and Brownson, 1964; Raiffa, 1961), preference ranking of premiums (MacCrimmon and Larsson, 1979), payoff made proportional to the number of balls of a winning color (Becker and Brownson, 1964), minimum selling prices (Curley, Yates and Abrams, 1986; Yates & Zukowski, 1976), desired amount of payoffs (Yokoyama and Shigemasu, 1992), probability estimates of winning (Larson, 1980; Yokoyama and Shigemasu, 1992) and confidence in their estimates (Yokoyama and Shigemasu, 1992). A fairly extensive review of literature is available in Camerer and Weber (1992).

Ambiguity arising from the multiplicity of possible states, it is natural for researchers to operationalize the concept in the urn model as the range of the unknown proportion of chips or balls. To the extent that the wider range induces more ambiguity, ambiguity aversion would lead people to choose an alternative with a smaller range. The prediction was supported by the empirical studies under various range arrangements (Becker &

Brownson 1964; Curley & Yates, 1985, 1989; Larson, 1980). It should be noted that Yates and Zukowski's (1976) claim against the range-sufficiency hypothesis is unwarranted in light of their procedure for creating a 'uniform' second-order distribution (see also Gärdenfors & Sahlin, 1983). Indeed, Larson (1980) showed the effectiveness of the range size controlled for the probability of winning.

Another interesting question about ambiguity, besides the range effect, is the influence of the perceived chance of winning. Note that in the two-urn settings of the Ellsberg's (1961) and Becker and Brownson's (1964) studies, the midpoint of the ranges was set at .5. By systematically varying the midpoint and the size of ranges, Curley and Yates (1985) found the increase of ambiguity aversion as a function the midpoint only in comparisons of non- and ambiguous options, but not between ambiguous pairs. The result is compatible with those reported by Larson (1980) and Curley and Yates (1989). Hence, the complete specificity about a chance of winning appears to exert a strong impact on choice. However, such specificity is rare in natural settings, on account of the nonavailability of perfect information relevant to events of interest. The point bears great importance when one considers the generality of findings about choice behaviors under ambiguity in wider natural circumstances.

Uncertainties in non-gambling, real world problems stem from ill-defined processes. Hence, our knowledge about a propensity (see Popper, 1959, for the propensity interpretation of probability) often falls short of perfection in contrast to the urn and other gambling models where generating mechanisms are usually well known. In addition to the limitation about propensity, there is also a limitation in frequentistic knowledge, since even similar cases rarely qualify formally required identicalness. On account of these limitations, probabilistic information in the so-called real world problems is inherently ambiguous or vague. This may explain the prevalence of verbal approximations (see Budescu, Weinberg and Wallsten, 1988; Kuipers, Moskowitz & Kassirer, 1988; Wallsten, Budescu, Rapoport et al, 1986; Zimmer, 1983). There are also occasions where no approximations are possible either numerically or verbally, leading to total nonspecificity like the Ellsberg's ambiguous urn. Stemming from our general interest in everyday interpretations of uncertainty information, we will test the generality of ambiguity reactions beyond the urn-related settings, by incorporating four issues that have great psychological importance: 1) the relativity of evidential impact, 2) belief, preference and willingness in a choice scheme, 3) the framing effect, and 4) the externality-internality of uncertainty control.

Firstly, there seems to be two camps of researchers that are divided by their relativistic and non-relativistic views about the impact of uncertainty information. Those range-oriented researchers cited above belong to the latter on account of their tacit assumption that the 'decision weights' in computing the attractiveness of an alternative be influenced by the associated range and unaffected by the range of other alternatives. Non-relativistic views also appear in more formal models of Ellsberg's (1961), Einhorn and Hogarth's (1985, 1986,1990), Gärdenfors and Sahlin's (1982, 1983) in which decision makers are held to engage in mental simulations over possible or reasonable probability distributions. Beyond the differences in the decision strategies and the manner in which individuals' attitudes are incorporated, the common underlying premise is that a decision weight is uniquely determined by the uncertainty/ambiguity evidence, given ambiguity attitudes and, perhaps as Einhorn and Hogarth posit, the size and sign of the payoff.

The relativistic position has become popular since the publication of Schmeidler's theory (1982, later published in 1989) of non-additive probabilities and expected utilities by the use of Choqet integral. Other interesting applications include the works of Quiggin's (1982), Segal's (1987) and Tversky & Kahneman's (1992). The gist of the new approach is that the utility of a given outcome is not weighted solely by the associated uncertainty level, but rather by the cumulative (or *de*cumulative) uncertainty in a given prospect, provided that outcomes are rank ordered. Although this within-prospect relativity merits further explication, of no less importance is an inquiry into the between-prospect relativity for which we employ three attitudinal measures discussed below. For the sake of simplicity, we add a third alternative to the Ellsberg's two-urn setting translated into a real world problem. Then, reactions to ambiguity are expected to vary depending on the uncertainty level of a third alternative under the between-prospect relativity.

Secondly, when faced with alternatives to choose, a decision maker estimates how likely an event occurs in his or her own case, taking into account the provided information about uncertainty. The subjects in Larson's study (1980) expressed their own chances that were close to, but still at variance with the informed probabilities (see also Yokoyama & Shigemasu, 1992). Our interest is in analyzing ambiguity reactions from the 'beliefs' about one's own chance along with two other attitudinal measures--preference and willingness. Similarly to the conventional notion, we assume that one can rank order alternatives according to the overall evaluation of each alternative in terms of preference (see Sarin & Winkler, 1992, for an outcome-value related view of preference). Given three alternatives, the problem of ties in ranking can be avoided by asking the most and the least preferred ones as was done by Fischhoff (1983) for the evaluation of naturalness. The total ranking can be drawn form these responses with the assumption of transivity: One which is neither most nor least preferred falls second in the preference ordering. Beliefs and preferences may not provide sufficient force for the final decision. What is needed more is willingness to commit oneself to one's own choice, taking attributional concerns into account (e.g., Heath and Tversky, 1991; Winkler, 1991). Suffice it say here that preference is basically personal in nature, while willingness is social. To the extent that these attitudes reflect separate sets of factors, we can expect different patterns of reactions to ambiguity in them.

The third issue pertains to the effect of framing of a decision scheme by positive or negative phrasing. In their series of work, Kahneman & Tversky (1979) and Tversky and Kahneman (1981, 1986) demonstrated a reflection effect to result from 'framing' or 'phrasing' formally identical problems in positive and negative terms such 60% of lives saved vs. 40% lives lost. By this manipulation, risk-aversive and risk-seeking behaviors were induced in the domains of gains and losses, respectively. Although the universality or robustness of the framing effects is still controversial (see Cohen, Jaffray and Said, 1987; Fagley & Miller, 1987; Fischhoff, 1983; Hershey & Schoemaker, 1980; Rybash & Roodin, 1989), there are supportive findings from Kahn & Sarin's (1988) and McNeil, Pauker, Sox and Tversky's (1982) studies conducted in the contexts of consumer choice and medical treatment, respectively. Of the two, the former is particularly relevant to the present work because of its identification of a reversal of ambiguity reactions under positive and negative framing: In the positive domain, ambiguity aversion was dominant at high probabilities, while ambiguity seeking was observed at low probabilities. The tendency was reversed in the negative domain. Then, it is of interest to ask if the reflection effect observed in ambiguity premiums is generalizable to the attitude measures discussed above. They also observed contrasting ambiguity reactions across the problem contexts--e.g., ambiguity aversion in the context of pregnancy and seeking for radio warranty. The result raises a question about a factor or factors responsible for the context effect. The design of the present work is intended to offer a clue to the answer.

Finally, we address the issue of externality and internality of uncertainty, or more specifically the locus of control of uncertainty. The distinction was noted in the classification of uncertainty by Howell and Burnett (1978) as well as Kahneman and Tversky (1982). The context effect reported by Kahn and Sarin (1988) appears to illustrate the relevance of this distinction: The items with uncertainties people have little control are conducive to ambiguity aversion, while those with room for own efforts lead to ambiguity seeking. However, the overall tendency was not statistically confirmed due to a methodological reason. Our study is planed to provide a firmer test on the importance of externality-internality distinction. To this end, we selected the choice of a hospital and a driving school. In the former choice, it is not the patient but a doctor (or a medical team) in charge of the surgery who possesses the control over uncertainties about the outcome. Hence, the control is external to a patient. The latter choice also involves great amount of uncertainties in achieving a goal of obtaining a license, if time is very limited. (In Japan, unlike U.S., virtually everyone needs to go to a driving school from the very beginning to the end.) Nonetheless, one can hope that one's own skills and efforts help reduce the uncertainties.

It is interesting to note that the past ambiguity studies on the real world problems employed betting procedures of one form or another (e.g., Curley, Eraker & Yates, 1984; Goldsmith & Sahlin, 1983; Heath & Tversky, 1991; MacCrimmon & Larsson, 1979), with an exception of Jagacinski and Faber's (1978) analysis. Put modestly, this is an important step toward understanding the nature of epistemological, or judgmental uncertainties in contrast to aleatory ones (see Hacking, 1975). Still, we wonder how often we encounter with natural situations that involve betting. Many volunteers in our pilot study indeed complained the difficulty with such procedures. The problems we selected offer a new experimental setting in this regard, being devoid of betting elements. This allows subjects to concentrate on epistemological uncertainties. The details of the present design is explained in the next section.

Method

Subjects

179 subjects were recruited from the introductory psychology courses at the University of Tsukuba and Gakushuin University, in Japan. They participated in the experiment as a part of class assignments.

Design

The subjects were randomly assigned to a group in a 2 x 2 (Framing x B-level) design and were asked to answer two sets of choice problems. The two framing conditions differed in the manner in which the outcome uncertainty was phrased, either positively (PF: positive framing) or negatively (NF: negative framing). Under the PF condition, subjects were informed of chances of successful outcome as opposed to chances of unsuccessful outcome under the NF condition.

The second factor, B-level, pertains to the uncertainty level associated with one of three alternative sites to make judgments: hospitals for a treatment of a tennis elbow (H-Problem); and driving schools (D-Problem). The sites will be referred to as Sites A, B and C below. They differed in the level or specificity of the outcome uncertainty derived from the past records and expertise. Subjects were informed that they would have, at Site A, 50% chance of success (PF) or failure (NF). The informed chance about Site C was ambiguous: The outcome of their cases was contingent upon many factors, and, hence, no specific estimate could be provided. The figures for Site B, i.e., B-level, varied across groups in two levels: 70% or 55% for PF, and 30% or 45% for NF. For the ease of presentation, the four groups will be denoted hereinafter as PF70, PF55, NF30 and NF45 to denote the framing condition and the B-level. To summarize, the subjects in all groups received a chance list [50%, (50+x)%, *%] for Sites A, B and C, in which the value of x was either 20 and 5 for PF (-20 and -5 for NF) and * denotes nonspecificity.

Procedure

The subjects in all groups first answered questions in the hospital problem with the following description of the situation (for Site A of PF70 and PF55):

> *You have developed a tennis elbow which causes occasional pains, though not severely acute. With the current medical technique, complete cure is not guaranteed and you may suffer from an aftereffect that could cause trouble in driving a car. At the hospital you first visited you are told that the chance of successful treatment is 50%, estimated on the basis of the past records of similar cases and expertise.*

The chance levels for Site B and C were modified accordingly. The outcome was phrased as the unsuccessful treatment for groups NF30 and NF45.

In Question 1.1 they were asked to evaluate their own odds of successful against unsuccessful outcome at this hospital (Site A) expressed in two numbers, i.e. success : failure. Also asked were their willingness in a binary form (yes/no) of undergoing an operation at this site (Question 1.2). After answering the same questions about Sites B (Questions 2.1, 2.2) and C (Questions 3.1, 3.2), they were requested to express their willingness in Question 4 about the three sites on a continuous scale, in the interval of [0.0,1.0]. Finally, subjects were requested to name the most (Question 5) and least (Question 6) preferable sites among the three. The two questions were prepared to avoid the possibility of ties in ranking.

Following the hospital (H-) problem, the subjects answered the driving-school (D-) problem in which the situation was about obtaining a license within a month. The Questions 1 through 6 were essentially identical to those used in H-Problem, except minor adjustments in wording. The cost at the three alternatives was described in both problems to be approximately 200 thousand yen (about 1.6 times the monthly income of a new college graduate then).

Measures of Ambiguity Attitudes

Since our primary interest is in testing ambiguity attitudes between alternatives that are normatively equivalent, we will focus on the comparison of responses to Sites A and C about which 50:50 and ambiguous chances were informed, respectively. The assumption of normative equivalence is based on the principle of insufficient reason (see Hacking, 1975), and is expected to hold collectively. Ambiguity attitude measures explained below reflect the degree or pattern of ambiguity seeking relative to aversion.

Belief --The odds estimates of success to failure for Site A in Question 1.1 was first converted into a single score by Success/(Success + Failure). The same was done to the estimates for Site C in Question 3.1. The ambiguity measure in belief was then computed by subtracting the score for Site A from that to Site C. For convenience, the measures in the H- and D-Problems will be respectively referred to as Bel.HCA and Bel.DCA hereinafter. One is ambiguity seeking in belief concerning H-Problem, if Bel.HCA is positive. Similarly, one is ambiguity aversive if Bel.HCA is negative, and neutral or indifferent, if it is zero. The same holds for Bel.DCA in D-Problem.

Willingness--There are two kinds of willingness measures, one being binary and the other continuous. Because of similarity to the belief measures, the latter will be explained first.

Ambiguity attitudes in willingness on a continuous scale were derived by subtracting the score of Site A from that of Site C in Question 4. The measures in H- and D-Problems will be referred to as Will.HCA and Will.DCA, respectively. Like Bel.HCA and Bel.DCA, they reflect one's ambiguity seeking relative to aversion.

Binary responses (yes/no) to Sites A and C in Question 1.2 and 3.2 were jointly recoded to create qualitative ambiguity attitudes in willingness: One was classified as ambiguity seeking if a positive response ('yes') to Site C was accompanied by a negative ('no') one to Site A. The reverse response pattern was coded as ambiguity aversion. Responses of the same kind were treated as ambiguity indifference. The measures in H- and D-Problems will be called Will#.HCA and Will#.DCA, respectively.

Preference Ranking--Preference rankings were constructed from responses to Questions 5 and 6 on the assumption that an unreported Site would have fallen second in ranking. For instance, a report of Sites C and A as the most (Question 5) and the least (Question 6) preferable ones was coded as ranking CBA. One is ambiguity seeking (or aversive) in preference, if C (or A) precedes A (or C) in ranking. Since no ties occur in this ranking, indifference needs no consideration.

AIC and Model Selection

A note seems in order about a model selection criterion called *AIC* , or Akaike Information Criterion, (Akaike, 1973, 1974, 1977; Sakamoto, Ishiguro and Kitagawa, 1986) which is fairly popular in Japan and is to be used in the present analysis of qualitative willingness measures (Will#.HCA and Will#.DCA) and preference rankings.

The basic tenet of *AIC* is that one can increase the fit of a model by increasing the number of parameters, but at a cost of the bias in the maximum likelihood estimate of the model. The simplicity of the idea is nicely reflected in its form:

$$AIC = -2 \log_e LR + 2\, df$$

where LR and df respectively denote the maximum likelihood of a model and the number of parameters comprising it. Also simple is the selection principle on account of the smallness of bias: the smaller the AIC, the better the model. Further attractiveness accrues from its relation to the likelihood-ratio chi-square statistics (LR). In short, LR associated with a model corresponds to the difference of AIC of the (partial) model and that of a fully specified model. More specifically:

$$LR + 2\, df = AIC_{\text{Partial}} - AIC_{\text{Full}}$$

where df denotes the degree of freedom of the (partial) model.

Now, we will illustrate the applicability of AIC to log-linear modeling (see Matsuda, 1988 as well as Sakamoto, Ishiguro and Kitagawa, 1986). The technique is suitable for analyzing the relationships of variables in a multiway contingency tables. It expresses the log of the expected frequency of an internal cell as a linear function of effect parameters. For instance, a full (or saturated) model for a three-way (V x W x Z) table becomes:

$$\log m_{ijk} = u + u^{V}_{i} + u^{W}_{j} + u^{Z}_{k} + u^{VW}_{ij} + u^{VZ}_{ik} + u^{WZ}_{jk} + u^{VWZ}_{ijk}$$

where m_{ijk} denotes the expected frequency of the cell (i,j,k). The first u-parameter without a subscript serves the reference level for measuring the subsequent effects, i.e., the main effects (u^{V}_{i}, u^{W}_{j}, u^{Z}_{k}), the first- and the second-order interactions (u^{VW}_{ij}, u^{VZ}_{ik}, u^{WZ}_{jk}; u^{VWZ}_{ijk}) (e.g., Bishop, Fienberg and Holland, 1975; Goodman, 1978; Matsuda, 1988). A partial model to be tested can be constructed by eliminating unessential effects under a given hypothesis. The hierarchy principle in model construction makes substantive interpretations easy, although it is not mandatory. We will use LR for the selection of competing candidate models from which a final one is to be chosen on the basis of AIC .

Results

This section presents analysis of ambiguity attitudes in belief, willingness and preference, using the measures explained in the method section.

Belief

As shown in Table 3.1.1, the mean scores of Bel.HCA (PF70: 0.022, NF30: 0.037, NF45: 0.063) and Bel.DCA (PF70: 0.022, NF30: 0.093, NF45: 0.067) are all positive except for PF55 (-0.021, -0.003), indicating the general tendency toward ambiguity seeking in both H- and D-Problems. The tendency is more pronounced under the negative than the positive framing conditions: Bel.HCA of NF45 ($p < .01$) as well as Bel.DCA of NF30 ($p < .01$) and NF45 significantly differ from zero ($p < .05$), whereas none of the scores of PF70 and PF55 are significant at the 5% level. This is reflected in the 95% confidence intervals listed in Table 3.1.1. Those associated with the significant mean scores fall entirely in the positive domain, while nonsignificant ones extend over both domains (see also Figure 3.1.1 which also shows the mean scores of willingness). Although Bel.DCA is greater than or equal to Bel.HCA in all groups, the highly over-lapping confidence intervals suggest a weak or a lack of the problem effect. The framing and problem effects are to be examined more closely below in light of the uncertainty level about Site B (B-level).

Table 3.1.1

Mean Belief Scores and 95% Confidence Intervals by Problem and Group

Belief	Group (N)	Mean	Standard Deviation	95% Confidence Interval	
Bel.HCA					
	PF70 (45)	0.022	0.178	-0.031	0.076
	PF55 (46)	-0.021	0.146	-0.064	0.022
	NF30 (40)	0.037	0.180	-0.020	0.095
	NF45 (45)	0.063**	0.138	0.021	0.104
Bel.DCA					
	PF70 (45)	0.022	0.168	-0.029	0.072
	PF55 (46)	-0.003	0.172	-0.054	0.048
	NF30 (40)	0.093**	0.172	0.038	0.148
	NF45 (45)	0.067*	0.201	0.007	0.128

Note: * ($p < .05$), ** ($p < .01$).

MANOVA was conducted on Bel.HCA and Bel.DCA as the within-subject factor in order to test the problem effect with Framing and B-level as the between-subjects factors. The homogeneity hypothesis about variance is not to be rejected (Box's M = 11.677, p = .247). The test results of between-subjects effects are summarized in Table 3.1.2. Only the main effect Framing is significant ($F(1,172)=10.20$, $p=.002$): The negative framing induced higher mean ambiguity seeking (0.065) than positive framing (0.005). The problem effect ($F(1,172)=1.23$, $p=.268$) and none of its interactions with the between-subjects factors are significant ($p \geq .268$).

Table 3.1.2

Between-Subjects Effects on Belief

Source of Variance	*SS*	*df*	*MS*	*F*	*p*
Within Cells	5.34	172	0.03		
Framing	0.32	1	0.32	10.20	.002
B-level	0.03	1	0.03	0.81	.369
Framing x B-level	0.02	1	0.02	0.80	.371

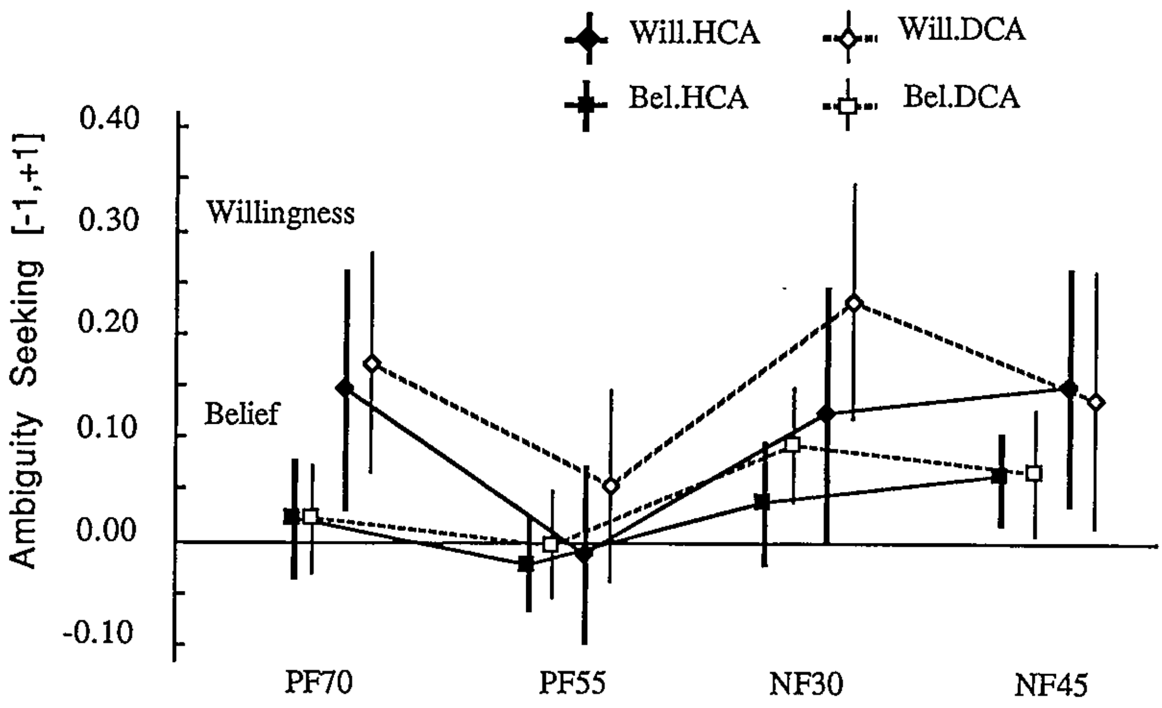

Figure 3.1.1 Ambiguity attitudes in belief and willingness by problem and group: Mean scores with the 95% confidence intervals

The lack of problem effect, however, does not necessarily mean that the subjects were inattentive to the substantive differences of the problems. Their responses to the two problems are rather independent as reflected in the small and nonsignificant correlation coefficients between Bel.HCA and Bel.DCA (.230, .241, -.038 and -.137 in order for PF70, PF55, NF30 and NF45).

Willingness

Shown in Table 3.2.1 are the mean scores of Will.HCA and Will.DCA with the associated standard deviations and the 95% confidence intervals. The overall pattern resembles that of belief (see Figure 3.1.1), but the tendency toward ambiguity seeking is more pronounced. All the positive mean scores of Will.HCA (0.148, 0.125, 0.152 in order for PF70, NF30 and NF45) and Will.DCA (0.174, 0.235, 0.139 in order for PF70, PF55, NF30 and NF45) significantly differ from zero ($p < .05$), as reflected in their associated 95% confidence intervals that lie entirely in the positive domain. Neither of the negative means of PF55 (Will.HCA: -0.011, Will.DCA: -0.054) is statistically significant, indicating ambiguity indifference.

MANOVA was conducted on Will.HCA and Will.DCA as the within-subject factor in order to test the problem effects in light of the between-subjects factors--Framing and the B-level. The homogeneity hypothesis about variance is not to be rejected (Box's M = 10.095, $p = .360$). The test results of between-subjects factors are summarized in Table 3.2.2. In contrast to the belief measures, only the main effect B-level is significant $F(1,172)=3.96$, $p=.048$): The relatively nonuniform set [50,50±20,*] induced greater ambiguity seeking (0.170) than the uniform set [50,50±5,*]. Neither Framing ($F(1,172)=2.68$, $p=.104$) nor the interaction Framing x B-level are ($F(1,172)=1.44$, $p=.232$). The problem effect ($F(1,172)=2.19$, $p=.141$) and none of its interactions with the between-subjects factors are significant ($p \geq .201$).

The correlation coefficients between Will.HCA and Will.DCA were computed to examine whether the lack of problem effect was attributable to the similarity of responses to the two problems. The low and nonsignificant coefficients ($p > .5$) for PF70 (.287), PF55 (.276) and NF30 (.191) indicated independent responses in general. Even the significant one for NF45, the dependency is only intermediate in degree (.452, $p=.002$).

Belief and Willingness

The patterns in Figure 3.1.1 suggests a higher degree of ambiguity seeking in willingness than in belief. However, the absence of homogeneity of variance among the four groups when these measures were considered together (Box's M=57.362, $p=.004$) made it necessary to put limitations on the comparison of the means. Among other possibilities, four pair-wise comparisons were conducted by MANOVA in view of our interest in the framing (PF70 vs. NF30, PF55 vs. NF45) and B-level effects (PF70 vs. PF55, and NF30 vs. NF45). The hypothesis of homogeneity-of-variance is not to be rejected concerning the first and the last pairs (Box's M=6.359, $p=.813$; 16.820, .102, respectively) in which the between-subjects factor Belief-Willingness is significant (PF70 vs. NF30: $F(1, 82)=22.43$, $p=.000$, and NF30 vs. NF45: $F(1, 81)=11.42$, $p=.001$). No other main or interaction effects are significant ($p \geq .175$).

Table 3.2.1

Mean Willingness Scores with 95% Confidence Intervals by Problem and Group

Willingness Group (N)	Mean	Standard Deviation	95% Confidence Interval	
Will.HCA				
PF70 (46)	0.148*	0.385	0.034	0.262
PF55 (46)	-0.011	0.281	-0.094	0.072
NF30 (40)	0.125*	0.379	0.004	0.246
NF45 (44)	0.152**	0.369	0.040	0.265
Will.DCA				
PF70 (46)	0.174*	0.357	0.068	0.280
PF55 (46)	-0.054	0.309	-0.037	0.146
NF30 (40)	0.235**	0.359	0.120	0.350
NF45 (44)	0.139**	0.407	0.015	0.262

Note: * ($p < .05$), ** ($p < .01$).

Table 3.2.2

Between-Subjects Effects on Willingness

Source of Variance	*SS*	*df*	*MS*	*F*	*p*
Within Cells	28.73	172	0.17		
Framing	0.45	1	0.45	2.68	.104
B-level	0.66	1	0.66	3.96	.048
Framing x B-level	0.24	1	0.24	1.44	.232

Qualitative-Willingness

Among the three categories (Ambiguity Seeking, Indifference and Ambiguity Aversion), indifference is the modal response, in both problems and for all groups, ranging from 52.5% (Will#.DCA of NF30) to 80% (Will#.HCA of NF45) of the observed cases. An indication of the limited problem effect is found in the relative dominance between ambiguity seeking and aversion in the nonmodal responses. In Will#.DCA ambiguity seeking exceeds aversion in proportion across groups, whereas the reverse is true for Will#.HCA in PF70, PF55 and NF30. The exception to the latter is Will#.HCA in PF45 that showed ambiguity seeking. Log-linear analysis was performed on the four-way table (Will#.HCA x Will#.DCA x Framing x B-level) to examine the tendencies in population.

In the initial model fitting, we first tested a model with two principal main effects, Will#.HCA and Will#.DCA. Then, other main-effects-only models were tested, by adding either Framing or B-level, or both. For the sake of simplicity, the ambiguity attitudes will be referred to as HCA and DCA below.

The fit of these models, listed in Table 3.3.1, is reasonable in terms of the probabilities ($p \geq .478$) for the likelihood-ratio chi-square statistics ($LR(31)=28.98$, $LR(30)=28.958$, $LR(30)=28.773$ and $LR(30)=28.750$ in order of Models 1, 2, 3 and 4). The *AIC* of Model 1, [HCA DCA], is smallest (128.650). Removal of either HCA or DCA from these models greatly reduce the fit ($p < .01$). On the other hand, addition of an interaction term HCA x DCA to Model 1 improves the fit ($p = .728$), but at the cost of increase in *AIC* (129.816). Since there is no compelling reason to test more complicate models with other interactions, the model [HCA DCA] is to be selected as the final model on the basis of minimal *AIC*.

Table 3.3.1

Fit of Loglinear Models for Qualitative Willingness

Model	*LR*	*df*	*p*	*AIC*
1: HCA DCA*	28.982	31	.570	128.650
2: HCA DCA Framing	28.959	30	.520	130.532
3: HCA DCA B-level	28.773	30	.530	130.441
4: HCA DCA Framing B-level	28.750	29	.478	132.381

Note: The accepted model is marked by *.
LR = likelihood-ratio chi-square.
AIC=Akaike Information Criterion

Table 3.3.2

Qualitative Willingness in Model [HCA DCA]

Ambiguity Attitudes	Relative Frequency	Effect Parameter	Standard Error.	Standardized Value
HCA	(N=172)	($\hat{u}^{HCA}$)		
(1) Aversive	12.8%	.000[a]	---	---
(2) Indifferent	74.4	1.761*	.231	7.630
(3) Seeking	12.8	.000	.302	.000
DCA	(N=172)	($\hat{u}^{DCA}$)		
(1) Aversive	6.4	.000[a]	---	---
(2) Indifferent	67.4	2.356*	.315	7.467
(3) Seeking	26.2	1.409*	.336	4.188

Note: Significant parameters are marked by * ($p < .01$).
a: The reference category is 1 for both HCA and DCA, i.e.,
$u^{HCA}_1 = u^{DCA}_1 = 0$ under the present dummy coding.

Under the accepted model, the log of the expected frequency of cell (i,j,k,l) is expressed as

$$\log m_{ijkl} = u + u^{HCA}_{i} + u^{DCA}_{j}$$

where i,j = 1,2,3 correspond to ambiguity aversion, indifference and seeking, respectively. By setting cell (1,1,1,1) as the reference point,

$$\log m_{1111} = u; \;\; u^{HCA}_{1} = u^{DCA}_{1} = 0$$

obtain under the dummy coding scheme (see Matsuda, 1988). Thus, the main effect parameters for the second and the third categories, (u^{HCA}_{i} , u^{DCA}_{j} , i=2,3, j=2,3), express the extent of indifference and ambiguity seeking relative to aversion.

Shown in Table 3.3.2 are the main effect parameters and the marginal (relative) frequencies for HCA and DCA from which internal expected frequencies in population are reproducible. As noted earlier about the observed frequencies, indifference is predominant in both H- and D-Problems (74.4, 67.4%). The two problems, however, greatly differ in the ratio of ambiguity seeking to aversion as reflected in the main effect parameters ($\hat{u}^{HCA}_{3}$=.000, $\hat{u}^{DCA}_{3}$=1.409): While aversion and seeking are equally likely in the H-Problem, the latter is 4.092 (=exp($\hat{u}^{DCA}_{3}$)) times more likely than the former in the D-Problem.

Preference

Ambiguity seeking rather than aversion is found in the observed ranking patterns of the primary choice of sites: The combined proportions of CAB and CBA are greater than those of ABC and ACB in all groups. The former ranges from 19.5% of cases (NF30 in H-Problem) to 60.5% (NF45 in D-Problem), whereas the latter is less than 10% in all groups. In order to examine the tendency in population, log-linear analysis was performed on the four-way table (Ranking x Problem x Framing x B-level). Preference ranking being our major concern, the main effect Ranking was included in all the tested model. To simplify the presentation, the variables will be denoted by their initial letters below: R, P, F and B for ranking, problem, framing and B-level, respectively.

The fit of the main-effects-only models was poor ($p < .01$). Hence, a single interaction was alternately added in the second testing stage to the four main effects. Reasonable fit was obtained only from the model with the interaction R x B (LR (34)=32.272, $p = .552$). The fit of the other single-interaction models was poor ($p < .01$). In search of a simpler model, three were created by eliminating either P or F, or both, from the above one [R P F B RxB] in compliance with the hierarchy principle. Their fit was reasonable ($p \geq .552$) as shown in Table 3.5.1, making it necessary to rely on *AIC* for the selection of a final model. The model [R B RxB] is most parsimonious in terms of both *AIC* (209.253) and the number of constituent terms. Hence, this is to be accepted as the final model which requires only B-level for explaining the ranking pattern. We revert to full naming of the variables below for the ease of exposition.

Table 3.5.1

Fit of the Candidate Models for Preference Ranking

Model		*LR*	*df*	*p*	*AIC*
1: R P F B	R x B	32.272	34	.552	212.800
2: R F B	R x B	32.279	35	.600	210.779
3: R P B	R x B	32.717	35	.579	211.287
4: R B	R x B*	32.723	36	.625	209.253

Note: The accepted model is marked by *.
LR = likelihood-ratio chi-square. *AIC*=Akaike Information Criterion. R=Ranking, P=Problem , F=Framing and B=B-level .

Table 3.5.2

Proportion of Ranking-Patterns by B-level

Ranking Pattern	B-level ±20 (PF70, NF30)	B-level ±5 (PF55, NF45)
(1) ABC	2.84%	4.84%
(2) ACB	3.13	2.28
(3) BAC	26.14	36.47
(4) BCA	35.80	7.41
(5) CAB	5.40	9.12
(6) CBA	26.70	39.89
Total	100.00 (N=176.0)	100.00 (N=175.5)

Note: The frequencies (N) differ from the actually observed counts due to 0.5 added to the empty cells in the four-way table.

The accepted model requires only the marginal table Ranking x B-level in explaining the internal patterns of the original four-way table (Ranking x Problem x Framing x B-level) in population. That is, the proportions of the six ranking patterns listed in Table 3.5.2 for the two levels about Site B is sufficient for the analysis. In what follows, we will first examine the attractiveness of A and C as the most and least preferred sites.

First, the proportions of the patterns CBA (26.70, 39.89% for the ±20 and ±5 levels) and CAB (5.40, 9.12%) are greater than those of ABC (2.84, 4.84%) and ACB (3.13, 2.28%), reflecting the general tendency of ambiguity seeking rather than aversion in the primary choice. The sharp contrast between the high level of CBA and the low level of CAB is attributable to the attractiveness of Site B as a secondary choice relative to A. Between the two uncertainty sets [50, 50±20,*] and [50, 50±5,*], the latter that is relatively uniform induced the higher proportions of the primary choice of C (49.01%), i.e., ambiguity seeking, than the former (32.1%). It must be noted that the reverse pattern of the relativity effect was observed in the willingness measure.

Ambiguity aversion exceeds seeking in proportion only in the comparison of BAC (36.47%) and BCA (7.41) for the '±5' B-level groups (PF55, NF45). The reverse (26.14, 35,8%) holds for the '±20' B-level groups (PF70, NF30), but the difference is much smaller. The odds of choosing A against C given B as the primary choice are 4.922 and 0.730 for the '±5' and '±20' B-level groups, respectively. The odds-ratio (-1.908) is the only significant interaction at the 5% level when the reference cell is set to (3,1,1,1) in the four-way table (Ranking x Problem x Framing x B-level).

Finally, the proportions of CBA (26.7, 39.89% for the ±20, ±5 B-levels, respectively) and BAC (26.14, 36.47%) are comparable and the estimated interaction parameter for them ($\hat{u}^{RB}_{62}$ =0.068) is nonsignificant ($p > .05$) with the same setting of the reference cell as above. This is an indication that the attractiveness of Site C as the primary choice could reach that of Site B at both B-levels.

Discussion

The results of the present analysis are surprising in view of the law-like ambiguity aversion established in the past studies most of which employed betting procedures. Our subjects displayed, in response to real-world-like choice problems, either indifference or inclination for ambiguity (or, more precisely, for ambiguous alternatives) in the belief, preference and willingness measures. Even the qualitative willingness measure in which indifference was predominant revealed a high ratio of ambiguity seeking pattern to aversion in the driving school problem. However, the results were not uniform across the experimental conditions, i.e., framing, the problem type, and the relativity of informed chance levels: The effects of outcome framing and the problem type were found in belief and qualitative willingness measures, respectively; and the relativity effects were observed in willingness and preference, but in different directions. In other words, each measure was sensitive to a single effect, eliciting a particular mode of information processing. This provides, in a broad sense, further evidence for the constructive nature of human judgment that has been revealed especially in studies on preference reversals (Lichtenstein and Slovic, 1971; Slovic and Lichtenstein, 1968, 1983; Slovic, Griffin and Tversky, 1990; Tversky, Sattath and Slovic, 1988). The phenomenon originally (see, Lichtenstein and Slovic, 1971) referred to the reversals of preferences that were inferred from choice and bidding. Normatively one is expected to show consistency in these behavior when presented with a pair of bets, provided that they both correctly reflect one's underlying preference. However, people often choose a bet with a higher probability of winning, but assign a larger amount of money to the unchosen bet.

Of particular interest to us is an early finding that choice is under the influence of probabilities (Slovic and Lichtenstein, 1968). The preference ranking in the present work was constructed from responses to the most and least preferable sites, i.e., positive and negative choices. In the preference-reversal research, not only choice but preference or attractiveness ratings are also known to cause the phenomenon (see Slovic and Lichtenstein, 1968; Goldstein and Einhorn, 1987; Johnson, Payne and Bettman, 1988; Schkade and Johnson, 1989). We also found that the quantitative measure of ambiguity attitudes, i.e., willingness, captured the relativity of the probabilistic information along with the preference ranking. To the extent they reflect the same underlying mental states or processing, the relativity should have taken effect in the same direction. However, as reported in the previous section, the nearly uniform set [50,50±5,*] induced a higher proportion of ambiguity seeking in the primary choice, whereas the nonuniform set [50,50±20,*] yielded greater seeking in the mean willingness scores. There are two possible reasons for this discrepancy, one substantive and the other methodological.

A substantive account concerns the mental processes in expressing choice and willingness. We had an implicit assumption in designing the present study a) that a subject would form for each alternative willingness to commit him/herself to (or accept) it and belief about the likeliness of good outcome in his/her case, and b) that he/she makes a choice by comparing alternatives evaluated in these dimensions. That is, willingness and belief would reflect consideration of a single alternative, while multiple alternatives would be considered for the final choice. This account may apply to the lack of relativity effect on belief, but not to the its presence on willingness. Before we change our assumption about willingness, a methodological account must be sought. The willingness measure analyzed here was actually derived from the related responses about Sites A and C. Hence, the observed tendency could have been resulted from mild, or no relativity effect on one constituent response and strong one on the other. This possibility is currently examined and will be reported subsequently.

Framing effect is perhaps one of the most popular phenomenon in the current psychology, but at the same time its generality has remained controversial as discussed in introduction. Our findings about its presence in belief and lack in preference suggest that preference or choice is not the sole mode of behavior where it takes effect. Thus, belief need to be inspected when framing is tested. Nonetheless, in our approach, the same methodological issue raised about willingness need to be clarified in future, i.e., the analyzed belief measure was derived from the related responses to Sites A and C. It is of great interest to examine the framing effect on its constituents.

The major motive to have incorporated belief here was to inspect how one perceives ones own chance in a single-time trial to which a piece of probabilistic best serves the starting point of anchoring and adjustment in judgment, the process first suggested by Lichtenstein and Slovic (1971) and subsequently studied mostly in research on preference reversals. Taking into account the distributional nature or reasons mentioned earlier in discussion about uncertainties, one may arrive at an estimate that deviates from the provided information. In fact, only 22% to 36% of our subjects equated their estimates with the provided one concerning Site A (i.e., .50). How about the estimates about Site C for which no specific figure was given? Interestingly, the proportion of subjects who assigned .50 to Site C, (or more correctly odds of 1) ranged from 11% to 54%. Moreover, the proportions about C were comparable to those about A in most conditions. Hence, Sites A and C were collectively equivalent in terms of these proportions. An experiment is presently designed to shed light on the anchoring and adjustment processes, using the HyperScaling technique (Matsuda, 1993).

Finally, the most fundamental question has not been explored yet. That is, why did our subject show the lack of ambiguity aversion? Some of them commented that they valued room for own-efforts and luck in the nonspecific uncertainties. Apparently, this is not the general attitudes in our sample in view of the limited problem effect pertaining to the external-and-internal control of uncertainty. A cultural-bias is an easy hypothesis, but the last resort to us, since nothing is eventually clarified unless one extends an explanation to the components of cultures with respect to information processing in particular. Also, it is incompatible with the findings in betting (Yokoyama and Shigemasu, 1992). In order to make an answer applicable to the abundant findings of ambiguity aversion in the past research, we need to explicate more about the human perceptions and evaluations of uncertainties in betting and non-betting situations. One of the key elements to be considered in such comparisons is the control of attractiveness between the objects or states per se, such as color indifference in the Ellsberg's experiments, that is hard to implement in non-betting experiments.

REFERENCES

Akaike,H. (1973) Information theory and an extension of the maximum likelihood principle. In Proceedings of the 2nd International Symposium on Information Theory (N. Petrov and F. Csàdki (eds.)). Budapest:Akadémiai Kiadó, pp.267-281.

Akaike, H. (1974) A New look at the statistical model idetification. *IEEE Transactions on Automatic Control*, AC-19, 716-723.

Akaike, H. (1977) On entropy maximization principle. In P.R. Krishnaiah (ed.) *Aplication of statistics*. North-Holland, pp.27-41.

Becker, S.W., & Brownson, F.O. (1964) What price ambiguity? Or the role of amibiguity in decision-making. *Journal of Political Economy, 72*, 62-73.

Budescu, D.V., Weinberg, S., & Wallsten, T.S. (1988) Decisions based on numerically and verbally expressed uncertainties. *Journal of Experimental Psychology*, **14**, 281-294.

Cohen, M., Jaffray, J-Y., & Said, T. (1987) Experimental comparison of individual behavior under risk and under uncertainty for gains and for losses. *Organizational Behavior and Human Decision Processes*, **39**, 1-22.

Curley, S.P., Eraker, S.A. & Yates, J.F. (1984) An investigation of patients' reactions to therapuetic uncertainty. *Medical Decision Making*, **4**, 501-511.

Curley, S.P., Yates, J.F., & Abrams, R.A. (1986) Psychological sources of ambiguity avoidance. *Organizational Behavior and Human Performance*, **38**, 230-256.

Ellsberg, D. (1961) Risk, ambiguity, and the Savege axioms. *Quarterly*

Journal of Economics, 75, 643-669.

Einhorn, H.J., & Hogarth, R.M. (1985) Amibiguity and uncertainty in probabilisti inference. *Psychological Review, 92*, 433-461.

Einhorn, H.J., & Hogarth, R.M. (1986) Decision making under ambiguity. *Journal of Business*, **59**, S225-S260.

Einhorn, H.J., & Hogarth, R.M. (1976) Confidence in judgement: Persistence of the illusion of validity. *Psychological Review*, *85*, 395-416.

Fishburn, P.C. (1983) Ellsberg revisited: A new look at comparative probability. *The Annals of Statistics*, **11**, 1047-1059.

Fischhoff, B. (1983) Predicting frames. *Journal of Experimental Psychology*, **9**, 103-116.

Fagely, N.S., & Miller, P.M. (1987) The effects of decision framing on choice of risky vs. certain options. *Organizational Behavior and Human Decision Processes*, **39**, 264-277.

Gärdenfors, P., & Sahlin, N. (1982) Unreliable probabilities, risk taking, and decision making. *Synthese, 53*, 361-386.

Gärdenfors, P., & Sahlin, N. (1983) Decision making with unreliable probabilities. *British Journal of Mathematical and Statistical Psychology*, **36**, 240-251.

Goldstein, W.M., & Einhorn, H.J. (1987) Expression theory and the preference reversal phenomenon. *Psychological Review*, **69**, 623-638.

Goldsmith, R.W., & Sahlin, N-E. (1983) The role of second-order probabilities in decision making. In P. Humphreys, O. Svenson, A. Vari (eds.) Analysing and aiding decision processes. Amsterdam: North-Holland, pp. 455-467.

Goldstein, W.M., & Einhorn, H.J. (1987) Expression theory and the preference reversal phenomenon. *Psychological Review*, **69**, 623-638.

Hacking, (1975) *Emergence of probability.* London: Cambridge University Press.

Hazen., G.B., & Lee, J-S. (1991) Ambiguity aversion in the small and in the large for weighted linear utility. *Journal of Risk and Uncertainty,* **4**, 177-212.

Hershey, J.C., & Shoemaker, P.J.H. (1980) The reflection hypothesis. *Organizational Behavior and Human Performance,* **25**, 395-418.

Hogarth, R.M. (1975) Cognitive processes and the assessment of subjective probability distributions. *Journal of the American Statistical Association,* ***70***, 271-289.

Howell, W.C., & Burnett, S,A. (1978) Uncertainty measurement: A cognitive taxonomy. *Organizational Behavior and Human performance,* ***22***, 45-68.

Johnson, E.J., Payne, J.W., & Bettman, J.R. (1988) Information displays and preference reversals. *Organizational Behavior and Human Processes,* **42**, 1-21.

Kahn, B. E., & Sarin, R.K. (1988) Modeling ambiguity in decisions under uncertainty. *Journal of Consumer Research,* **15**, 265-272.

Kahneman, D., & Tversky, A. (1979) Prospect theory: An analysis of decision under risk. *Econometrikca,* **47**, 263-291.

Kahneman, D., & Tversky, A. (1982a) Evidential impact of base rate. In D. Kahneman, P. slovic, and A. Tversky (eds.) *Judgment under uncertainty: Heuristics and biases.* Cambridge University Press. pp.153-160.

Kahneman, D., & Tversky, A. (1982b) Variance of uncertainty. *Cognition,* ***11***, 143-157.

Kuipers, B., Moskowitz, A.J. & Kassirer, J.P. (1988) Critical decisions under uncertainty: Representation and structure. *Congnitive Science,*

12, 177-210.

Langer, E.J. (1975) The illusion of control. *Journal of Personality and Social Psychology*, **32**, 311-328.

Langer, E.J., & Roth, J. (1975) Heads I win, tails it's chance: The illusion of control as a function of the sequence of outcomes in a purely chance task. *Journal of Personality and Social Psychology*, **32**, 951-955.

Lichtenstein, S., & Slovic, P. (1971) Reversals of preference between bids and choices in gambling decisions. *Journal of Experimental Psychology*, **89**, 46-55.

Ludke, R.L., Stauss, F.F., & Gustafson, D.H. Comparison of five methods for estimating subjective probability distributions. *Organizational Behavior and Human Performance*, 1977, **19**, 162-179.

MacCrimmon, K. (1965) An experimental study of the decision making behavior of business executives. *Unpublished dissertation*, University of California, Los Angeles.
Cited in MacCrimmon, K., & Larsson, S. (1979).

MacCrimmon, K., & Larsson, S. (1979) Utility theory: Axioms versus paradoxes. In Allais & Hagen (Eds.) *Expected utility hypothesses and the Allais paradox.* Dordrecht: D. Reidel, 1979.

Matsuda, N. (1988) Multivariate analysis of qualitative information (in Japanese). Tokyo: Asakura.

Matsuda, N. (1993) HyperScaling: The Reference manual. *Discussion Paper Series No. 550.* Institute of Socio-Economic Planning

McNeil, B.J., Pauker, S.G., Sox, H.C. and Tversky, A. (1982) On the elicitation of prefernces for alternative therapies. *New England Journal of Medicine*, **27**, 1259-1262.

Popper, K. R. (1959) The propensity interpretation of probability. *British*

Journal for the Philosophy of Science, **10**, 25-42.

Raiffa, H. Risk, ambiguity, and the Savage axioms: Comment. *Quarterly Journal of Economics*, 1961, **75**, 690-694.

Ramsey, F. Truth and probability. In F.P. Ramsey, *The Foundations of mathematics and other logical essays.* New York: Harcourt, Brace and Co.

Rothbart, M., & Snyder, M. (1970) Confidence in the prediction and postdiction of an uncertain outcome. *Canadian Journal of Behavioral Sciences*, *2*, 38-43.

Rybash, J.M., & Roodin, P.A. (1989) *Aplied Cognitive Psychology*, **3**, 171-180.

Sakamoto, Y., Ishiguro, M., & Kitagawa, G. (1986) *Akaike Information Criterion Statistics.* Dordrecht, Netherland: D. Reidel.

Sarin, R., & Winkler, R.L. (1992) Ambiguity and decision modeling: A preference-based approach. *Journal of Risk and Uncertainty*, **5**, 389-407.

Savage, L.J. (1954) *The foundations of statistics.* Wiley.

Schkade, D.A., & Johnson, E.J. (1989) Cognitive processes in preference reversals. *Organizational Behavior and Human Processes*, **44**, 203-231

Schmeidler, D. (1989) Subjective probability and expected utility without additivity. *Econometrica*, **57**, 571-587.

Slovic, P., & Lichtenstein, S. (1983) Preference reversals: A broader perspective. *American Economic Review*, **73**, 596-605.

Slovic, P., & Tversky, A. (1974) Who accepts Savage's axioms? *Behavioral Science, 19*, 368-373.

Slovic, P., Fischhoff, B., & Lichtenstein, S. (1977) Behavioral decision theory. *Annual Review of Psychology*, *28*, 1-39.

Slovic, P., Griffin, D., & Tversky, A. (1990) Compatibility effects in judgment and choice. In R.M. Hogarth (ed.), *Insights in decision making: A tribute to Hillel J. Einhorn.* Chicago, IL: University of Chicago Press.

Suppes, P. (1974) The measurement of belief. *Journal of Royal Statistical Society, Series B*, **36**, 160-175.

Tversky, A., & Kahneman, D. (1981) The framing of decisions and the psychology of choice. *Science*, **211**, 453-458.

Tversky, A., & Kahneman, D. (1986) Rational choice and the framing of decisions. *Journal of Business*, **59**, 251-278.

Tversky, A., & Kahneman, D. (1992) Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, **5**, 297-323.

Tversky, A., Sattath. S., & Slovic, P. (1988) Contingent weighting in judgment and choice. *Psychological Review*, **95**, 371-384.

Yates, J.F., Jagacinski, C.M., & Faber, M.D. (1978) Evaluation of partially described multiattribute options.

Yates, J.F., & Zukowski, L.G. (1976) Characterization of ambiguity in decision making. *Behavioral Science*, **21**, 19-25

Yokoyama, A., & Shigemasu, M. (1992) Evaluation of subjective probabilities in the Ellsberg-type urn problems. A paper presented at the 34th Annual Meeting of the Japanese Association of Educational Psychology.

Wallsten, T.S., Budescu, D.V., Rapoport, A., Zwick, R., & Forsyth, B. (1986) Measuring the vague meanings of probability terms. *Journal of Experimental Psychology, General*, **115**, 348-365.

Winkler, R. (1991) Ambiguity, probability, prefernce and decision analysis.

Journal of Risk and Uncertainty, **4**, 285-297.

Zimmer, A.C., (1983) Verbal vs. numerical processing of subjective probabilities. In R.W. Scholtz (ed.) *Decision making under uncertainty.* Amsterdam: North-Holland, pp159-182.